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# Variant 4D supergravities and membranes

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**Abstract.** We review the dynamical generation of coupling constants in supergravity by means of gauge three-forms. The latter are introduced as components of particular variant chiral multiplets and can be coupled to membranes preserving local supersymmetry. We present generic  $\mathcal{N} = 1$ ,  $D = 4$  supergravity models with variant multiplets and study domain wall solutions that interpolate between vacua with different values of the cosmological constant.

## 1. Introduction

One of the most challenging problems in modern theoretical physics is to uncover the origin of dark energy. Assuming that it is sourced by a cosmological constant will require a mechanism that can explain its extremely small value compared to the Planck scale ( $\Lambda_{\text{obs}} \sim 10^{-120} M_{\text{P}}^4$ ). To avoid undesired fine-tuning, Brown and Teitelboim [1, 2] proposed a mechanism where the cosmological constant is dynamically neutralized by membrane nucleation from a gauge three-form field. Such mechanism however had the setback of requiring membranes with finely-tuned charges, and conflicted with late-time cosmology. A possible resolution was investigated by Bousso and Polchinski in [3]. By introducing multiple three-forms, within the context of string/M-theory compactifications, one could accommodate the observational data and avoid fine-tuning the membrane charges. Simple supergravity models with supermembranes were readily studied in [4, 5, 6], however, the realization of the Bousso–Polchinski mechanism within 4D supergravity requires the general coupling of gauge three-forms and membranes to be thoroughly explored.

In this article, we will review the main results of [7] and [8] (see also [9]) regarding the description of  $\mathcal{N} = 1$ ,  $D = 4$  supergravity models with three-forms and membranes. Such setup provides an effective description for Type II string compactifications on Calabi-Yau three-folds with orientifolding and fluxes and, therefore, may serve as a basis for a concrete realization of the Bousso–Polchinski scenario. In the final part of this article we briefly discuss domain wall solutions that interpolate between distinct Minkowski or AdS vacua.

## 2. The Brown–Teitelboim and Bousso–Polchinski mechanisms

The Brown–Teitelboim mechanism [1, 2] is realized by coupling a membrane to a single real gauge three-form  $A_3$  and considering the system within gravitation. Let us denote with  $A_{mnp}$



the components of  $A_3$  and with  $F_{mnpq} = 4\partial_{[m}A_{npq]}$  those of  $F_4$ , namely its four-form field-strength<sup>1</sup>. The membrane worldvolume  $\mathcal{C}$  is parametrized by the coordinates  $\xi^\mu$  ( $\mu = 0, 1, 2$ ). The membrane dynamics are captured by the embedding of  $\mathcal{C}$  into the ambient space-time, parametrized by the coordinates  $x^m$  ( $m = 0, 1, 2, 3$ ), namely:  $\xi^\mu \mapsto x^m(\xi)$ . The action describing the interactions among gravity, the gauge-three form and the membrane is given by

$$S = - \int d^4x e \left[ \frac{R}{2} - \frac{1}{2}(*F_4)^2 \right] + \frac{1}{3!} \int d^4x e \nabla_m (A_{npq} F^{mnpq}) + \int_{\mathcal{B}} d^3x \sqrt{-h} K \quad (1)$$

$$- T_M \int_{\mathcal{C}} d^3\xi \sqrt{-h} + \frac{q}{3!} \int_{\mathcal{C}} d^3\xi \varepsilon^{\mu\nu\rho} \frac{\partial x^m}{\partial \xi^\mu} \frac{\partial x^n}{\partial \xi^\nu} \frac{\partial x^p}{\partial \xi^\rho} A_{mnp}.$$

The gravitational sector of the first line of (1) contains the scalar curvature and the Gibbons–Hawking boundary term, where  $K$  is the extrinsic curvature and  $\mathcal{B}$  denotes the boundary. The kinetic term for the gauge three-form is also appearing in the first line of (1), and is expressed in terms of the Hodge-dual of its field strength  $*F_4 = -\frac{e}{4!}\varepsilon_{mnpq}F^{mnpq}$ . Moreover, the proper gauge invariant boundary conditions for the gauge three-forms are  $\delta F_{mnpq}|_{\text{bd}} = 0$  (see e.g. [1, 2, 11]), rather than  $\delta A_{mnp}|_{\text{bd}} = 0$ . The additional boundary term appearing in (1) ensures that variations of the gauge three-form are compatible with these boundary conditions. The second line of (1) describes the dynamics of the membrane. The first term is the Nambu–Goto action with  $T_M$  being the membrane tension and  $h_{\mu\nu}$  being the worldvolume pullback of the space-time metric  $g_{mn}$ , namely

$$h_{\mu\nu}(\xi) \equiv \frac{\partial x^m}{\partial \xi^\mu} \frac{\partial x^n}{\partial \xi^\nu} g_{mn}(x(\xi)), \quad h \equiv \det h_{\mu\nu}(\xi). \quad (2)$$

The last term in the second line of (1) describes the minimal coupling of the membrane of charge  $q$  to the pull-back of the three-form. Such term can be also written as  $-q \int_{\mathcal{C}} A_3$ .

