Study of Non-Standard Interactions in Rare Mesonic Decays

By

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I Shakeel Mahmood, **CIIT/FA10-PPH-001/ISB**, hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to a disciplinary action under the plagiarism rules of the HEC.

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It is certified that Mr. Shakeel Mahmood, Registration number CIIT/FA10-PPH-001/ISB has carried out all the work related to this thesis under my supervision at the Department of Physics, COMSATS Institute of Information Technology, Islamabad and the work fulfills the requirement for the award of PhD degree.

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To My Parents and Beloved Wife

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List of thesis Publications

This thesis is based on the following papers.

1) Hints of new physics from $D_s^+ \to D^+ \nu\bar{\nu}$ **,** $D^0 \to \pi^0 \nu\bar{\nu}$ **, and** $D_s^+ \to K^+ \nu\bar{\nu}$ **decays**

Shakeel Mahmood, *Farida Tahir*, *Azeem Mir*, *Modern Physics Letters A* Vol. 30, No. 01, 1550004 (2015).

2) NSIs in semileptonic rare decays of mesons induced by second generation of quarks (c-quark and s-quark)

Shakeel Mahmood, *Farida Tahir*, *Azeem Mir*, International Journal of Modern Physics A Vol. 30, No. 04n05, 1550024 (2015).

3) Study of pure and semileptonic decays of Ds meson within R-parity violating supersymmetric model

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4) A search of New Physics with $D_s^+ \to D^+ \nu\bar{\nu}$ **,** $B_s^0 \to B^0 \nu\bar{\nu}$ **,** $K^+ \to \pi^+ \nu\bar{\nu}$ **,** $D^+ \to \pi^+ \nu\bar{\nu}$ **,** $D^0 \to \pi^0 \nu\bar{\nu}$ **and** $D_s^+ \to K^+ \nu\bar{\nu}$

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ABSTRACT

Study of Non-Standard Interactions in Rare Mesonic Decays

The study of rare decays and search for new physics are entangled with each other. The importance of these decays highly increases if the decay products contain dineutrinos in their final state due to its theoretically cleanest nature. In this scenario, for detailed illustration, we will use the pure and semileptonic rare decays of pseudoscalar mesons with missing energy and study the role of NSIs on Br and non standard parameters $\epsilon^{qL}_{\alpha\beta}$and $\epsilon^{qR}_{\alpha\beta}$ by using model-independent analysis. We investigate the long distance dominated (in the standard model) processes $D^+_s \rightarrow K^+ \upsilon\overline{\upsilon}$, $D^0 \rightarrow \pi^0 \upsilon\overline{\upsilon}$ and short distance dominated (in the standard model) $D^+_s \rightarrow D^+ \upsilon\overline{\upsilon}$ decays for the purpose of non-standard neutrino interactions (NSIs). The branching ratios of $D^+_s \rightarrow K^+ \upsilon\overline{\upsilon}$, $D^0 \rightarrow \pi^0 \upsilon\overline{\upsilon}$ and $D^+_s \rightarrow D^+ \upsilon\overline{\upsilon}$ decays are calculated in the framework of NSIs. The values of non-standard parameters $\epsilon^{uL}_{\tau\tau}$, $\epsilon^{dL}_{\tau\tau}$ and $\epsilon^{dL}_{\alpha\beta}$ for $\alpha = \beta = e$ or μ are found. Analysis of NSIs are extended by incorporating the second and third generations of quarks. We investigate that why the only available non-standard parameter constraints in the literature are $\epsilon^{uL}_{\alpha\beta}$ and $\epsilon^{dL}_{\alpha\beta}$,and why we are unable to find bounds on non-standard parameters, pertaining to second and third generation, i.e. $\epsilon^{bL}_{\alpha\beta}$, $\epsilon^{sL}_{\alpha\beta}$, $\epsilon^{cL}_{\alpha\beta}$ and $\epsilon^{tL}_{\alpha\beta}$. Contrary to quark sector, in charged lepton sector, non-standard parameters $\epsilon^{eL}_{\alpha\beta}$, $\epsilon^{\mu L}_{\alpha\beta}$and $\epsilon^{\tau L}_{\alpha\beta}$, relevant to second and third generations, are good constraints. We investigate $D^+_s \rightarrow K^+ \upsilon\overline{\upsilon}$, $D^0 \rightarrow \pi^0 \upsilon\overline{\upsilon}$ and $D^+ \rightarrow \pi^+ \upsilon\overline{\upsilon}$ decays, in which flavor changing neutral current (FCNC) involve only up type quarks, i.e. $c \longrightarrow u\upsilon\overline{\upsilon}$ as an external particles and down type (d, s, b) quarks propagating in the loop. While in $K^+ \rightarrow \pi^+ \upsilon\overline{\upsilon}$, $D^+_s \rightarrow D^+ \upsilon\overline{\upsilon}$ and $B^0_s \longrightarrow B^0 \upsilon\overline{\upsilon}$ FCNC involves down type quarks, i.e. $s \longrightarrow d\upsilon\overline{\upsilon}$ as an external lines, and up type (u, c, t) quark propagating inside the loop. The comparative study of the processes is done to check the generation sensitivity of the parameters of NSIs. We show that the dominant and comparable contribution of NSI is due to the first and second generation, i.e. (u, d) and (c, s) quarks, while contribution of $(t, b$) quarks is highly suppressed at radiative level, which is contrary to the SM. Furthermore, We present the comparative study of semileptonic and leptonic decays of $D_s, D^{\pm}$ ($D \rightarrow l_\alpha \nu_\alpha$, $D \rightarrow l_\alpha \bar{l}_\beta$, $D^{\pm}_s \rightarrow K^{\pm} l^+_\alpha l^-_\beta (v_\alpha)$ along with $D \rightarrow M l_\alpha \bar{l}_\beta$; $M = \pi, K$ and $\alpha, \beta = e, \mu$) within the framework of R-parity violating Minimal Supersymmetric Standard Model (MSSM). The comparison shows that combination($\lambda_{\beta i\alpha} \lambda^{*\prime}_{ijq}$) and product couplings ($\lambda^{\prime}_{\beta qk} \lambda^{*\prime}_{\alpha jq}$), contributing to the branching fractions of the processes $D^0 \rightarrow \mu^+\mu^-$, $D^+ \rightarrow l_\alpha \nu_\beta$, $D^+_s \rightarrow l_\alpha \nu_\beta$, $D^0 \rightarrow K^- e^+ \nu_e$, $D^{\pm}_s \rightarrow K^{\pm} l^+_\alpha l^-_\beta$ and $D^{\pm} \rightarrow \pi^{\pm} l^+_\alpha l^-_\beta$ (both for $\alpha = \beta$ and $\alpha \neq \beta$), are either consistent or comparable with the existing experimental data, when calculating in the R-parity violating SUSY model. Hence the golden channel for the study of new physics is provided. Contrary to that, processes like $D^0 \rightarrow e^+e^-$, $D^0 \rightarrow \pi^- l^+_\alpha \nu_\alpha$, $D^+ \rightarrow \pi^0 l^+_\alpha \nu_\alpha$ and $D^0 \rightarrow K^- l^+_\alpha \nu_\alpha$ are accommodated well in SM, but unfavorable for the study of new physics. We identify such type of processes in our analyses and single out the important ones, suitable for exploring in the current and future experiments.

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Chapter 1

Introduction

The reduction principle [1, 2] is the principle that plays a pivotal role in enabling us to study the properties of the very large variety of macroscopic forms of matter in terms of a few microscopic particles that interact through electromagnetic, weak and strong interactions. The gradual diminution of world from macroscopic range to particle physics scale, enables us to explore such structures by means of atomic and nuclear physics. In this sense, particle physics accommodates the basic laws of nature with the help of the standard model (SM).

The SM is a remarkable theory of fundamental particles and their interactions; developed by Salam, Glashow and Weinberg [3]. This theory not only incorporated all those sub-atomic particles known at that time, but it also predicted the existence of additional particles as well. These predicted particles were later on discovered experimentally as $W^{\pm}$ and Z bosons in 1983 [4, 5], top quarks in 1995 [6], the tau neutrino in 2000 [7] and Higgs boson in 2012 [8, 9]. The unification of electromagnetic and weak interaction is provided by the electroweak part of the SM. The experimental varifications have given it the status of a highly precise model. The SM is based on gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, where the associated lagrangian remains invariant under local gauge transformation. The requirement of local gauge invariance gives birth to different gauge bosons (gluons, weak vector bosons and photons) and provides reason for the existence of strong, weak and electromagnetic forces. Here, $SU(3)_C$ is a non-abelian group which deals with chromodynamics and $SU(2)_L \times U(1)_Y$ groups are associated with the electroweak part of the standard model. $SU(2)_L$ is a non-abelian group representing electroweak isospin having triplet W_μ^1, W_μ^2 and W_μ^3, while $U(1)_Y$ is an abelian group of weak hypercharge associated with gauge boson B_μ. W_μ^3 and B_μ are mixtures of physical gauge bosons ($A_\mu$and Z_μ),which are propagators of electromagnetic and weak interactions respectively. Under $SU(2)_L \times U(1)_Y$ gauge groups, all fermions(chiral particles) transform as left handed doublets and right handed singlets. That is the very reason why masses of all fermions are protected by chiral symmetry and the masses of all gauge bosons are protected by gauge symmetry. In order to retain the demand of local gauge invariance and and at the same time give masses to both gauge bosons and fermions, we have to introduce Higgs mechanism [10, 11], which is responsible for breaking $SU(2)_L \times U(1)_Y$ symmetry into $U(1)_{em}$ one. In Higgs mechanism, the weak gauge bosons ($W^\pm$ and Z^0) acquire their masses through spontaneous symmetry breaking mechanism (SSB), whereas the gauge particle of electromagnetic interaction, i.e. photon(γ), remains massless: While all the charged leptons and the quarks generate their masses through Yukawa interactions (interaction of fermions with the minimum of the Higgs field). In the Yukawa interactions, each fermion flips its handedness ($L \leftrightarrow R$). As there are no right-handed neutrinos in the SM, Yukawa interaction, in the SM, is therefore unable to generate the masses of neutrinos. Hence, neutrinos are strictly massless in the SM. Consequently, lepton flavor is conserved, i.e. each neutral lepton (ν_e, ν_μ, ν_τ) is produced in conjecture with charge leptons(e^-, μ^-, τ^-), hence leading to the lepton universality, which depicts the fact that $e^- \leftrightarrow \nu_e$, $\mu^- \leftrightarrow \nu_\mu$ and $\tau^- \leftrightarrow \nu_\tau$ can proceed through lepton flavour conserving universal charge current ($W^\pm$) interaction ($LFCUCC$), while lepton flavour violating charge inteactions ($e^- \not\leftrightarrow \nu_\mu(\nu_\tau), \mu^- \not\leftrightarrow \nu_e\nu_\tau$ and $\tau^- \not\leftrightarrow \nu_e\nu_\mu$) are not allowed. Similarly, we have lepton flavour conserving ($\nu_e \leftrightarrow \nu_e, \nu_\mu \leftrightarrow \nu_\mu$ and $\nu_\tau \leftrightarrow \nu_\tau$) universal neutral current (Z^0) interaction ($LFVUNC$), but flavour changing current transitions ($\nu_e \not\leftrightarrow \nu_\mu(\nu_\tau), \nu_\mu \not\leftrightarrow \nu_e(\nu_\tau)$ and $\nu_\tau \not\leftrightarrow \nu_e(\nu_\mu)$)are not allowed. Hence, we can conclude that the flavour changing charge as well as neutral current are not allowed in lepton sector of the SM at any level.

However, the observational fact of neutrino oscillation provides the evidence that neutrinos are massive and they do mix. Consequently, we have non-universal (rather than universal) flavour changing(violating) charge ($NUFVCC$) as well as neutral ($NUFVNC$) current interaction, which cannot be accomodated in the SM. For this we have to look at some new model, which can accomodate these new kind of interactions, hence the so called non-standard interaction (NSI). In general there are many extentions of the SM, which not only include masses and mixing, but also generate these new NSI; and supersymmetry ($SUSY$) is one of these extentions. The main focus of our research is to study the effect of these NSIs in semileptonic rare decays of mesons having neutrinos in their final state, both in the model independent and model dependent ways. In the model dependent approach, we use R-parity violating SUSY model.

SUSY (the symmetry between bosons and fermions) is the most fascinating, renormalizable, anomaly free and gauge invariant theory based on the extended quantum field theoretical structure of the SM. This theory not only accomodates the masses of neutrinos, but also helps to achieve the one of the biggest goals of particle physics, i.e. grand unification of all types of interactions. Furthermore, it also solves the problem of instability of Higgs mass due to quardratic divergent radiative correction, the so called Hierarchy problem. The cost which we have to pay to achieve these big goals is to introduce the new particles, called supersymmetric particles. The minimal version of SUSY is the one in which we can associate only one SUSY particle with each SM particle, known as minimal supersymmetric standrad model ($MSSM$). Due to double particle spectrum, the number of interactions also increases, as a consequence of which we get most generalized lagrangian, having Lepton and Baryon number conserving term (analogous to the SM) as well as the violating (not present in the SM) one. This simultaneous presence of lepton and baryon number violating terms has a dangerous impact on matter, i.e. matter is no more stable (leading to fast proton decay), which is contradictory to the observational fact.

The standard way to cope with the problem of fast proton decay is to introduce a new adhoc symmetry, called R-parity (R_p), defined as $R = (-1)^{3B+2S+L}$, where B is the baryon number, L is the lepton number and S is the intrinsic spin of the particle. According to this definition, all SM particles have(R_p) = +1 and all SUSY particles have (R_p) = −1. This symmetry dictates that SUSY particles are always produced in pair, and lightest SUSY particle (LSP) is stable, which is the candidate of cold dark matter. This is what we call the R_p conserving version of SUSY. But, on the other hand, introducing certain symmetry just by hand to protect proton decay is somewhat artificial, as it does not endure any internal inconsistency. Therefore, other symmetries can be presented to prevent proton decay without any obligation of R-parity. These symmetries lead to the violation of either lepton or baryon number, but not both simultaneously.

Consequently, we can get production of single SUSY particle, the LSP can decay to the SM particles and the ordinary SM particles can decay through SUSY particles as their resonance state. In the last case, constraining R_p violating Yukawa coupling is an important task, which is one of the aims of our research. The glimpse of chapterwise plan of our research work is organised as follows:

In **Chapter 1,** we give the overall introduction of the thesis. **Chapter 2** is fully devoted to the SM. In this chapter, we develop the historical background of the SM in particular and gauge structure of the SM in general. In sections 2.3 and 2.4, we discuss the theoretical bases and formulation of the Electroweak part of the SM. In **Chapter 3,** we carry out the phenomenological implication of the SM. In this chapter, our focus is on pure leptonic, semileptonic and non-leptonic interactions, which lead towards the Cabibbo theory [12], where we discuss universality of weak interactions and suppression of $\triangle S = 1$ weak interactions in section 3.1. This naturally leads to the non existence of flavour changing neutral current, the so-called GIM mechanism [13], which we discuss in section 3.2. Section 3.3 deals with the generalization of GIM mechanism for three generations, i.e. CKM matrix [14], which ensures the absence of flavour changing neutral current ($FCNC$) at tree level and incorporates CP violation: there by providing the reason for matter dominant universe. In sections 3.4 and 3.5, we discuss the need for "operator product expansion" and "Effective Hamiltonian". With this, we develop sufficient background for the uniform treatment of the study of rare weak decays of two and three bodies, pure and semileptonic pseudoscalar meson decays, (especially with neutrinos in the final state). Sections 3.6 and 3.7 are devoted to this purpose. In section 3.8, we discuss the limitations of the SM, which prompt us to go beyond the SM, i.e. new physics (NP).

The need for NP pertaining to neutrino is explored in **Chapter 4.** In sections 4.1-4.3, we study the leading (oscillation) and subleading (non-standard neutrino oscillation) mechanism in order to explain neutrino flavour transition. Whereas sections 4.4 and 4.5 address the semileptonic decays of charm mesons in the framework of NSIs. We perform our analysis in model independent (MI) way. The results obtained by this study are discussed in section 4.6. In **Chapter 5,** we extend our analysis of NSI (MI) by incorporating second and third generation of quarks in the loop and its experimental status. Section 5.4 is devoted to exploration of NSI, using $D_s^+ \rightarrow K^+\nu_\alpha\overline{\nu}_\beta$, $D^+ \rightarrow \pi^+\nu_\alpha\overline{\nu}_\beta$ and $D^0 \rightarrow \pi^0\nu_\alpha\overline{\nu}_\beta$ as probe, and results are discussed in section 5.5.We opt for model dependent approach in **Chapter 6.** In this chapter, we develop necessary background for SUSY model in general, and R-parity violating model in particular. As a phenomenological implication, we study tree and loop level pure leptonic decays of charm mesons, $(D^{+}, D_{s}^{+} \rightarrow l_{\alpha}^{+}\nu_{\beta})$ and $\left(D^{0} \rightarrow l_{\alpha}^{\pm} l_{\beta}^{\mp}\right)$, in sections 6.8 and 6.10, respectively. While sections 6.9 and 6.11 are devoted to the study of semileptonic decays of charm mesons $D \rightarrow (\pi, K^{+})\, l_{\alpha}^{+}\nu_{\beta}$ and $D_{s} \rightarrow K l_{\alpha}^{\pm} l_{\beta}^{\mp}$ in R -parity violating SUSY model. Results obtained from this analysis are discussed in section 6.12. Brief summary of our main results and conclusion are presented in **Chapter 7.**

Chapter 2

The Standard Model of Particle Physics

2.1 History of the Standard Model

In 1961, Glashow proposed [15] the symmetry group, $SU(3) \times SU(2) \times U(1)$, for electrorweak theory. His aim was to unify weak and electromagnetic interactions by using a symmetry group which contained $U(1)_{em}$. There was a prediction of four physical vector bosons states, $W^{\pm}, Z$ and γ obtained by the rotation of weak eigen states. Especially, θ_W rotation defines Z weak boson which was already included in the theory. The inclusion of Z boson mediated current provided the exact structure of weak neutral current. The gauge symmetry is the custodian of masslessnees of gauge bosons. The insertion of mass term for the gauge bosons was also making the theory non-renormalizable.

The Goldstone theorem introduced by Nambu in 1961 [16] and further developed and generalized by Goldstone, Salam and Weinberg also contributed to its generalization in 1962 [3]. This theorem states that spinless massless particles.are produced if the global symmetries are spontaneously broken.

The electroweak theory was made by Weinberg and Salam independently in 1968 by using $SU(2)\times U(1)$ gauge group introduced by Glashow. This remarkable theory is known as Glashow-Weinberg-Salam model or the standard model (SM). Although, it was constructed with the help of gauge principle but it was equally capable of incorporating all the known phenomenological properties of pregauged theories of weak interactions, especially, intermediate vector boson (IVB) theory. It was developed with the idea of spontaneous symmetry breaking (SSB). The heart of SSB is the introduction of a Higgs doublet which gives masses to gauge bosons with out destroying the renormalizability of gauge theory. In the SM, IVB are the associated gauge bosons, $W^{\pm}, Z$ and γ. The gauge boson $W^{\pm}, Z$ acquire masses and electroweak theory still respects unitarity at all energies as well as renormalizability.

In 1971, t'hooft provided the proof of renormalizability of gauge theories with and without spontaneous symmetry breaking.

The experimental discovery of weak neutral current in 1983 [4, 5], as predicted by the model, made it a successful theory of electroweak interactions. The experiment also provided the first measurement of $sin(\theta_W)$. By using θ_W and weak coupling g_W the SM provided the first estimate of M_W and M_Z which were discovered experimentally in 1983 at the predicted masses.

The discovery of $W^{\pm}$ and Z bosons was made at SpS collider at CERN in 1983 [4, 5]. There have been plenty of tests of the SM even at quantum level and all of them have been successful so far. In 2012, discovery of the Higgs particle at LHC has further strengthened the model [8, 9].

2.2 The Need of $SU(2) \times U(1)$ Gauge Group for Electroweak Unification

The gauge group required for the electroweak unification is $SU(2)\times U(1)$. In order to understand the choice of this group, it is sufficient to take only $e^{-}\nu$ component of charge weak current. this can be written in the form:

$$J_\mu = \overline{\nu}\gamma_\mu(\frac{1-\gamma_5}{2})e \ . \tag{2.2.1}$$

By using $(\frac{1-\gamma_5}{2})^2 = \frac{1-\gamma_5}{2}$ and $\gamma_\mu\gamma_5 = -\gamma_5\gamma_\mu$,it can be written as

$$J_\mu = \overline{\nu}_L\gamma_\mu e_L \ . \tag{2.2.2}$$

In doublet form

$$\Psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \ \overline{\Psi}_L = (\ \overline{\nu}_L \quad \overline{e}_L \ ), \tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2) \tag{2.2.3}$$

$\tau_i (i = 1, 2, 3)$ are Pauli matrices.

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The generators I_i of $SU(2)$ satisfy the

$$[I_i, I_j] = i\epsilon_{ijk} I_k, \text{ and } I_i = \frac{\tau_i}{2}.$$

These (unitary and unimodular) form the fundamental representation of $SU(2)$. By using τ_+ and doublet Ψ_L,eq.(2.2.2) becomes

$$J_\mu = \overline{\Psi}_L \gamma_\mu \tau_+ \Psi_L, \tag{2.2.4}$$

and similarly

$$J_\mu^\dagger = \overline{\Psi}_L \gamma_\mu \tau_- \Psi_L. \tag{2.2.5}$$

For charge current, only two generators are used but $SU(2)$ algebra is not complete. To complete it we have to use the third generator,

$$J_\mu^3 = \overline{\Psi}_L \gamma_\mu \tau_3 \Psi_L = \frac{1}{2}(\overline{\nu}_L \gamma_\mu \nu_L - \overline{e}_L \gamma_\mu e_L). \tag{2.2.6}$$

But,this can not be J_μ^{em} which is $-\overline{e}\gamma_\mu e$. This implies that $SU(2)$ is not sufficient for electroweak unification. So, the group must be extended and the simplest extension is $SU(2) \times U(1)$.

The Gell-Mann Nishijima relation is given as:

$$Q = I_3 + \frac{Y}{2} \tag{2.2.7}$$

where I_3=weak isospin, Y=weak hypercharge and Q=elecrtic charge.

The corresponding relation among the currents can be made as

$$J_\mu^{em} = J_\mu^3 + \frac{1}{2} J_\mu^Y, \tag{2.2.8}$$

and

$$J_\mu^Y = 2(J_\mu^{em} - J_\mu^3), \tag{2.2.9}$$

$$J_\mu^{em} = (-1)\overline{e}_L \gamma_\mu e_L + (-1)\overline{e}_R \gamma_\mu e_R, \tag{2.2.10}$$

$$J_\mu^3 = \overline{\Psi}_L \gamma_\mu \tau_3 \Psi_L = \frac{1}{2}(\overline{\nu}_L \gamma_\mu \nu_L - \overline{e}_L \gamma_\mu e_L). \tag{2.2.11}$$

By using eqs.(2.2.10) and (2.2.11) as input, we obtain the output

$$J_\mu^Y = (-1)\overline{e}_L \gamma_\mu e_L + (-2)\overline{e}_R \gamma_\mu e_R + (-1)\overline{\nu}_L \gamma_\mu \nu_L, \tag{2.2.12}$$

which gives hypercharge of doublet $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ as -1 and for singlet e_R as -2.

When the symmetry is broken, the two neutral currents J_μ^{em} and J_μ^3 will mix to form two physical currents, out of which one must be electromagnetic current and other will be a new neutral current. These currents will couple to physical vector bosons A_μ and Z_μ as follow;

$$g_2 J_\mu^3 W_\mu^3 + \frac{1}{2} g^{'} J_\mu^Y B_\mu = e J_\mu^{em} A_\mu + g_z J_\mu^z Z_\mu, \tag{2.2.13}$$

where

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3, \tag{2.2.14}$$

and

$$\begin{aligned} Z_\mu &= -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3; \qquad (2.2.15) \\ \theta_W &= \text{Weinberg or weak angle} \end{aligned}$$

or

$$W_\mu^3 = \sin\theta_W A_\mu + \cos\theta_W Z_\mu, \quad B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu \tag{2.2.16}$$

Thus we get

$$g_2 J_\mu^3(\sin\theta_W A_\mu + \cos\theta_W Z\mu) + \frac{1}{2}g^{'} J_\mu^Y(\cos\theta_W A_\mu - \sin\theta_W Z\mu) = eJ_\mu^{em} A_\mu + g_z J_\mu^z Z_\mu \quad (2.2.17)$$

So

$$eJ_\mu^{em} = g_2 J_\mu^3 \sin\theta_W + \frac{1}{2}g^{'}\cos\theta_W J_\mu^Y = -e(\overline{e}_L\gamma_\mu e_L + \overline{e}_R\gamma_\mu e_R) \quad (2.2.18)$$

but using eq.(2.2.16), we have

$$eJ_\mu^{em} = \frac{1}{2}g_2\sin\theta_W(-\overline{e}_L\gamma_\mu e_L + \overline{\nu}_L\gamma_\mu\nu_L) + \frac{1}{2}g^{'}\cos\theta_W((-1)\overline{e}_L\gamma_\mu e_L + (-2)\overline{e}_R\gamma_\mu e_R + (-1)\overline{\nu}_L\gamma_\mu\nu_L). \quad (2.2.19)$$

Comparing eq (2.2.18), and eq.(2.2.19), we get the following relations

$$\begin{aligned} g_2\sin\theta_W &= e \text{ and } g^{'}\cos\theta_W = e \\ \tan\theta_W &= \frac{g_2}{g^{'}}, \end{aligned}$$

and similarly

$$\begin{aligned} g_z J_\mu^z &= g_2 J_\mu^3\cos\theta_W - \frac{1}{2}g^{'}\sin\theta_W & (2.2.20) \\ &= \frac{g_2}{\cos\theta_W}(-\overline{e}_L\gamma_\mu e_L(\frac{1}{2} - \sin^2\theta_W) + \frac{1}{2}\overline{\nu}_L\gamma_\mu\nu_L + \sin^2\theta_W\overline{e}_R\gamma_\mu e_R), & (2.2.21) \end{aligned}$$

J_μ^z with corresponding coupling g_z was the new current. This was the main indication of electroweak unification.

2.3 The Electroweak Standard Model

The Electroweak Standard Model (SM) is the most accepted model for fundamental electroweak interactions [18]. It is a model whose foundation lies on the principle of symmetry. The gauge group associated with its symmetry is $SU(2) \times U(1)$. This symmetry is broken down spontaneously by introducing the Higgs Mechanism [17]. There are three components that makes the electroweak part of the SM.

1) Quaks and Leptons: The three generations of quarks and leptons are currently thought to

$$\begin{pmatrix} u \\ d \end{pmatrix}_{1st} \begin{pmatrix} c \\ s \end{pmatrix}_{2nd} \begin{pmatrix} t \\ b \end{pmatrix}_{3rd}$$

Table 2.3.1: Three generations of Quarks

$$\begin{pmatrix} \upsilon_e \\ e^- \end{pmatrix}_{1st} \begin{pmatrix} \upsilon_\mu \\ \mu^- \end{pmatrix}_{2nd} \begin{pmatrix} \upsilon_\tau \\ \tau^- \end{pmatrix}_{3rd}$$

Table 2.3.2: Three generations of Leptons

be the ultimate constituents of matter. The discovery of two heavy quarks charm and bottom along with tau lepton (τ) in 1970s presented a mystery known as generation puzzle

where all up have charge $+\frac{2}{3}e$ and all down have $-\frac{1}{3}e$. A quark can only change its flavour through weak interactions. For leptons we have exactly the three generations, just like quarks.

Similarly we also have three generations of anti-quarks and anti-leptons.

2) Force Carriers: Quarks and leptons interact through four types of forces. The gauge bosons of spin 1 are mediating these interactions. The electromagnetic and the weak interactions are unified as electroweak interactions in the SM. $\gamma, W^{\pm}$ and Z are the gauge bosons for electroweak interactions.

3) Higgs Mechanism: It is the third component of the SM, and is responsible for the mass acquisition of gauge bosons. In this regard, we need some theoretical bases for the complete understanding of the SM.

2.4 The Theoretical Bases

The SM is a gauge theory which should accommodate the massive as well as massless gauge bosons. But the inclusion of the mass term will destroy the symmetry of the theory. So, an alternative approach of putting the mass term by hand instead, should be adopted. That approach is known as Higgs mechanism which breaks the symmetry spontaneously and gives masses to gauge bosons.

(a) Gauge invariance

Gauge invariant theories remain invariant under gauge transformations of the fermion fields:

$$\psi \to U\psi. \tag{2.4.1}$$

U is taken as a phase factor for abelian transformations or unitary matrix for non-abelian transformations. The transformations are acting on the multiplets of the fermion field ψ. Now, if we demand that the theory is local gauge invariant then U depends on the space time point x, the usual space-time. We should also replace ∂_μ with covariant derivative D_μ which contains a new vector field V_μ:

$$i\partial_\mu \to iD_\mu = i\partial_\mu - gV_\mu, \tag{2.4.2}$$

g is representing a universal gauge coupling of the system. Local gauge transformations are also transforming the gauge field by a rotation and a shift:

$$V_\mu \to UV_\mu U^{-1} + ig^{-1}\left[\partial_\mu U\right]U^{-1}. \tag{2.4.3}$$

But curl F of V_μ,

$$F_{\mu\nu} = -ig^{-1}\left[D_\mu, D_\nu\right] \tag{2.4.4}$$

is only rotated.

