Prepared for submission to JINST

ICFA Beam Dynamics Newsletter #83 — Advanced Accelerator Concepts / ICFA ANA
FERMILAB-PUB-22-137
Submitted: March 14, 2022

On Possibility of Low-emittance High-energy Muon Source Based on Plasma Wakefield Acceleration

Vladimir Shiltsev [a,1]

[a] *Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA*

E-mail: shiltsev@fnal.gov

Abstract: Plasma wakefield acceleration (PWA) channels are characterized by very high accelerating gradients and very strong focusing fields. We propose to employ these properties for effective production of low emittance high energy muon beams, consider muon beam dynamics in the PWFA cell and analyze various options and potential of the PWA-based muon sources.

Keywords: Wake-field acceleration (laser-driven, electron-driven), Targets (spallation source targets, radioisotope production, neutrino and muon sources), Beam dynamics, Accelerator subsystems and technologies

[1]This manuscript has been authored by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics.

Contents

1 Introduction

The muon was discovered almost hundred years ago, but still remains of a profound and growing interest for particle physics (see, e.g. [1]). About 200 times heavier than electron, the muon is now part of the second generation of particles of the Standard Model of elementary particles, together with muon neutrino and the strange s and the charm c quarks. The Standard Model is proven to describe processes at the distances $O(10^{-19}$ m) but is predicted to break at the Planck scale $O(10^{-35}$ m). Muons are used to search for processes beyond the Standard Model, such as possible rare muon decays and muon to electron conversion, possible muonium-antimuonium conversion, decays of the B-meson into a muon pair. Muons are in the heart of other puzzles, like the *proton radius puzzle*, anomalous magnetic and dipole muon moments and rare processes initiated by muon.

Applications of muon beams range from chemistry (where the muonium atom is a true light version of hydrogen) to condensed matter physics, where the muon spin spectroscopy, also known as μSR, helps to study properties semiconductors, magnetism, superconductivity and quantum diffusion; to exploration of alternative energy source within the framework of muon catalyzed fusion (μCF) and discoveries at the particle physics energy frontier envisioned with future muon colliders.

All these research and applications greatly depend on effective generation of muon beams, some requiring high intensity, others high energy or low emittances, while, for example, neutrino factories and muon colliders requiring it all - high intensity, low emittance, high-energy muon beams [2, 3].

1.1 Challenges of muon production

The main challenge to produce muons is that they are not stable particles having lifetime of 2.2 μs in the muon rest frame before decaying into an electron, a muon-type neutrino, and an electron-type antineutrino ($\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$). Therefore, other stable particles, such as electrons, photons or protons are required to generate muons as either secondary or even tertiary reaction product. The most

common is production chain $p \rightarrow$ target $\rightarrow \pi^+(\pi^-) \rightarrow \mu^+(\mu^-)$ in which the pions (and subsequent muons) are produced with large transverse angular spread $\theta \simeq m_\pi / P_\pi \; O(1)$ and efficient collection requires strong downstream focusing elements that can keep the secondaries confined within limited aperture. Spreads of longitudinal momenta of the pions and muons are quite large, too.