From (1), we can derive the equations of motion for the gauge three-form  $A_{mnp}$ , which are given by

$$\partial_m *F_4 = \frac{q}{3!} \int d^3\xi \delta(x - x(\xi)) \varepsilon_{mnpq} \varepsilon^{\mu\nu\rho} \frac{\partial x^n(\xi)}{\partial \xi^\mu} \frac{\partial x^p(\xi)}{\partial \xi^\nu} \frac{\partial x^q(\xi)}{\partial \xi^\rho}. \quad (3)$$

Away from the membrane, this equation is simply solved by  $*F_4 = E$ , with  $E$  being a real constant. Plugging this value back into the action (1), and taking into account the contribution from the boundary terms, a cosmological constant term of the form  $-eE^2/2$  is *dynamically generated*.

If the membrane surface is closed, it will divide the ambient space-time into an *outside* region and an *inside* region. The constant value of the four-form flux  $E$  will be different in the two regions, and the difference between the two values can be readily computed from (3). Let us call  $E_I$  the constant inside the membrane and  $E_O$  the constant outside, and see how they are related with the membrane charge  $q$ , once we go to the *static gauge*. We adopt a local coordinate system (in the vicinity of a point on the membrane) such that three of the space-time coordinates coincide with the worldvolume coordinates, namely we set  $x^\mu = \xi^\mu$ . The worldvolume is then described by the equation  $x^3 = x^3(\xi) = 0^2$ . Now, in this neighbourhood, the  $m = 3$  component of Eq. (3) reads

$$\partial_3 *F_4 = q \delta(x^3) \int d^3\xi \delta^3(x^\mu - \xi^\mu) = q \delta(x^3). \quad (4)$$

<sup>1</sup> In the conventions of [10], one has  $A_3 = \frac{1}{3!}A_{mnp}dx^p \wedge dx^n \wedge dx^m$  and  $F_4 \equiv \frac{1}{4!}F_{mnpq}dx^m \wedge dx^n \wedge dx^p \wedge dx^q$ . Our exterior derivative acts from the right so that  $F_4 = dA_3$ . As in [8], we define the components of the Hodge dual of a bosonic  $p$ -form  $\omega_p$  as  $(*\omega_p)_{m_{p+1}\dots m_4} = -\frac{e}{p!(4-p)!}\epsilon_{m_1\dots m_4}\omega^{m_1\dots m_p}$ , where  $\epsilon^{0123} = -\epsilon_{0123} = 1$ .

<sup>2</sup> Such a gauge can be fixed by target-space general coordinate transformations which is a gauge symmetry of our dynamical system (1). This reflects the Goldstone nature of the membrane coordinate functions, which transform as Stückelberg fields in the presence of dynamical gravity. See [12] for a discussion and references on this issue.

Once equation (4) is integrated on  $dx^3$  over a small (infinitesimal) interval, say from  $-\epsilon$  to  $+\epsilon$ , it delivers

$$E_O - E_I = q. \quad (5)$$

Hence, the value of the cosmological constant in the outside region compared to its value in the inside region changes as

$$\Lambda_O = \frac{E_O^2}{2} \rightarrow \Lambda_I = \frac{E_I^2}{2} = \frac{(E_O - q)^2}{2}. \quad (6)$$

Therefore, the subsequent nucleation of membranes induces a relaxation of the cosmological constant.

However, in the classical framework, the simple model (1) could not solve the problem of the smallness of the cosmological constant [1, 2, 13]. The issue could be better addressed in the context of string or M-theory compactifications, where models as (1) naturally arise [3]. In fact, after reducing higher  $p$ -forms of ten (or eleven) dimensional supergravity, a plethora of gauge three-forms  $A_{3I}$  ( $I = 1, \dots, N$ ) appear in the effective four dimensional theory. Their quantized four-form field strengths, once set on-shell, here lead to a *discretum* for the values of the cosmological constant. Accordingly, multiple membranes, coupled with generic charges  $q_I$  to the gauge three-forms  $A_{3I}$ , may be nucleated. After a single membrane nucleation the cosmological constant changes as follows

$$\Lambda_O = \lambda + \frac{1}{2} \sum_{I=1}^N n_I^2 q_I^2 \rightarrow \Lambda_I = \lambda + \frac{1}{2} \sum_{i=1}^N (n_i - 1)^2 q_i^2, \quad (7)$$

where  $n_I$  are units of flux quanta and  $\lambda$  is a *bare* cosmological constant which may include all the other contributions. Because of the dependence of the cosmological constant on multiple  $q_I$  and  $n_I$ , a value close to  $\Lambda_{\text{obs}}$  can be easily achieved. For example, assuming a bare cosmological constant  $\lambda$  of order  $\mathcal{O}(1)$ ,  $\Lambda_{\text{obs}}$  can be obtained for  $N \sim 100$  with  $|\lambda| \sim \mathcal{O}(1)$  and  $|q| \sim \mathcal{O}(10^{-1})$ .