The Lagrangian of the system of spin$\frac{1}{2}$ fermions and gauge bosons for massless particles can be written as:

$$L\left[\psi, V\right] = \bar{\psi} iD\psi - \frac{1}{2}TrF^2. \tag{2.4.5}$$

It incorporates the following interactions:

Fermion-gauge bosons couplings

$$-g\bar{\psi}V\psi. \tag{2.4.6}$$

Three bosons couplings

$$igTr\left(\partial_\nu V_\mu - \partial_\mu V_\nu\right)\left[V_\mu, V_\nu\right] \tag{2.4.7}$$

Four boson couplings

$$\frac{1}{2}g^2 Tr\left[V_\mu, V_\nu\right]^2. \tag{2.4.8}$$

(b) Higgs Mechanism

The Higgs mechanism uses the idea of the spontaneous symmetry breaking to generate the masses of vector bosons. The $SU(2) \times U(1)$ gauge invariance keeps massless gauge bosons, since the mass term for the gauge bosons violates gauge invariance. The Higgs mechanism incorporates this requirement by beginning with a gauge invariant theory and massless gauge bosons. Z^0 and $W^\pm$ were acquiring their masses by the breakup of the local gauge symmetry $SU(2)_L \times U(1)_Y$ spontaneously i.e.

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}. \tag{2.4.9}$$

It is done by the introduction of a self-interacting complex scalar field, Φ, which transforms as an $SU(2)$ doublet. The field Φ and its complex conjugate contain four independent fields. Spontaneous symmetry breaking (SSB) was achieved by providing a nonzero vacuum expectation value to one of the neutral fields,

$$\langle\phi\rangle \equiv \langle 0\left|\phi\right|0\rangle = \frac{v}{\sqrt{2}} \neq 0.$$

Out of the four fields in the Lagrangian before SSB, three fields are giving the longitudinal degrees of freedom for the $W^\pm$and Z^0 (vector bosons) ; the photon still remains massless, being attached with the remaining symmetry group $U(1)_{em}$-generators.

This theory predicted a neutral scalar particle for the physical sector. This is so-called Salam-Weinberg Higgs particle, which $SU(2) \times U(1)$ model predicts to exist. This long awaited (predicted in 1964) particle was discovered in 2012 at LHC.

All bosons and fermions take their masses by interacting with Higgs doublet through Yukawa couplings. Although, the $SU(2) \times U(1)$ model gives prediction for the Higgs particle couplings with all the existing particles but it does not give any hint regarding its own mass. This could lie in the foundation of Weinberg-Salam theory, because Higgs particle mass was taken as a function of the unknown quartic Higgs-boson coupling constant.

States	Weak isospin T	T_3	Weak hypercharge	Charge $\frac{Q}{e}$ of lepton/quark
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	$+\frac{1}{2}$	-1	0
e^-_L, μ^-_L, τ^-_L	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
e^-_R, μ^-_R, τ^-_R	0	0	-2	-1
u_L, c_L, t_L	$\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$
u_R, c_R, t_R	0	0	$+\frac{4}{3}$	$+\frac{2}{3}$
d'_L, s'_L, b'_L	$\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{3}$
d'_R, s'_R, b'_R	0	0	$-\frac{2}{3}$	$-\frac{1}{3}$

Table 2.5.1: Hypercharge and Isospin Relation with electric Charge

2.5 Formulation of the Electroweak Standard Model

The Matter Sector

The fundamental fermions appear as left handed weak isospin doublets and right handed weak isospin singlets in the fundamental representation of the group $SU(2) \times U(1)$. It is also realized that the symmetry pattern remains same in the generations of leptons,

$$\begin{bmatrix} \nu_e \\ e^- \end{bmatrix}_L e^-_R; \begin{bmatrix} \nu_\mu \\ \mu^- \end{bmatrix}_L \mu^-_R; \begin{bmatrix} \nu_\tau \\ \tau^- \end{bmatrix}_L \tau^-_R. \tag{2.5.1}$$

Here we do not have right handed neutrinos. Just like leptons, we have three generations for quark sector,

$$\begin{bmatrix} u \\ d \end{bmatrix}_L u_R, d_R; \begin{bmatrix} c \\ s \end{bmatrix}_L c_R, s_R; \begin{bmatrix} t \\ b \end{bmatrix}_L t_R, b_R. \tag{2.5.2}$$

This symmetry structure cannot be derived by the SM. It is also an experimental fact that in weak interactions the parity is not conserved. The different isospin assigned to the left handed and right handed field will produce maximal parity violation in the weak interactions. So the experimental fact is incorporated in the natural way.

The Gell-Mann-Nishijima relationship is links the electric charge Q with basic quantum numbers given by the following equation and the numbers are provided in table 2.4.1.

$$Q = I_3 + \frac{Y}{2}.$$

Interactions

The interactions of the SM are summarized by the three terms in the fundamental Lagrangian:

$$L = L_G + L_F + L_H \tag{2.5.3}$$

which are quantified as;

Gauge fields

$SU(2)_L \times U(1)_Y$ is a non-Abelian group which is generated by the isospin operators I_1, I_2, I_3 and the hypercharge Y. Each of these is associated with a vector field: a triplet of vector fields $W_\mu^{1,2,3}$ with $I_{1,2,3}$ and a singlet field B_μ with Y. The iso-triplet W_μ^a, $a = 1, 2, 3$ and iso-singlet B_μ make the field strength tensors

$$\begin{aligned} W_{\mu\nu}^a &= \partial_\mu W_\nu^a + \partial_\nu W_\mu^a + g_2 \varepsilon_{abc} W_\mu^b W_\nu^c, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \tag{2.5.4}$$

g_2 is defined as the coupling constant for $SU(2)$.

Using the equation (2.5.4) the pure gauge field Lagrangian can be written as

$$L_G = -\frac{1}{4} W_{\mu\nu}^a W^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}. \tag{2.5.5}$$

It is invariant under the $SU(2)_L \times U(1)_Y$ transformation.

Fermion fields and fermion-gauge interactions

The left-handed fermion fields of each lepton family can be written as:

$$\psi_j^L = \begin{pmatrix} \psi_{j+}^L \\ \psi_{j-}^L \end{pmatrix}$$

Where the family index j are grouped into $SU(2)$ doublets with component index $\sigma = \pm$, and the right-handed fields into singlets

$$\psi_j^R = \psi_{j\sigma}^R.$$

Each left and right-handed multiplet is an eigenstate of the weak hypercharge Y such that the relation (2.5.4) is fulfilled. The covariant derivative,

$$D_\mu = \partial_\mu - ig_2 I_a W_\mu^a + ig_1 \frac{Y}{2} B_\mu, \tag{2.5.6}$$

induces the fermion-gauge field interaction via the minimal substitution rule

$$L_F = \sum_j \bar{\psi}_j^L i\gamma^\mu D_\mu \psi_j^L + \sum_{j,\sigma} \bar{\psi}_{j\sigma}^R i\gamma^\mu D_\mu \psi_{j\sigma}^R. \tag{2.5.7}$$

g_1 represents the coupling constant of $U(1)$.

Higgs field and Higgs interaction

The spontaneous breaking of the $SU(2)_L \times U(1)_Y$ symmetry leaves the electromagnetic gauge group $U(1)_{em}$ unbroken. A single complex scalar doublet field with hypercharge $Y = 1$

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} \tag{2.5.8}$$

is coupled to the gauge fields through

$$L_H = (D_\mu \phi)^+ (D^\mu \phi) - V(\phi) \tag{2.5.9}$$

with the covariant derivative

$$D_\mu = \partial_\mu - ig_2 I_a W_\mu^a + ig_1 \frac{B_\mu}{2}.$$

The Higgs field potential

$$V(\phi) = \frac{1}{2}\mu^2 \phi^+ \phi + \frac{\lambda}{4} \left(\phi^+ \phi\right)^2 \tag{2.5.10}$$

is constructed in such a way that vacuum expectation value of ϕ becomes massless, i.e.

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix},$$

with

$$v = \frac{2\mu}{\sqrt{\lambda}}. \tag{2.5.11}$$

The field in eq (2.5.8) can be written as

$$\phi\left(x\right) = \begin{pmatrix} \phi^{+}\left(x\right) \\ \left(v + H\left(x\right) + i\chi\left(x\right)\right)/\sqrt{2} \end{pmatrix}, \tag{2.5.12}$$

where the field components ϕ^{+}, H, χ have zero vacuum expectation values. The pictorial view of Higgs potential is given in 2.5.

Using the invariance of the Lagrangian, the components ϕ^{+}, χ are gauged away; this means that they are unphysical (Higgs ghosts or would be Goldstone bosons). In this particular gauge, the unitarity gauge, the Higgs field takes the simple form

$$\phi\left(x\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H\left(x\right) \end{pmatrix}.$$

The real field $H\left(x\right)$ which describes small perturbations about the ground state and it describes the physical Higgs field.The Higgs field has triplet and quartic self couplings arising from V and couplings to the gauge fields via the kinetic term of eq.(2.5.9).

In addition to this, Yukawa couplings of the fermions are introduced to make the fermion massive. The Lagrangian for the Yukawa term is written as:

$$L_{Yukawa} = g_l\left(\nu_L\phi^{+}l_R + \bar{l}_R\phi^{-}\nu_L + \bar{l}_L\phi^{0}l_R + \bar{l}_R\phi^{0*}l_L\right) \tag{2.5.13}$$

And similarly for quarks, we have

$$L_{Yukawa} = -g_d(\overline{u}_L\phi^+ d_R + \overline{d}_R\phi^- u_L + \overline{d}_L\phi^0 d_R + \overline{d}_R\phi^{0*} d_L)$$
$$-g_u(-\overline{u}_R\phi^+ d_L - \overline{d}_L\phi^- u_R + \overline{u}_R\phi^0 u_L + \overline{u}_L\phi^{0*} u_R). \quad (2.5.14)$$

The fermion mass terms follow from the v- part of ϕ^0 and in the unitary gauge [18] we have

$$L_{Yukawa} = -\sum_f m_f \overline{\psi}_f \psi_f - \sum_f \frac{m_f}{v} \overline{\psi}_f \psi_f H. \quad (2.5.15)$$

.

The Lagrangian 2.5.3, describes not only the laws of physics for the electroweak interactions between the leptons, but also provides the self-interaction between the gauge fields. Moreover, the specific form of the Higgs interaction generates the mass of the particles and the Higgs boson itself [19].

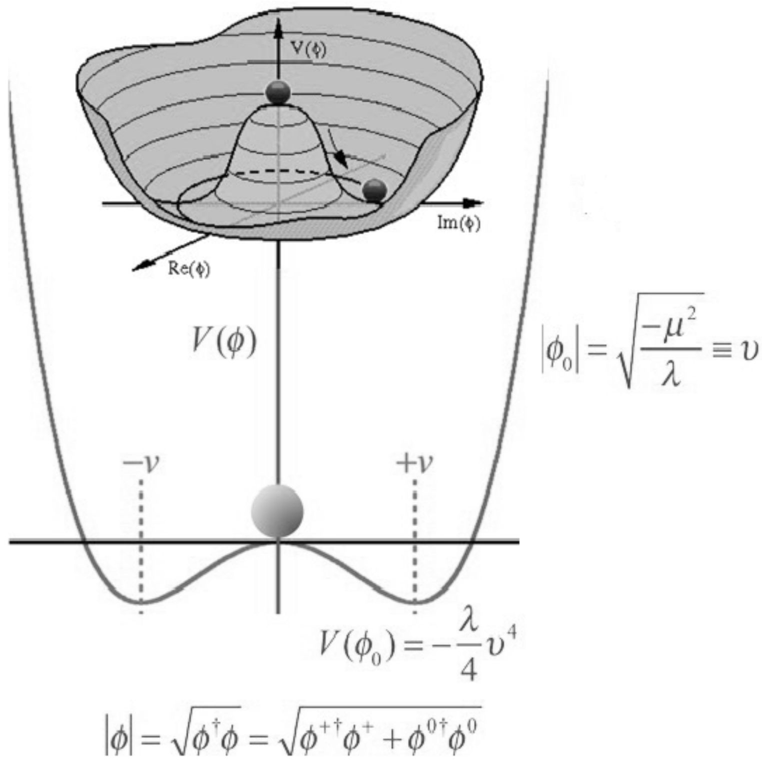
Masses and eigenstates of the particles

If we use the unitary gauge then the mass terms are obtained by this substitution $\phi \rightarrow 0, \frac{v}{\sqrt{2}}$, in the basic Higgs Lagrangian (2.5.9). The $SU(2)$ symmetry appears to be gone, but it is still there in hidden form ; the resulting Lagrangian contains an apparent local gauge symmetry, $U(1)$, which can be recognized as the electromagnetic gauge symmetry: $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ [19].

Gauge Vector Bosons

The mass matrix of the gauge bosons, In the basis $\left(\vec{W}, B\right)$, mass matrix of the gauge bosons has the following form:

$$M_V^2 = \frac{1}{4}v^2 \begin{pmatrix} g_W^2 & & & \\ & g_W^2 & & \\ & & g_W^2 & g_W \acute{g}_W \\ & & g_W \acute{g}_W & \acute{g}_W^2 \end{pmatrix}. \quad (2.5.16)$$



$$V\left(\phi\right) = \frac{1}{2}\mu^2\phi^+\phi + \frac{\lambda}{4}\left(\phi^+\phi\right)^2$$

Figure 2.5.1: Higgs Potential

This gives the mass of the vector boson in non diagonal form. The mass of the charged weak bosons is obvious

$$M^2_{W^\pm} = \frac{1}{4} g^2_W v^2.$$

Since eigenstates are related to the two masses $M^2_{W^\pm}$, the charged $W^\pm$ boson state can be defined as

$$W^\pm_\mu = \frac{1}{\sqrt{2}} \left[W^1_\mu \mp W^2_\mu \right]. \tag{2.5.17}$$

For the neutral bosons (γ, Z), the mass term from the matrix

$$M^2_{V_N} = \frac{1}{4} \begin{pmatrix} g^2_W & g_W \acute{g}_W \\ g_W \acute{g}_W & \acute{g}^2_W \end{pmatrix} v^2, \tag{2.5.18}$$

Since $\det(M^2_{V_N}) = 0$, therefore one of the eigenvalue of $M^2_{V_N}$ is zero. The above matrix is diagonalized by defining the fields A_μ, Z_μ:

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W^3_\mu, \tag{2.5.19}$$

$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W^3_\mu. \tag{2.5.20}$$

In matrix form the above equations can be written as follows;

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix}. \tag{2.5.21}$$

Thus we get

$$M^2_A = 0 \qquad A_\mu : \text{ photon} \tag{2.5.22}$$

$$\begin{aligned} M^2_Z &= \frac{1}{4} \left(g^2_W + \acute{g}^2_W \right) v^2 \\ &= \frac{1}{4} g^2_W v^2 \left(\frac{1}{\cos^2\theta_W} \right), \end{aligned} \tag{2.5.23}$$

where

$$\tan\theta_W = \frac{\acute{g}_W}{g_W} \tag{2.5.24}$$

i.e. the electroweak mixing angle θ_W (Weinberg angle) is defined by the ratio of the $SU(2)$ and $U(1)$ couplings.

Introducing a parameter

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W},$$

and using the value of M_Z^2, we get

$$\rho = 1.$$

This is due to the fact that Higgs field is a doublet under $SU(2)_L$ [21].

Experimental value of the mixing angle, $\sin^2 \theta_W \simeq 0.23$ [22], shows that the mixing effects are large. This fact strengthens the argument that the weak and the electromagnetic interactions produce a unified electroweak interaction in spite of the fact that the $SU(2) \times U(1)$ symmetry is not so simple. This provides us with an evidence that the weak and the electromagnetic interactions are combined in the Salam-Glashow-Weinberg theory of the electroweak interactions.

The ground-state or vacuum expectation value of the Higgs field is also linked with the Fermi coupling constant. When the mass relation $M_{W^\pm}^2 = \frac{1}{4} g_W^2 v^2$, is combined with the β decay low-energy relation $G_F/\sqrt{2} = g_W^2/8M_W^2$, the value of v becomes:

$$\begin{aligned} v &= [1/\sqrt{2}G_F]^{1/2} \\ &\simeq 246GeV. \end{aligned} \quad (2.5.25)$$

The typical range for electroweak phenomena, defined by the weak masses M_W and M_Z, is of the order $100GeV$.

Fermions

The fermions acquire their masses by interacting with the Higgs ground state through Yukawa couplings:

$$M_f = g_f \frac{v}{\sqrt{2}}. \quad (2.5.26)$$

Although the chiral fermions acquire their masses via Higgs mechanism, the Standard Model is unable to give the experimental values of Yukawa couplings g_f, and as a result the masses are not predicted. The true theory of the masses is yet to develop.

The Higgs Bosons

The real field $H(x)$ which is just quantum oscillations about the minimum energy state provides us the mass of the physical neutral scalar particle:

$$M_H = \mu\sqrt{2} = \sqrt{\lambda}v. \tag{2.5.27}$$

As the quartic coupling λ is an unknown parameter, so the mass of the Higgs can not be calculated in the SM.

We conclude this session with these remarks:

A definite prediction of electroweak unification is the existence of weak neutral currents with the same effective couplings as charged currents. This current has been found experimentally. The existence of the vector bosons $W^{\pm}$, Z with definite masses have also been discovered. The theory has one free parameter: $\sin^2\theta_W$.

2.6 Quantum Chromodynamics

For the completeness of discussion on the SM, we give a very brief introduction of the strong interaction (which is not of our interest here). The third interaction accommodated in the SM is strong interaction. There are additional quantum numbers given to each quark (known as colors), namely, Red,Green and Blue. For anti-quarks, we have anti colors (anti-Red, anti-Green, anti-Blue) . The fact that quarks carry colors as well as electric charge means that they participate in all types of interactions. The quantum field theory describing strong interactions is known as Quantum Chromodynamics (QCD). The SM incorporates QCD via $SU(3)$ gauge symmetry. Similar to $SU(2)$,having $2^2-1=3$ gauge bosons, QCD have 3^2-1 gauge bosons known as gluons. There are eight gluons and each carries the color combination of

$$r\overline{g}, r\overline{b}, g\overline{r}, g\overline{b}, b\overline{r}, b\overline{g}, \frac{(r\overline{r}-g\overline{g})}{\sqrt{2}} \text{ and } \frac{(r\overline{r}+g\overline{g}-2b\overline{b})}{\sqrt{6}}. \tag{2.6.1}$$

Gluons are the carriers of strong force and they keep the quarks only in the bound states and these bound states are color singlets. If the bound state is made up of a quark and anti-quark, it is named as meson; and if they are made up of three quarks then they are known as baryons. Baryons and mesons are collectively known as hadrons.

Chapter 3

Weak decays in The Standard Model

Slow decay of unstable particles is due to weak interactions. Weak force is the only force which exists between all leptons and quarks and is mediated by $W^{\pm}$ and Z^0. Typical decay time for strong interactions is $10^{-23}Sec$; for electromagnetic it is 10^{-16} but weak interaction takes $\approx 10^{-8}Sec$. Weak interactions can be classified into three types:

(a) Pure Leptonic Interactions

Pure leptonic interactions involve only leptons in their initial and final state. The examples are

$$\begin{aligned} \mu^- &\rightarrow e^-\overline{\upsilon}_e\upsilon_\mu \\ \mu^+ &\rightarrow e^+\upsilon_e\overline{\upsilon}_\mu \end{aligned} \tag{3.0.1}$$

and

$$\begin{aligned} \tau^- &\rightarrow \mu^-\overline{\upsilon}_\mu\upsilon_\tau \\ \tau^+ &\rightarrow e^+\upsilon_e\overline{\upsilon}_\tau \end{aligned} \tag{3.0.2}$$

These interactions obey pure vector minus axial vector current $(V-A)$ theory.

(b) Semileptonic Interactions

In semileptonic interactions, hadrons decay into hadrons and leptons. The hadrons may be flavored (carrying s, c or b quarks flavour) or non-flavored (carrying only u or d quarks). So semileptonic decays can be further divided into two categories:

(1) Strangeness Conserving Interactions ($\Delta S = 0$)

Strangeness Conserving Interactions are without strange hadrons in the initial or final state. These can be divided into three types:

1) Neutron decay

$$n \rightarrow p + e^- + \overline{\upsilon}_e. \tag{3.0.3}$$

2) Neutrino reaction (elastic and inelastic)

$$\upsilon_\mu + n \rightarrow p + \mu^-, \tag{3.0.4}$$

$$\pi^- \rightarrow e^-\overline{\upsilon}_e \text{ or } \mu^-\overline{\upsilon}_\mu,\ \Sigma^- \rightarrow \Lambda + e^- + \overline{\upsilon}_e. \tag{3.0.5}$$

3) Lepton capture

$$e^- + p \rightarrow n + \upsilon_e, \tag{3.0.6}$$

$$\mu^- + p \rightarrow n + \upsilon_\mu. \tag{3.0.7}$$

(2) Flavour Changing Interactions

a) Strangeness changing $|\Delta S = 1|$

$$\Lambda^0 \rightarrow p + e^- + \overline{\upsilon}_e, \tag{3.0.8}$$

$$K^+ \rightarrow \mu^+ + \upsilon_\mu, \tag{3.0.9}$$

$$K^+ \rightarrow \pi^0 + \mu^+ + \upsilon_\mu. \tag{3.0.10}$$

b) Charmness changing $|\Delta c = 1|$, $\Delta S = 0$

$$\Lambda_c^+ \to \Lambda^0 + e^+ + \upsilon_e, \tag{3.0.11}$$

Here Λ_c^+ is a charmed baryon

c) Bottomness changing $|\Delta b = 1|$, $\Delta S = 1$

$$\Lambda_b^0 \to \Lambda_c^+ + e^- + \overline{\upsilon}_e. \tag{3.0.12}$$

(c) Non-leptonic Weak Processes

Non-leptonic Weak Processes involve only the hadrons (mesons and baryons) in initial and final states. Example are

1) $|\Delta S = 1|$

$$\Delta^0 \to p + \pi^-,\ K^- \to \pi^- + \pi^0,\ K_s^0 \to \pi^+ + \pi^-, \tag{3.0.13}$$

2) $|\Delta c = 1|$

$$D^0 \to K^- + \pi^+, \Lambda_c^+ \to \Lambda^0 + \pi^+, \Lambda_c^+ \to \Lambda^0 + K^+ + \pi^0, \tag{3.0.14}$$

3) $|\Delta b = 1|$

$$\Lambda_b^0 \to \Lambda_c^+ + \pi^-. \tag{3.0.15}$$

Leptonic and semileptonic decays of hadrons provide a unique way of studying the rich and diverse phenomenology of weak interactions. These decays can have a charge lepton in the final state which is the cleanest experimental signature for W mediated process. These are also simple theoretically, and provide a means to measure standard model parameters as well as detailed studies of dynamics of the decay.

Historically, β decay was the first semileptonic decay that led not only to a new era in weak interactions but also to introduction of a new particle named neutrino in the particle physics dictionary. In beta decay weak transition $d \to uW^-$ and then $W^- \to e^-\overline{\upsilon}_e$, was responsible for the β decay. It was the only weak process discovered before the discovery of muons, pions and kaons in cosmic rays in late 1930s and 1940s. Drastic change in studies of weak decays came with the invention of modern accelerators. The decay process $k \to \pi e^-\overline{\upsilon}_e$, showed that kaons could decay exactly in a similar manner as β decay: $s \to uW^-$ and then $W^- \to e^-\overline{v}_e$. Such decays in which no hadrons are present in the final state, have also played an important role in revealing the underlying secrets of weak interactions. The amazing 10^{-4} suppression of $\pi^- \to e^-\overline{v}_e$ as compared to $\pi^- \to \mu^-\overline{v}_\mu$ was well explained by weak interactions, and universal weak couplings of leptons were confirmed by precision measurements.

3.1 Cabibbo Theory

At the time when there were only three quarks (u, d and s), known, there were two phenomena motivated Cabibbo [12] to present his theory for weak decays. That is:

(a) Universality of Weak Coupling

When the value of weak coupling G_F was calculated experimentally for the reaction

$$n \to p + e^- + \overline{v}_e \tag{3.1.1}$$

represented by quark level process

$$(udd) \to (\text{ uu}d) + e^- + \overline{v}_e,$$

it was $G_F^\beta \equiv (1.136 \pm 0.003) \times 10^{-5}\ GeV^{-2}$(in natural units). But when it was calculated for pure leptonic reaction

$$\mu^- \to e^- + \overline{v}_e + v_\mu \tag{3.1.2}$$

it was $G_F^\mu \equiv (1.6632 \pm 0.00002) \times 10^{-5} GeV^{-2}$. So experimental fact was $G_F^\mu > G_F^\beta$ which did not match with theory.

(b) Suppression of $\Delta S = 1$ Weak Interactions

When the strangness changing decays

$$K^+ \to \mu^- + \overline{v}_\mu \text{ or } (u\overline{s}) \to \mu^- + \overline{v}_\mu \tag{3.1.3}$$

and

$$\Sigma^- \to n + e^- + \overline{v}_e \ or \ (dds) \to (ddu) + e^- + \overline{v}_e, \tag{3.1.4}$$

were compared with

$$\pi^+ \to \mu^- + \overline{v}_\mu, \tag{3.1.5}$$

and

$$n \to p + e^- + \overline{v}_e, \tag{3.1.6}$$

respectively, then it was found that K^+ decay was suppressed 20% as compared to π^+. The only difference in the above mentioned decays is s and d. This was also the case with hadronic and semileptonic decays of hadrons.

The solutions to these problems were provided by Cabibbo. He presented the idea of rotated states and argued that weak eigen states were rotated (by an angle θ known as cabibbo angle $\theta_c \approx 12^\circ$) with mass eigen states.Weak coupling for these rotated states were exactly equal to that for leptonic doublets. For beta decay, the coupling was $G_F^\mu \cos(\theta_c)$. The experimental difference between two couplings was removed theoretically with the help of θ_c. In this way the doublet of quarks

$$\begin{pmatrix} u \\ d' \end{pmatrix},$$

where

$$d' = d\cos(\theta_c) + s\sin(\theta_c), \tag{3.1.7}$$

restored the universality of weak coupling. Now the cross generation interaction among the mass eigen states of quarks was possible but with less strength. For $s \to u$, the coupling was $G_F^\mu \, \mathrm{Si}\, s(\theta_c)$ instead of $G_F^\mu \cos(\theta_c)$ which was responsible for 20% suppression of $\Delta S = 1$ processes.

3.2 GIM Mechanism

Although the Cabbibo theory successfully explained the relationship between different branching ratios (Br) of the decays, but, it was silent for very small decay rate of

$$K_L^0 \to \mu^+ + \mu^- \tag{3.2.1}$$

This silence was broken by Glashow, Iliopous and Maiani in 1970 when they collectively introduced GIM mechanism [13]. It can be understood by taking two $\Delta S = 1$ decays

$$K^+ \to \mu^+ + \upsilon_\mu \text{ or } (u\bar{s}) \to \mu^+ + \upsilon_\mu \text{with Br } 64\% \tag{3.2.2}$$

and

$$K_L^0 \to \mu^+ + \mu^- \text{ or } (d\bar{s}) \to K_L^0 \to \mu^+ + \mu^- \text{with Br } 7.37 \times 10^{-7} \tag{3.2.3}$$

. Even for $K^+ \to \pi^+ + \upsilon + \overline{\upsilon}$ and $K^+ \to \pi^0 + \mu^+ + \upsilon_\mu$ semileptonic decays the ratio is

$$\frac{K^+ \to \pi^+ + \upsilon + \overline{\upsilon}}{K^+ \to \pi^0 + \mu^+ + \upsilon_\mu} \le 10^{-5}. \tag{3.2.4}$$

In the $K_L^0 \to \mu^+ + \mu^-$ and $K^+ \to \pi^+ + \upsilon + \overline{\upsilon}$ reactions, two down quarks (strangeness changing neutral current, SCNC) were involved and cabbibo suppression mechanism or any other theory could not provide any satisfactory answer for such small Br. Such small Br demanded the cancellation of SCNC at tree level. Cabbibo theory with three quarks gave the current:

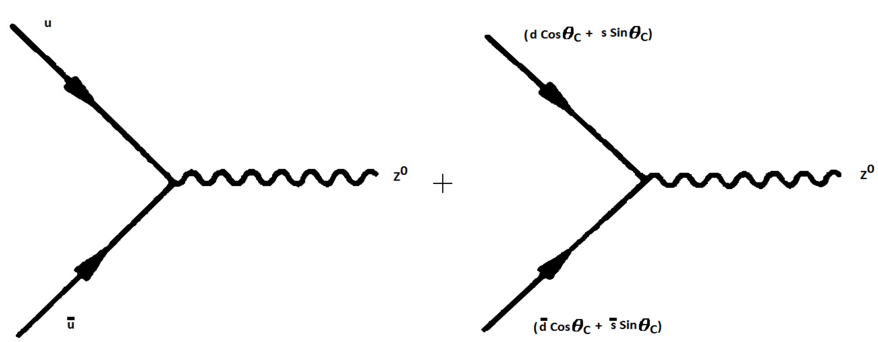


Figure 3.2.1: u and d bar weak neutral current diagrams

$$\underbrace{(u\overline{u} + d\overline{d}\cos^2\theta_c + s\overline{s}\sin^2\theta_c)}_{\Delta S=0} + \underbrace{(d\overline{s} + \overline{d}s)\cos\theta_c\sin\theta_c}_{\Delta S=1}. \tag{3.2.5}$$

With the proposal of fourth quark 'c' and $s^{'} = s\cos\theta_c - d\sin\theta_c$, Glashow, Iliopous and Maiani showed that the current became

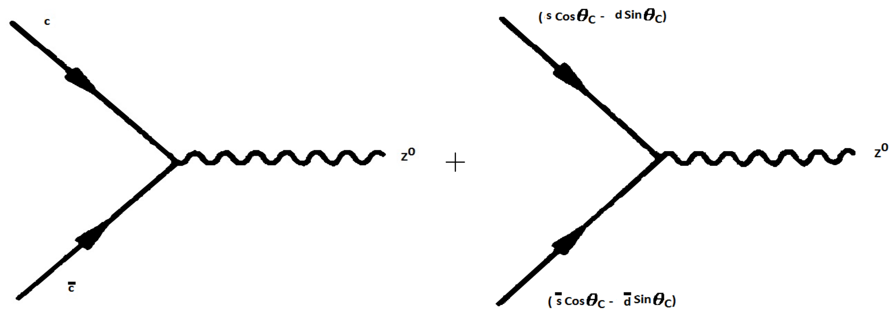


Figure 3.2.2: c quark and s bar neutral current diagrams

$$\underbrace{(c\overline{c} + s\overline{s}\cos^2\theta_c + d\overline{d}\sin^2\theta_c)}_{\Delta S=0} - \underbrace{(d\overline{s} + \overline{d}s)\cos\theta_c\sin\theta_c}_{\Delta S=1}. \tag{3.2.6}$$

Over all, there were no tree level contributions for $\Delta S = 1,$ however, $\Delta S = 0$ reactions occurred at tree level; and for these cabibbo angle was not contributing.