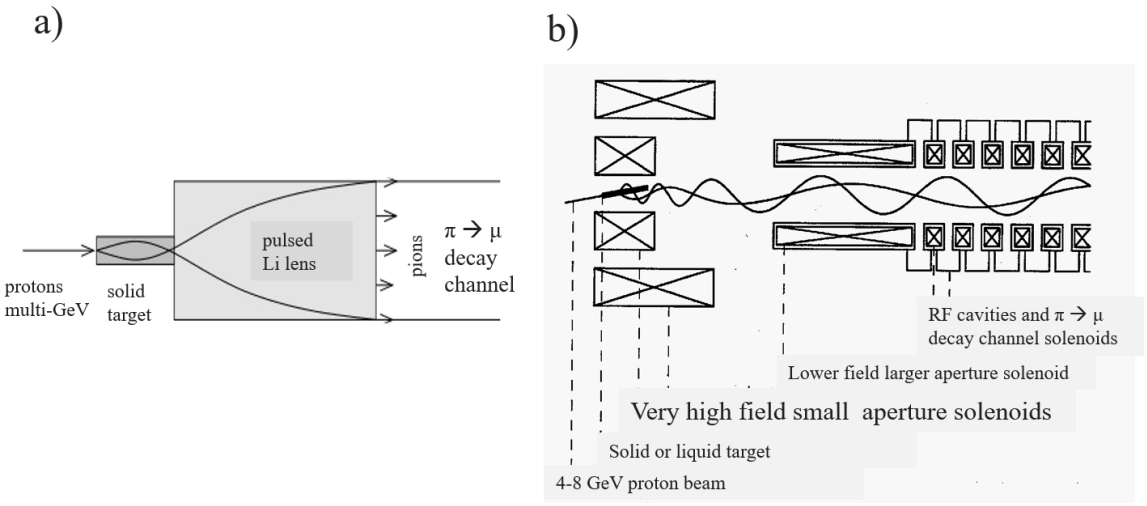

Figure 1. Muon production schemes: a) using pulsed short-focus Lithium lens for collection of pions, which later decay into muons; b) using very high field SC solenoids for pion capture, followed by RF acceleration section.

Fig.1 illustrates schematically two options. In the first one, pulsed short-focus Lithium lens is used for collection of pions, which later decay into muons. The method is employed, e.g., to get high energy beam of polarized muons for the muon $g-2$ experiment [4] and has an efficiency of conversion of 8 GeV protons into 3.1 GeV muons of about $2{\cdot}10^{-7}$ for the downstream accelerator beamline acceptance $O(1\%)$ [5]. The production system of the Fermilab Muon Campus includes Inconel target, followed by Lithium Lens with pulsed 116 kA current, producing a magnetic field gradient of 232 T/m located 0.3 m downstream the target. The Lithium Lens has the advantage over conventional quadrupoles in that it focuses in both transverse planes and produces an extremely strong magnetic field.

The most challenging are requirements of a future muon collider. Its muon source uses very high field SC solenoids for pion capture, followed by RF acceleration section. A high power proton beam out of 4-8 GeV 2–4 MW SRF H^- linac goes through pre-target accumulation and compressor rings where high-intensity 1–3 ns long proton bunches are formed. At $O(5$ Hz) rate, one of several of such bunches hit liquid mercury target for converting the proton beam into a tertiary muon beam with energy of about 200 MeV. The $p \rightarrow \mu$ conversion efficiency is very high $O(10\%)$ but resulting transverse normalized emittance of the muon beams are huge $O(0.1$ m rad) and a sophisticated multi-stage ionization cooling section is required to reduce the transverse and longitudinal emittances by a factor of 10^6 (for 6D emittance) and to make the beams suitable for a multistage acceleration and, eventually, high luminosity $\mu^+\mu^-$ collisions.

A complete cooling channel for high luminosity muon collider would be $O(1$ km) long and consist of $O(30)$ cooling stages, each yielding about a factor of 2 in 6D phase space reduction.

The ionization cooling method, though relatively straightforward in principle, faces some practical implementation challenges. These include RF breakdown suppression and attainment of high accelerating gradients in relatively low frequency NC RF cavities immersed in strong magnetic fields. The International Muon Ionization Cooling Experiment (MICE) [6] at RAL (UK) has recently demonstrated effective $O(10\%)$ reduction of transverse emittance of initially dispersed 140 MeV/c muons passing through an ionization cooling channel cell consisting of a sequence of LiH or liquid hydrogen absorbers within a lattice of up to 3.5 T solenoids that provide the required particle focusing.

2 PWA-based muon source

Plasma wakefield acceleration (PWA) channels are characterized by very high accelerating gradients and very strong focusing fields. These properties can be used for effective production of low emittance high energy muon beams.