As it is clear, the Bousso-Polchinski proposal crucially relies on the existence of multiple gauge three-forms and the corresponding membranes, which are indeed expected for generic string compactifications. Still, a model like (1), even if extended to include multiple three-forms, cannot be the final answer. In typical compactifications, plenty of scalar moduli are generically present, which non-trivially interact with the three-form fluxes. We should then expect that also these scalar fields experience some *jumps* across the membranes.

In the following, we shall summarize how all these ingredients can be realized in a large class of supergravity models of the kind arising in string compactifications. In fact, as long as the supersymmetry breaking scale is low enough, the effective theory should exhibit a supersymmetric structure. Therefore, a first step is to find a proper supersymmetric embedding of gauge three-forms; later we shall include membranes manifestly preserving supersymmetry.

### 3. The bulk sector: gauge three-form supergravity

To incorporate gauge three-forms in 4D supergravity a general method was proposed in [7] for trading ordinary chiral multiplets for the so-called *double* (and *single*) *three-form* multiplets [4, 5, 14, 15, 16, 17, 18, 19, 20, 21]. These multiplets are also chiral, but they differ from the ordinary ones in their highest components. Instead of auxiliary complex scalar fields, they contain particular combinations of real gauge three-forms. In the following, we summarize the results of [7], to which we address the reader for further details and references.<sup>3</sup>

Let us consider a generic matter-coupled  $\mathcal{N} = 1$  (minimal) supergravity. We divide its matter content into two distinct sets of chiral multiplets:  $T^r$ , with  $r = 1, \dots, m$ , which are ordinary

<sup>3</sup> Gauge three-forms can also be introduced as auxiliary fields of vector multiplets, see e.g. [22]. Notice also that the Hamiltonian formalism for three form fields interacting with gravity was developed recently in [23].

chiral multiplets having zero scaling dimension  $\Delta_T = 0$ , and  $S^I$ , with  $I = 0, \dots, n$ , which have scaling dimension  $\Delta_S = 3$  and are double three-form multiplets. We also assume that the manifolds parametrized by  $S^I$  and  $T^r$  factorize and that  $S^I$  describe a special Kähler manifold locally specified by a prepotential  $\mathcal{G}(S)$  which is holomorphic in  $S^I$  and homogeneous of order two  $\mathcal{G}(\mu S) = \mu^2 \mathcal{G}(S)$ <sup>4</sup>. More explicitly, the double three-form superfields  $S^I$  are defined by the following non-linear relations

$$S^I \equiv \frac{1}{4}(\bar{\mathcal{D}}^2 - 8\mathcal{R}) [\mathcal{M}^{IJ}(S)(\Sigma_J - \bar{\Sigma}_J)] , \quad (8)$$

where  $\mathcal{M}^{IJ} \equiv (\mathcal{M}_{IJ})^{-1}$ ,  $\mathcal{M}_{IJ} \equiv \text{Im}\mathcal{G}_{IJ}$  and  $\Sigma_I$  are complex linear superfields obeying the constraint  $(\bar{\mathcal{D}}^2 - 8\mathcal{R})\Sigma_I = 0$ . The superfields  $\Sigma^I$  accommodate two sets of real gauge three-forms  $A_3^I$  and  $\tilde{A}_{3I}$  as their components<sup>5</sup>

$$\bar{\sigma}_m^{\dot{\alpha}\alpha} [\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}] \Sigma_I | = -2(*\tilde{A}_{3I} - \mathcal{G}_{IJ}(s)*A_3^J)_m . \quad (9)$$

The bosonic components of  $S^I$  are  $s^I = S^I|$ , and

$$F_S^I \equiv -\frac{1}{4}\mathcal{D}^2 S^I | = \bar{M}s^I - \frac{i}{2}\mathcal{M}^{IJ}*\mathcal{F}_{4J} , \quad (10)$$

where  $M$  is the complex scalar auxiliary field of the supergravity multiplet and we have defined

$$\mathcal{F}_{4I} \equiv \tilde{F}_{4I} - \bar{\mathcal{G}}_{IJ}F_4^J . \quad (11)$$

These are complex four-forms that depend on the field strengths of the real three-forms  $F_4^I \equiv dA_3^I$  and  $\tilde{F}_{4I} \equiv d\tilde{A}_{3I}$ .