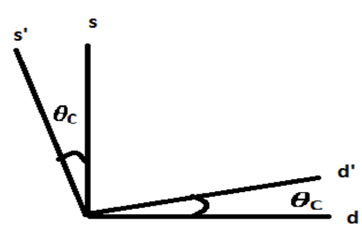


Figure 3.2.3: Cabbibo Rotated states

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}. \tag{3.2.7}$$

It does not mean that $\Delta S = 1$ processes can not occur; they could occur at loop level given by fig (3.2.4),

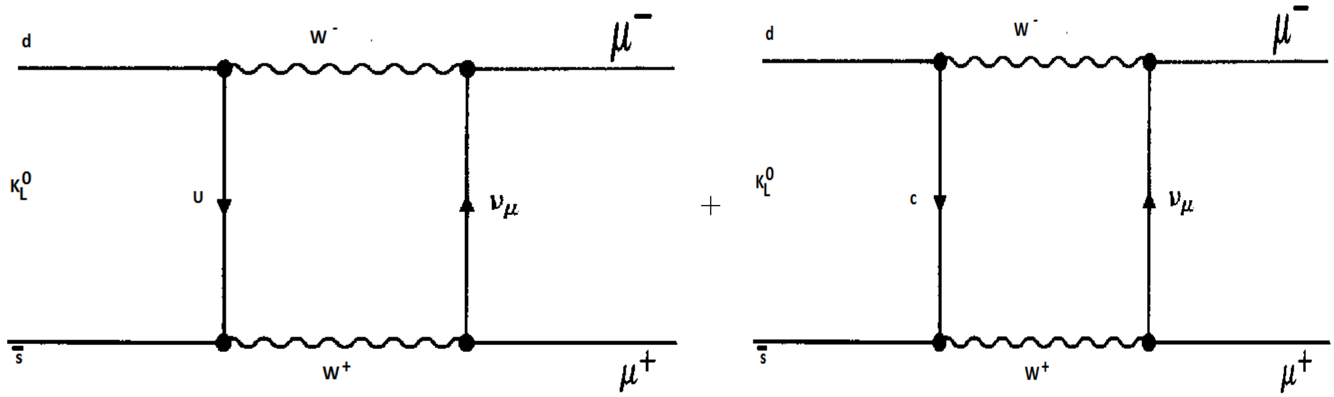


Figure 3.2.4: K zero long decaying to mu mu bar

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}.$$

Table 3.3.1: CKM matrix

3.3 The Cabibbo-Kobayashi-Maskawa (CKM) Matrix

The generalization of GIM mechanism for three generations was CKM matrix [12, 14]. Here instead of only SCNC cancellation at tree level, Flavour Changing Neutral Currents (FCNC) cancellation also occurred at tree level,

$$\begin{aligned}
V_{ud} &= C_{12}C_{13}, V_{us} = S_{12}C_{13}, V_{ub} = S_{13}\exp(-i\delta_{13}), \\
V_{cd} &= -S_{12}C_{23} - C_{12}S_{23}S_{13}\exp(i\delta_{13}), \\
V_{cs} &= C_{12}C_{23} - S_{12}S_{23}S_{13}\exp(i\delta_{13}), \\
V_{cb} &= S_{23}C_{13}, \\
V_{td} &= S_{12}S_{23} - C_{12}C_{23}S_{13}\exp(i\delta_{13}), \\
V_{ts} &= S_{12}S_{23} - C_{12}C_{23}S_{13}\exp(i\delta_{13}), \\
V_{tb} &= C_{23}C_{13}.
\end{aligned}$$

Where $C_{ij} = \cos\theta_{ij}, S_{ij} = \sin\theta_{ij}, i, j = 1, 2, 3$. The angle $\theta_{12} = \theta_C$, phase angle δ_{13} was responsible for CP violation.

Latest experimental [24] values of CKM elements are

$$\begin{pmatrix} 0.974 & 0.225 & 0.003 \\ 0.225 & 0.973 & 0.0412 \\ 0.008 & 0.040 & 0.999 \end{pmatrix},$$

when $\theta_{13} = \theta_{23} = 0$

$$\begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Consequences of GIM and CKM

(1) Cross generation interaction among the three generations was made possible

(2) Absence of flavour changing neutral current (FCNC) at tree level.

(3) CP violation was incorporated.

(4) Mass eigen states of quarks are different from flavour eigen states and weak doublets are;

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \begin{pmatrix} t \\ b' \end{pmatrix},$$

or

$$\begin{pmatrix} u' \\ d \end{pmatrix} \begin{pmatrix} c' \\ s \end{pmatrix} \begin{pmatrix} t' \\ b \end{pmatrix},$$

instead of

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}.$$

3.4 Operator Product Expansion Approach

The most commonly used theoretical tool for the calculation of decay rates of a FCNC decay is the operator product expansion (OPE) approach. The idea behind this approach is that the energy scale for the relevant interaction (weak decays of hadrons having u,d,s,c and b quark) is very small as compared to the mass of W boson (propagator in this case). The propagator of W boson has the form $\frac{1}{q^2-m_W^2}$, here q is the momentum transferred by the W boson. The amplitude can be expressed as an expansion of $\frac{q^2}{m_W^2}$ term:

$$M = \frac{-4G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu) \langle f|Q_i(\mu)|M\rangle [1 + O(\frac{q^2}{m_W^2})]. \tag{3.4.1}$$

Here, μ is taken as renormalization scale. Short distance physics (shorter than μ^{-1}) is contained in the Wilson coefficients C_i, while long distance physics (Longer than μ^{-1}) is element in the hadronic matrix element $\langle f|Q_i(\mu)|M\rangle$ of the local operator Q_i. There are many infinite terms in OPE , but, higher dimension operators are contributing less as they are expressed by the powers of $\frac{q^2}{m_W^2}$. So, the contribution of higher powers in the expansion can be ignored easily. This is exactly the same thing as H_{eff}, where the short range interactions of massive gauge bosons can be replaced by the point like interactions. In order to obtain H_{eff}, the product of two charge-current operators is expanded as a series of local operators. The contribution of these operators Q_i, is weighted by effective coupling known as Wilson coefficients C_i

$$H_{eff} = \frac{-4G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu) Q_i. \tag{3.4.2}$$

Small distance contribution can be calculated by using perturbative theory because of small α_s. But for long distance QCD, uncertainties will be very large and we have to use some model. μ is taken $1GeV$ for kaons decay and few GeV for D and B decays [26].

3.5 Effective Hamiltonian

All FCNC processes have same set of basic effective vertices. The effective Hamiltonian (H_{eff}),for FCNC decay is an expansion in terms of four fermion local operators which describe the effective vertices. These local operators can be categorized into six classes [29].

3.6 Rare Weak Decays of Pseudoscalar mesons

Such decays are not very common and their branching ratios are very small and can occur at one loop level or higher. Such decays are very useful for the search of new physics. If we have lepton in final state then these are divided into two general types

(1) Leptonic Decay

$$M \to l_\alpha l_\beta,$$

(2) Semileptonic Decay

$$M \to M' l_\alpha l_\beta,$$

where leptons l_α and $l_\beta$can be charged or neutral, $M \ > M'$, $(M, M' = \pi, K, D$ and B mesons)

Standard model only allows $\alpha = \beta$ but we are interested in weak neutral current, so leptons are only neutrinos. In this case, only short range force is responsible for the connection between hadronic and leptonic currents. So, perturbation theory can be used easily for the calculation and these serve as test for quantum mechanics. These processes can be represented by quark level process

$$q_i \to q_j \upsilon_\alpha \overline{\upsilon}_\alpha.$$

In the study of rare decays of mesons we use effective Hamiltonian (**EH**) which is a low energy approximation of the whole theory. **EH** is obtained by the use of operator product expansion (**OPE**) and renormalization group (**RG**). By this approach we can easily separate short-distance contributions and study them within perturbative QCD. The long distance contributions are encoded in the matrix elements of the operators. These matrix elements require non-perturbative methods for their calculation and hence they are model dependent and carry uncertainties. In our case Hamiltonian can be written as a product of hadronic and leptonic currents and matrix element can be obtained from an experimentally measured tree level process. Hence, these theoretically clean processes are used for the search of new physics beyond Standard Model. Such processes can be inclusive and exclusive, but inclusive are difficult to measure experimentally, so we concentrate only on exclusive processes. The feynman diagrams for these processes are shown in the fig. 3.6.1.

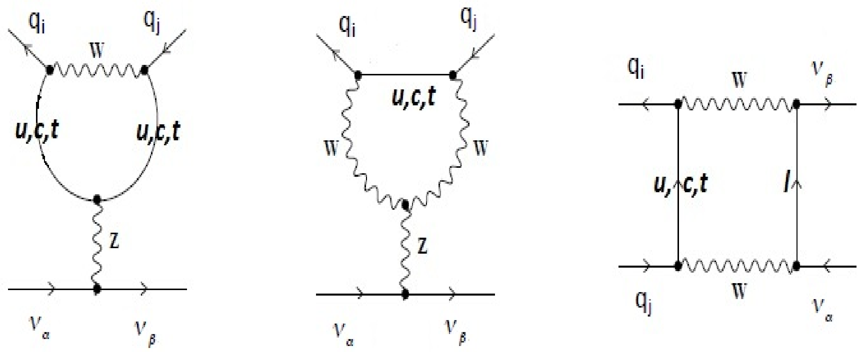


Figure 3.6.1: Standard Model FCNC diagrams having two neutrinos in the final state

3.7 Theory of $q_i \to q_j \upsilon\overline{\upsilon}$

As shown in fig. (3.6.1), these decays proceed through an effective FCNC induced by penguin and box diagrams. Inclusive $q_i \to q_j \upsilon\overline{\upsilon}$ decays are considered as free of non-perturbative effects due to quark-hadron duality. For exclusive decays there are necessary final state corrections. Due to the fact that we have only form factors of hadronic currents, special attentions are given to $B \to (K,\pi)\upsilon\overline{\upsilon}$, $D \to (K,\pi)\upsilon\overline{\upsilon}$ and $K \to \pi\upsilon\overline{\upsilon}$. Form factors are eliminated by using experimentally found tree level decays. That is why these decays are thought to be theoretically clean.

(a) $b \to q\upsilon\overline{\upsilon}$ Decays

The decay $b \to (s,d)\upsilon\overline{\upsilon}$,both inclusive and exclusive, are thought to be very clean rare decays. These are extremely sensitive to the new physics, even though it appears at very high energy scale. Since neutrinos can only interact through weak interactions, (short range interactions), thus perturbation theory is fully applicable for calculation. Here, QCD is affecting only hadronic side of the interactions, making these effects, almost controllable. The decay rates of such decays are small and these would be of prime interest for a super B factory.

The inclusive decays are free from non-perturbative effects because of quark-hadron duality, but for exclusive decays, final state correction must be included. The effective Lagrangian for

$\overline{B} \to X_{s,d}\nu\overline{\nu}$ is given by

$$H_{eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{2\pi \sin^2\theta_W} V_{tb} V_{ts}^* X(\frac{m_t^2}{m_W^2})(\overline{q}\gamma_\mu P_L b)(\overline{\nu}\gamma^\mu P_L \nu) + h.c. \quad (3.7.1)$$

Here $q = s, d$. The GIM mechanism comes into play for the short distance Wilson coefficient X and produce

$$\frac{(\frac{m_c^2}{m_W^2})}{(\frac{m_t^2}{m_W^2})} = O(10^{-3}).$$

Despite the fact that the CKM factor is unable to enhance charm quark contribution, it nevertheless gives a way to the dominant contribution of the top quark only. After summing up neutrino flavours, the branching ratio (Br) of $\overline{B} \to X_{s,d}\nu\overline{\nu}$ is obtained as:

$$Br(\overline{B} \to X_s\nu\overline{\nu}) = Br_{\exp}(\overline{B} \to X_c\nu\overline{\nu}) \frac{\alpha_{em}^2}{4\pi^2 \sin^4\theta_W} \frac{|V_{ts}|^2}{|V_{cb}|^2} \frac{X^2(\frac{m_t^2}{m_W^2})\overline{\eta}}{f(\frac{m_c^2}{m_b^2})\kappa(\frac{m_c^2}{m_b^2})}$$

where $f(z) = 1 - 8z + 8z^3 - z^4 - 12z^2\ln(z)$ with $z = \frac{m_c^2}{m_b^2}$
and $\kappa(z) = 0.88$, $\eta = \kappa(0) = 0.83$.

A useful discussion about the factors can be found in [25] and [26]. With the latest values of the constants, we have the Br

$$Br(B \to X_c\nu\overline{\nu})_{SM} = 3.6 \times 10^{-5}.$$

For exclusive reaction, the effective Hamiltonian will be

$$H_{eff}^{SM} = \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{2\pi \sin^2\theta_W} \sum_{\alpha,\beta=e,\mu,\tau} V_{tb}^* V_{td} X(x_t)) \times (\bar{d}b)_{V-A}(\nu_\alpha\overline{\nu}_\beta)_{V-A},$$

where $V-A$ in the subscript represents the vector and axial vector current respectively. For such reactions, charm quark contribution in the loop is negligible in contrast to K decay due to smallness of off-diagonal CKM element; and $X(x_t)$ is the loop integral of top-quark exchange [27]. For this reaction, we have two penguin and one box diagrams [26]; and sum of all gives the contribution

$$X(x_t) = \eta_X \frac{x_t}{8}[\frac{x_t+2}{x_t-1} + \frac{3x_t-6}{(x_t-1)^2}\ln x_t].$$

Here $x_t = \frac{m_t^2}{m_w^2}$ and $\eta_X = 0.985$ is the QCD small distance correction. By using the above Hamiltonian, we can obtain Br as

$$Br(B^+ \to \pi^+\nu\overline{\nu})_{SM} = r_{iso}\frac{3\alpha_{em}^2}{|V_{ub}|^2 2\pi^2 \sin^4\theta_W}|V_{tb}^* V_{td} X(x_t)|^2 Br(B^+ \longrightarrow \pi^0 l^+ \nu_l),$$

$r_{iso} \simeq 0.94$ is the isospin breaking effect for B. It is discussed for K mesons in [28] which depends on, at least, three things: (1) mass effect (2) a suppression of about 4% in neutral form factor comes from $\eta - \pi$ mixing and (3) about 2% suppression due to absence of log leading correction.

(b) $s \to d\upsilon\overline{\upsilon}$ Decays

For strangeness changing FCNC, we have only one possibility: $s \to d$. The effective hamiltonian is given by

$$H_{eff}^{SM} = \frac{G_F}{\sqrt{2}}\frac{\alpha_{em}}{2\pi\sin^2\theta_W}\sum_{\alpha,\beta=e,\mu,\tau}(V_{cd}^* V_{cs} X_{NL}^l + V_{td}^* V_{ts} X(x_t)) \times (\overline{s}d)_{V-A}(\nu_\alpha\overline{\nu}_\beta)_{V-A}, \qquad (3.7.2)$$

Where $x_t = \frac{m_t^2}{m_W^2}$. Here, charm quark contribution cannot be ignored due to the large effects of CKM elements. The whole process is on equal footing with top quark. Such processes are dominated by short distance physics, and reliable perturbation calculation are possible: but short distance QCD effects are the source of uncertainties. For some processes, we can have tree level processes, which are linked with these processes by some symmetry. Such tree level processes are used to absorb the hadronic uncertainties, making these processes theoretically clean. For example, $K^+ \to \pi^+\nu\overline{\nu}$ has isospin linked $K^+ \to \pi^0 e^+\nu$.

Branching ratio (Br) of $K^+ \longrightarrow \pi^+\overline{\upsilon}\upsilon$ is

$$\frac{Br(K^+ \to \pi^+\overline{\upsilon}\upsilon)}{Br(K^+ \to \pi^0 e^+\upsilon)} = r_{K^+}\frac{\alpha_{em}^2}{|V_{us}|^2 2\pi^2\sin^4\theta_W}\sum_{\alpha,\beta=e,\mu,\tau}|V_{cd}^* V_{cs} X_{NL}^l + V_{td}^* V_{ts} X(x_t)|^2 \qquad (3.7.3)$$

As $\langle \pi^+ |(\overline{s}d)_{V-A}| K^+\rangle = \sqrt{2}\langle \pi^0 |(\overline{s}u)_{V-A}| K^+\rangle$

$r_{K^+} = 0.901$ is isospin effect given in [28];

Using $V_{us} = 0.2252$; $V_{ud} = 0.97425$; $\theta_w = 28.7°$; $BR(K^+ \longrightarrow \pi^0 e^+ \nu_e) = 5.07 \times 10^{-2}$[78]

Br of $K^+ \longrightarrow \pi^+ \overline{v} v$ becomes

$$(7.8 \pm 0.8) \times 10^{-11}[30].$$

Similarly other processes having same quark level processes can be calculated:

$\frac{Br(D_s^+ \to D^+ \overline{v} v)}{Br(D_s^+ \to D^0 e^+ v)} = \frac{\alpha_{em}^2}{|V_{us}|^2 2\pi^2 \sin^4\theta_W} \sum_{\alpha,\beta=e,\mu,\tau} |V_{cd}^* V_{cs} X_{NL}^l + V_{td}^* V_{ts} X(x_t)|^2,$

and

$\frac{Br(B_s^0 \to B^0 \overline{v} v)}{Br(B_s^0 \to B^+ e^+ v)} = \frac{\alpha_{em}^2}{|V_{us}|^2 2\pi^2 \sin^4\theta_W} \sum_{\alpha,\beta=e,\mu,\tau} |V_{cd}^* V_{cs} X_{NL}^l + V_{td}^* V_{ts} X(x_t)|^2.$

Although the $Br(D_s^+ \to D^0 e^+ v)$ and $Br(B_s^0 \to B^+ e^- v)$ are yet to be observed experimentally but we have very elegantly calculated values for BES-III given in [31], which can be used. Here we are ignoring effects of isospin breaking D and B mesons. A useful information about the isospin breaking effects can be found in [32, 28].

Using

$Br(D_s^+ \to D^0 e^+ v) = 5 \times 10^{-6}$, $Br(B_s^0 \to B^+ e^- v) = 4.46 \times 10^{-8}$ [31],

we calculate SM Br as:

$Br(D_s^+ \to D^+ \overline{v} v)_{SM} = 7.72 \times 10^{-15}$.

$Br(B_s^0 \to B^0 \overline{v} v)_{SM} = 6.86 \times 10^{-17}$.

(c) $c \to u v \overline{v}$ Decays

This is the only reaction available to study FCNC in the up sector for the bound states. Top quark does not make any bound state and principal decay mod is to decay to b-quark.

Such processes are dominated by the long distance contributions (Figure 3.7.2) [33], so perturbation theory is not available for the calculations. The effective hamiltonian for short distance is given by

$$H_{eff}^{SM} = \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{2\pi \sin^2\theta_W} \sum_{\alpha,\beta=e,\mu,\tau} [V_{cs}^* V_{us} X(x_s) + V_{cb}^* V_{ub} X(x_b)] \times (\overline{u}c)_{V-A} (\nu_\alpha \overline{\nu}_\beta)_{V-A} \quad (3.7.4)$$

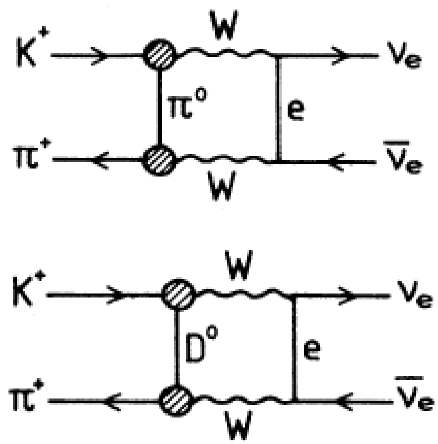


Figure 3.7.1: Long distance contribution of K decay into Pi neutrino antineurino

but, the dominant contribution for such reactions comes from long distance physics. The examples are $D_s^+ \to K^+ \upsilon_\alpha \overline{\upsilon}_\beta$, $D^+ \longrightarrow \pi^+ \nu_\alpha \overline{\nu}_\beta$ and $D^0 \longrightarrow \pi^0 \nu_\alpha \overline{\nu}_\beta$. These are dominated by long distance effects and hence, new physics might improve the Br of these reactions tremendously.

3.8 Limitations of the SM

The SM is a very elegant and remarkable theory and its validity has been tested to very high level of precision. But still, it is unable to answer many [23] questions.

(a) Phenomena not incorporated in the SM

1) Gravity. Gravity is not included in the SM. The inclusion of just "graviton" in the SM will not serve the purpose because experimental observations are not compatible with the SM calculations. General relativity is the successfull theory that explains the gravitational phenomena.

2) Dark energy and dark matter. The matter explained by the SM is the visible matter which is only 5% of the total. Cosmological calculations reveals that 26% is dark matter and the remaining 69% is dark energy about which the SM is silent.

3) Neutrino masses. Neutrinos are treated as massless in the SM but the results from neutrino oscillations provide the evidence for mass of neutrinos.

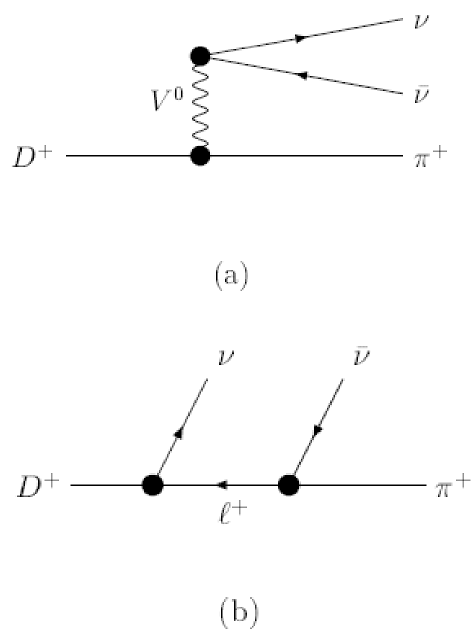


Figure 3.7.2: Long distance contribution of D+ decays to Pi+ neutrino antineutrino

4) Matter-antimatter asymmetry. The dominance of matter over anti-matter in the universe is not explained by the SM. According to the SM there should be a symmetry between matter and anti-matter.

(b) Theoretical problems with the SM

1) Hierarchy problem in

(i) strength of forces

(ii) masses of particles

(iii) quadratic divergence in Higgs mass

All the particles in the SM acquire their mass by the SSB caused by the Higgs particle. In the SM, quantum correction to the Higgs mass from the virtual particles loops becomes very large even more than the mass of Higgs itself. So fine-tuning must be introduced to cancel these quatum correction and this fine tuning is unnatural to many theorists.

2) Strong CP violation. Theoretically, there should be a term for CP violation in the strong sector which is relating matter to anti-matter. There is no evidence for such term from the

experiments.

3) Large number of parameters. There are at least 19 parameters in the SM. The values of these parameters are taken from the experiments. But the origin of these parameter is completely unknown in the SM.

4) Generations of quarks and leptons. Why do we have only three generation of leptons and quarks?

5) Masses of particles. Masses are found from the experiments; but in the SM, we do not have the answer for these masses.

May be these problems are due to the fact that it was developed for the energy scale of $O(100GeV)$. We can have some more phenomena at very high energies as compared to the previous scale. It is generally believed that standard model is a low energy approximation of a more fundamental theory.

(c) Universality in the SM

Charge bosons $W^{\pm}$ have universal couplings with the leptons and quarks in the SM..And this is known as "weak universality" Similarly, all three charge leptons have same coupling strength for Z^0bosons and all three flavors of neutrino have same coupling with Z^0. This phenomenon is called "lepton universality". For the massive neutrinos we can have mixing in the leptonic sector too, just like quark sector and there will be non universal weak interactions.

Chapter 4

Non Standard Neutrino Interactions

4.1 Introduction

Super-Kamiokande [34] was the first experiment that established the oscillations of neutrinos as a leading phenomenon behind flavour transitions of neutrinos. Later on, SNO, KamLand, MINOS, MINIBooNE and K2K results further put this phenomenon on firm grounds. These oscillations predict non zero mass for the neutrinos. Neutrino mass is the only concrete fact against the SM. There are many dedicated experiments that (like, Daya Bay, ICARUS, IceCube, KATRIN, Double Chooz, NovA, RENO, OPERA and T2K) are in search of missing neutrinos parameters.

We can have new physics which may appear as unknown couplings of neutrinos. These couplings are taken as non-standard neutrino interactions (NSIs). NSIs could effect the product, propagation and detection of neutrinos.

4.2 Neutrino Oscillations

Neutrinos interact through weak interactions only and for many years it was treated as massless particle. After the discovery of neutrino flavor transition mechanism it has become evident that neutrinos have mass. With this development, it was proved that the flavor eigen states of neutrinos are different from mass eigen states, which is analogous to quarks. The difference from quarks doublet comes from charge leptons for which mixed states do not occur. In leptonic sector we have PMNS (Pontecorvo–Maki–Nakagawa–Sakata) matrix just like CKM matrix [35]

$$\begin{pmatrix} v_e \\ v_\mu \\ v_\tau \end{pmatrix} = U \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \tag{4.2.1}$$

U is a unitary matrix establishing relationship between weak eigen states (v_e, v_μ, v_τ) and mass eigen states (v_1, v_2, v_3). It depends on mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$, the Dirac CP- violating phase δ, and majorana CP-violating phases (ρ and σ). In the standard parameterization [37], U is written as

$$\begin{aligned} U \;=\; & \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \\ & \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned} \tag{4.2.2}$$

$c_{ij} \equiv \cos(\theta_{ij})$, $s_{ij} \equiv \sin(\theta_{ij})$.

The time evaluation eq. for neutrinos, like Shrodinger eq. is given as

$$i\frac{d\nu}{dt} = \frac{1}{2E}[MM^\dagger + diag(A, 0, 0)]\nu \equiv H\nu. \tag{4.2.3}$$

Here, E is neutrino energy, $M = Udiag(m_1, m_2, m_3)U^\dagger$ is representing the neutrino mass matrix, and $A = 2\sqrt{2}EG_F N_e$ is the effective potential due to the charge-current weak interactions with electrons [38, 39]. m_1, m_2, m_3 are the masses of neutrinos and $\mathrm{G}_F = (1.663787 \pm 0.0000006) \times 10^{-5}\ GeV^{-2}$ is the Fermi coupling constant [37], N_e is the electron density along the neutrino path. The neutrino states are not pure states but quantum mixed states, thus there can be a flavour transition during propagation. As an example, if we take two flavour mixed state of ν_e and ν_μ, then probability of conversion during oscillation along a path L is given by

$$P(\nu_e \to \nu_\mu; L) = \sin^2(2\theta)\sin(\frac{\Delta m^2 L}{4E}) \tag{4.2.4}$$

and similarly survival probability is given by

$$P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_\mu) = 1 - P(\nu_\mu \to \nu_e) = P(\nu_\mu \to \nu_\mu). \quad (4.2.5)$$

In 4.2.4, θ is mixing angle and Δm^2 is mass square difference. For the case of three flavours, we have the following formula

$$P(\nu_\alpha \quad \to \quad \nu_\beta; L) = \delta_{\alpha\beta} - 4\sum_{i\rangle j} \mathrm{Re}(U_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \times \sin^2(\frac{\Delta m^2_{ij} L}{4E})$$
$$+2\sum_{i\rangle j} \mathrm{Im}(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\frac{\Delta m^2_{ij} L}{4E}) \quad (4.2.6)$$

$(\alpha, \beta = e, \mu, \tau)$. There are many open questions for neutrinos. e.g. nature of neutrino (Dirac or Majorana), absolute masses of neutrinos, leptonic CP-violation, sterile neutrinos etc. But, in literature there is a widely asked question, are there NSIs (non standard neutrino interactions)?