2.1 Acceleration by plasma wake-fields

Ionized plasmas can sustain electron plasma density waves with electric fields in excess of $E_0 = cm_e\omega_p/e$ or

$$E_0 \approx 96\,[\mathrm{V/m}]\,\sqrt{n_0[\mathrm{cm}^{-3}]}, \tag{2.1}$$

(so-called *cold nonrelativistic wave-breaking field* [7]) where n_0 denotes the ambient electron number density, $\omega_p = k_p c = \sqrt{e^2 n_0/(m_e\varepsilon_0)}$ the electron plasma frequency, m_e and e electron rest mass and charge, respectively, c the speed of light in vacuum, and ε_0 the electrical permittivity of free space. For example, a plasma density of about 10^{18} cm^{-3} yields $E_0 \sim$100 GV/m, approximately three orders of magnitude greater than ∼100 MV/m obtained in conventional breakdown limited RF structures. Such gradients can be effectively excited by either powerful external pulses of laser light or by electron bunches if they are shorter than the plasma wavelength $\lambda_p = c/\omega_p \approx 33\mu\mathrm{m}\times\sqrt{10^{18}\,\mathrm{cm}^{-3}/n_0}$, or by longer beams of charged particles if their charge density is modulated with the period of λ_p.

There are several regimes of laser-driven plasma acceleration that may be accessed based on the intensity of the short laser pulses or electron bunches. *Quasi-linear regime* takes place in the case of low intensity drivers, while for the high intensities the plasma channel sustains, so called *bubble regime*, where (almost) all the electrons are expelled by either the drive e^- beam of by the laser ponderomotive force, forming an ion cavity co-propagating behind the laser. The structure of wakefield forces inside the ion cavity in the bubble regime is (see, eg [8]

$$F_z = eE_0k_p\zeta/2\,, \qquad F_r = e(E_r - B_\theta) = -eE_0k_pr/2 \tag{2.2}$$

Here $\zeta = z - ct$ is co-moving longitudinal coordinate (ζ=0 marks the center of the ion cavity) and r the radial coordinate. One can see that the accelerating force F_z is independent of the transverse position and the focusing force F_r linear with r. The transverse size of the bubble is about $r_{max} \simeq 3/k_p \approx \lambda_p/2$. The focusing forces of the plasma bubble in 10^{18} cm^{-3} plasma are equivalent to those for $G = E_0k_p/2 \simeq 0.3\mathrm{MT/cm}(n_p/10^{18}) \sim 300$ kT/cm gradients.

Naturally, the transverse fields in the ion cavity are defocusing for positively charge particle; hence, stable acceleration of, e.g., e^+ of μ^+ particles can not occur in the bubble regime and either the quasi-linear regime of operation or wakefield excitation in hollow plasma channels is needed.

So far, the primary advantage that a plasma wakefield accelerator could present was considered to be its compactness and, correspondingly, short length of, e.g., high energy e^+e^- collider. Collisions of intrinsically short bunches (a fraction of the plasma wavelength) are also advantageous for the reduction of beamstrahlung effects in plasma-based $O(1\ \mathrm{TeV})$ cme e^+e^- colliders.

Strong focusing forces of the PWA channels have been proposed for use in the final focus systems of the future linear e^+e^- colliders (*plasma lenses* [9–11]), but they also can help in production and efficient collection of secondary particles, such as muons.

2.2 Muons in PWA channel

Let us consider a muon born in plasma wake-field channel - see Fig.2. Equation of motion in the longitudinal degree of motion:

$$\frac{dp_z}{dt} = F_z\,, \tag{2.3}$$

where the muon momentum $p_z = \gamma_\mu \beta_\mu m_\mu$, naturally yields approximately linear growth of the muon's energy

$$\gamma_\mu = 1 + z\kappa e E_0/m_\mu c^2 \approx z\kappa k_p (m_e/m_\mu)\,. \tag{2.4}$$

For simplicity one can assume that particles quickly get relativistic ($\beta \approx 1$ and $dt = dz/c$, and optimal position ζ for the fastest acceleration corresponding to $\kappa \simeq 2$.