We will consider the following super-Weyl invariant Lagrangian (with no superpotential)

$$\mathcal{L} = -3 \int d^4\theta E \Omega(S, \bar{S}; T, \bar{T}) + \text{c.c.} + \mathcal{L}_{\text{bd}} . \quad (12)$$

Here, as in (1), owing to the presence of gauge three-forms, boundary terms should be included to ensure the correct variation of the Lagrangian and  $\Omega(S, \bar{S}; T, \bar{T})$  is the kinetic function which has scaling dimension  $\Delta_\Omega = 2$ . In our setup,  $\Omega$  factorizes as

$$\Omega(S, \bar{S}; T, \bar{T}) = \Omega_0(S, \bar{S})e^{-\frac{1}{3}\hat{K}(T, \bar{T})} , \quad \Omega_0(S, \bar{S}) = [i\bar{S}^I\mathcal{G}_I(S) - iS^I\bar{\mathcal{G}}_I(\bar{S})]^{\frac{1}{3}} . \quad (13)$$

In order to compute the bosonic components of the Lagrangian (12), we first have to fix the super-Weyl invariance. To this aim, we write  $S^I$  in terms of a chiral compensator  $Y$ , carrying scaling dimension  $\Delta_Y = 3$  and the ‘physical’ chiral superfields  $\Phi^i$ , with  $i = 1, \dots, n$ , having  $\Delta_\Phi = 0$ . We set

$$S^I = Y f^I(\Phi) , \quad (14)$$

where  $f^I(\Phi)$  are holomorphic functions of  $\Phi^i$  such that  $\text{rank}(\partial_i f^I) = n$ . We may now gauge fix the super-Weyl symmetry by setting  $Y = 1$  and integrate over the fermionic coordinates. Once we perform a Weyl rescaling to pass to the Einstein frame and, after having integrated out the auxiliary fields, we arrive at the action whose bosonic part is

$$S_{\text{SG, bos}} = - \int d^4x e \left( \frac{1}{2}R + \partial\phi^i\partial\bar{\phi}^{\bar{j}} + \hat{K}_{p\bar{q}}\partial t^p\partial\bar{t}^{\bar{q}} - \mathcal{T}^{IJ}*\bar{\mathcal{F}}_{4I}*\mathcal{F}_{4J} \right) + S_{\text{bd}} . \quad (15)$$

<sup>4</sup> In the following we will denote  $\mathcal{G}_I(S) \equiv \partial_I\mathcal{G}(S) = \mathcal{G}_{IJ}S^J$ ,  $\mathcal{G}_{IJ} \equiv \partial_I\partial_J\mathcal{G}(S)$ .

<sup>5</sup> We will use simply “|” to refer to “ $|_{\theta=\bar{\theta}=0}$ ”, except otherwise noted.

The boundary terms in (15) are

$$S_{\text{bd}} = -2\text{Re} \int d^4x e \nabla_m \left[ \mathcal{T}^{IJ} (*\tilde{A}_{3I} - \mathcal{G}_{IK} *A_3^K)^m *F_{4J} \right]. \quad (16)$$

The space-time boundary is understood to be at infinity. The quantities appearing in (15) are all given in terms of the prepotential  $\mathcal{G}(S)$  and the Kähler potential  $\hat{K}(T, \bar{T})$  as follows

$$K_{i\bar{j}} \equiv G_{IJ} f^I_i \bar{f}^{\bar{J}}_{\bar{j}}, \quad (17a)$$

$$G_{IJ} \equiv -\frac{\mathcal{M}_{IJ}}{(f\mathcal{M}\bar{f})} + \frac{(\mathcal{M}\bar{f})_I (\mathcal{M}f)_J}{(f\mathcal{M}\bar{f})^2}, \quad (17b)$$

$$\mathcal{T}^{IJ} \equiv \frac{1}{4} e^{-\kappa} \left[ \mathcal{M}^{IK} G_{LK} \mathcal{M}^{LJ} + \frac{f^I \bar{f}^J}{\gamma (f\mathcal{M}\bar{f})^2} \right], \quad (17c)$$

$$\gamma \equiv \hat{K}_{\bar{q}} \hat{K}^{\bar{q}p} \hat{K}_p - 3, \quad (17d)$$

where  $(\mathcal{M}f)_I \equiv \mathcal{M}_{IJ} f^J$ ,  $(\mathcal{M}\bar{f})_I \equiv \mathcal{M}_{IJ} \bar{f}^{\bar{J}}$  and  $(f\mathcal{M}\bar{f}) \equiv f^I \mathcal{M}_{IJ} \bar{f}^{\bar{J}}$ .