In the neutrino oscillation experiments, some of the neutrino parameters are found with great precision [34, 40, 41] i.e., $\Delta m^2_{12}, |\Delta m^2_{31}|, \theta_{12}, \theta_{13}$ and θ_{23}. Other parameters like the sign of Δm^2_{31} along with Majorana CP-violating phases (δ) and absolute scale for neutrino masses are completely unknown. Current and future experiments might probe these parameters. New physics such as NSIs might have effects for these parameters.

4.3 Other Mechanism for Neutrino Flavor Transition

Initially, non standard neutrino interactions (NSIs) were presented as an alternative to the oscillations for flavour transition. But due to careful analysis of experimental data, this assumption is ruled out. NSIs are still present as a sub leading effect along with oscillations. NSIs can produce resonance condition [42], which will be the modified version of Mikheyev-Smirnove-Wolfenstein effect [40, 41, 43].

(a) Non Standard Neutrino Interactions (NSIs)

The operator used for NSIs can be written as [36]

$$L_{eff}^{NSI} = -2\sqrt{2}G_F \left[\sum_{\alpha=\beta} \epsilon_{\alpha\beta}^{ff^{/}P} (\overline{\nu}_\alpha \gamma_\mu L \nu_\beta)(\overline{f}\gamma^\mu P f) + \sum_{\alpha\neq\beta} \epsilon_{\alpha\beta}^{fP} (\overline{\nu}_\alpha \gamma_\mu L \nu_\beta)(\overline{f}^{'}\gamma^\mu P f) \right] \quad (4.3.1)$$

Here $\epsilon_{\alpha\beta}^{fP}$ have the information about the dynamics, $P = L = \frac{1-\gamma_5}{2}$ or $R = \frac{1+\gamma_5}{2}$and f and $f^{'}$ are usually the fermions (quark or lepton) from the $1st$ generation. If $f \neq f^{'}$, then these are charged-current like NSIs, and if $f = f^{/}$ then these are neutral-current like NSIs, and $\epsilon_{\alpha\beta}^{ff^{/}P} \equiv \epsilon_{\alpha\beta}^{fP}$. It is important to note that operators 4.3.1 are neither gauge invariant nor re-normalizable. Thus we get dimension 6 operator for NSIs after integrating out heavy degrees of freedom. In the effective Lagrangian 4.3.1, S,P and T Lorentz stuctures can also be considered, but this work assumes that only V and A stuctures are the most important. Now by using the famous relation

$$\frac{G_F}{\sqrt{2}} \simeq \frac{g_W^2}{8m_W^2},$$

we obtain the effective NSIs parameters [44, 45, 46]

$$\epsilon \propto \frac{m_W^2}{m_x^2}.$$

Here $m_W = (80.385 \pm 0.015)GeV \simeq 0.1TeV$ [37] is mass of W boson and m_x is the mass scale at which NSIs may be generated [47]. NSIs can effect production, propagation and detection of neutrinos.

4.4 Semi-leptonic Decays of Mesons in NSIs

These are the decays involving two neutrinos in their final state. As discussed earlier, these are suppressed in SM and can occur only at loop level in the SM. If the neutrino flavour is also violated then we have to use two loops instead of one in SM. But, we will keep ourself only at one loop level in the SM. NSIs are considered to be well-matched with the oscillation effects along with new features in neutrino searches [48, 49, 50, 51, 52, 53, 54]. NSIs may conserve flavor $\alpha = \beta$,for this we have ϵ_{ee}^{fP}, $\epsilon_{\mu\mu}^{fP}$ and $\epsilon_{\tau\tau}^{fP}$ known as flavour diagonal (FD). It can violate flavor conservation $\alpha \neq \beta$, for which we have $\epsilon^{fP}_{e\mu}$, $\epsilon^{fP}_{e\tau}$, $\epsilon^{fP}_{\mu e}$, $\epsilon^{fP}_{\mu\tau}$, $\epsilon^{fP}_{\tau e}$, and $\epsilon^{fP}_{\tau\mu}$ known as Flavor non diagonal (FND). Constraints on NSIs parameter $\epsilon^{fP}_{\alpha\beta}$ have been studied in References [55, 56, 57, 58]. From scattering in leptonic sectors (f is lepton), constraints are determined for first two generations ϵ^{fP}_{ll} ($l = e$,μ) by tree level processes and could be limited at $O(10^{-3})$ by future $\sin^2\theta_W$ experiments. For third generation (τ) we study decays which occur at loop level. KamLAND data [59] and solar neutrino data [60, 61] can improve the third generation (τ) limit to (0.3) [55] . Although, the constraints on $\epsilon^{fP}_{\tau l}$ are given by the precision experiments but they are bounded by $O(10^{-2})$ [55, 62].

(a) NSIs in Charm Rare Decays

Semileptonic decays of K and B mesons have and will continue their role for exploring NP. But for D sector due to smallness of the branching ratios in SM and lack of experimental data, semileptonic charm physics is difficult to study. But now the data from BES-III, B factory, Super-B and LHC-b for the rare decays will improve our knowledge of charm physics. A theoretical estimate for CC (charge currents) decays $D^+_s \to D^0 e^+ \upsilon_e, B^0_s \to B^+ e^- \overline{\upsilon}_e, D^+_s \to D^+ e^+ e^-$and $B^0_s \to B^0 e^+ e^-$ is given in [63], for future data at different luminosities of these machines. Theoretical values of NSIs could also be calculated for FCNC in charm decays. We select D ($D^+_s \to K^+ \upsilon\overline{\upsilon},\ D^+ \to D^0 \upsilon\overline{\upsilon}, D^0 \to \pi^0 \upsilon\overline{\upsilon}$) for this purpose and analyzes them in the frame work of NSIs.

$c \to u\upsilon\overline{\upsilon}$ Decays in NSIs

The NSIs diagram of process the $c \longrightarrow u\nu_\alpha\overline{\nu}_\beta$ is given in Figure 4.4.1 and represented by the Hamiltonian (eq.4.4.1)

$$H^{NSI}_{c\longrightarrow u\nu_\alpha\overline{\nu}_\beta} = \frac{G_F}{\sqrt{2}}(\frac{\alpha_{em}}{4\pi\sin^2\theta_W} V_{cd} V^*_{ud} \epsilon^{dL}_{\alpha\beta} \ln\frac{\Lambda}{m_W})(\overline{\nu}_\alpha\nu_\beta)_{V-A}(\overline{c}u)_{V-A}. \tag{4.4.1}$$

For $D^+ \longrightarrow \pi^+\nu_\alpha\overline{\nu}_\beta$ decay NSIs is calculated in [55]

$$BR(D^+ \longrightarrow \pi^+\nu_\alpha\overline{\nu}_\beta)_{NSI} = |V^*_{ud}\frac{\alpha_{em}}{4\pi\sin^2\theta_W}\epsilon^{dL}_{\alpha\beta}\ln\frac{\Lambda}{m_W}|^2 BR(D^+ \longrightarrow \pi^0 e^+\nu_e) \tag{4.4.2}$$

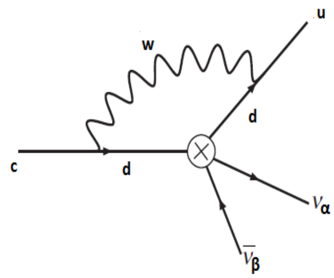


Figure 4.4.1: NSIs c decays to u neutrino antineutrino

$BR(D^+ \longrightarrow \pi^+\nu_\alpha\overline{\nu}_\beta)_{NSI} = 2\times 10^{-8}|\epsilon^{dL}_{\alpha\beta}\ln\frac{\Lambda}{m_W}|^2$and it is mentioned that as α and β could represent any lepton, we take $\epsilon^{dL}_{\tau\tau}\sim 1, \epsilon^{dL}_{ll/}\langle 1$ for $l = l^/ \neq \tau$. Here $\ln\frac{\Lambda}{m_W}\sim 1$.

We point out that the same is applicable to two other processes $D^+_s \longrightarrow K^+\nu_\alpha\overline{\nu}_\beta$ and $D^0\rightarrow\pi^0\nu_\alpha\overline{\nu}_\beta$.

$$BR(D^+_s \longrightarrow K^+\nu_\alpha\overline{\nu}_\beta)_{NSI} = |V^*_{ud}\frac{\alpha_{em}}{4\pi\sin^2\theta_W}\epsilon^{dL}_{\alpha\beta}\ln\frac{\Lambda}{m_W}|^2 BR(D^+_s \longrightarrow K^0 e^+\nu_e), \quad (4.4.3)$$

$$BR(D^0 \longrightarrow \pi^0\nu_\alpha\overline{\nu}_\beta)_{NSI} = |V^*_{ud}\frac{\alpha_{em}}{4\pi\sin^2\theta_W}\epsilon^{dL}_{\alpha\beta}\ln\frac{\Lambda}{m_W}|^2 BR(\overline{D}^0 \longrightarrow \pi^- e^+\nu_e). \quad (4.4.4)$$

Using [37] Values $BR(D^+_s \longrightarrow K^0 e^+\nu_e) = (3.7\pm 1)\times 10^{-3}$, $V_{ud} = 0.97425\pm 0.00022$, $\alpha_{em} = \frac{1}{137}$,we get

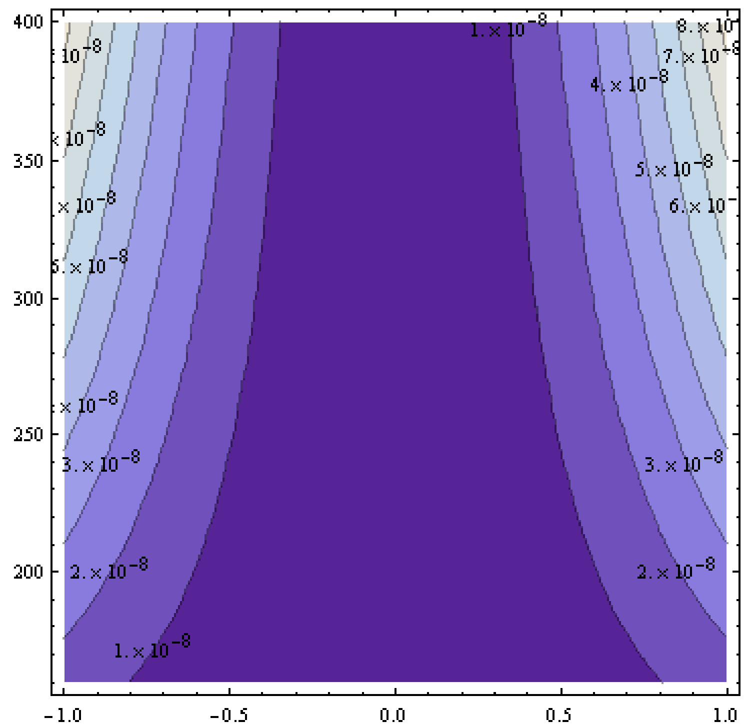
$$BR(D^+_s \longrightarrow K^+\nu_\alpha\overline{\nu}_\beta)_{NSI} = 2.22796\times 10^{-8}(\epsilon^{dL}_{\alpha\beta})^2|\ln\frac{\Lambda}{m_W}|^2. \quad (4.4.5)$$

For $\epsilon^{dL}_{\tau\tau}\sim 1$ and $\ln\frac{\Lambda}{m_W}\sim 1$,we get $BR(D^+_s \longrightarrow K^+\nu_\alpha\overline{\nu}_\beta)_{NSI} = 2.22796\times 10^{-8}$.

Similarly for $BR(\overline{D}^0 \longrightarrow \pi^- e^+\nu_e) = 2.89\times 10^{-3}$, we have

$$BR(D^0 \longrightarrow \pi^0\nu_\alpha\overline{\nu}_\beta)_{NSI} = 3.21068\times 10^{-8}(\epsilon^{dL}_{\alpha\beta})^2|\ln\frac{\Lambda}{m_W}|^2 \quad (4.4.6)$$

10^{-8}will be in the range of BES-III. If it is not detected then useful limits for new physics can be suggested.



Contour Plot $(D^0 \longrightarrow \pi^0\nu_\tau\overline{\nu}_\tau)_{NSI}$ *as a function of* $\epsilon^{dL}_{\tau\tau}$, *shown on* horizontal axis *and new energy scale* Λ (verticle axis)

Figure 4.4.2: Contour plot of NSIs showing dependence of NSIs on new physics scale and NSIs parameter

Reaction	SM			NSIs	$\epsilon^{dL}_{\tau\tau}$	$\epsilon^{dL}_{ll'}$ $l = l' \neq \tau$
$BR(D^+ \longrightarrow \pi^+\nu_\alpha\overline{\nu}_\beta)$	Long Distance	$< 8 \times 10^{-16}$	[65]	2×10^{-8} [55]	~ 1 [55]	$\langle 1$
	Short Distance	3.9×10^{-16}				
$BR(D^+_s \longrightarrow K^+\nu_\alpha\overline{\nu}_\beta)$	Long Distance	$< 4 \times 10^{-16}$[64]		2.23×10^{-8} [66]	~ 1 [66]	$\langle 1$
	Short Distance	1.5×10^{-16}				
$BR(D^0 \longrightarrow \pi^0\nu_\alpha\overline{\nu}_\beta)$	Long Distance	$< 6 \times 10^{-16}$	[65]	3.21×10^{-8} [66]	~ 1 [66]	$\langle 1$
	Short Distance	4.9×10^{-16}				

Table 4.4.1: Summary table of NSIs and bounds with d in the loop

$Process$	SM	$NSIs$	$\epsilon^{uL}_{\tau\tau}$
$D^+_s \to D^+\upsilon\overline{\upsilon}$	6×10^{-15} [66]	2×10^{-15} [66]	$O(10^2)$ [66]
$K^+ \to \pi^+\upsilon\overline{\upsilon}$	$(8 \pm 1.1) \times 10^{-11}$ [55]	5×10^{-11}	$O(10^2)$ [55]

Table 4.5.1: Comparison of SM and NSIs Branching Ratios and bounds on NSIs for u quark exchange

4.5 NSIs in $D^+_s \to D^+\nu_\alpha\overline{\nu}_\beta$

It is short distance dominant process represented by quark level process $s \to d\ \nu_\alpha\overline{\nu}_\beta$ just like $K^+ \to \pi^+\nu_\alpha\overline{\nu}_\beta$ for which $\epsilon^{uL}_{\tau\tau} \leq \frac{8.8\times10^{-3}}{\ln\frac{\Lambda}{m_W}}$ is pointed out by [55] and ploted in 3D in fig.4.5.1. NSIs diagram for $s \to d\ \nu_\alpha\overline{\nu}_\beta$ is given in fig 4.5.2

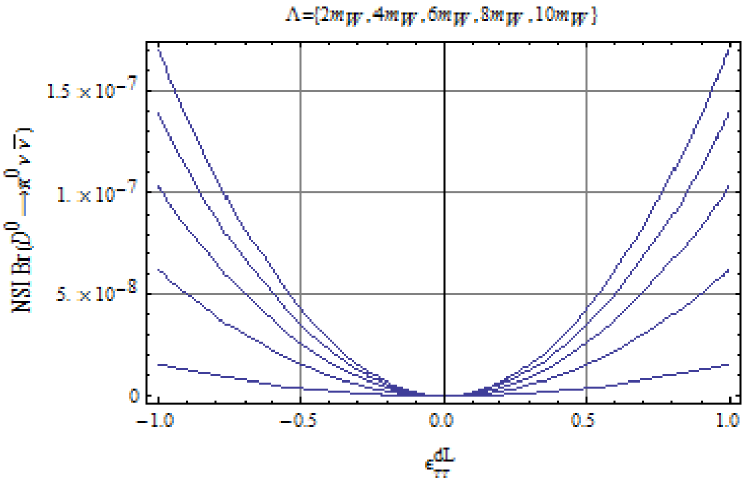
The effective Hamiltonian for such reaction is given by 4.5.1

$$H^{NSI}_{eff} = \frac{G_F}{\sqrt{2}}(V^*_{us}V_{ud}\frac{\alpha_{em}}{2\pi\sin^2\theta_W}\epsilon^{uL}_{\alpha\beta}\ln\frac{\Lambda}{m_W}) \times (\nu_\alpha\overline{\nu}_\beta)_{V-A}(\overline{s}d). \tag{4.5.1}$$

From this branching ratio of $D^+_s \to D^+\nu_\alpha\overline{\nu}_\beta$, NSIs Br becomes

$$Br(D^+_s \to D^+\nu_\alpha\overline{\nu}_\beta)_{NSI} = |\frac{\alpha_{em}}{4\pi\sin^2\theta_W}V_{ud}\epsilon^{uL}_{\alpha\beta}\ln\frac{\Lambda}{m_W}|^2 BR(D^+_s \longrightarrow D^0e^+\nu_e) \tag{4.5.2}$$

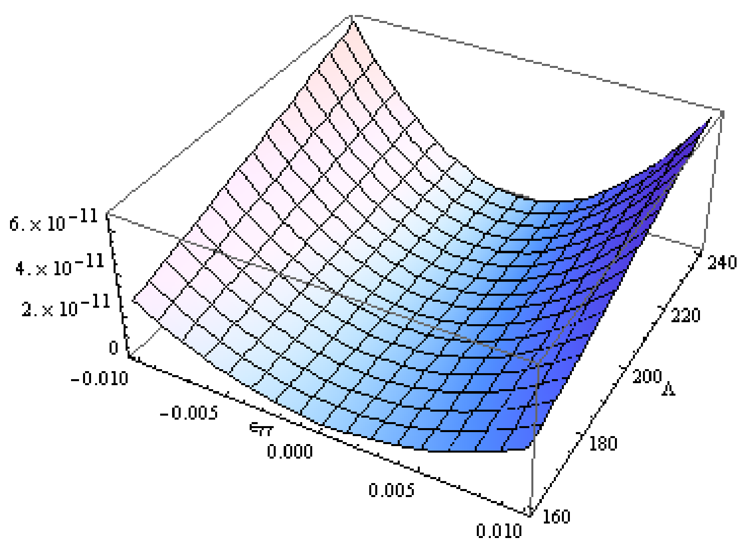
Using estimated $BR(D^+_s \longrightarrow D^0e^+\nu_e) = 5 \times 10^{-6}$ for BES in [63], we get NSIs $Br(D^+_s \to D^+\nu_\tau\overline{\nu}_\tau) = 2.33153 \times 10^{-15}$ which could enhance SM value ($\sim 6 \times 10^{-15}$) even at electroweak scale.



NSIs Branching Ratio of $D^0 \to \pi^0 \nu_\tau \overline{\nu}_\tau$

Λ *is new phyics scale,* $\epsilon_{\tau\tau}^{dL}$ *new physics parameter*

Figure 4.4.3: 3-D plot of NSIs



NSIs Br of $K^+ \to \pi^+ \nu \overline{\nu}$ as a function of new physics scale Λ and parameter $\epsilon_{\tau\tau}^{uL}$

Figure 4.5.1: 3D plot of NSIs with u quark in the loop

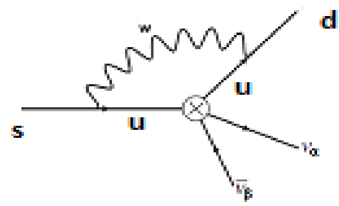


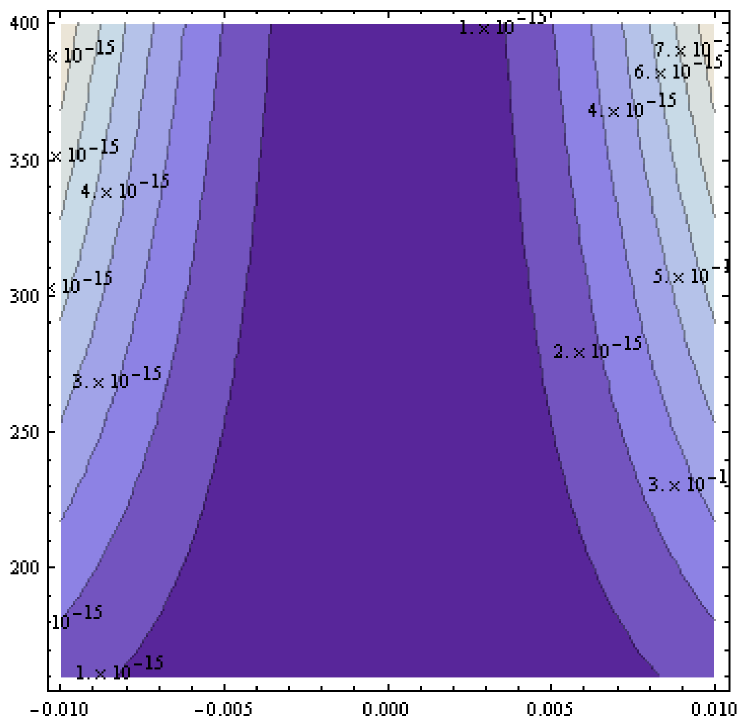
Diagram representing the NSIs of $s \to d\ \nu_\alpha \overline{\nu}_\beta$
Blob is representing the New Physics contribution

Figure 4.5.2: NSIs with u quark in the loop

This cannot be detected in BES-III but there is a chance for them in B-factories or in a future accelerator. The contour plot of Br ratio as a function of new energy scale Λ and $\epsilon^{uL}_{\tau\tau}$ is given in fig 4.5.3.

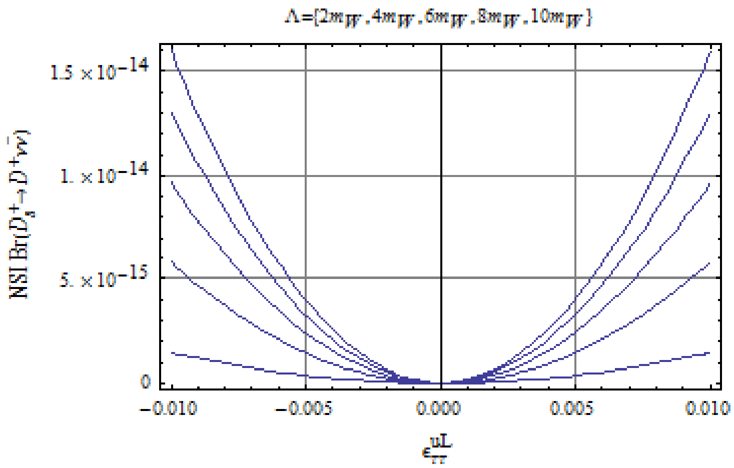
4.6 Summary and Discussion

We have investigated $D_s^+ \to K^+ \upsilon\overline{\upsilon}$, $D^0 \to \pi^0 \upsilon\overline{\upsilon}$. These are long distance dominated processes and are model dependent. We have found in this case that the contribution from NSIs is very large as compared to the SM, so the SM contribution can easily be ignored, as depicted in table 4.4.1. Whereas, $D_s^+ \to D^+ \upsilon\overline{\upsilon}$ is short distance (SD) dominant process, here SM contribution can not be ignore, but NSIs can improve SD dominated contribution, as it appears as an additive term, evident from table 4.5.1. This fact is depicted by the analysis of only experimentally measured process i.e. $K^+ \to \pi^+ \upsilon\overline{\upsilon}$ .as provided in the table 4.5.1 and fig 4.5.1. The information (value of NSI) obtained by this process can be used in $D_s^+ \to D^+ \upsilon\overline{\upsilon}$ to get the contribution of NSI in total branching fraction. Thus branching ratios of $D_s^+ \to K^+ \upsilon\overline{\upsilon}$, $D^0 \to \pi^0 \upsilon\overline{\upsilon}$ and $D_s^+ \to D^+ \upsilon\overline{\upsilon}$ decays are 2.23×10^{-8}, $3.21 \times 10^{-8}$and 2.33×10^{-15} respectively in the frame work of NSIs. From these calculations bounds on $\epsilon^{uL}_{\tau\tau}$ and $\epsilon^{dL}_{\tau\tau}$ are $O(10^{-2})$ and ~ 1 respectively, $\epsilon^{dL}_{\alpha\beta} < 1$ for $\alpha, \beta = e, \mu$. Hence, in the rare decays of charm meson, the long distance dominated processes are dominated by NSIs, whereas there is a considerable enhancement in the Br of short distance processe due to NSIs. The bounds on $\epsilon^{dL}_{\alpha\beta}$ are weak as compared to



Contour Plot $(D_s^+ \longrightarrow D^+\nu_\tau\overline{\nu}_\tau)$ *NSI as a function of* $\epsilon_{\tau\tau}^{uL}$ *and new energy scale* Λ

Figure 4.5.3: Contour plot



Branching Ratio of NSI of $D_s^+ \to D^+ \upsilon_\tau \overline{\upsilon}_\tau$

Λ *is new phyics scale,* $\epsilon_{\tau\tau}^{uL}$ *is new physics parameter*

This shows a strong dependence of Br on New Physics parameter and New Energy Scale

Figure 4.5.4: Plot for NSIs of Ds decay to D+, netrino and antineutrino

$\epsilon_{\alpha\beta}^{uL}$, but we do not have experimental values for either of these. This fact is providing a room for the new physics esepecially for the reactions involving $\epsilon_{\alpha\beta}^{dL}$.

Chapter 5

Extension of Non Standard Neutrino Interactions to Second and Third Generation of Quarks

Rare decays of mesons having two neutrinos in the final state are thought to be a clean signal for the new physics (NP). These decays provide us with a unique opportunity to study "non standard neutrino interactions" known as NSIs. NSIs is a very well understood phenomena now. The effective Lagrangian for it in model independent way is given in [36]

$$L_{eff}^{NSI} = -2\sqrt{2}G_F\left[\sum_{\alpha=\beta}\epsilon_{\alpha\beta}^{fP}(\overline{\nu}_\alpha\gamma_\mu L\nu_\beta)(\overline{f}\gamma^\mu Pf) + \sum_{\alpha\neq\beta}\epsilon_{\alpha\beta}^{fP}(\overline{\nu}_\alpha\gamma_\mu L\nu_\beta)(\overline{f}\gamma^\mu Pf)\right] \tag{5.0.1}$$

Here NSIs parameter $\epsilon_{\alpha\beta}^{fP}$ carries information about the dynamics. NSIs are assumed to be well-matched with the oscillation effects along with some new features in neutrino searches [48, 49, 50, 51, 52, 53, 54]. It is believed that NSIs can have their effects on neutrinos at production, propagation and detection level. Constraints on NSIs parameter $\epsilon_{\alpha\beta}^{fP}$ have been studied in many references, i.e., [67, 57, 69]. These interactions are loop induced interactions in standard model (SM), having charge as well as neutral vertices but NSIs will affect neutral vertices only [68]. From scattering in leptonic sectors constraints are determined for first two generations ϵ_{ll}^{fP} ($l = e,\mu$) by tree level processes and could be limited at $O(10^{-3})$ by future

$\sin^2\theta_W$ experiments. For third generation (τ) decays which occur at loop level are studied; the limit of $O(0.3)$ is expected from KamLAND data [70] and solar neutrino data [60, 71]. Although, the constraints on $\epsilon^{fP}_{\tau l}$ are given by the precision experiments but they are bounded by $O(10^{-2})$ [72]. It is pointed out in reference [55] that by using $K^+ \longrightarrow \pi^+\overline{v}v$ the $\epsilon^{uL}_{\tau\tau}$ constraints could be $O(10^{-2})$. Mostly f is taken as a lepton or quark from first generation (u or d). If we take f from second and third generation of quark we have almost same constraints as for first generations, $\epsilon^{uP}_{\alpha\beta}$. Similar thing happen to the other partners of c, t quarks s and b. Leptons and quarks generations are playing on equal footings and in leptonic sector we have $\epsilon^{eP}_{\alpha\beta}$, $\epsilon^{\mu P}_{\alpha\beta}$ and even $\epsilon^{\tau P}_{\alpha\beta}$,. Although, nobody is talking about these types of effects for the second generation simply due to the fact that the ordinary matter consist of only of first generation of quarks but we point out that just like second generation of leptons, NSIs are also affected by second generation of quarks at the production of neutrinos from rare decays of mesons. These could be responsible for the flavor violating neutrino production.

We investigate $K^+ \longrightarrow \pi^+\overline{v}v, D^+_s \rightarrow D^+\overline{v}v, B^0_s \rightarrow B^0\overline{v}v, D^+ \longrightarrow \pi^+\overline{v}v, D^0 \longrightarrow \pi^0\overline{v}v$ and $D^+_s \longrightarrow K^+\overline{v}v$ processes to show that three generations of quarks are affecting the NSIs. These processes can give us NP contributions in terms of NSIs $D^+_s \rightarrow D^+\overline{v}v$ and $B^0_s \rightarrow B^0\overline{v}v$ are searched for NSIs with c and t quarks. Just like these three other processes $D^+ \longrightarrow \pi^+\overline{v}v, D^0 \longrightarrow \pi^0\overline{v}v$ and $D^+_s \longrightarrow K^+\overline{v}v$ are calculated with s and b quarks instead of d quark in the loop. The results and comparison are provided and conclusion is given at the end of the chapter.