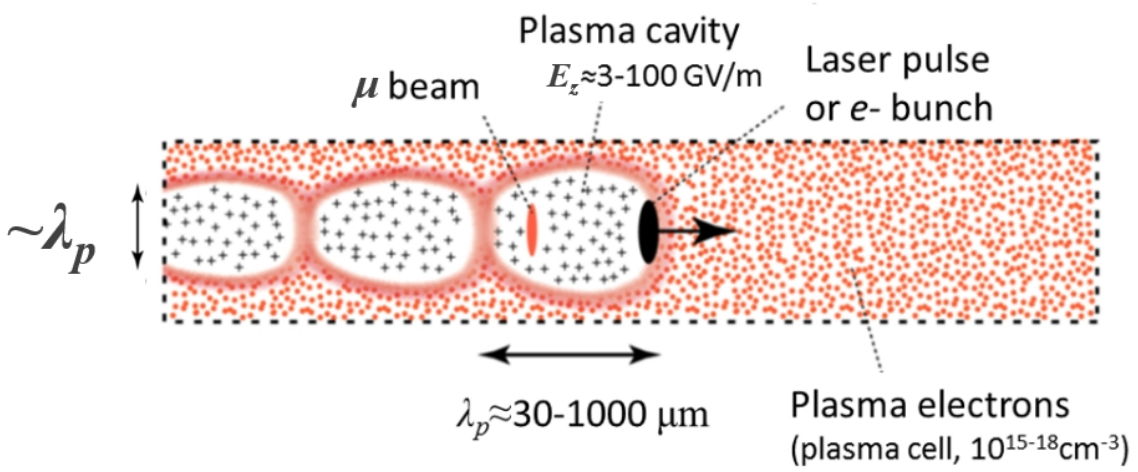

Figure 2. Conceptual scheme of the plasma-wakefield acceleration muon source.

In the transverse plane, the equation reads:

$$\frac{dp_r}{dt} = m_\mu c^2 \frac{d}{dz}\left(\gamma_\mu(z)\frac{dr}{dz}\right) = -eE_0\frac{k_p r}{2}. \tag{2.5}$$

The exact solution is:

$$r(z) = c_1 J_0\left(2\sqrt{\frac{z\pi}{\kappa\lambda_p}}\right) + c_2 Y_0\left(2\sqrt{\frac{z\pi}{\kappa\lambda_p}}\right). \tag{2.6}$$

It describes betatron oscillations with diminishing amplitude $r(z) \propto \exp(izk_\mu(z))$. Of importance for us is that the betatron oscillations are fast compared to the rate of acceleration. Indeed, the betatron-function $\beta_\mu = 1/k_\mu \approx \lambda_p\sqrt{2\gamma\mu m_\mu/m_e}$ (assuming negligible beam loading) and the fractional energy increase over the betatron oscillation period $\Delta\gamma_\mu/\gamma_\mu$ is small:

$$\frac{\Delta\gamma_\mu}{\gamma_\mu} = \frac{2\pi\beta_\mu\kappa k_p(m_e/m_\mu)}{\gamma_\mu} = 2\pi\kappa\sqrt{\frac{2m_e}{\gamma_\mu m_\mu}} \lesssim \frac{0.1}{\sqrt{\gamma_\mu}}. \tag{2.7}$$

Therefore, the transverse motion is adiabatic, the period of the muon oscillations grows as $\sqrt{\gamma_\mu} \propto \sqrt{z}$ while the amplitude slowly diminishes as $|r| \approx r_{max}\sqrt{\beta_\mu/\gamma_\mu} \propto r_{max}/\gamma_\mu^{1/4}$. The maximum angular deviation $\theta = dr/dz$ and the maximum amplitude relate to each other as $\theta_{max} = r_{max}/\beta_\mu$ and if the latter is limited by the transverse size of the bubble $r_{max} \simeq \lambda_p/2$, then

$$\theta_{max} = r_{max}/\beta_\mu = \pi\sqrt{\frac{m_e}{2\gamma_\mu m_\mu}} \approx \frac{0.15}{\sqrt{\gamma_\mu}}, \tag{2.8}$$

that defines the maximum normalized acceptance ϵ_μ^{max} of the muons captured and accelerated in the PWA channel:

$$\epsilon_\mu^{max} = \gamma_\mu\theta_{max}r_{max} = \lambda_p\gamma_\mu^{1/4}\sqrt{\frac{m_e\pi^2}{8m_\mu}} \approx 0.078\lambda_p\,. \tag{2.9}$$