As in Section 2, we can now obtain the potential by setting the gauge three-forms on-shell. By solving their equations of motion, we get

$$2\text{Re}(\mathcal{T}^{IJ} *F_{4J}) = m^I, \quad 2\text{Re}(\mathcal{G}_{IJ} \mathcal{T}^{JK} *F_{4K}) = e_I, \quad (18)$$

where  $e_I$  and  $m^I$  are real quantized constants. The Lagrangian (12) then becomes

$$S_{\text{bos}} = - \int d^4x e \left( \frac{1}{2} R + G_{IJ} f^I_i \bar{f}^{\bar{J}}_{\bar{j}} \partial \phi^i \partial \bar{\phi}^{\bar{j}} + \hat{K}_{p\bar{q}} \partial t^p \partial \bar{t}^{\bar{q}} + V(\phi, \bar{\phi}, t, \bar{t}; e, m) \right), \quad (19)$$

where  $V(\phi, \bar{\phi}, t, \bar{t}; e, m) = \mathcal{T}^{IJ} *F_{4I} *F_{4J}|_{\text{on-shell}}$  is the potential for the scalar fields  $\phi^i$  and  $t^a$ . It can be shown that this potential matches with the one obtained from ordinary chiral models with superpotential

$$W = e_I f^I(\phi) - m^I \mathcal{G}_I(\phi), \quad (20)$$

i.e.

$$V = \mathcal{T}^{IJ} *F_{4I} *F_{4J}|_{\text{on-shell}} = e^{\mathcal{K}} (K^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W} + \gamma |W|^2). \quad (21)$$

Gauge three-forms dynamically generate the parameters  $e_I$  and  $m^I$  of  $W$ , promoting them to vacuum expectation values of the fluxes.

#### 4. Supergravity coupled to membranes

To fully embed the action (1) within 4D supergravity, one should couple membranes to three-forms described by (12) in such a way that the local supersymmetry is preserved. Membranes can be promoted to objects residing in the whole superspace [4, 5, 6, 24, 25, 26, 27, 28, 29]. Their dynamics is described by an embedding of the supermembrane worldvolume parametrized by  $\xi^\mu$  into target superspace  $z^M$

$$\xi^\mu \mapsto \mathcal{C} : z^M(\xi) = (x^m(\xi), \theta^\alpha(\xi), \bar{\theta}^{\dot{\alpha}}(\xi)), \quad (22)$$

governed by the superspace action

$$S_{\text{M}} \equiv S_{\text{WZ}} + S_{\text{NG}}. \quad (23)$$

The Wess-Zumino term

$$S_{\text{WZ}} = \int_{\mathcal{C}} (q_I A_3^I - p^I \tilde{A}_{3I}), \quad (24)$$

supersymmetrizes the bosonic minimal coupling of (1) and combines with the Nambu–Goto term

$$S_{\text{NG}} = -2 \int_{\mathcal{C}} d^3\xi \sqrt{-h} |q_I S^I - p^I \mathcal{G}_I(S)|, \quad (25)$$

which is basically fixed by  $\kappa$ -symmetry – see below. In (25) it is understood that the superfields  $S^I$  are evaluated on the membrane worldvolume and  $h_{\mu\nu}$  is the induced worldvolume metric defined in terms of the pull-backs of target-space supervielbeins

$$h_{\mu\nu}(\xi) \equiv \eta_{ab} E_\mu^a(\xi) E_\nu^b(\xi) \quad \text{with} \quad E_\mu^a(\xi) \equiv \partial_\mu z^M(\xi) E_M^a(z(\xi)). \quad (26)$$

The supervielbeins  $E_M^A(z)$  obey the minimal supergravity constraints [10].

Let us explore the symmetry properties of the action (23).

**Worldvolume reparametrization invariance.** Action (23) is invariant under reparametrizations of the membrane worldvolume  $\xi \rightarrow \xi'(\xi)$ . A way to fix this freedom is to set

$$\xi^\mu \equiv x^\mu. \quad (27)$$

This gauge choice leaves the fourth coordinate  $x^3 \equiv y(\xi)$  as the only bosonic physical field describing the dynamics of the membrane. The field  $y(\xi)$  can then be interpreted as the transverse displacement of the membrane from its static position. However, the static configuration breaks the translation invariance of the background, and  $y(\xi)$  plays the role of the Goldstone field associated with this spontaneous breaking.

**The  $\kappa$ -symmetry.** The action (23) also enjoys a peculiar local fermionic symmetry, called  $\kappa$ -symmetry, which acts on the space-time coordinates as follows

$$\delta z^M(\xi) = \kappa^\alpha(\xi) E_\alpha^M(z(\xi)) + \bar{\kappa}^{\dot{\alpha}}(\xi) E_{\dot{\alpha}}^M(z(\xi)). \quad (28)$$