5.1 Experimental Status

It is expected that at the end of this decade we will be able to detect rare decays of meson involving neutrinos in the final state just like $K^+ \longrightarrow \pi^+\overline{v}v$ [73]. But so far, it is the only semileptonic reaction involving two neutrinos in the final state whose experimental value is $(1.7 \pm 1.1) \times 10^{-10}$[47]. So by using this reaction we can point out exact region for the new physics. $D^+_s \rightarrow D^+\overline{v}v$, $B^0_s \rightarrow B^0\overline{v}v, D^+ \longrightarrow \pi^+\overline{v}v, D^0 \longrightarrow \pi^0\overline{v}v$ and $D^+_s \longrightarrow K^+\overline{v}v$ are yet to be detected. In BES -III, super b-factories and in future super collider, we will have an opportunity to detect them in a clean environment.

5.2 NSIs in $K^+ \longrightarrow \pi^+\overline{\upsilon}\upsilon$, $B_s^0 \to B^0\overline{\upsilon}\upsilon$ and $D_s^+ \to D^+\overline{\upsilon}\upsilon$

The NSIs effective Hamiltonian for u- quak in the loop is given by

$$H_{eff}^{NSI} = \frac{G_F}{\sqrt{2}}(V_{us}^* V_{ud} \frac{\alpha_{em}}{4\pi \sin^2 \theta_W} \epsilon_{\alpha\beta}^{uL} \ln \frac{\Lambda}{m_w}) \times (\nu_\alpha \overline{\nu}_\beta)_{V-A}(\overline{s}d), \quad (5.2.1)$$

from which the NSIs Br

$$Br(K^+ \longrightarrow \pi^+\overline{\upsilon}\upsilon)_{NSI} = r_{K^+} \frac{\alpha_{em}^2}{|V_{us}|^2 2\pi^2 \sin^4 \theta_W} |V_{us}^* V_{ud} \frac{1}{2} \epsilon_{\alpha\beta}^{uL} \ln \frac{\Lambda}{m_w}|^2 \times BR(K^+ \longrightarrow \pi^0 e^+ \nu_e).$$

This was calculated in [55] and the writers claimed that $\epsilon_{\tau\tau}^{uL} \leq \frac{8.8\times10^{-3}}{\ln \frac{\Lambda}{m_W}}$ .With latest values $\epsilon_{\tau\tau}^{uL} \leq \frac{6.7\times10^{-3}}{\ln \frac{\Lambda}{m_W}}$.

When we insert this value for our processes, we have

$$Br(B_s^0 \to B^0\overline{\upsilon}\upsilon)_{NSI} = \frac{\alpha_{em}^2}{|V_{us}|^2 2\pi^2 \sin^4 \theta_W} |V_{us}^* V_{ud} \frac{1}{2} \epsilon_{\alpha\beta}^{uL} \ln \frac{\Lambda}{m_w}|^2 \times Br(B_s^0 \to B^+ e^- \upsilon).$$

Numerically we get

$Br(B_s^0 \to B^0\overline{\upsilon}\upsilon)_{NSI} = 2.17 \times 10^{-17}$

The $Br(D_s^+ \to D^+\overline{\upsilon}\upsilon)_{NSI} = 2.70 \times 10^{-15}$ is given in [66]

5.3 NSIs with c and t quarks in the loop

Now we generalize the process and take Q in the loop which can be any up-type quark in the loop instead of u quark

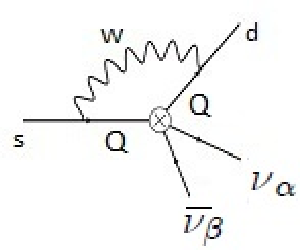


Figure 5.3.1: c,t quark induced NSIs diagram

The NSIs effective hamiltonian is given by

$$H_{eff}^{NSI} = \frac{G_F}{\sqrt{2}}(V_{Qs}^{*}V_{Qd}\frac{\alpha_{em}}{4\pi\sin^2\theta_W}\epsilon_{\alpha\beta}^{cL}\ln\frac{\Lambda}{m_w}) \times (\nu_\alpha\overline{\nu}_\beta)_{V-A}(\overline{s}d). \quad (5.3.1)$$

Here, it is same as that of u quark in the loop and we are simply replacing c with u.

The NSIs Br with c quark becomes

$$Br(K^+ \longrightarrow \pi^+\overline{\upsilon}\upsilon)_{NSI} = r_{K^+}\frac{\alpha_{em}^2}{|V_{us}|^2 2\pi^2\sin^4\theta_W}|V_{Qs}^{*}V_{Qd}\tfrac{1}{2}\epsilon_{\alpha\beta}^{cL}\ln\tfrac{\Lambda}{m_w}|^2\times$$
$$BR(K^+ \longrightarrow \pi^0 e^+\nu_e)$$

Putting the values from [78] we get $\epsilon_{\tau\tau}^{cL} \leq \frac{6.2\times10^{-3}}{\ln\frac{\Lambda}{m_W}}$.

When we insert this value for our processes, we have

$$Br(D_s^+ \to D^+\overline{\upsilon}\upsilon)_{NSI} = \frac{\alpha_{em}^2}{|V_{us}|^2 2\pi^2\sin^4\theta_W}|V_{Qs}^{*}V_{Qd}\tfrac{1}{2}\epsilon_{\alpha\beta}^{cL}\ln\tfrac{\Lambda}{m_w}|^2\times$$
$$Br(D_s^+ \to D^0 e^+\upsilon)$$

$$Br(B_s^0 \to B^0\overline{\upsilon}\upsilon)_{NSI} = \frac{\alpha_{em}^2}{|V_{us}|^2 2\pi^2\sin^4\theta_W}|V_{Qs}^{*}V_{Qd}\tfrac{1}{2}\epsilon_{\alpha\beta}^{cL}\ln\tfrac{\Lambda}{m_w}|^2\times$$
$$Br(B_s^0 \to B^+ e^-\upsilon)$$

The results are given in the table 5.5.1.

5.4 NSIs in $D_s^+ \to K^+\upsilon_\alpha\overline{\upsilon}_\beta$, $D^+ \longrightarrow \pi^+\nu_\alpha\overline{\nu}_\beta$ and $D^0 \longrightarrow \pi^0\nu_\alpha\overline{\nu}_\beta$

The quark level process $c \longrightarrow u\nu_\alpha\overline{\nu}_\beta$ is representing all above processes. For $D^+ \longrightarrow \pi^+\nu_\alpha\overline{\nu}_\beta$, NSIs with u quark in the loop is calculated in [55]

$$Br(D^+ \longrightarrow \pi^+\nu_\alpha\overline{\nu}_\beta)_{NSI} = |V_{ud}^{*}\frac{\alpha_{em}}{4\pi\sin^2\theta_W}\epsilon_{\alpha\beta}^{dL}\ln\frac{\Lambda}{m_W}|^2 BR(D^+ \longrightarrow \pi^0 e^+\nu_e) \quad (5.4.1)$$

$Br(D^+ \longrightarrow \pi^+\nu_\alpha\overline{\nu}_\beta)_{NSI} = 4.49\times10^{-8}|\epsilon_{\alpha\beta}^{dL}\ln\frac{\Lambda}{m_W}|^2$ and it is mentioned that as α and β could represent any lepton, we take $\epsilon_{\tau\tau}^{dL} \sim 1, \epsilon_{ll^/}^{dL}\langle 1$ for $l = l^/ \neq \tau$. Here $\ln\frac{\Lambda}{m_W} \sim 1$.

NSIs diagram with d, s and b quarks is shown in fig. 5.4.1, here $q = d, s$ and b

$$H_{c\longrightarrow u\nu_\alpha\overline{\nu}_\beta}^{NSI} = \frac{G_F}{\sqrt{2}}(\frac{\alpha_{em}}{4\pi\sin^2\theta_W}V_{cq}V_{uq}^{*}\epsilon_{\alpha\beta}^{sL}\ln\frac{\Lambda}{m_W})(\overline{\nu}_\alpha\nu_\beta)_{V-A}(\overline{c}u)_{V-A} \quad (5.4.2)$$

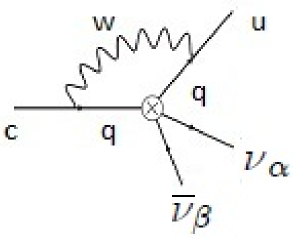


Figure 5.4.1: d,s,and b quark induced NSIs diagram

and Br becomes

$$Br(D^+ \longrightarrow \pi^+\nu_\alpha\overline{\nu}_\beta)_{NSI} = |\frac{V^*_{us}V_{cs}}{V_{cd}} \times \frac{\alpha_{em}}{4\pi\sin^2\theta_W}\epsilon^{sL}_{\alpha\beta}\ln\frac{\Lambda}{m_W}|^2 BR(D^+ \longrightarrow \pi^0 e^+\nu_e) \quad (5.4.3)$$

$Br(D^+ \longrightarrow \pi^+\nu_\alpha\overline{\nu}_\beta)_{NSI} = 4.55\times10^{-8}|\epsilon^{sL}_{\alpha\beta}\ln\frac{\Lambda}{m_W}|^2$and it is mentioned that as α and β could represent any lepton, we take $\epsilon^{sL}_{\tau\tau} \sim 1, \epsilon^{sL}_{ll'}\langle 1$ for $l = l' \neq \tau$. Here $\ln\frac{\Lambda}{m_W} \sim 1$.We further see that same is applicable to two other processes $D^+_s \longrightarrow K^+\nu_\alpha\overline{\nu}_\beta$ and $D^0 \rightarrow \pi^0\nu_\alpha\overline{\nu}_\beta$.

$$Br(D^+_s \longrightarrow K^+\nu_\alpha\overline{\nu}_\beta)_{NSI} = |\frac{V^*_{uq}V_{cq}}{V_{cd}}\frac{\alpha_{em}}{4\pi\sin^2\theta_W}\epsilon^{qL}_{\alpha\beta}\ln\frac{\Lambda}{m_W}|^2 BR(D^+_s \longrightarrow K^0 e^+\nu_e), \quad (5.4.4)$$

$$Br(D^0 \longrightarrow \pi^0\nu_\alpha\overline{\nu}_\beta)_{NSI} = |\frac{V^*_{uq}V_{cq}}{V_{cd}}\frac{\alpha_{em}}{4\pi\sin^2\theta_W}\epsilon^{qL}_{\alpha\beta}\ln\frac{\Lambda}{m_W}|^2 BR(\overline{D}^0 \longrightarrow \pi^- e^+\nu_e). \quad (5.4.5)$$

Here q is representing any quark of down type, Using [78] Values $BR(D^+_s \longrightarrow K^0 e^+\nu_e) = (3.7\pm1)\times10^{-3}$, $V_{ud} = 0.97425\pm0.00022$, $\alpha_{em} = \frac{1}{137}$,we get

$$Br(D^+_s \longrightarrow K^+\nu_\alpha\overline{\nu}_\beta)_{NSI} = 2.28\times10^{-8}(\epsilon^{sL}_{\alpha\beta})^2|\ln\frac{\Lambda}{m_W}|^2. \quad (5.4.6)$$

For $\epsilon^{sL}_{\tau\tau} \sim 1$ and $\ln\frac{\Lambda}{m_W} \sim 1$,we get $BR(D^+_s \longrightarrow K^+\nu_\alpha\overline{\nu}_\beta)_{NSI} = 2.28\times10^{-8}$.

Similarly for $Br(\overline{D}^0 \longrightarrow \pi^- e^+\nu_e) = 2.89\times10^{-3}$ we have

$$Br(D^0 \longrightarrow \pi^0\nu_\alpha\overline{\nu}_\beta)_{NSI} = 3.25\times10^{-8}(\epsilon^{sL}_{\alpha\beta})^2|\ln\frac{\Lambda}{m_W}|^2 \quad (5.4.7)$$

10^{-8}will be the reach of BES-III, so it is hoped that we might observe these decays there. If not, even then useful limits for new physics can be suggested. NSIs with d quark are discussed for $D_s^+ \longrightarrow K^+\nu_\alpha\overline{\nu}_\beta$, and $D^0 \longrightarrow \pi^0\nu_\alpha\overline{\nu}_\beta$ in [66]. All the values are summarized, in the table 5.5.2, for comparison along with the values for three generations of down type quark (d, s and b quarks).

5.5 Results and Summary

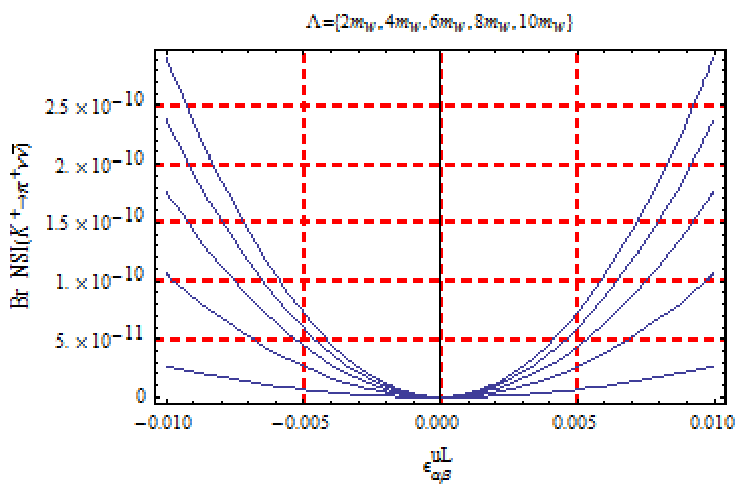


Figure 5.5.1: Plot for NSIs with u quark in the loop

It is evident from the plots given in figures 5.5.1-5.5.3 and table 5.5.1 that $\epsilon_{\alpha\beta}^{tL} = \epsilon_{\alpha\beta}^{cL} = \epsilon_{\alpha\beta}^{uL} \leq 10^{-2}$. As we have both experimental and theoretical values for K^+ decay so we can specify exact region for new physics. But for other two reactions only expected contribution from NSIs can be given. The $D_s^+ \longrightarrow D^+\overline{\upsilon}\upsilon$ and $B_s^\circ \longrightarrow B^\circ\overline{\upsilon}\upsilon$ are decays of B and charm mesons respectively but the quark decay processes is similar to K meson decay. These are very heavy mesons and decaying again into heavy mesons so there is a lot of energy required for their observation. These are sensitive to t and c quarks just like u quark. We know that we have second and even third generation constraints on free parameter of NSIs for charge leptons, like $\epsilon_{\alpha\beta}^e, \epsilon_{\alpha\beta}^\mu$and $\epsilon_{\alpha\beta}^\tau$ but we had only $\epsilon_{\alpha\beta}^{uL}$ and $\epsilon_{\alpha\beta}^{dL}$. From the other three reactions $D_s^+ \rightarrow K^+\upsilon_\alpha\overline{\upsilon}_\beta$, $D^+ \longrightarrow \pi^+\nu_\alpha\overline{\nu}_\beta$ and $D^0 \longrightarrow \pi^0\nu_\alpha\overline{\nu}_\beta$ we find $\epsilon_{\alpha\beta}^{bL}$and $\epsilon_{\alpha\beta}^{dL}$ and we come to know that $\epsilon_{\alpha\beta}^{bL}$ =$\epsilon_{\alpha\beta}^{dL} = \epsilon_{\alpha\beta}^{sL} \sim 1$. The NSIs Br for these decays is given in table 2. So, all generations of quarks

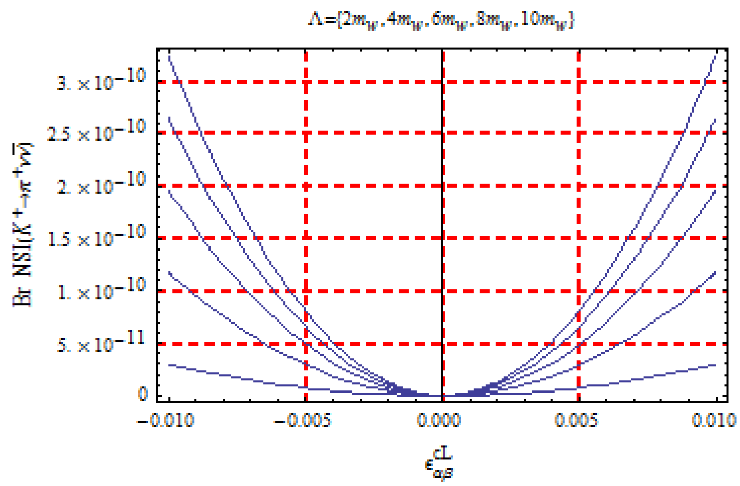


Figure 5.5.2: Plot for NSIs with c quark in the loop

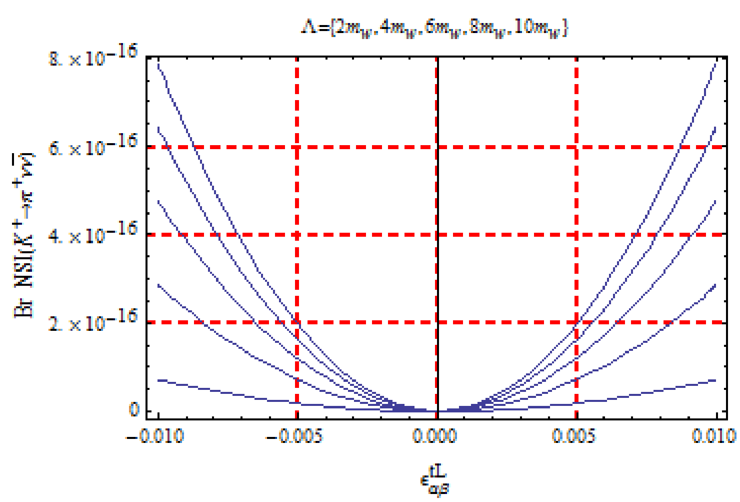


Figure 5.5.3: Plot for NSIs with t quark in the loop

Reaction	Standard Model Branching Ratio	Experimental Branching Ratio	NSIs with u	NSIs with c	NSIs with t
$K^+ \longrightarrow \pi^+\overline{\upsilon}\upsilon$ $(u\bar{s})\longrightarrow(u\bar{d})\overline{\upsilon}\upsilon$	$(7.8 \pm 0.8) \times 10^{-11}$ [30]	$(1.7 \pm 1.1) \times 10^{-10}$ [78]	2.46×10^{-11} [66]	2.42×10^{-11} [79]	$\sim 10^{-16}$ [80]
$D_s^+ \longrightarrow D^+\overline{\upsilon}\upsilon$ $(c\bar{s})\longrightarrow(c\bar{d})\overline{\upsilon}\upsilon$	7.69×10^{-15}	not known	2.70×10^{-15} [66]	2.57×10^{-15} [79]	$\sim 10^{-20}$ [80]
$B_s^\circ \longrightarrow B^\circ\overline{\upsilon}\upsilon$ $(s\bar{b})\longrightarrow(\bar{b}d)\overline{\upsilon}\upsilon$	6.86×10^{-17}	not known	2.17×10^{-17} [66]	2.0×10^{-17} [?]	$\sim 10^{-23}$ [80]

Table 5.5.1: Comparision of u,c and t quark dependent parameter

Reaction	Standard Model Branching Ratio		NSIs with d	NSIs with s	NSIs with b
	Long Distance	Short Distance			
$D^+ \longrightarrow \pi^+\nu_\alpha\overline{\nu}_\beta$ $(c\bar{d})\longrightarrow(u\bar{d})\overline{\upsilon}\upsilon$	$< 8 \times 10^{-16}$ [65]	3.9×10^{-16}	4.49×10^{-8} [66]	4.55×10^{-8} [80]	$\sim 10^{-14}$ [80]
$D_s^+ \longrightarrow K^+\nu_\alpha\overline{\nu}_\beta$ $(c\bar{s})\longrightarrow(u\bar{s})\overline{\upsilon}\upsilon$	$< 4 \times 10^{-16}$ [64]	1.5×10^{-16}	2.23×10^{-8} [66]	2.28×10^{-8} [66]	$\sim 10^{-14}$ [80]
$D^0 \longrightarrow \pi^0\nu_\alpha\overline{\nu}_\beta$ $(c\bar{u})\longrightarrow(u\bar{u})\overline{\upsilon}\upsilon$	$< 6 \times 10^{-16}$ [65]	4.9×10^{-16}	3.21×10^{-8} [66]	3.25×10^{-8} [80]	$\sim 10^{-14}$ [80]

Table 5.5.2: Comparison of d, s and b quark dependence of NSIs parameter

$\begin{pmatrix} u \\ d \end{pmatrix}$, $\begin{pmatrix} c \\ s \end{pmatrix}$ and $\begin{pmatrix} t \\ b \end{pmatrix}$ could affect NSIs of the rare decays of mesons. The constraints are summarized in table 5.5.3.

$\epsilon_{\tau\tau}^{QL} \sim O(10^{-2})$ where $Q = u, c, t$
$\epsilon_{\tau\tau}^{qL} \sim 1$ where $q = d, s, b$
$\epsilon_{ll'}^{qL}\langle 1$, for $l = l' \neq \tau$

Table 5.5.3: Constraints are summarized

Chapter 6

Elementary Theory of Supersymmetry

The standard model (SM) of electroweak interactions is thought to be a low-energy approximation of a more fundamental theory. In the SM, we have to take care of the conservation of lepton number, which although has been tested very precisely, but is not a requirment of an existing gauge theory. Therefore, many extensions to the SM have been studied and supersymmetry is one of them [81].

Supersymmetry (SUSY) provides an elegant way to link fermions with bosons. The most important thing attached to SUSY is that it can effectively tackle the problem of the divergence in the mass of Higgs. In SUSY theories, there is always a loop of super-partners accompanying the loop of normal SM particles. The supersymmetric relations between couplings and masses, along with the the extra minus that comes with any fermionic loop, guarantee the vanishing of the divergence. The SUSY is also a concrete worked example of the physics beyond the SM. One of the advantages of the extension of the SM by using SUSY is that we are hopeful of discovering the new spectrum of particles at the next energy scale, and the break down of the electroweak symmetry occurs in SUSY models at the level of perturbation of theory, without any demand for a new strong interactions. The SUSY naturally accomodates a complex and large spectrum of new particles. These particles can have some interesting properties which can test the proficiencies of the existing experiments. As SUSY has weak coupling, these signatures can be solved quite easliy. Due to large number of undetermined parameters in SUSY, it can show a great diversity of physical effects. Thus, SUSY enables us to foresee the pictures of experiments on physics beyond the SM, and we can make preparation for these experiments.

6.1 Nomenclature

SUSY transformation changes a fermionic state into a bosonic state, and vice versa. Q operator which is responsible for such transformations should be anticommuting and carring spin-$\frac{1}{2}$:

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle; \qquad Q|\text{Fermion}\rangle = |\text{Boson}\rangle. \tag{6.1.1}$$

Due to intrinsically complex nature of the spinors, both $Q^\dagger$ and Q (Hermitian conjugate) should be symmetry generators. Based on the fermionic nature of both Q and $Q^\dagger$ operators, it is clear that SUSY must have a space-time symmetry. The Haag-Lopuszanski-Sohnius extension of the Coleman-Mandula theorem[84] imposes restrictions on the specific forms for such symmetries in an interacting quantum field theory. For realistic theories which, like the SM, have chiral fermions, the parity-violating interactions, demand that the Q and $Q^\dagger$ must fulfill an algebra of commutation and anticommutation relations given as:

$$\{Q_A, Q^\dagger_{\dot{A}}\} = P^\mu \tag{6.1.2}$$

$$\{Q_A, Q_A\} = \{Q^\dagger_{\dot{A}}, Q^\dagger_{\dot{A}}\} = 0 \tag{6.1.3}$$

$$[P^\mu, Q_A] = [P^\mu, Q^\dagger_{\dot{A}}] = 0 \tag{6.1.4}$$

P^μ is the generator of space-time translations and μ is Lorentz index, while Q_A are Weyl spinors and A is representing spinor index. .

The single-particle states of the supersymmetry algebra are called *supermultiplets*.(For details see section 6.2). These super-multiplets contain both boson and fermion states, called *super-partners of* each others. If $|\Omega\rangle$ and $|\Omega'\rangle$ are taken in the same super-multiplet; then $|\Omega'\rangle$ should be proportional to some combination of Q and $Q^\dagger$ operators which are acting on $|\Omega\rangle$, up to a rotation or space-time translation. $-P^2$ commutes with the operators Q, $Q^\dagger$as well as with all space-time rotation and translation operators, so that particles lying in the same irreducible supermultiplet carrying equal eigenvalues of $-P^2$, equal masses of particles in a supermultiplets as a consequence. SUSY generators (Q, $Q^\dagger$) are also commuting with the generators of gauge transformations. Thus the particles in the same supermultiplet get the same value of electric charges, weak isospin and color.

To prove that each supermultiplet has same number of bosonic and fermionic degrees of freedom, let us consider the operator $(-1)^{2s}$(s is the spin). From the application of spin-statistics theorem, we obtain a value of -1 for a fermionic state and $+1$ for a bosonic state. As any fermionic operator will change a fermionic state into a bosonic state and vice versa, $(-1)^{2s}$ must anticommute with every fermionic operator, especially with Q and $Q^\dagger$. Now we take the states $|i\rangle$ in a supermultiplet with the same eigenvalue p^μ. According to eq. (6.1.4), any combination of SUSY operators acting on $|i\rangle$ will produce another state $|i'\rangle$ which has the same eigenvalue of four-momentum . Therefore one can use the completeness relation $\sum_i |i\rangle\langle i| = 1$ within subspace of states. Now we can take trace over all such states of the operator $(-1)^{2s}P^\mu$:

$$\begin{aligned}
\sum_i \langle i|(-1)^{2s}P^\mu|i\rangle &= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_i \langle i|(-1)^{2s}Q^\dagger Q|i\rangle \\
&= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_i \sum_j \langle i|(-1)^{2s}Q^\dagger|j\rangle\langle j|Q|i\rangle \\
&= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_j \langle j|Q(-1)^{2s}Q^\dagger|j\rangle \\
&= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle - \sum_j \langle j|(-1)^{2s}QQ^\dagger|j\rangle \\
&= 0. \qquad (6.1.5)
\end{aligned}$$

The 1st equality is obtained by using the supersymmetry algebra relation of eq. (6.1.2); the 2nd and 3rd from the use of the completeness relation; and the 4th from the anticommutation of $(-1)^{2s}$ with Q. As

$$\sum_i \langle i|(-1)^{2s}P^\mu|i\rangle = p^\mu Tr[(-1)^{2s}] \qquad (6.1.6)$$

is just proportional to the number of bosonic degrees of freedom n_B minus the number of fermionic degrees of freedom n_F in the trace. This implies that we have equal number of boson and fermion degrees of freedom, i.e.,

$$n_B = n_F \qquad (6.1.7)$$

for a given $p^\mu \neq 0$ for every supermultiplet.

6.2 Supermultiplets

A supermultiplet contains a single Weyl fermion (2 helicity states make up two fermionic degrees of freedom) as well as two real scalars, each with one bosonic degree of freedom. It is possible in SUSY algebra to combine the two real scalar degrees of freedom into a complex scalar field, Feynman rules, supersymmetry violating effects, etc. A combination of a complex scalar field and Weyl fermion is making as a *matter* or *scalar or chiral* supermultiplet.

The next possibility for a supermultiplet is to carry a spin-1 vector boson. For a renormalizable theory it is recongnized as a gauge boson without any mass, at least before the spontaneously broken of the gauge symmetry. A massless spin one boson has only two allowed helicity states (so $n_B = 2$). Super-partner of this boson must be a massless spin one half Weyl fermion carrying two helicity states, making an equal number of fermionic degrees of freedom. (The use of a massless spin three half fermion is giving a theory which is not renormalizable.) Both Gauge bosons and their fermionic partners should transform as the adjoint representation of the gauge group. This combination of spin-one gauge bosons and spin-half gauginos is known as a *gauge* or *vector* supermultiplet. Any other combination of particles, allowed by eq. (6.1.7), then such combinations if demonstrate some renormalizable interactions, are always reducible to chiral and gauge supermultiplets.

So, every fundamental particle should should therefore either be a chiral or gauge supermultiplet and have a super-partner with spin differing by half unit in the SUSY extension of the SM [85, 86, 87]. The names given to the spin-zero partners of the leptons and quarks are baptized by prepending an "*s*", hence making *sleptons* and *squarks* (short for "scalar lepton" and "scalar quark"). The symbols used for the squarks and sleptons are the same as for the corresponding fermion, but with a tilde for the super-partner. The super-partners of the left-handed and right-handed electron of the Dirac field are called left-haned and right-handed selectrons, $\widetilde{e}_L$ and $\widetilde{e}_R$. It is essential to note that the "handedness" is not of the selectrons as they are spin-zero particles, but of their super-partners. The same nomenclature is applicable for smuons and staus: $\widetilde{\mu}_L$, $\widetilde{\mu}_R$, $\widetilde{\tau}_L$, $\widetilde{\tau}_R$. In the SM, we only have the left handed neutrinos, so the sneutrinos are $\widetilde{\nu}$, with a possible subscript indicating the lepton flavour: $\widetilde{\nu}_e$, $\widetilde{\nu}_\mu$, $\widetilde{\nu}_\tau$. The list for squarks is $\widetilde{q}_L$, $\widetilde{q}_R$ with $q = u, d, s, c, b, t$. The gauge interactions of these sfermion fields remain the same as for the corresponding SM fermion.