This equation indicates that the limit is set by the dynamics at low energies $\gamma_\mu \sim 1$ and gives the estimated normalized emittance of the accelerated muon beam $\epsilon_\mu^{max} \approx 2.6$ μm for dense gaseous plasma $n_e = 10^{18}$ cm^{-3} and about 8-20 nm for solid state plasma $n_e = 10^{22-23}$ cm^{-3}. In the first case, acceleration of muons to, say, 10 GeV, would require $O(10$ cm) PWA channel, while in the case of a crystal it can be done over $O(1$ mm). To be noted, some emittance growth may occur by elastic scattering of the beam muon against the background ions in the plasma, but as shown elsewhere – see, e.g., Ref.[8] - the strong focusing in the plasma bubble suppresses the growth. The effect is almost independent on the plasma density and is very small even for electrons, while for muons it is negligible to due additional suppression factor $(m_e/m\mu)$.

Of course, these estimates look extremely attractive if compared to the most sophisticated muon production, ionization cooling and initial acceleration complex of a muon collider that requires kilometers of magnets and RF cavities to get $O(10$ GeV) muons with one to three orders of magnitude higher normalized transverse emittances $O(30$ μm).

2.3 Possible production schemes

Practical applications of the proposed PWA-based low-emittance high-energy muon sources critically depend on the muon intensity. Obviously, to take advantage of the fast acceleration and strong focusing, the muons should be produced inside the plasma bubble or in its vicinity $O(\lambda_p)$. One can think of three process that can make that:

a) photoproduction: high energy γ's or electrons on target $\rightarrow \mu^+(\mu^-)$;

b) "standard" production scheme described above in the introduction: protons $\rightarrow$ target $\rightarrow$ pions $\rightarrow \mu^+(\mu^-)$;

c) generation of "prompt" muons: protons $\rightarrow$ target $\rightarrow$ vector mesons $\rightarrow \mu^+(\mu^-)$.

In all these schemes the external primary beam can be injected into the plasma channel together

(simultaneously and in the same location) with external plasma drivers (electron bunch or laser pulse) - see Fig.3. Note, that plasma wakefield oscillations are essentially low-Q, so only one of few plasma periods/bubble can be employed to generate few short muon bunches. Also, the bubble wake field might be distorted (loaded) by high intensity primary beams and secondary particles (e^+/e^-, pions, kaons, etc).

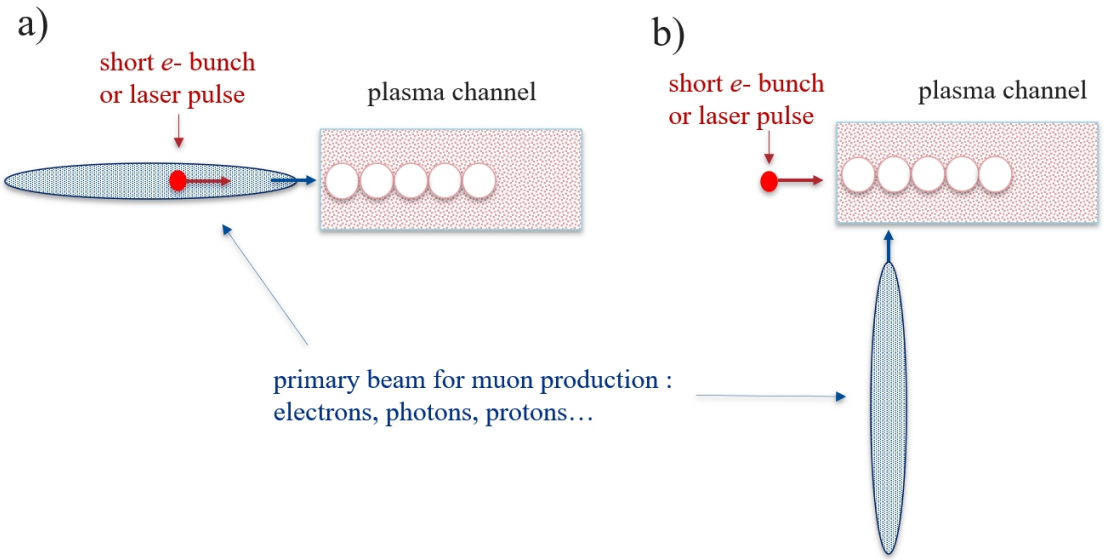

Figure 3. Muon PWA source : possible schemes for simultaneous injection of plasma wakefield drivers (laser pulses or short electron bunches) and primary beams needed for muon generation: a) collinear; b) orthogonal.