Here  $\kappa^\alpha(\xi)$  (with  $\bar{\kappa}^{\dot{\alpha}}(\xi) \equiv \overline{\kappa^{\dot{\alpha}}}(\xi)$ ) is a local fermionic parameter satisfying the projection condition

$$\kappa_\alpha = \frac{q_I S^I - p^I \mathcal{G}_I}{|q_I S^I - p^I \mathcal{G}_I|} \Gamma_{\alpha\dot{\alpha}} \bar{\kappa}^{\dot{\alpha}}, \quad \text{with} \quad \Gamma_{\alpha\dot{\alpha}} \equiv \frac{i\epsilon^{\mu\nu\rho}}{3!\sqrt{-h}} \epsilon_{abcd} E_\mu^b E_\nu^c E_\rho^d \sigma_{\alpha\dot{\alpha}}^a. \quad (29)$$

This condition reduces the number of four real independent components of  $\kappa^\alpha, \bar{\kappa}^{\dot{\alpha}}$  to two. The  $\kappa$ -invariance is indeed a fancy realization of a conventional local worldvolume supersymmetry of the membrane which becomes manifest in the superembedding approach [30, 31]. It can be used to put to zero half of the fermionic coordinates  $\theta^\alpha(\xi), \bar{\theta}^{\dot{\alpha}}(\xi)$ , while the other half are dynamical worldvolume fermionic fields playing the role of the goldstini of the partially broken bulk supersymmetry. In this sense, the membranes described by the action (23) are  $\frac{1}{2}$ -BPS objects.

**Bulk super-diffeomorphism invariance.** Finally, the action (23) is invariant under bulk super-diffeomorphisms. When (23) is included in the action of interacting system including also dynamical supergravity (like in our (32)), super-diffeomorphism invariance is a gauge symmetry. Therefore we may use it to choose a membrane embedding such that the oscillations of the membrane in the transverse bosonic direction and along the fermionic direction look frozen, that is  $y(\xi) = 0$  and  $\theta(\xi) = 0, \bar{\theta}(\xi) = 0$ . Therefore, in the interacting system including dynamical (super)gravity we may assume the membrane to be static and located at  $y_0 = 0$  without any loss of generality. In this gauge the dynamics of the membrane is encoded in the worldvolume pull-backs of the bulk supergravity fields [12, 32].

The super-membrane action (23)-(25) is also super-Weyl invariant. Fixing the super-Weyl invariance and performing the Weyl rescaling to pass to Einstein frame, we reduce (23) to an action with bosonic sector

$$S_{\text{M, bos}} = -2 \int_{\mathcal{C}} d^3\xi \sqrt{-h} e^{\frac{1}{2}\mathcal{K}} |q_I f^I(\phi) - p^I \mathcal{G}_I(\phi)| + q_I \int_{\mathcal{C}} A_3^I - p^I \int_{\mathcal{C}} \tilde{A}_{3I}. \quad (30)$$

From the Nambu-Goto term (25) we immediately read the expression for the effective tension of the membrane

$$T_{\text{M}} = 2 e^{\frac{1}{2}\mathcal{K}} |q_I f^I(\phi) - p^I \mathcal{G}_I(\phi)|, \quad (31)$$

which, rather than being a constant, depends on the values of the scalar fields on the membrane worldvolume.

The full locally supersymmetric action we consider is then

$$S = S_{\text{SG}} + S_{\text{M}}, \quad (32)$$

whose bosonic components are (15) and (30). The coupling to membranes inevitably influences the equations of motion of the gauge three-forms which become

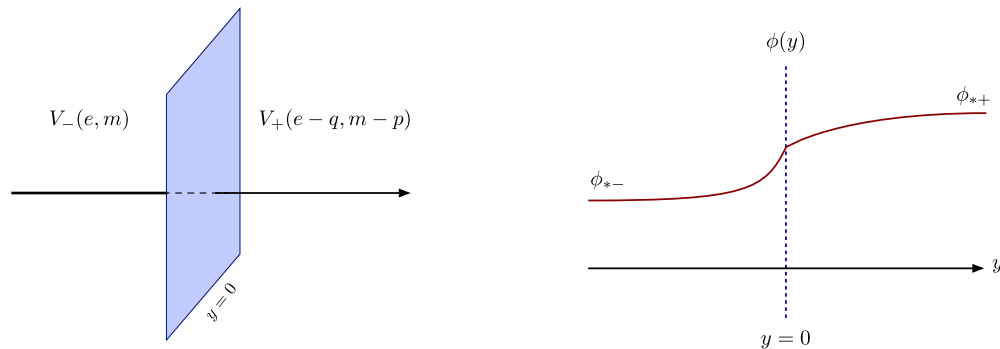
$$d\text{Re}(\mathcal{T}^{IJ*}\mathcal{F}_{4J}) = -\frac{1}{2}p^I \delta_1(\mathcal{C}), \quad d\text{Re}(\mathcal{G}_{IJ}\mathcal{T}^{JK*}\mathcal{F}_{4K}) = -\frac{1}{2}q_I \delta_1(\mathcal{C}). \quad (33)$$