One chiral super-multiplet is not considered good enough for Higgs scalar boson. If we have only one Higgs chiral super-multiplet, the electroweak gauge symmetry will have a triangle gauge anomaly, which leads to an inconsistent quantum theory. It happens due to the fact that for cancellation of gauge anomalies we must have

$$\mathrm{Tr}[Y^3] = \mathrm{Tr}[T_3^2 Y] = 0 \tag{6.2.1}$$

, where T_3 and Y are the third component of weak isospin and the weak hypercharge, respectively, and

$$Q_{\mathrm{EM}} = T_3 + Y \tag{6.2.2}$$

In the SM, these requirments are fulfilled automatically by the existing leptons and quarks. In SUSY, a fermionic partner for a Higgs chiral supermultiplet should be in a weak isodoublet with weak hypercharge $Y = -1/2$ or $Y = 1/2$. Such a fermion will always provide a non-zero contribution to the traces and hence it will destroy the anomaly cancellation. This situation can be eliminated if we have two Higgs super-multiplets, with $Y = \pm 1/2$. So, the anomaly traces from the two fermionic partners of the Higgs chiral super-multiplets will be zero. These can also be satisfied because of the structure of SUSY theories. Here Higgs with $Y = +1/2$ can only couple through Yukawa couplings to give masses to charge $+2/3$ up-type quarks (up, charm, top), and only a Higgs with $Y = -1/2$ can have the Yukawa couplings which are essential for masses of charge $-1/3$ down-type quarks (down, strange, bottom) and also for charged leptons. $SU(2)_L$-doublet complex scalar fields for these two cases (H_u and H_d) will serve the purpose. The doublet of H_u has weak isospin components $T_3 = (+1/2, -1/2)$, electric charges 1, 0 representing H_u^+, H_u^0. Similar thing happens with $SU(2)_L$-doublet complex scalar H_d which have $T_3 = (+1/2, -1/2)$ components and denoted by (H_d^0, H_d^-). The linear combination of H_u^0 and H_d^0 of the neutral component of the scalar represents the physical SM Higgs boson. The name of a spin-half super-partner is appended with "-ino" to the name of the SM particle, so

Names		Spin 0	Spin 1/2	$SU(3)_c$, $SU(2)_L$, $U(1)_Y$
squarks, quarks ($\times$ 3 families)	Q	$\left(\tilde{u}_L\, \tilde{d}_L\right)$	$(u_L\, d_L)$	$\left(3,\, 2,\, \frac{1}{6}\right)$
	$\bar{u}$	$\tilde{u}_{R^*}$	$u_R^\dagger$	$\left(\bar{3},\, 1, -\frac{2}{3}\right)$
	$\bar{d}$	$\tilde{d}_{R^*}$	$d_R^\dagger$	$\left(\bar{3},\, 1,\, \frac{1}{3}\right)$
sleptons, leptons ($\times$ 3 families)	L	$(\tilde{\nu}_L\, \tilde{e}_L)$	$(\nu_L\ e_L)$	$\left(1,\, 2, -\frac{1}{2}\right)$
	e	$\tilde{e}_{R^*}$	$e_R^\dagger$	$(1,\, 1, 1)$
Higgs, Higgsinos	H_u	$\left(H_u^+\, H_u^0\right)$	$\left(\tilde{H}_u^+\, \tilde{H}_u^0\right)$	$\left(1,\, 2, +\frac{1}{2}\right)$
	H_d	$\left(H_d^0\, H_d^-\right)$	$\left(\tilde{H}_d^0\, \tilde{H}_d^-\right)$	$\left(1,\, 2, -\frac{1}{2}\right)$

Table 6.2.1: Supersymmetric mutiplets

the super-partners (fermionic) of the Higgs scalars are known as higgsinos. They are denoted by $\widetilde{H}_u$, $\widetilde{H}_d$ for the $SU(2)_L$-doublet left-handed Weyl spinor fields, the weak isospin components are $\widetilde{H}_u^+$, $\widetilde{H}_u^0$ and $\widetilde{H}_d^0$, $\widetilde{H}_d^-$.

All of the chiral super-multiplets needed for a minimal extension of the SM are summarized in Table 5.1.1. These are also classified under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, which combines $\nu, e_L$and u_L, d_L into $SU(2)_L$ doublets. Here we are also following the standard convention and putting all chiral super-multiplets in terms of left-handed Weyl spinors,and their *conjugates* are the right-handed quarks and leptons as shown in Table 5.1.1. This convention turns out to be very useful for constructing supersymmetric Lagrangians. Here Q stands for the $SU(2)_L$-doublet chiral super-multiplet containing $\widetilde{u}_L, u_L$ (weak isospin component $T_3 = +1/2$), and $\widetilde{d}_L, d_L$ (with $T_3 = -1/2$), while $\overline{u}$ represents the $SU(2)_L$-singlet super-multiplet having $\widetilde{u}_R^*, u_R^\dagger$. There are three families for each of the quark and lepton in the super-multiplets, but only the first-family is used in Table 5.1.1. Here, a family index $i = 1, 2, 3$ is given which must be read as. $(\overline{e}_1, \overline{e}_2, \overline{e}_3) = (\overline{e}, \overline{\mu}, \overline{\tau})$. The bar on top ($\overline{u}$, $\overline{d}$, $\overline{e}$) of fields is part of the name, not a conjugate state.

It is worth noting that the Higgs chiral super-multiplet H_d (consist of H_d^0, H_d^-, $\widetilde{H}_d^0$, $\widetilde{H}_d^-$) has the same SM gauge quantum numbers as the $(\widetilde{\nu}, \widetilde{e}_L, \nu, e_L)_L$. One can assume that we could have been more economical by adopting a neutrino and a Higgs scalar to be the super-partners of each other. This would make the Higgs boson and a sneutrino the same particle. It would be welcomed as it served a role in making a connection between SUSY and phenomenology,[85]

Given Names	Spin 1/2	Spin 1	$SU(3)_c$, $SU(2)_L$, $U(1)_Y$
gluino, gluon	$\tilde{g}$	g	(8, 1, 0)
winos, W bosons	$\tilde{W}^{\pm}$, $\tilde{W}^0$	$W^{\pm}$, W^0	(1, 3, 0)
bino, B boson	$\tilde{B}^0$	B^0	(1, 1, 0)

Table 6.2.2: Supersymmetric patners of gauge bosons

but, unfortunately it is now known not to work. Even keeping the anomaly cancellation problem apart, many other phenomenological problems would arise, e.g., lepton number violation and a mass of the neutrinos have a violation to experimental bounds. Hence, all the super-partner should be taken as new particles.

The vector bosons of the SM reside in gauge super-multiplets along with their fermionic super-partners known as gauginos. The gluons are QCD mediators, whose spin-half SUSY partner is the gluino. The symbols for the gluon and gluino are g and $\widetilde{g}$ respectively. The electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ has associated bosons of spin-one, W^+, W^0, W^- and B^0, whose spin-half super-partners are $\widetilde{W}^+, \widetilde{W}^0, \widetilde{W}^-$ and $\widetilde{B}^0$, called *winos* and *binos.* After the break up of electroweak symmetry, the gauge eigenstates mix to give W^0, B^0 mixtures represent mass eigenstates of Z^0 and γ, having super-partner $\widetilde{W}^0$ and $\widetilde{B}^0$ which are called zino ($\widetilde{Z}^0$) and photino ($\widetilde{\gamma}$). Table 5.1.2 gives the gauge super-multiplets of a minimal supersymmetric extension of the SM.

6.3 Ingredients for Supersymmetric Lagrangian

Now after the introduction of the nomenclature of the supersymmetry and the supersymmetric algebra we are in a position to go forward and construct a supersymmetric Lagrangian. But before doing this, first of all we discuss the transformations under which this supersymmetric Lagrangian will be invariant and then, vector and chiral superfields.

(a) Superspace

The superspace formalism

Superspace gives a geometric picture of supersymmetry and provides representations of the supersymmetry algebra which are not restricted by any mass shell conditions. This is taken care of, just like, Lorentz invariance which inherently manifests in the 4-dimensional Minkowsky space. The superspace formalism, originally introduced by Salam and Strathdee [89], is also constructed such that supersymmetry is inherently manifest in the formalism. Since the supersymmetry algebra has anticommuting elements we should extend Minkowsky space-time with four-independent anticommuting, Grassmann number coordinates. These coordinates can be represented by a Majorana spinor or, using the two component Weyl formalism,

$$\{\theta_A\}_A = 1, 2, \qquad \mathit{and} \qquad \{\bar{\theta}_{\dot{B}}\}_{\dot{B}} = \dot{1}, \dot{2},$$

which satisfy the following anti-commutation relations:

$$\begin{aligned} \{\theta_A, \theta_B\} &= 0 \\ \{\bar{\theta}_{\dot{A}}, \bar{\theta}_{\dot{B}}\} &= 0 \\ \{\theta_A, \bar{\theta}_{\dot{B}}\} &= 0 \end{aligned} \tag{6.3.1}$$

The elements of superspace are the super-coordinates $(x^A, \theta^A, \bar{\theta}_{\dot{A}})$. Using the anticommuting Grassmann numbers, the graded Lie algebra of super symmetry is transformed into an ordinary Lie algebra by the following relations:

$$[\theta^A Q_A, \bar{\theta}_{\dot{B}} \bar{Q}^{\dot{B}}] = 2\theta^A \sigma^{\mu}_{A\dot{B}} \bar{\theta}^{\dot{B}} P_{\mu} \tag{6.3.2}$$

$$[\theta^A Q_A, \theta^B Q_B] = 0 = [\bar{\theta}_{\dot{A}} \bar{Q}^{\dot{A}}, \bar{\theta}_{\dot{B}} \bar{Q}^{\dot{B}}] \tag{6.3.3}$$

where θ^A and $\bar{\theta}_{\dot{A}}$ anti-commute also with spinor Q_A and $\bar{Q}_{\dot{A}}$.

Fierz and Spinor identities

After the super coordinates $(x^\mu, \theta^A, \bar{\theta}_{\dot{A}})$, we prove a number of spinor and Fierz identities involving variables and $\varphi, \bar{\varphi}, \chi, \bar{\chi}$ fields:

$$\begin{aligned} \chi\sigma^\mu\bar{\varphi} &\equiv \chi^A\sigma^\mu_{A\dot{A}}\overline{\varphi}^{\dot{A}} = -\bar{\varphi}^{\dot{A}}\sigma^\mu_{A\dot{A}}\chi^A & (6.3.4) \\ &= -\bar{\varphi}\overline{\sigma}^\mu\chi & (6.3.5) \end{aligned}$$

$$\begin{aligned} \varphi\sigma^{\mu\nu}\chi &= \frac{i}{4}\epsilon^{AB}\varphi_B(\sigma^\mu_{A\dot{A}}\bar{\sigma}^{\nu\dot{A}C} - \sigma^\nu_{A\dot{A}}\overline{\sigma}^{\mu\dot{A}C})\epsilon_{CD}\chi^D \\ &= -\frac{i}{4}\epsilon^{AB}\chi^D\sigma^\mu_{A\dot{E}}\epsilon^{\dot{E}\dot{F}}\epsilon_{\dot{F}\dot{A}}\overline{\sigma}^{\dot{A}C}\epsilon_{CD}\varphi_B + \frac{i}{4}(\mu \to \nu) \\ &= \frac{i}{4}\chi(\sigma^\mu\overline{\sigma}^\nu - \sigma^\mu\overline{\sigma}^\nu)\varphi \end{aligned}$$

which provide

$$\varphi\sigma^{\mu\nu}\chi = -\chi\sigma^{\mu\nu}\varphi \qquad (6.3.6)$$

Similarly

$$\bar{\varphi}\bar{\sigma}^{\mu\nu}\overline{\chi} = -\bar{\chi}\bar{\sigma}^{\mu\nu}\varphi$$

and one can also write

$$\begin{aligned} \theta^A\theta^B &= -\frac{1}{2}\epsilon^{AB}\theta\theta \;,\; \theta_A\theta_B = \frac{1}{2}\epsilon_{AB}\theta\theta & (6.3.7) \\ \overline{\theta}^{\dot{A}}\overline{\theta}^{\dot{B}} &= \frac{1}{2}\epsilon^{\dot{A}\dot{B}}\overline{\theta\theta} \;,\; \overline{\theta}_{\dot{A}}\overline{\theta}_{\dot{B}} = -\frac{1}{2}\epsilon_{\dot{A}}\epsilon_{\dot{B}}\overline{\theta\theta} & (6.3.8) \end{aligned}$$

Also, we have the relations

$$\begin{aligned} \theta\varphi\theta\chi &\equiv \theta^A\varphi_A\theta^B\chi_B \\ &= -\frac{1}{2}\theta\theta\chi\varphi \\ \bar{\theta}\bar{\varphi}\bar{\theta}\bar{\chi} &= -\frac{1}{2}\bar{\varphi}\bar{\chi}\bar{\theta}\bar{\theta} & (6.3.9) \end{aligned}$$

and

$$
\begin{aligned}
\varphi\psi\bar{\chi}\bar{\tau} &= \psi\varphi\bar{\chi}\bar{\tau} \\
&= -(\bar{\chi}\bar{\sigma}_{\mu}\varphi)(\psi\sigma^{\mu}\bar{\tau})
\end{aligned}
\tag{6.3.10}
$$

where upon using (3) we get

$$
\varphi\psi\bar{\chi}\bar{\tau} = \frac{1}{2}(\varphi\sigma_{\mu}\bar{\chi})(\psi\sigma^{\mu}\bar{\tau}) \tag{6.3.11}
$$

Similarly

$$
\bar{\varphi}\bar{\psi}\chi\tau = \frac{1}{2}(\bar{\varphi}\bar{\sigma}_{\mu}\chi)(\psi\bar{\sigma}^{\mu}\tau)
$$

Fierz identities are general infinitesimal global supersymmetric transformations of the component fields.

The General Superfields

The superfield is a function in superspace and the supersymmetry algebra transforms as a scalar under the following infinitesimal transformations

$$
\delta x^{\mu} = i\sigma^{\mu}\overline{\theta} - i\theta\sigma^{\mu}\overline{\epsilon} \tag{6.3.12}
$$

$$
\delta\theta = \epsilon, \delta\overline{\theta} = \overline{\epsilon} \tag{6.3.13}
$$

which are given as an infinitesimal two-component spinorial parameters $\epsilon^{A}, \overline{\epsilon}_{\dot{A}}$.

A general superfield Φ is an operator-valued function defined on superspace. It is a power series expansion in θ and $\overline{\theta}$. Since θ and $\overline{\theta}$ are anticommuting, this power series expansion is infinite

$$
\begin{aligned}
\Phi(z) &= \Phi(x,\theta,\bar{\theta}) = f(x) + \theta^{A}\varphi_{A}(x) + \bar{\theta}_{\dot{A}}\bar{\chi}^{\dot{A}}(x) + (\theta\theta)m(x) + (\bar{\theta}\bar{\theta})n(x) \\
&\quad + (\theta\sigma^{\mu}\overline{\theta})V_{\mu}(x) + (\theta\theta)\overline{\theta}_{\dot{A}}\lambda\overline{\lambda}^{\dot{A}}(x) + (\overline{\theta\theta})\theta^{A}\varphi_{A}(x) + (\theta\theta)(\overline{\theta\theta})d(x)
\end{aligned}
\tag{6.3.14}
$$

where, $(\theta\theta) = \theta^{A}\theta_{A}$ and $(\bar{\theta}_{\dot{A}}\bar{\theta}^{\dot{A}}) = \theta\theta$. Only the above mentioned combinations will survive, since

$i)$Any combination of more than two θ's must disappear:

$$(\theta\theta)\theta^1 = \theta^B\theta_B\theta^1 = (\theta^1\theta^2 - \theta^2\theta^1)\theta^1 = -(\theta^1\theta^2 - \theta^2\theta^1)\theta^1 = -(\theta\theta)\theta^1 \tag{6.3.15}$$

because

$$2(\theta\theta)\theta^1 = 0$$

$\Rightarrow$

$$(\theta\theta)\theta^1 = 0$$

similarly for θ^2 and $(\overline{\theta\theta})\overline{\theta}_{\dot{A}}$

$ii)$ Any higher Lorentz tensor term must vanish, since $\sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)$

$$\Phi\sigma^{\mu\nu}\chi = -(\chi\sigma^{\mu\nu}\varphi) \tag{6.3.16}$$

where

$$(\theta\sigma^{\mu\nu}\theta) = 0$$

$iii)$ $(\bar{\theta}\overline{\sigma}^\mu\theta)$ does not appear since

$$\theta\sigma^\mu\bar{\theta} = -(\bar{\theta}\overline{\sigma}^\mu\theta) \tag{6.3.17}$$

and finally, we have only the Lorentz scalar or pseudoscalar allowed by these conditions.

$f(x), \varphi(x), \bar{\chi}(x), m(x), n(x), V_\mu\ (x), \bar{\lambda}(x), \varphi(x)$ and $d(x)$ are known as component fields. Provided that $\Phi(x, \theta, \bar{\theta})$ is a Lorentz scalar or pseudoscalar of Lorentz group, the properties of the component fields are.

- $m(x), n(x), f(x)$ are complex scalar or pseudoscalar fields.
- $\varphi(x), \psi(x)$ are left handed Weyl spinor fields.
- $\bar{\lambda}, \bar{\chi}$, are right handed Weyl spinor fields.
- $V_\mu(x)$ is a Lorentz four-vector field.
- $d(x)$ is a scalar field.

Weyl equations need not be conserved under parity transformations $f(x)$; and $m(x)$ may be sums of pseudoscalar and scalar contributions as long as superfield gets a well-defined parity when we rewrite it in 4×4 Dirac formulation.

Supersymmetric Transformation

A finite SUSY transformation is written as

$$\exp\left[i\left(\theta Q + \overline{Q\theta} - x_\mu p^\mu\right)\right];$$

which can be compared with a non-abelian gauge transformation $\exp\left(i\phi_a T^a\right)$: T^a are the generators. The objects of these SUSY transformations should depend on θ and $\overline{\theta}$. So, we have an introduction of superfields, which are the functions of θ and $\overline{\theta}$ and the superspace coordinate x_μ . Since θ and $\overline{\theta}$ are definitely the two-component spinors, hence supersymmetry doubles the dimension of space-time. The new dimensions are fermionic.

The infinitesimal SUSY transformations can be written as

$$\delta_S\left(\xi,\overline{\xi}\right)\Phi\left(x,\theta,\overline{\theta}\right) = \left[\xi\frac{\partial}{\partial\theta} + \overline{\xi}\frac{\partial}{\partial\theta} - i\left(\xi\sigma_\mu\overline{\theta} - \theta\sigma_\mu\overline{\xi}\right)\frac{\partial}{\partial x_\mu}\right]\Phi\left(x,\theta,\overline{\theta}\right). \tag{6.3.18}$$

Here $\alpha, \overline{\alpha}$ are Grassmann variables and Φ is a superfield. This implies that SUSY generators can be written as:

$$Q_A = \frac{\partial}{\partial\theta^A} - i\sigma^\mu_{A\dot{B}}\overline{\theta}^{\dot{B}}\partial_\mu; \tag{6.3.19}$$

$$\overline{Q}_{\dot{A}} = -\frac{\partial}{\partial\overline{\theta}^{\dot{A}}} + i\theta^B\sigma^\mu_{B\dot{A}}\partial_\mu \tag{6.3.20}$$

The SUSY-covariant derivative will anti-commute with the transformation in the eq. 6.3.18:

$$D_A = \frac{\partial}{\partial\theta^A} + i\sigma^\mu_{A\dot{B}}\overline{\theta}^{\dot{B}}\partial_\mu; \tag{6.3.21}$$

$$\overline{D}_{\dot{A}} = -\frac{\partial}{\partial\overline{\theta}^{\dot{A}}} - i\theta^B\sigma^\mu_{B\dot{A}}\partial_\mu. \tag{6.3.22}$$

The eqs. 6.3.18-6.3.21 provide mass dimension $-\frac{1}{2}$, to A and θ, while Q and D have dimension $+\frac{!}{2}$.

Eqs 6.3.18-6.3.20 treat θ and $\overline{\theta}$ on equal footing . The chiral representations treat θ and $\overline{\theta}$, slightly differently. (The spinor indices are suppressed from now on):

$$L - represenation$$

$$\delta_S \Phi_L = \left(\xi \frac{\partial}{\partial \theta} + \overline{\xi} \frac{\partial}{\partial \overline{\theta}} = 2i\theta\sigma^\mu \overline{\xi}\partial_\mu\right) \Phi_L; \tag{6.3.23}$$

$$D_L = \frac{\partial}{\partial \theta} + 2i\sigma^\mu \overline{\theta}\partial_\mu; \tag{6.3.24}$$

$$\overline{D}_L = -\frac{\partial}{\partial \overline{\theta}} \tag{6.3.25}$$

and

$$R - represation$$

$$\delta_S \Phi_R = \left(\xi \frac{\partial}{\partial \theta} + \overline{\xi} \frac{\partial}{\partial \theta} - 2i\xi\sigma^\mu \overline{\theta}\partial_\mu\right) \Phi_R;$$

$$\overline{D}_L = -\frac{\partial}{\partial \overline{\theta}} - 2i\theta\sigma^\mu \partial_\mu; \tag{6.3.26}$$

$$D_R = \frac{\partial}{\partial \theta} \tag{6.3.27}$$

By using the following identity we can switch between the representations:

$$\Phi\left(x, \theta, \overline{\theta}\right) = \Phi_L\left(x_\mu, i\theta\sigma_\mu\overline{\theta}, \theta.\overline{\theta}\right) = \Phi_R\left(x_\mu - i\theta\sigma_\mu\overline{\theta}, \theta, \overline{\theta}\right). \tag{6.3.28}$$

Here, we need two types of superfields of SUSY algebra ; vector-superfields.and chiral-superfields

Chiral Superfields

The chiral superfields are derived by the fact that in the SM fermions are chiral particles. We, therefore, require such superfields which cannot only accommodate the two fermionic degrees of freedom, but then describe components (the left- or right-handed) of a SM fermion, along with bosonic partners and the S-fermions.

Such superfields can be constructed by imposing the condition

$$\overline{D}\Phi_L \equiv 0 \tag{6.3.29}$$

or

$$D\Phi_R \equiv 0. \tag{6.3.30}$$

These conditions are fulfilled by SUSY-covariant derivatives and chiral representations of SUSY generators. We can expand Φ_L as :

$$\Phi_L\left(x,\theta\right) = \phi\left(x\right) + \sqrt{2}\theta^{\alpha}\psi_{\alpha}\left(x\right) + \theta^{\alpha}\theta^{\beta}\epsilon_{\alpha\beta}F(x) \tag{6.3.31}$$

$\epsilon_{\alpha\beta}$ is a two dimensional anti-symmetric tensor. Given mass dimension +1 to scalar field ϕ are then provides the mass dimension $+\frac{3}{2}$ for ψ (fermionic field); and fix the unusual mass dimension +2 for scalar field F. The Φ itself carries mass dimension +1. Since the square of each of the components vanishes, so expansion of eq.6.3.31 is exact and θ only has two components. The F and ϕ are complex scalar fields, ψ is a Weyl spinor. It appears that Φ_L contains four bosonic degrees of freedom and only two of them are fermionic ones; however, it will be clear later on that not all of them happen to be bosonic fields (represent physical degrees of freedom). The expansion of Φ_R is very similar; just replace θ by $\overline{\theta}$.

Applying eq. 6.3.24, is allowing the SUSY transformation for the left-chiral superfield 6.3.31 results in;

$$\begin{aligned} \delta_S\Phi_L &= \sqrt{2}\xi^{\alpha}\psi_{\alpha} + \xi^{\alpha}\theta^{\alpha} \in_{\alpha\beta} F + 2i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\beta}}\overline{\alpha}^{\dot{\beta}}\partial_{\mu}\phi && (6.3.32)\\ &\quad +2\sqrt{2}i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\beta}}\overline{\xi}^{\dot{\beta}}\theta^{\beta}\partial_{\mu}\psi_{\beta}, && (6.3.33)\\ &= \delta_S\phi + \sqrt{2}\theta\delta_S\psi + \theta\theta\delta_S F. && (6.3.34) \end{aligned}$$

The first two terms in eq. 6.3.33 comes from the application of $\frac{\partial}{\partial\theta}$ part of δ_S, while the last two come from the ∂_{μ} part. The last term in eq.6.3.33 survives and there are only three factors of θ in it. There are no $\xi\overline{\theta}$ terms in eq 6.3.33, so an expansion, like in the eq 6.3.31, should be applicable to it. Now, the following terms remain:

(boson-fermion)

$$\delta_S\phi = \sqrt{2}\xi\psi \tag{6.3.35}$$

(fermion-boson)

$$\delta_S \psi = \sqrt{2}\xi F + i\sqrt{2}\sigma^\mu \overline{\xi}\partial_\mu \phi \tag{6.3.36}$$

(F is the total derivative)

$$\delta_S F = -i\sqrt{2}\partial_\mu \psi \sigma^\mu \overline{\xi} \tag{6.3.37}$$

Notice that the result 6.3.37 implies that

$$\int d^4 x F(x)$$

is invariant under $SUSY$ transformations.

Vector Superfields

The chiral superfields describe spin-0 bosons and spin $-\frac{1}{2}$ fermions, e.g. the quarks and leptons of SM and the Higgs bosons. However, we need the spin-1 gauge bosons of the SM, for that we should introduce vector self-conjugate fields V:

$$V(x,\theta,\overline{\theta}) \equiv V^\dagger(x,\theta,\overline{\theta}). \tag{6.3.38}$$

The representation of V in the component form:

$$\begin{aligned} V(x,\theta,\overline{\theta)} &= (1+\frac{1}{4}\theta\theta\overline{\theta\theta}\partial_\mu\partial^\mu)C(x) + (i\theta + \frac{1}{2}\theta\theta\sigma^\mu\overline{\theta}\partial_\mu)\chi(x) \\ &+ \frac{i}{2}\theta\theta\left[M(x)+iN(x)\right] + (-i\overline{\theta} + \frac{1}{2}\overline{\theta\theta}\sigma^\mu\theta\partial_\mu)\overline{\chi}(x) \\ &- \frac{i}{2}\overline{\theta\theta}\left[M(x)-iN(x)\right] + \theta\sigma_\mu\overline{\theta}A^\mu(x) + i\theta\theta\overline{\theta\lambda}(x) \\ &- i\overline{\theta\theta}\theta\lambda(x) - i\theta\theta\overline{\theta\theta}D(x), \end{aligned} \tag{6.3.39}$$

where C, M, N and D are the scalars and χ and λ are Weyl spinors. A^μ describes a gauge boson and V transforms as an adjoint representation of the gauge group. We have many more gauge degrees of freedom in the supersymmetric theories, since the gauge parameters are themselves representing the superfields. A non abelian supersymmetric gauge transformation of V is:

$$e^{gV} \longrightarrow e^{-ig\Lambda^{\dagger}} e^{gV} e^{ig\Lambda} \tag{6.3.40}$$

here, $\Lambda(x, \theta, \overline{\theta})$ is a chiral superfield and g is coupling. This transformation can be simply:

$$V \longrightarrow V + i(\Lambda - \Lambda^{\dagger})(\text{abelian case}). \tag{6.3.41}$$

The chiral superfield has four degrees of freedom (bosonic) and a Weyl spinor. One can use transformation eq.6.3.40 or eq.6.3.41 to choose

$$\chi(x) = C(x) = M(x) = N(x) \equiv 0. \tag{6.3.42}$$

This is so-called "Wess-Zumino" (W-Z) gauge. It is understood as the SUSY analogue of the unitary gauge, since it absorbs un-physical degrees of freedom. In Λ only three out of four bosonic degrees of freedom are used. So, we are still using the ordinary gauge freedom,

$$A_{\mu}(x) \longrightarrow A_{\mu}(x) + \partial_{\mu}(x) + \partial_{\mu}\phi(x). \tag{6.3.43}$$

We can say that the W-Z gauge can be combined with any of the usual gauges. The dimension of A^{μ} is $+1$ assigning the canonical mass dimension $+\frac{3}{2}$ for the field λ , and with the field D has the unusual mass dimension $+2$, just as the the case of F-component of the chiral superfield in the eq.6.3.31. The superfield V has no mass dimension.

Only the important result after applying SUSY transformation to equation 6.3.39,

$$\delta_S D = -\xi\sigma^{\mu}\partial_{\mu}\overline{\lambda} + \overline{\xi}\sigma^{\mu}\partial_{\mu}\lambda \tag{6.3.44}$$

is quoted. This gives the D component of a vector superfield that transforms into the total derivative. These results will be used in the next section.