All these schemes have serious challenges, some of them considered in [12]. The photoproduction via Bethe-Haitler (BH) reaction $\gamma + Z \rightarrow \mu^+\mu^- + Z$ has small cross-section. Photons need to have energy far exceeding the muon production threshold of 210 MeV and can be obtained via bremstrahlung of multi-GeV electron beam in the target. Similar BH-channel efficiency for the positron production is $O(1\%)$ e^+/e^-. For the muon pair production, the BH cross-section is $(m_e/m_\mu)^2$=1/40000 times smaller - so, not only that muon efficiency will be $O(10^{-(5-6)}$ but also one should expect significant loading of the plasma bubble by much more numerous electrons and positrons.

Even smaller are cross-sections of the muon pair production via e^+e^- annihilation σ = 87nb/s(GeV^2) and even with most optimistic luminosity assumptions the production rate will be in sub-Hz regime [13].

Muon production cross-sections via photo-meson reaction and proton-nucleon reactions can be quite high O(100 mb) and conversion efficiencies can indeed be high, but the problem is two fold: a) proton bunches are usually much longer $O(1-10cm)$ and only a small fraction of proton beam can interact with plasma when the short sequence of bubbles exist in the plasma after the passage of drive laser pulse or short electron bunch; b) also, the immediate products of these reactions are mesons (pions and kaons) and takes long time and distance for them to decay into muons. For charge pions it is τ=26 ns ($c\tau$=7.8 m) and, correspondingly, these pions should stay in the plasma

channel for that long to decay that creates additional difficulties.

There are fast decay options, eg $\pi^0 - - > \gamma\gamma$ (τ =85 as with follow up photons possibly generation muons is of high enough energy. That would require very high energy primary proton beams O(100 GeV). Similarly, fast decays via vector bosons (e.g., D's) and generation of so called *prompt muons* offer another opportunity [14–16]. These reactions might have relatively high (still small) cross-section O(1 *mu*b) and might require very high energy of primary protons O(100-1000 GeV). Practicality of these and other possible muon generation schemes for the PWA-based muon sources requires further, more detail examination.