Here  $\delta_1(\mathcal{C})$  is a delta-like one-form localized on the membrane worldvolume  $\mathcal{C}$ . From the left side of the membrane to the right, the quantized constants defined in (18) change as

$$m_-^I \rightarrow m_+^I \equiv m_-^I - p^I, \quad e_{-I} \rightarrow e_{+I} \equiv e_{-I} - q_I. \quad (34)$$

As a result, the membrane divides the space-time into two regions where the scalar fields feel different potentials,  $V_-(e, m)$  on the left and  $V_+(e - q, m - p)$  on the right (as depicted in Fig. 1). In view of (21) and (20), this means that we can define a superpotential over the whole space-time as

$$W(\phi, y) = e_{-I} f^I(\phi) - m_-^I \mathcal{G}_I(\phi) - \Theta(y) (q_I f^I(\phi) - p^I \mathcal{G}_I(\phi)). \quad (35)$$



**Figure 1.** Left panel: two different potentials on the membrane sides. Right panel: a domain wall-like solution interpolating between two supersymmetric vacua localized at  $\phi_{*-}$  (on the left of the membrane) and  $\phi_{*+}$  (on the right). Although  $\phi(y)$  is continuous, its derivative might be discontinuous at  $y = 0$ .



## 5. Domain wall solutions and the flow equations

We now pass to the study of domain wall solutions which connect vacua on each side of the membrane. We consider a setting with a single flat membrane located at  $y = 0$  and we look for domain walls which are  $\frac{1}{2}$ -BPS.

We split the coordinates  $x^m$  in  $x^\mu$ ,  $\mu = 0, 1, 2$  and  $y \equiv x^3$ . The latter is the coordinate transverse to the membrane. We consider the following domain wall ansatz for the space-time metric

$$ds^2 = e^{2D(y)} dx^\mu dx_\mu + dy^2, \quad (36)$$

and assume that the scalar fields  $\phi^i$  depend only on the transverse coordinate  $y$ . The study of the domain wall solutions now proceeds along the same lines as in [33, 34, 35, 36, 37, 38] for standard supergravity. By imposing that the variations of the fermions vanish for the metric ansatz (36), we arrive at the following equations

$$\dot{\phi}^i = e^{\frac{1}{2}\mathcal{K}(\phi, \bar{\phi}) + i\vartheta(y)} K^{i\bar{j}} (\bar{W}_{\bar{j}} + K_{\bar{j}} \bar{W}), \quad (37a)$$

$$\dot{D} = -e^{\frac{1}{2}\mathcal{K}(\phi, \bar{\phi})} |W|, \quad (37b)$$

$$\dot{\vartheta} = -\text{Im} \left( \dot{\phi}^i K_i \right). \quad (37c)$$

The *dot* in (37a)-(37c) corresponds to the derivative with respect to  $y$ , and  $\vartheta(y) = \vartheta(\phi(y), \bar{\phi}(y))$  is the phase of  $W$ , that is  $W = e^{i\vartheta} |W|$ . Note that  $W$ , although generically depends on the field strengths  $\mathcal{F}_{4I}$ , will reduce to (35) when setting the gauge three-forms on-shell.

The equations (37) are *flow equations*, which describe how the scalars, the warp factor  $D(y)$  and the phase of the superpotential vary along the direction transverse to the membrane and determine the domain wall solution. In order to better characterize the flow, we can introduce a ‘flowing’ covariantly holomorphic superpotential [38]

$$\mathcal{Z}(\phi, y) \equiv e^{\frac{1}{2}\mathcal{K}(\phi, \bar{\phi})} W = e^{\frac{1}{2}\mathcal{K}(\phi, \bar{\phi})} [\Theta(y) W_+(\phi) + \Theta(-y) W_-(\phi)]. \quad (38)$$

Using (38), the equations (37) are recast to the form

$$\dot{\phi}^i = 2K^{i\bar{j}} \partial_{\bar{j}} |\mathcal{Z}|, \quad (39a)$$

$$\dot{D} = -|\mathcal{Z}|, \quad (39b)$$

$$\dot{\vartheta} = -\text{Im} \left( \dot{\phi}^i K_i \right). \quad (39c)$$

From (39a), we see that the fixed points for the flow of the scalars are those for which  $\partial_{\bar{j}} |\mathcal{Z}| = 0$  (or, equivalently,  $D_{\bar{j}} \bar{W} = 0$ ). The domain wall solution that we seek interpolates between two supersymmetric vacua on the left and the right of the membrane. These are specified by the field configurations  $\phi_{*-}$  on the left, reached in the limit  $y \rightarrow -\infty$ , and  $\phi_{*+}$  on the right, reached as  $y \rightarrow +\infty$ . An example of the flow of the scalars is illustrated in Fig. 1. The vacua on the sides of the membrane are generically AdS, since asymptotically from (39b) it follows that  $D_{\pm} = -|\mathcal{Z}_{*\pm}| y$  (where  $\mathcal{Z}_{*\pm} \equiv \lim_{y \rightarrow \pm\infty} \mathcal{Z}(y)$ ), which correspond to AdS spaces with radii  $1/\mathcal{Z}_{*\pm}$ .