(b) Allowed Terms in the Lagrangian

Now we are equipped to construct a supersymmetric Lagrangian for field theory. As usual, we want the action under supersymmetric transformations to remain invariant, i.e.

$$\delta_S \int d^4x \mathcal{L}(x) = 0. \tag{6.3.45}$$

This is only possible when $\mathcal{L}$ transforms into a total derivative. We have already mentioned that the highest components (those maximum number of θ and $\overline{\theta}$ factors) of vector and chiral superfields satisfy this requirement; so they can be used for the construction of the Lagrangian. The action S is written schematically as

$$S = \int d^4x (\int d^2\theta \mathcal{L}_F + \int d^2\theta d^2\overline{\theta} \mathcal{L}_D) \tag{6.3.46}$$

where the integration is defined (over Grassmann variable) as

$$\int d\theta_\alpha = 0, \int \theta_\alpha d\theta_\alpha = 1 \tag{6.3.47}$$

and no summation over α is taken.

where $\mathcal{L}_F$ and $\mathcal{L}_D$ in eq. 6.3.46 represents the chiral and vector superfields and give rise to F and D-terms respectively. Now, let us calculate the product of two left-chiral superfields;

$$\begin{aligned} \Phi_{1,L}\Phi_{2,L} &= (\phi_1 + \sqrt{2}\theta\psi_1 + \theta\theta F_1)(\phi_2 + \sqrt{2}\theta\psi_2 + \theta\theta F_2 \\ &= \phi_1\phi_2\sqrt{2}\theta(\psi_1\phi_2 + \phi_1\psi_2) + \theta\theta(\phi_1 F_2 + \phi_2 F_1 \\ &\quad -\psi_1\psi_2) \end{aligned} \tag{6.3.48}$$

as ($\theta\theta\theta = 0$). It can be taken as a candidate which is contributing to $\mathcal{L}_F$, the term in the action, eq.6.3.46. The last term in eq 6.3.48 seems to be a mass term for fermion. So we have identified a first term in the Lagrangian.

We can compute the highest component involved in the product of three such fields:

$$\int d^2\theta \Phi_{1,L}\Phi_{2,L}\Phi_{3,L} = \phi_1\phi_2F_3 + \phi_1F_2\phi_3 + \phi_2\phi_3F_1$$
$$-\psi_1\phi_2\psi_3 - \phi_1\psi_2\psi_3 - \psi_1\psi_2\phi_3 \quad (6.3.49)$$

Last three terms in eq. 6.3.49 represents the Yukawa interactions, and give masses to quarks and leptons. So we have identified first interaction term in the SUSY Lagrangian too. If $\phi_1$is the Higgs field , and ψ_2 and ψ_3 are the left-and right handed components of top quark respectively, eq 6.3.49 will not only produce the desired top-top-Higgs interaction, but will also produce interactions between a fermionic "higgsino" $\tilde{h}$ and the scalar top $\tilde{t}$ and top quark with equal strength . This is the first example of relationship between couplings introduced by SUSY. We have not yet found any kinetic energy terms involving in the derivatives . If we multiply more and more left-chiral superfields with each other then this will give rise to terms with mass dimension > 4 in the Lagrangian, making the interactions, non-renormalizeable. Since, we are forced to use the same representation of the SUSY generators everywhere, we have to write the chiral superfield in the Lrepresentation, using eq. 6.3.28;

$$[\Phi_L(x,\theta)]^\dagger = \phi^* - 2i\theta\sigma_\mu\overline{\theta}\partial^\mu\phi^* - 2(\theta\sigma_\mu\theta^*)(\theta\sigma_\nu\overline{\theta})\partial^\mu\partial^\nu\phi^*$$
$$+\sqrt{2}\overline{\theta\psi} - 2\sqrt{2}i(\theta\sigma_\mu\overline{\theta})\partial^\mu(\overline{\theta\psi}) + \overline{\theta\theta}F^* \quad (6.3.50)$$

No doubt, $\Phi_L\Phi_L^\dagger$ is a self conjugate vector superfield. So it contributes to the D-terms in the action.6.3.44 :

$$\int d^2\theta d^2\overline{\theta}\Phi_L\Phi_L^\dagger = FF^* - \phi\partial_\mu\partial^\mu\phi^* - i\overline{\psi}\sigma_\mu\partial^\mu\psi \quad (6.3.51)$$

The above equation gives kinetic energy terms for the scalar ϕ and fermionic component ψ,but no kinetic energy terms for F. It means F field is non-propagating, an auxiliary field which can be integrated out by using equations of motion. It has equal numbers of propagating fermionic and bosonic degrees of freedom.

Let us see how the F-fields can be removed from the Lagrangian by using superpotential f:

$$f(\phi_i) = \sum_i K_i \Phi_i + \frac{1}{2}\sum_{i,j} m_{ij}\Phi_i\Phi_j + \frac{1}{3}\sum_{i,j,k}\Phi_i\Phi_j\Phi_k \tag{6.3.52}$$

where ϕ_i are all left-chiral superfields, and g_{ijk}, m_{ij} and K_i are constants with mass dimension $0, 1, 2$ respectively. So far we have identified the following contribution in the Lagrangian:

$$\mathcal{L} = \sum_i \int d^2\theta d^2\overline{\theta}\Phi_i\Phi_i^* + \left[\int d^2\theta f(\Phi_i) + h.c\right] \tag{6.3.53}$$

$$\begin{aligned}\mathcal{L} &= \sum_i (F_i F_j^* + |\partial_\mu \phi|^2 - i\overline{\psi}\sigma_\mu\partial^\mu\psi_i) \\ &+ \left[\sum_j \frac{\partial f(\phi_i)}{\partial \phi_j}F_j - \frac{1}{2}\sum_{j,k}\frac{\partial^2 f(\phi_i)}{\partial\phi_j\partial\phi_k}\psi_j\psi_k + h.c]\right].\end{aligned} \tag{6.3.54}$$

In eq.6.3.54 f is a function of scalar fields ϕ_i, not of the superfield. The equations of motion for auxiliary fields F_j, are simply given by $\frac{\partial\mathcal{L}}{\partial\phi_j}$,

$$F_j = -\left[\frac{\partial f(\phi_i)}{\partial\phi_j}\right]^*. \tag{6.3.55}$$

The insertion of this value in equation 6.3.54 gives,

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{kin} - \left[\sum_{j,k}\frac{\partial^2 f(\phi_i)}{\partial\phi_j\partial\phi_k}\psi_j\psi_k + h.c\right] \\ &-\sum_j\left|\frac{\partial f(\phi_i)}{\partial\phi_j}\right|^2,\end{aligned} \tag{6.3.56}$$

where $\mathcal{L}_{kin}$ is the kinetic part given by the first line of equation 6.3.54. The second term in equation 6.3.56 is providing the masses for fermions and Yukawa interactions. The last term gives scalar mass and scalar interactions. There should be many relationships between the coupling constants, as both terms.are determined by the single function f

Now, let us introduce guage interaction. A SUSY version of the familiar “minimal coupling” is:

$$\int d^2\theta d^2\overline{\theta}\Phi\Phi^\dagger \longrightarrow \int d^2\theta d^2\overline{\theta}\Phi^\dagger e^{2gV}\Phi$$
$$= |D_\mu\phi|^2 - i\overline{\psi}\sigma_\mu D^\mu\psi + g\phi^* D\phi$$
$$+ig\sqrt{2}(\phi^*\lambda\psi - \overline{\lambda\psi}\phi) + |F|^* \quad (6.3.57)$$

The W-Z gauge 6.3.42 had been used in the second step, and we have introduced the usual gauge-covariant derivative

$$D_\mu = \partial_\mu + igA^a_\mu T_a \quad (6.3.58)$$

this Lagrangian not only describes the interactions, but also provides the gauge-strength.

Finally, with the help of superfield we can describe the kinetic energy terms of gauge fields as;

$$W_\alpha = (\overline{D}_{\dot{\alpha}}\overline{D}_{\dot{\beta}} \in^{\dot{\alpha}\dot{\beta}})e^{-gV}D_\alpha e^{gV} \quad (6.3.59)$$

Where $D, \overline{D}$ SUSY-covariant derivatives, carry spinor subscripts with themselves. For abelian symmetries, this will reduce to

$$W_\alpha = (\overline{D}_{\dot{\alpha}}\overline{D}_{\dot{\beta}} \in^{\dot{\alpha}\dot{\beta}})D_\alpha V \quad (6.3.60)$$

As $\overline{D}_{\dot{\alpha}}\overline{D}_{\dot{\beta}} \equiv 0, \overline{D}_{\dot{\alpha}}W_\alpha \equiv 0$, so W_α is a left-chiral superfield. Now we show that product $W_\alpha W^\alpha$ is also a gauge invariant term;

$$\frac{1}{32g^2}W_\alpha W^\alpha = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \frac{1}{2}D_a D^a$$
$$+\left(-\frac{i}{2}\lambda^a\sigma_\mu\partial^\mu\overline{\lambda}_a + \frac{1}{2}gf^{abc}\lambda_a\sigma_\mu A^\mu_b\overline{\lambda}_c\right) + h.c \quad (6.3.61)$$

This has a kinetic energy term for gauginos λ_a, along with canonical couplings of the gauginos to the gauge fields, which is contained in the structure constants f^{abc}.

As the equation 6.3.61 does not contain a kinetic energy term for D_a fields, therefore these are auxiliary fields. It is apparent from equations 6.3.61 and 6.3.57 that their equation of motion is

$$D_a = -g \sum_{i,j} \phi_i^* T_a^{ij} \phi_j \tag{6.3.62}$$

where we have used group indices explicitly. The field D in equation 6.3.61 is equal to $\sum_a D_a T^a$ which is in exact analogy with gauge fields. We can combine the third term in the second line in equation 6.3.57 and second term in equation 6.3.61 as follows:

$$-V_D = -\frac{1}{2} \sum_a \left| \sum_{i,j} g \phi_i^* T_{ij}^a \phi_j \right|^2 \tag{6.3.63}$$

The scalar interactions in the Lagrangian are explicitly fixed by the gauge couplings. This completes the terms in a Lagrangian for the renormalizable supersymmetric field theory.

(c) Supersymmetry Breaking

The supersymmetric Lagrangian satisfies the equation

$$m_{\tilde{f}} = m_f \tag{6.3.64}$$

which provides equal masses of the super-partners and their SM particles. As there is no selectron with mass 511 KeV, nor a smuon with mass 106 MeV etc, so condition given in 6.3.64 is not possible. The super-parteners are yet to be discovered and the searches at e^+e^- collider LEP gives us information that these must be heavier than 60 to 80 GeV. The Tevatron $p\overline{p}$ collider results also provide bounds on squark and gluino masses between 150 and 220 GeV [90, 91]. For these reasons, supersymmetry must be broken.

The great success of the SM with its broken $SU(2) \times U(1)_Y$ symmetry, we are well aware of the usefulness of broken symmetries, especially spontaneous symmetry breaking. Unfortunately it is not so easy to break SUSY, spontaneously. The definition of the SUSY algebra implies that

$$\frac{1}{4} \left(\overline{Q}_1 Q_1 + Q_1 \overline{Q}_1 + \overline{Q}_2 Q_2 + Q_2 \overline{Q}_2 \right) = P^0 \equiv H \geqslant 0 \tag{6.3.65}$$

H is the Hamiltonian (energy operator). Being the sum of perfect squares, it cannot be negative.

If the vacuum state $|0\rangle$ is also supersymmetric,then, we have

$$Q_\alpha |0\rangle = \overline{Q}_{\dot{\alpha}} |0\rangle = 0$$

and

$$E_{vac} \equiv \langle 0| H |0\rangle = 0 \tag{6.3.66}$$

If the vacuum state is not supersymmetric, then at least one generator does not annihilate the vacuum, so we have,

$$E_{vac} > 0 \tag{6.3.67}$$

The above equation 6.3.67 gives the condition that if the global supersymmetry is not broken spontaneously, then we have a definite positive vacuum energy. But, all this will result in a troublesome cosmological constant [92].

In most of the phenomenological analysis, we need not to understand the dynamical breaking of SUSY; however, it is sufficient to parametrize it by inserting some soft breaking term into the Lagrangian. Here the word "soft" means the cancellation of quadratic divergences. It can be shown that all the quadratic divergencies still cancel even if we insert (at least up to one-loop level)

i) scalar mass terms $-m^2_{\phi_i} |\phi_i|$ and

ii) trilinear scalar interactions $-A_{ijkj}\phi_i\phi_j\phi_k + h.c$

into the Lagrangian .Girardells and Grisaru [94] have calculated this in all orders in perturbation theory. They identified three additional types of soft breaking terms .

a) gaugino mass terms -$\frac{1}{2}m_l\overline{\lambda}_l\lambda_l$,where l again labels the group factors;

b) bilinear terms $-B_{ij}\phi_i\phi_j + h.c$; and

c) linear terms $-C_i\phi_i t$.

The linear terms are only gauge invariant for gauge singlet fields as shown in [95]. It is important to note that additional masses for Chiral fermions beyond those contained in the superpotential are forbidden. The relations between dimensionless couplings imposed by supersymmetry should not be broken as well.

6.4 Supersymmetric Lagrangian and Supersymmetric Potential

The Lorentz invariant and renormalizable Lagrangian of SUSY, in its simplest form can be written as [96, 97, 98]

$$\mathcal{L}_{int} = -\frac{1}{2}W^{ij}\psi_i\psi_j + c.c.,$$

here, W^{ij} is a function of some bosonic fields, and can also take the form

$$W^{ij} = \frac{\delta^2}{\delta\phi_i\delta\phi_j}W,$$

and

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k. \tag{6.4.1}$$

W is known as super-potential which is an analytical function of $\phi_{i's}$(the complex scalar fields) . M^{ij} is mass matrix for fermions and the Yukawa couplings are y^{ijk}. By using the chiral and the vector superfields, W can take the following form [97, 98]

$$\begin{aligned} W &= \varepsilon_{ab}[h^E_{ij}\widehat{H}^a_1\widehat{L}^b_i\widehat{E}^c_j + h^D_{ij}\widehat{H}^a_1\widehat{Q}^b_i\widehat{D}^c_j + h^U_{ij}\widehat{H}^a_2\widehat{Q}^b_i\widehat{U}^c_j - \mu\widehat{H}^a_1\widehat{H}^b_2] + \\ & \varepsilon_{ab}[\frac{1}{2}\lambda_{ijk}\widehat{L}^a_i\widehat{L}^b_j\widehat{E}^c_k + \lambda'_{ijk}\widehat{L}^a_i\widehat{Q}^b_j\widehat{D}^c_k] + \frac{1}{2}\lambda''_{ijk}\widehat{U}^c_i\widehat{D}^c_j\widehat{D}^c_k. \end{aligned} \tag{6.4.2}$$

ε_{ab} are antisymmetric and are used to raise and lower the spinors.indices

$$\varepsilon^{12} = \varepsilon_{21} = 1; \varepsilon_{12} = \varepsilon^{21} = -1; \varepsilon_{11} = \varepsilon_{22} = 0.$$

$\widehat{E}$, $\widehat{U}$ and $\widehat{D}$ denotes the right chiral superfields singlets and $\widehat{L}$, $\widehat{Q}$, $\widehat{H}_1$ and $\widehat{H}_2$ describe the left chiral superfields doublet . The term in the first bracket looks like SM Lagrangian. This describes the $SUSY$ extension of the SM. The term $\mu\widehat{H}^a_1\widehat{H}^b_2$ appears as Higgs mass but here it can be made to vanish after rotating the superfields $\widehat{H}^a_1$ and $\widehat{H}^b_2$ [98]. The second bracket introduces certain decays which are not allowed in the SM, like proton decay. Such decay processes involve baryon and lepton numbers violation. No such processes have yet been detected in the experiments. This means that additional symmetries are required to accommodate the conservation of these quantum numbers: R-parity is introduced, which serves the purpose [97, 99, 98].

6.5 Matter Parity or R-Parity

The second term in eq.6.4.2, is responsible for the rapid decay of proton, because it allows the following decay processes

$$p^+ \to \pi^0 e^+, \pi^0 \mu^+$$

$$\Gamma(p \to e^+\pi^0) \sim \frac{m^5_{proton} \sum_{i=2,3} \left|\lambda'_{11i}\lambda''_{11i}\right|^2}{m^4_{\tilde{d}_i}}, \tag{6.5.1}$$

if the Yukawa couplings ($\lambda''_{ijk}, \lambda'_{ijk}$) have a value greater than one. But, the experimental limits imposed on proton decay is 10^{34}.

To avoid this problem, a new symmetry is imposed (i.e., R-parity). This parity is not only taking care of lepton and baryon numbers but also the spin quantum number, in order to distinguish particles and their super-partners.It is defined as

$$R_P = (-1)^{3(B-L)+2S} \tag{6.5.2}$$

R-parity is $+1$ for SM particles and -1 for super-partners. At each vertex of SUSY Feynman diagram the product of R-parity should be equal to $+1$. It applications are:

- The lightest supersymmetric particle (LSP) remains stable, which can be identified as a candidate for the dark matter.
- Sparticles can only be produced in pairs.

R-parity is put by hand and can be relaxed by assuming that the $\lambda'_{ijk}\lambda''_{lmn}$ product can not survive, then this would be a case of R-parity violation.

6.6 R-parity violation

When R-paity is violated by keeping the Yukawa couplings very small, then we can have lepton and baryon number violating processes too. The potential for such interactions is given by, [97, 99, 98].

$$W_{\not{R}_p} = \varepsilon_{ab}[\frac{1}{2}\lambda_{ijk}\widehat{L}^a_i\widehat{L}^b_j\widehat{E}^c_k + \lambda'_{ijk}\widehat{L}^a_i\widehat{Q}^b_j\widehat{D}^c_k] + \frac{1}{2}\lambda''_{ijk}\widehat{U}^c_i\widehat{D}^c_j\widehat{D}^c_k. \tag{6.6.1}$$

Now we have sparticles as mediators in the processes which results in lepton number and flavour violation within MSSM [97]. This makes the flavor changing neutral currents, possible at tree level within MSSM [97, 98].

The Yukawa coupling (λ_{ijk}, λ'_{ijk} , λ''_{ijk}) are the parameters of MSSM. $\lambda_{ijk}$and λ''_{ijk} are antisymmetric in the first two and last two indices respectively.

$$\lambda_{ijk} = -\lambda_{jik};\ \lambda''_{ijk} = -\lambda''_{ikj}.$$

This makes 45 coupling parameters for R-parity violating SUSY model: 9 each from λ_{ijk} and λ'_{ijk}, and 27 from λ''_{ijk}. The operators $L_iL_jE^c_k$ and $L_iQ_jD^c_k$ contribute to leptonic and semi-leptonic decays of hadrons. Bounds on Yukawa coupling can be found by using the decays of D and Ds mesons. This analysis will enable us to specify the region for the new physics.

6.7 Decays of Mesons in R-Parity Violating Model

(a) $(D^+, D_s) \to l^+_\alpha \nu_\beta$ In $\not{R}_p$ SUSY

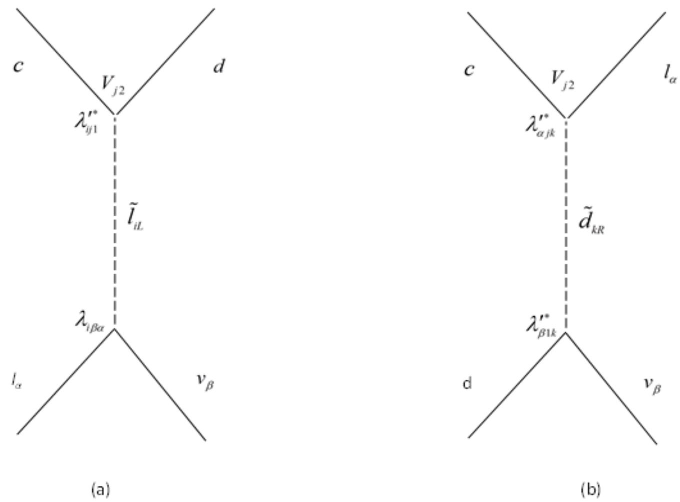
The effective Lagrangian for the decay of $(D^+, D_s) \to l^+_\alpha + \nu_\beta$ in the quark mass basis is given as:

$$L^{eff}_{\not{R}_P}\left(c \longrightarrow q + l^+_\alpha + \nu_\beta\right) = \frac{4G_F V_{cq}}{\sqrt{2}} \left[\begin{array}{c} A^{cq}_{\alpha\beta}\left(\bar{c}\gamma^\mu P_L q\right)\left(\bar{l}_\alpha \gamma_\mu P_L \nu_\beta\right) \\ -B^{cq}_{\alpha\beta}\left(\bar{c}P_R q\right)\left(\bar{l}_\alpha P_L \nu_\beta\right) \end{array} \right], \tag{6.7.1}$$

where $\alpha, \beta = e, \mu$ and $q = d, s$. The dimensionless coupling constants $A^{cq}_{\alpha\beta}$and $B^{cq}_{\alpha\beta}$ are given as

$$\begin{aligned} A^{cq}_{\alpha\beta} &= \frac{\sqrt{2}}{4G_F V_{cq}} \sum_{j,k=1}^{3} \frac{1}{2m^2_{\widetilde{d^c_k}}} V_{cj} \lambda'_{\beta qk} \lambda'^*_{\alpha jk} \\ B^{cq}_{\alpha\beta} &= \frac{\sqrt{2}}{4G_F V_{cq}} \sum_{i,j=1}^{3} \frac{1}{m^2_{\widetilde{l^c_i}}} V_{cj} \lambda_{\beta i\alpha} \lambda'^*_{ijq} \end{aligned} \tag{6.7.2}$$

Thus the decay rate of the flavor conserving process $D^+ \to l^+_\alpha \nu_\alpha$ is given by



$$D^0 \to \pi^- l_\alpha^+ \nu_\beta, D^+ \to l_\alpha^+ \nu_\beta, D^+ \to \pi^0 l_\alpha^+ \nu_\beta$$

Figure 6.7.1: Tree level diagram in R-parity violating model c decays to q, charge lepton and a neutrino

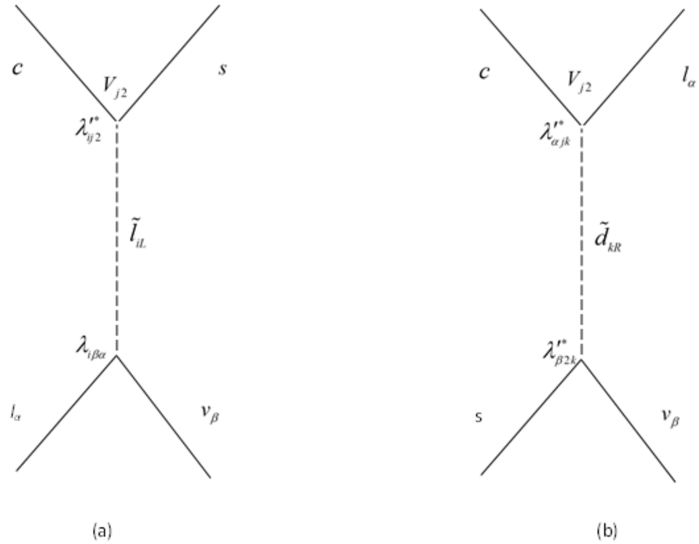
$$\Gamma\left(M^- \to l_\alpha \nu_\alpha\right) = \frac{1}{8\pi} G_F^2 \mid V_{cq} \mid^2 f_D^2 M_D^3 \left(1-\eta_\alpha^2\right)^2 \mid (1+A_{\alpha\alpha}^{cq})\eta_\alpha - \left(\frac{M_D}{m_c + m_{d,s}}\right) B_{\alpha\alpha}^{cq} \mid^2 \quad (6.7.3)$$

where $\eta_\alpha = \frac{m_\alpha}{M_D}$ is mass of charged lepton l, M_D is the mass of charm meson, and f_M is the pseudoscalar meson decay constant. Here, following PCAC (partial conservation of axial-vector current) relations have been used:

$$\begin{aligned} &< 0 \mid \bar{q}_c \gamma^\mu \gamma_5 q_q \mid M(p) >= i f_M p_M^\mu \\ &< 0 \mid \bar{q}_c \gamma_5 q_q \mid M(p) >= i f_M \frac{M_M^2}{m_{q_c} + m_{q_q}} \end{aligned} \quad (6.7.4)$$

(b) $D \to (\pi, K) l_\alpha^+ \nu_\beta$ decay in $\not{R}_p$ SUSY

The effective Lagrangian for the decay of $D \to (\pi, K) l_\alpha^+ + \nu_\beta$ in the quark mass basis is given as



$$D^0 \to K^- l_\alpha^+ \nu_\beta, D_s^+ \to l_\alpha^+ \nu_\beta, D^+ \to K^0 l_\alpha^+ \nu_\beta$$

Figure 6.7.2: Tree level diagram in R-parity violating model c decays to s, charge lepton and a neutrino

$$L^{eff}_{\not{R}_P}\left(c \longrightarrow q + l_\alpha^+ + \nu_\beta\right) = -\frac{4G_F V_{cq}}{\sqrt{2}} \left[\begin{array}{c} A^{cq}_{\alpha\beta}\left(\bar{c}\gamma^\mu P_L q\right)\left(\bar{l}_\alpha \gamma_\mu P_L \nu_\beta\right) \\ -B^{cq}_{\alpha\beta}\left(\bar{c} P_R q\right)\left(\bar{l}_\alpha P_L \nu_\beta\right) \end{array} \right], \tag{6.7.5}$$

where $\alpha, \beta = e, \mu$ and $q = d, s$. The dimensionless coupling constants $A^{cq}_{\alpha\beta}$and $B^{cq}_{\alpha\beta}$ are given as,

$$\begin{aligned} A^{cq}_{\alpha\beta} &= \frac{\sqrt{2}}{4G_F V_{cq}} \sum_{j,k=1}^{3} \frac{1}{2m^2_{\widetilde{d^c_k}}} V_{cj} \lambda'_{\beta qk} \lambda'^{*}_{\alpha jk} \\ B^{cq}_{\alpha\beta} &= \frac{\sqrt{2}}{4G_F V_{cq}} \sum_{i,j=1}^{3} \frac{1}{m^2_{\widetilde{l^c_i}}} V_{cj} \lambda_{i\beta\alpha} \lambda'^{*}_{ijq} \end{aligned} \tag{6.7.6}$$

Thus the decay rate of $D \to K l_\alpha^+ \nu_\beta$ induced by the quark level process $c \to q\ l_\alpha^+ v_\beta$ is given by[101]:

$$\Gamma\left[c \to q\ l_\alpha^+ v_\beta\right] = \frac{m_D^5}{192\pi^3} G_F^2 \mid V_{cq} \mid^2 (\mid A^{cq}_{\alpha\beta} \mid^2 + \mid B^{cq}_{\alpha\beta} \mid^2). \tag{6.7.7}$$

(c) $D^0 \to l_\alpha^\pm l_\beta^\mp$ In $R\!\!\!/_p$ SUSY

The effective Lagrangian for the decay of $D^0 \to l_\alpha^\pm l_\beta^\mp$ in the quark mass basis is given as

$$L^{eff}_{R\!\!\!/_p}\left(c \longrightarrow u + l_\alpha^\pm + l_\beta^\mp\right) = \frac{4G_F}{\sqrt{2}}\left[\ A^{cu}_{\alpha\beta}\left(\overline{l_\alpha}\gamma^\mu P_L l_\beta\right)\left(\overline{u}\gamma_\mu P_R c\right)\ \right], \tag{6.7.8}$$

where $\alpha, \beta = e, \mu$.The dimensionless coupling constants $A^{cu}_{\alpha\beta}$ is given by

$$A^{cu}_{\alpha\beta} = \frac{\sqrt{2}}{4G_F}\sum_{m,n,i=1}^{3}\frac{V^\dagger_{n2}V_{1m}}{2m^2_{\widetilde{d^c_i}}}\lambda'_{\beta ni}\lambda'^*_{\alpha mi} \tag{6.7.9}$$

The decay rate of the processes $M \to l_\alpha^\pm l_\beta^\mp$ is given by

$$\Gamma\left[M\,(cu) \to l_\alpha^\pm l_\beta^\mp\right] = \frac{1}{8\pi}G_F^2 f_M^2 M_M^3\sqrt{1+\left(\eta_\alpha^2+\eta_\beta^2\right)^2 - 2\left(\eta_\alpha^2+\eta_\beta^2\right)}\ \mid A^{cu}_{\alpha\beta}\mid^2\ [(\eta_\alpha^2+\eta_\beta^2)-(\eta_\alpha^2-\eta_\beta^2)^2] \tag{6.7.10}$$

where $\eta_\alpha \equiv \frac{m_\alpha}{M_M}$. m_α is mass of lepton, M_M is the mass of meson and f_M is the pseudoscalar meson decay constant which is extracted from the leptonic decay of each pseudoscalar meson.