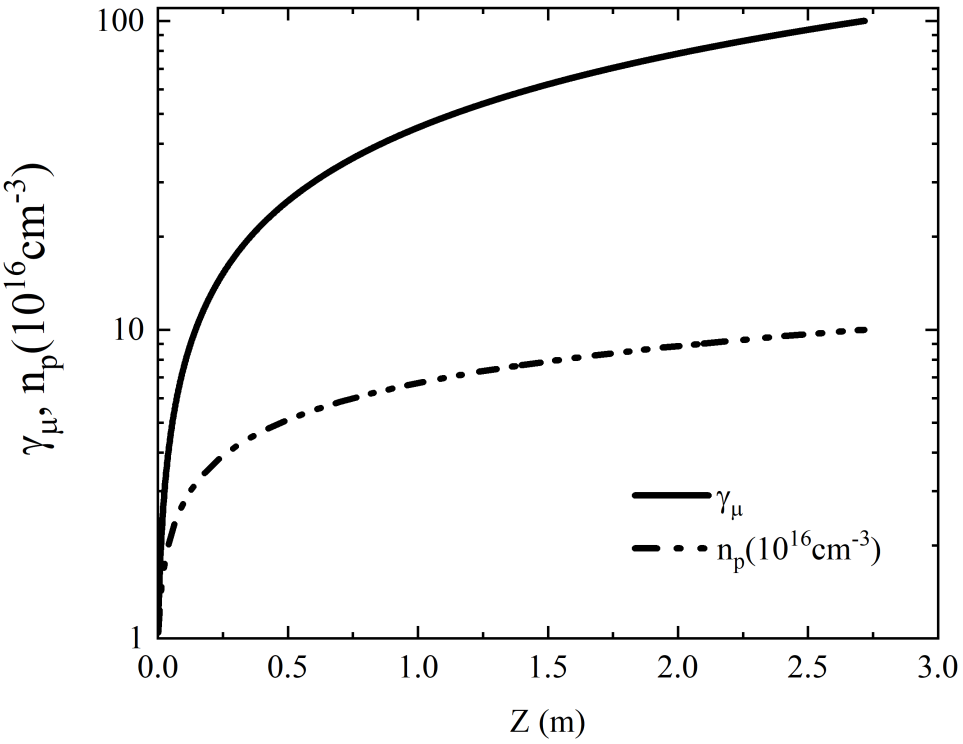

Figure 4. Plasma density and muon energy in tapered PWA-based 10 GeV muon source with normalized acceptance of 25 μm.

The PWA-based scheme can be modified to maximize the acceptance of the accelerated beam. For tha, a gradual increase of the plasma density $n_e(z)$ is needed. E.g., if one takes the desired normalized acceptance of ϵ_μ^{max} then according to Eq.2.9, in order to keep it constant along the channel, the plasma wavelength should scale as:

$$\lambda_p(z) = \lambda_p^0 \gamma_\mu(z)^{-1/4}, \tag{2.10}$$

where initial plasma density should correspond to initial wavelength of $\lambda_p^0 = \epsilon_\mu^{max}/0.078$. Solution

of (2.10) is trivial, taking into account Eq.(2.4):

$$n_p(z) = n_p^0\Big(1+\alpha z\Big)^{2/5}, \gamma_\mu(z) = \Big(1+\alpha z\Big)^{4/5}, \tag{2.11}$$

where $\alpha = (5\pi\kappa m_e)/(2\lambda_p^0 m_\mu)$. Fig.4 presents calculated density profile for a 2.5 m long 10 GeV muon source with $\epsilon_\mu^{max} = 25\mu$m (corresponding to $\lambda_p^0 = 0.33$ mm and $n_p^0 = 10^{16}$ cm$^{-}3$.

3 Discussion

The proposed muon source based on plasma wakefield acceleration looks very attractive because of the promise of very small emittance high energy muon beams in an ultimately small footprint. Our analysis above indicates normalized emittances possible in the range of μm rad to nm rad, depending on the plasma density (ϵ_n is proportional to the plasma wavelength λ_p). That surely would be very welcome development not only for advanced schemes (such as linear crystal channeling muon collider [17]) but also in traditional muon colliders as it would eliminate the most complex muon production, ionization cooling and early acceleration parts of multi-TeV muon collider. Of course, acceleration and focusing of positively charged muons μ^+ will be as challenging as for positrons in the advanced laser-plasma and beam-driven plasma wakefield accelerators.

Even the first look into the proposed muon sources indicates significant challenge as most potential applications require both small emittances and high intensity (i.e., high-brightness) muon beams, and that needs to be studied in detail and the most promising options identified and analyzed.

For completeness, one has to mention that due to growing interest in high brightness muon beams, several other novel schemes have been recently proposed, such as a) the *positron-driven muon production* scheme that envisions 45 GeV positrons hitting a target to generate muon pairs through e^+e^- annihilation just above threshold [18], allowing low emittance beams to be obtained directly, without any cooling [19]; and b) the *Gamma Factory* (GF) [20] that could potentially generate frequent bursts of gamma rays by repeatedly colliding a partially stripped heavy-ion beam circulating in the LHC, or in a future higher-energy hadron storage ring like the FCC-hh, with a conventional laser pulse, profiting from two Lorentz boosts. Being sent on a suitable target, the GF's intense bursts $O(1$ GeV) gamma could produce muons at an high rate. Thereby, the GF could deliver positrons at the rate required for the aforementioned positron-based muon production, or, alternatively, it could directly provide a low-emittance muon beam [21].