Let us notice that, combining the membrane equations of motion with the flow equations (39), we get

$$\frac{d|\mathcal{Z}|}{dy} = K_{i\bar{j}} \dot{\phi}^i \dot{\phi}^{\bar{j}} + \frac{1}{2} T_M \delta(y) = 4K^{i\bar{j}} \partial_i |\mathcal{Z}| \partial_{\bar{j}} |\mathcal{Z}| + \frac{1}{2} T_M \delta(y) \geq 0. \quad (40)$$

Hence,  $|\mathcal{Z}(y)| = -\dot{D}(y)$  is a monotonic increasing function and the flow is directed towards increasing values of  $|\mathcal{Z}|$ . In the following, we shall assume  $|\mathcal{Z}|_{y=+\infty} > |\mathcal{Z}|_{y=-\infty}$ , the other choice being obtained just by flipping  $y \rightarrow -y$ . Then, the flow starts from a supersymmetric vacuum

$\phi_{*-}$  to the left, at  $y = -\infty$ , crosses the membrane and a new supersymmetric vacuum  $\phi_{*+}$  is reached asymptotically to the right.

From (38) one recovers the tension of the membrane. In fact, close to the membrane

$$\Delta\mathcal{Z} \equiv \lim_{\varepsilon \rightarrow 0} (\mathcal{Z}|_{y=\varepsilon} - \mathcal{Z}|_{y=-\varepsilon}) = -e^{\frac{1}{2}\mathcal{K}} (q_I f^I - p^I \mathcal{G}_I)|_{y=0}, \quad (41)$$

and (31) can be expressed as

$$T_M \equiv 2 e^{\frac{1}{2}\mathcal{K}} |q_I f^I - p^I \mathcal{G}_I|_{y=0} = 2|\Delta\mathcal{Z}|. \quad (42)$$

However, we may also compute the tension of the domain wall which, although localized mainly close to the membrane, extends over the whole space-time. To this aim, we first plug the metric ansatz (36) into the full action (32) and then rewrite (32) in the *BPS-form*

$$S_{\text{red}} = \int d^3x \int dy e^{3D} \left[ 3(\dot{D} + |\mathcal{Z}|)^2 - K_{i\bar{j}}(\dot{\phi}^i - 2K^{i\bar{k}}\partial_{\bar{k}}|\mathcal{Z}|)(\dot{\phi}^{\bar{j}} - 2K^{l\bar{j}}\partial_l|\mathcal{Z}|) \right] - 2 \int d^3x [(e^{3D}|\mathcal{Z}|)|_{y=+\infty} - (e^{3D}|\mathcal{Z}|)|_{y=-\infty}]. \quad (43)$$

Its on-shell value is precisely the energy of the solitonic solution, that is the tension of the domain wall. Noticing that the first line contains (39a) and (39b), and therefore vanishes on-shell, we find

$$T_{\text{DW}} = 2(|\mathcal{Z}|_{y=+\infty} - |\mathcal{Z}|_{y=-\infty}). \quad (44)$$

It is important to stress that, generically, the tension of the membrane and of the domain wall are different, being related via

$$T_{\text{DW}} = 2(|\mathcal{Z}|_{y=+\infty} - \lim_{\varepsilon \rightarrow 0} |\mathcal{Z}|_{y=\varepsilon}) + 2(\lim_{\varepsilon \rightarrow 0} |\mathcal{Z}|_{y=-\varepsilon} - |\mathcal{Z}|_{y=-\infty}) + T_M. \quad (45)$$

They coincide only if the  $|\mathcal{Z}|$  is just a constant on the two sides. This is the case, for instance, for the *thin-wall* approximation, but for *thick* walls  $T_{\text{DW}} > T_M$  holds strictly.

## 6. Conclusions

In this article we have reviewed the main results of [7] and [8] on the construction of 4D supergravities containing gauge three-forms and membranes. These theories are appropriate for the description of type II string compactifications on Calabi–Yau manifolds with Ramond–Ramond fluxes. In particular, in [7, 8] it has been shown that the scalar potential obtained from compactifications of Type IIA string theory [39, 40, 41, 42, 43] is retrieved from (21) and that the tension of the membrane (31) is equal to that of membranes obtained from wrapping higher dimensional branes over special Lagrangian cycles in the internal CY. Moreover, we reviewed the domain wall solutions that connect different AdS regions divided by the membranes. We studied the equations that regulate the flow of the scalar fields and the metric warp factor.

In a more general context, the results reviewed above furnish an effective field-theoretical setup for studying string phenomenology and string cosmology.

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