Practicality of these schemes is under active exploration, which might be synergistically helpful to follow-up studies of the PWA-based high brightness high energy muon sources proposed in his paper.

Acknowledgments

Author greatly appreciates helpful discussions with M. Palmer, K. Yonehara and N. Mokhov on various aspects of the proposed scheme. This manuscript has been authored by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics.

References

[1] *Modern Muon Physics: Selected Issues*.

[2] S. Geer, *Muon colliders and neutrino factories*, *Annual Review of Nuclear and Particle Science* **59** (2009) 347.

[3] K.R. Long, D. Lucchesi, M.A. Palmer, N. Pastrone, D. Schulte and V. Shiltsev, *Muon colliders to expand frontiers of particle physics*, *Nature Physics* **17** (2021) 289.

[4] B. Abi, T. Albahri, S. Al-Kilani, D. Allspach, L. Alonzi, A. Anastasi et al., *Measurement of the positive muon anomalous magnetic moment to 0.46 ppm*, *Physical Review Letters* **126** (2021) 141801.

[5] D. Stratakis, M. Convery, J. Morgan, D. Still, M. Syphers and V. Tishchenko, *Simulated performance of the production target for the muon g-2 experiment*, *arXiv preprint arXiv:1709.07838* (2017) .

[6] M. Bogomilov et al., *Demonstration of cooling by the muon ionization cooling experiment*, *Nature* **578** (2020) 53.

[7] J.M. Dawson, *Nonlinear electron oscillations in a cold plasma*, *Physical Review* **113** (1959) 383.

[8] C. Schroeder et al., *Beam dynamics challenges in linear colliders based on laser-plasma accelerators*, *JINST* **this Special Issue** .

[9] P. Chen, *Grand disruption: A possible final focusing mechanism for linear colliders*, *Part. Accel.* **20** (1986) 171.

[10] J. Su, T. Katsouleas, J. Dawson and R. Fedele, *Plasma lenses for focusing particle beams*, *Physical Review A* **41** (1990) 3321.

[11] E. Chiadroni, M. Anania, M. Bellaveglia, A. Biagioni, F. Bisesto, E. Brentegani et al., *Overview of plasma lens experiments and recent results at SPARC_LAB*, *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* **909** (2018) 16.

[12] A.A. Sahai, T. Tajima and V.D. Shiltsev, *Schemes of laser muon acceleration: Ultra-short, micron-scale beams*, *International Journal of Modern Physics A* **34** (2019) 1943008.

[13] *dimus*, Tech. Rep.

[14] H. Kasha, R. Kellogg, M. Lauterbach, R. Adair, L. Leipuner, R. Larsen et al., *Evidence of pair origin of prompt muons*, *Physical Review Letters* **36** (1976) 1007.

[15] W. Morse, K.-W. Lai, R. Larsen, Y. Lee, L. Leipuner, D. Lowenstein et al., *Properties of prompt muons produced by 28 gev proton interactios*, in *AIP Conference Proceedings*, vol. 45, pp. 129–137, American Institute of Physics, 1978.

[16] R. Stroynowski, *Lepton pair production in hadron collisions*, *Physics Reports* **71** (1981) 1.

[17] V.D. Shiltsev, *High-energy particle colliders: past 20 years, next 20 years, and beyond*, *Physics-Uspekhi* **55** (2012) 965.

[18] M. Antonelli, M. Boscolo, R. Di Nardo and P. Raimondi, *Novel proposal for a low emittance muon beam using positron beam on target*, *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* **807** (2016) 101.

[19] M. Boscolo, J.-P. Delahaye and M. Palmer, *The future prospects of muon colliders and neutrino factories*, in *Reviews of Accelerator Science and Technology: Volume 10: The Future of Accelerators*, pp. 189–214, World Scientific (2019).

[20] M.W. Krasny, *The gamma factory proposal for CERN*, *arXiv preprint arXiv:1511.07794* (2015) .

[21] F. Zimmermann, *Future colliders for particle physics—"big and small"*, *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* **909** (2018) 33.