(d) $D_s \to K l_\alpha^- l_\beta^+$ decay in $R\!\!\!/_p$ SUSY

In MSSM the relevant effective Lagrangian for the decay process $D_s \to K l_\alpha^- l_\beta^+$ is given by[100]

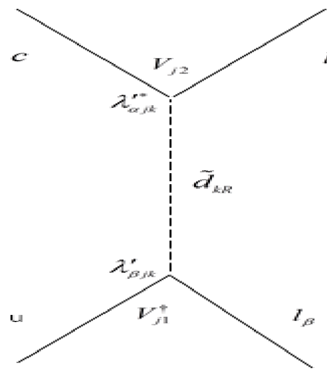
$$L^{eff}_{R\!\!\!/_p}\left(c \longrightarrow u + l_\alpha^- + l_\beta^+\right) = \frac{4G_F}{\sqrt{2}}\left[\ A^{cu}_{\alpha\beta}\left(\overline{l_\alpha}\gamma^\mu P_L l_\beta\right)\left(\overline{u}\gamma_\mu P_R c\right)\ \right]. \tag{6.7.11}$$

Where $\alpha, \beta = e, \mu$. The first term in eq.(2) comes from the up squark exchange (where c and u are up type quarks). The dimensionless coupling constant $A^{cu}_{\alpha\beta}$ is given by

$$A^{cu}_{\alpha\beta} = \frac{\sqrt{2}}{4G_F}\sum_{m,n,k=1}^{3}\frac{V^\dagger_{n2}V_{1m}}{2m^2_{\widetilde{d^c_k}}}\lambda'_{\beta nk}\lambda'^*_{\alpha mk}, \tag{6.7.12}$$

The inclusive decay rate of the process is given by[101]

$$\Gamma\left[c \to u\ l_\alpha^+ l_\beta^-\right] = \frac{m^5_{D^+}}{192\pi^3}G_F^2\mid A^{cu}_{\alpha\beta}\mid^2. \tag{6.7.13}$$



$D^o \rightarrow (e^+e^-, \mu^+\mu^-), D_s \rightarrow$
Ke^+e^-

Figure 6.7.3: c decays to u lepton lepton

6.8 Results and Discussions

We have plotted figs.6.8.1-6.8.8 by using data[78]. Tables 6.8.1,6.8.2 and 6.8.3 summarize the new bounds on the branching fraction of the given decay processes. In table 6.8.2, and 6.8.3, we have calculated branching fraction and Yukawa coupling bounds within 1σ error. These bounds on the Yukawa couplings can be compared with the bounds from [102, 103].

Fig.6.8.1, shows a comparison between different processes ($D^\pm \rightarrow \pi^\pm\mu^+\mu^-$, $D^0 \rightarrow \mu^+\mu^-$, $D_s \rightarrow K^\pm\mu^+\mu^-$) having common set of Yukawa coupling products($\lambda'^*_{231}\lambda'_{232}$). This comparison shows that $\not{R}_p$ MSSM contribution to $D^0 \rightarrow \mu^+\mu^-$ is 3 times smaller than the current experimental limits. This is significantly much better than the case of $D^0 \rightarrow e^+e^-$. This is because the branching fraction of the pure leptonic decay depends directly on the square of lepton to meson mass ratio. A comparison between $D^\pm \rightarrow \pi^\pm\mu^+\mu^-$ and $D_s^\pm \rightarrow K^\pm\mu^+\mu^-$ shows that $\not{R}_p$ MSSM contribution to these processes is comparable with the experimental limits: So this is a promising decay process to be explored at Fermilab and CLEO.

Fig.6.8.2, shows a comparison between different processes ($D^\pm \rightarrow \pi^\pm e^+e^-$, $D^0 \rightarrow e^+e^-$ and $D_s^\pm \rightarrow K^\pm e^+e^-$).having a common set of Yukawa coupling product ($\lambda'^*_{131}\lambda'_{132}$). This comparison shows that $\not{R}_p$ MSSM contribution to $D^0 \rightarrow e^+e^-$ is suppressed as compared to the current experimental limits. While a comparison between $D^\pm \rightarrow \pi^\pm e^+e^-$ and $D_s^\pm \rightarrow K^\pm e^+e^-$ shows that $\not{R}_p$ MSSM contribution to $D_s^{\pm} \to K^{\pm}e^{+}e^{-}$ and $D^{\pm} \to K^{\pm}e^{+}e^{-}$ is 10 times smaller than the current experimental limits.

Fig.6.8.3, shows a comparison between different processes ($D^{\pm} \to \pi^{\pm}e^{+}\mu^{-}$, $D^{0} \to e^{+}\mu^{-}$ and $D_s^{\pm} \to K^{\pm}e^{+}\mu^{-}$) having a common set of Yukawa coupling product ($\lambda'^{*}_{231}\lambda'_{132}$). This fig. shows a comparison between $D^{\pm} \to \pi^{\pm}e^{+}\mu^{-}$ and $D_s^{\pm} \to K^{\pm}e^{+}\mu^{-}$. This comparison shows that $\not{R}_p$ MSSM contribution to $D_s^{\pm} \to K^{\pm}e^{+}\mu^{-}$ is similar to the current experimental limits. Therefore, it is one forbidden process, promising enough to be explored at Fermilab and CLEO.

Fig.6.8.4, shows a comparison between ($D^{0} \to \pi^{-}e^{+}\nu_e$, $D^{+} \to e^{+}v_e$ and $D^{\pm} \to \pi^{0}e^{+}\nu_e$).having a common set of Yukawa coupling product ($\lambda'^{*}_{133}\lambda'_{113}$ and $\lambda'^{*}_{321}\lambda_{131}$). This comparison shows that $\not{R}_p$ MSSM contribution to $D^{0} \to \pi^{-}e^{+}v_e$ is solely by squark exchange Yukawa couplings ($\lambda'^{*}_{133}\lambda'_{113}$) while $\not{R}_p$ MSSM contribution to $D^{+} \to e^{+}v_e$ is mostly by sneutrino exchange Yukawa couplings ($\lambda'^{*}_{321}\lambda_{131}$). The contribution to Br($D^{+} \to e^{+}v_e$) from squark exchange Yukawa coupling products ($\lambda'^{*}_{133}\lambda'_{113}$) is comparable with SM contribution but negligible as compared to existing bounds. $D^{+} \to \pi^{0}e^{+}v_e$ receives negligible contribution from $\lambda'^{*}_{321}\lambda_{131}$ i.e. (10^{3}) times smaller than the experimentally measured branching fraction.

Fig.6.8.5, displays a comparison between processes ($D^{0} \to \pi^{-}\mu^{+}v_\mu$ and $D^{+} \to \mu^{+}v_\mu$) having a common set of Yukawa coupling product ($\lambda'^{*}_{233}\lambda'_{213}$ and $\lambda'^{*}_{321}\lambda_{232}$). This comparison shows that $\not{R}_p$ MSSM contribution to $D^{0} \to \pi^{-}\mu^{+}v_\mu$ is dominated by squark exchange. The contribution to Br($D^{+} \to \mu^{+}v_\mu$) from squark Yukawa couplings ($\lambda'^{*}_{233}\lambda'_{213}$) is comparable with SM, while slepton exchange Yukawa couplings ($\lambda'^{*}_{321}\lambda_{232}$) exchange Yukawa terms also contributes to $D^{+} \to \mu^{+}v_\mu$.

Fig.6.8.6, displays a comparison between $D^{0} \to K^{-}\mu^{+}v_\mu$ and $D_s^{+} \to \mu^{+}v_\mu$. This comparison shows that $\not{R}_p$ MSSM contribution to $D^{0} \to K^{-}\mu^{+}v_\mu$ and $D_s^{+} \to \mu^{+}v_\mu$ is consistent with available experimental data. The contribution to Br($D^{+} \to \mu^{+}v_\mu$) from squark Yukawa coupling products is comparable with SM.

Fig.6.8.7, displays a comparison between $D^{0} \to K^{-}e^{+}v_e$ and $D_s^{+} \to e^{+}v_e$ having a common set of couplings ($\lambda'^{*}_{123}\lambda'_{123}$). This comparison shows that $\not{R}_p$ MSSM contribution to $D^{0} \to K^{-}e^{+}v_e$ is solely by squark exchange Yukawa couplings ($\lambda'_{\beta qk}\lambda'^{*}_{\alpha jk}$), while $\not{R}_p$ MSSM contribution to $D_s^{+} \to e^{+}v_e$ is by slepton exchange Yukawa couplings ($\lambda_{\beta i\alpha}\lambda'^{*}_{ijq}$). Table 3 also shows that the contribution made by squark exchange Yukawa terms to the branching fraction of ($D_s^{+} \to e^{+}v_e$) is suppressed but is consistent with the prediction of the SM.

The comparison in table 6.8.1, shows that the branching fraction of some decay processes like $(D_s^{\pm} \to (\pi^{\pm}, K^{\pm}) l_{\alpha}^{\pm} l_{\beta}^{\mp}; \alpha, \beta = 1, 2)$ receives contribution from squark Yukawa coupling products $\lambda'_{\beta qk} \lambda'^{*}_{\alpha jk}$ that is comparable with experimental limits placed on the branching fraction of the these processes. Thus these processes can be explored for observing the effects of $\not{R}_p$Yukawa coupling products. $D^{\pm} \to \pi^{\pm} e^{+} e^{-}$, $D^0 \to \mu^{+} \mu^{-}$ and $D^0 \to e^{+} \mu^{-}$ receives slightly less contribution from $\not{R}_p$Yukawa coupling products i.e. ~ $(1 - 10)$. $D^0 \to e^{+} e^{-}$ is the most unfavorable process for the study of the effects of $\not{R}_p$Yukawa coupling products.

The comparison in table 6.8.2, shows that the branching fraction of some decay processes like $(D^{+} \to l^{+} \nu_l (l = e, \mu),\ D^0 \to K^{-} l_{\alpha}^{+} \nu_{\alpha} (\alpha = 1, 2), D_s^{+} \to \mu^{+} \nu_{\mu})$ receives contribution from squark Yukawa coupling products $\lambda'_{\beta qk} \lambda'^{*}_{\alpha jk}$ that is compareable to experimental limits placed on the branching fraction of these processes. Thus these processes can be explored for observing the effects of $\not{R}_p$Yukawa coupling products. $D^0 \to \pi^{-} e^{+} \nu_e$ and $D_s^{+} \to e^{+} \nu_e$ receive negligible contribution from $\not{R}_p$Yukawa coupling and are unfavorable processes for the study of the effects of $\not{R}_p$Yukawa coupling products $\lambda'_{\beta qk} \lambda'^{*}_{\alpha jk}$.

The comparison in table 6.8.3, shows that the branching fraction of some decay processes like $(D^{+} \to l_{\alpha}^{+} \nu_{\alpha}, D_s^{+} \to l_{\alpha}^{+} \nu_{\alpha})$ receives contribution from slepton Yukawa coupling products $\lambda_{\beta qk} \lambda'^{*}_{\alpha jk}$ that is comparable to experimental limits placed on the branching fraction of these processes. Thus these processes can be explored for observing the effects of $\not{R}_p$Yukawa coupling products. $(D^0 \to \pi^{-} l_{\alpha}^{+} \nu_{\alpha},\ D^0 \to K^{-} l_{\alpha}^{+} \nu_{\alpha}\ ; \alpha = 1, 2$ and $D^{+} \to \pi^0 e^{+} \nu_e)$ receives negligible contribution from $\not{R}_p$Yukawa coupling and are thus unfavorable processes for the study of the effects of $\not{R}_p$Yukawa coupling products $\lambda_{\beta qk} \lambda'^{*}_{\alpha jk}$.

In summary, we have analyzed decay processes $(D_s^{\pm} \to K^{\pm} l_{\alpha}^{+} l_{\beta}^{-} (v_{\alpha}),\ D^0 \to l_{\alpha}^{+} l_{\beta}^{-},\ D^{\pm} \to \pi^{\pm} l_{\alpha}^{+} l_{\beta}^{-} (v_{\alpha}))$ and compared their branching fractions against a common parameter $\lambda'_{\beta n1} \lambda'^{*}_{\alpha m2}$. The analysis distinguishes important processes to be studied at various accelerator facilities like Beijing Electron Positron Collider(BEPC), Fermilab and CLEO detector[104, 105, 106]. All the figures and tables in this chapter are used in [107]

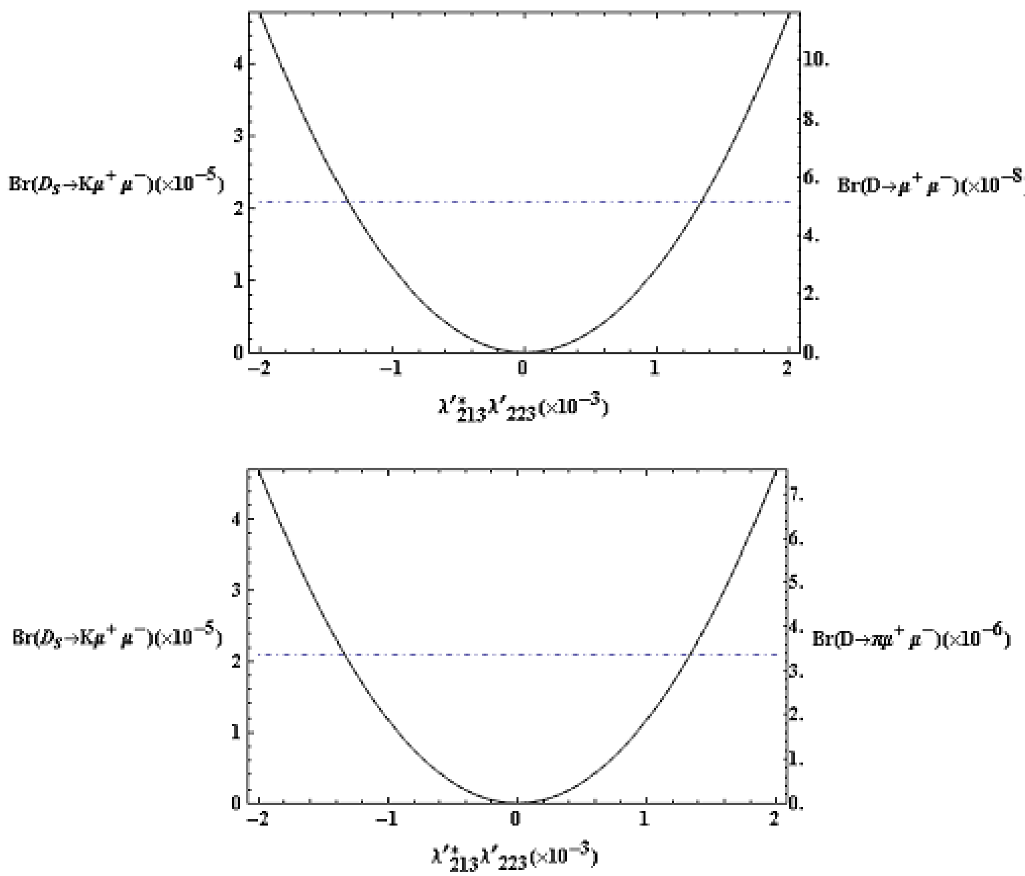
Process	Quark level Process	Branching Fraction	Branching Fraction	Branching Fraction
		(Experimental)	SM	($\not{R}_p$ contribution)
$D^0 \to e^+e^-$		$< 7.9 \times 10^{-8}$	1.52×10^{-24}	$\leq 2.1 \times 10^{-13}$
$D_s^\pm \to K^\pm e^+e^-$	$c \to u\ e^+e^-$	$< 3.7 \times 10^{-6}$	4.3×10^{-8}	$\leq 3.7 \times 10^{-6}$
$D^\pm \to \pi^\pm e^+e^-$		$< 1.1 \times 10^{-6}$	2×10^{-6}	$\leq 6 \times 10^{-7}$
$D^0 \to \mu^+\mu^-$		$< 1.4 \times 10^{-7}$	4.76×10^{-20}	$\leq 5 \times 10^{-8}$
$D_s^\pm \to K^\pm \mu^+\mu^-$	$c \to u\ \mu^+\mu^-$	$< 2.1 \times 10^{-5}$	4.3×10^{-8}	$\leq 2.1 \times 10^{-5}$
$D^\pm \to \pi^\pm \mu^+\mu^-$		$< 3.9 \times 10^{-6}$	1.9×10^{-6}	$\leq 3.4 \times 10^{-6}$
$D^0 \to e^+\mu^-$		$< 2.6 \times 10^{-7}$		$\leq 2.6 \times 10^{-8}$
$D_s^\pm \to K^\pm e^+\mu^-$	$c \to u\ e^+\mu^-$	$< 1.4 \times 10^{-5}$		$\leq 1.4 \times 10^{-5}$
$D^\pm \to \pi^\pm e^+\mu^-$		$< 2.9 \times 10^{-6}$		$\leq 2.4 \times 10^{-6}$

Table 6.8.1: A comparison of Ds, D+ and D zero leptonic and semi-leptonic decays

(a) Bounds on $\left|\lambda'^{*}_{113}\lambda'_{123}\right|$ ($< 5.61 \times 10^{-4}$) have been obtained from the experimental limits on Br($D_s^\pm \to K^\pm e^+e^-$

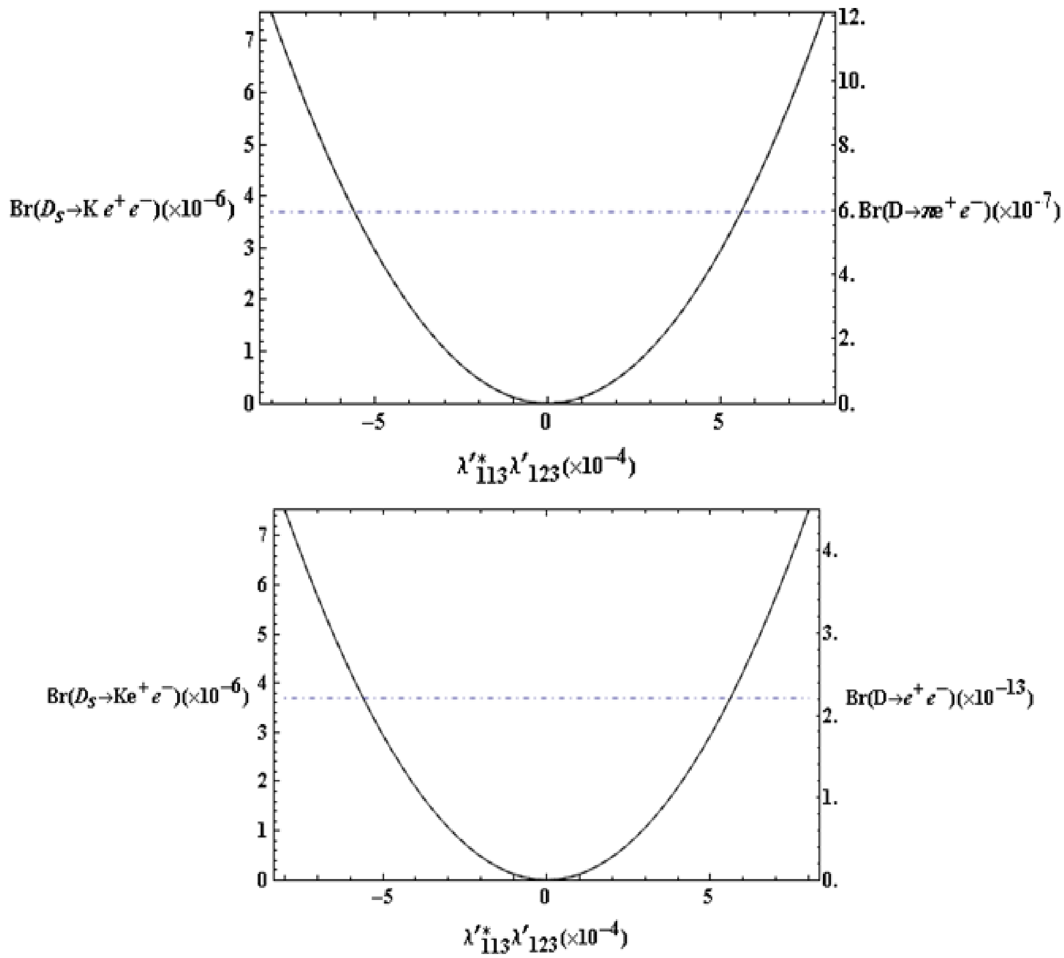
(b) Bounds on $\left|\lambda'^{*}_{213}\lambda'_{223}\right|$ ($< 1.34 \times 10^{-3}$) have been obtained from the experimental limits on Br($D_s^\pm \to K^\pm \mu^+\mu^-$)

(c) Bounds on $\left|\lambda'^{*}_{213}\lambda'_{123}\right|$ ($< 1.09 \times 10^{-3}$) have been obtained from the experimental limits on Br($D_s^\pm \to K^\pm e^+\mu^-$)



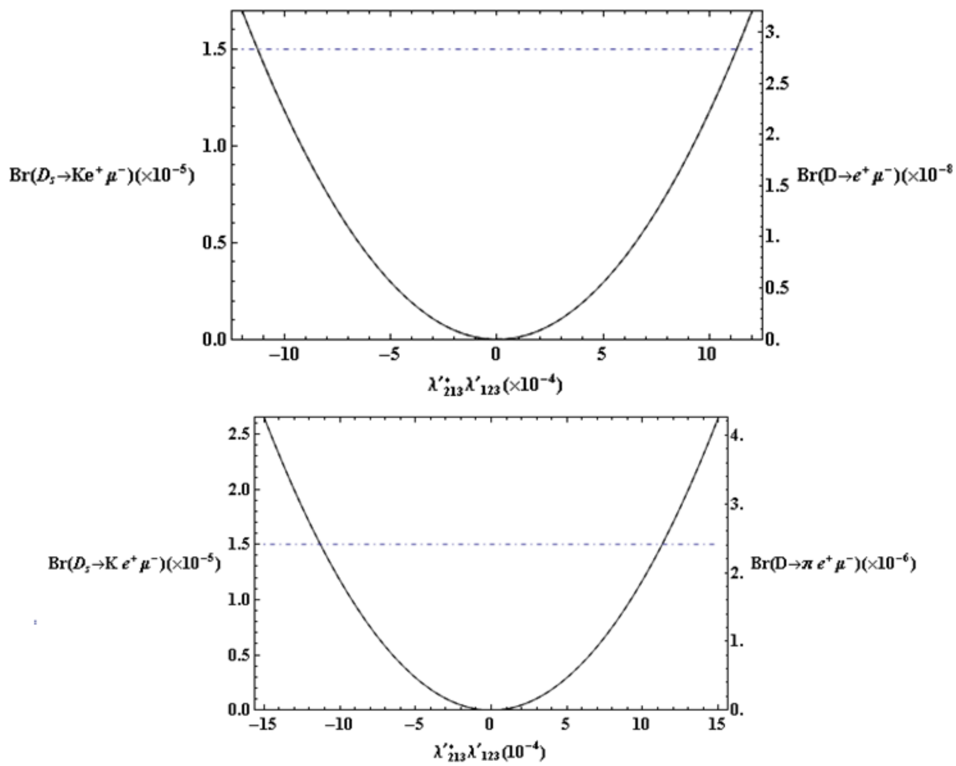
The dotted line shows the experimental bound on $D_s^{\pm} \to K^{\pm}\mu^+\mu^-.\lambda'_{232}\lambda'^*_{231}$ is expressed as $\frac{1}{(\widetilde{d}_L/100GeV^2)}$.

Figure 6.8.1: Graphs showing relation between branching fraction of leptonic and semileptonic decay of charm meson.



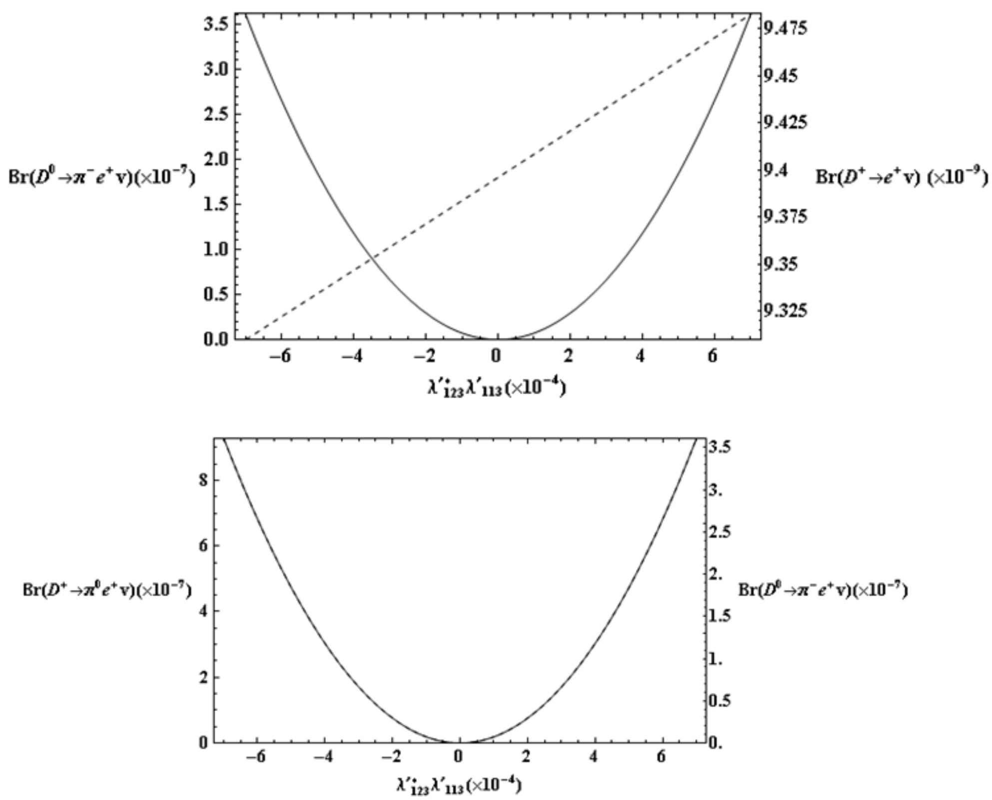
The dotted line shows the experimental bound on $D_s^{\pm} \rightarrow K^{\pm}e^{+}e^{-}$.$\lambda'_{123}\lambda'^{*}_{113}$ is expressed as $\frac{1}{(m_{\tilde{d}_L}/100GeV)^2}$.

Figure 6.8.2: Graphs showing relation between branching fraction of leptonic and semileptonic decay of charm meson.



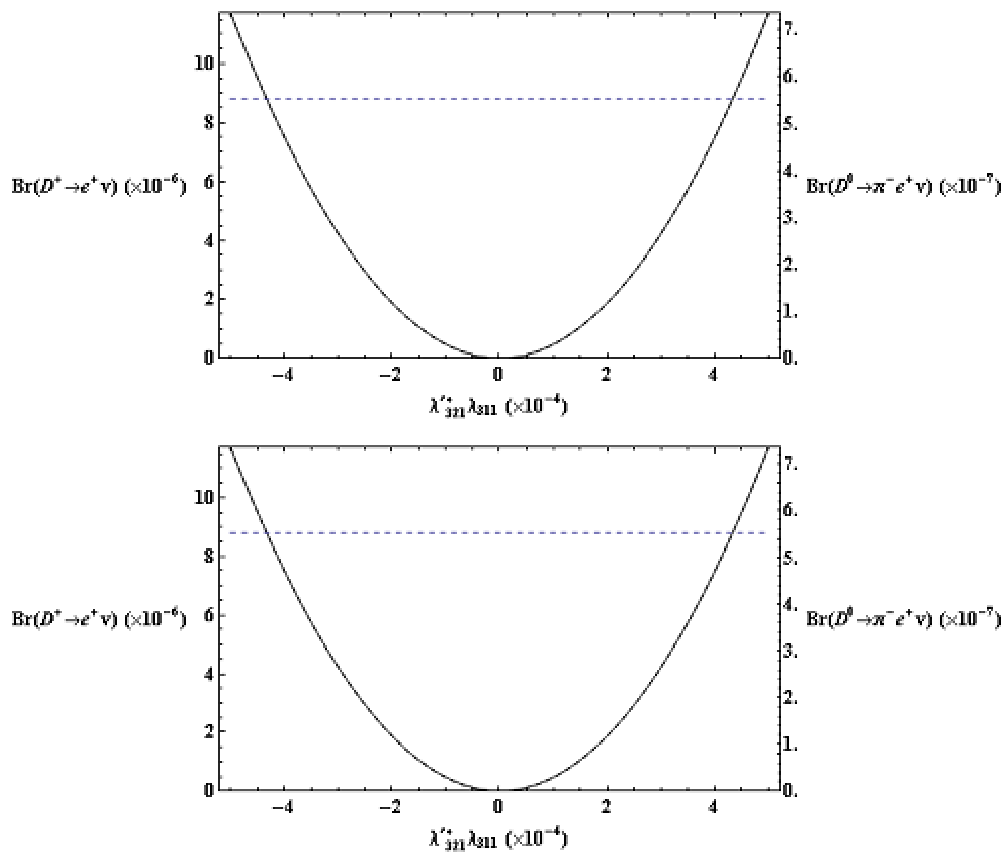
The dotted line shows the experimental bound on
$D_s^{\pm} \rightarrow K^{\pm}e^{+}\mu^{-}.\lambda'_{123}\lambda'^{*}_{213}$ is expressed as $\frac{1}{(m_{\tilde{d}_L}/100GeV)^2}$.

Figure 6.8.3: Graphs showing relation between branching fraction of leptonic and semileptonic decay of charm meson.



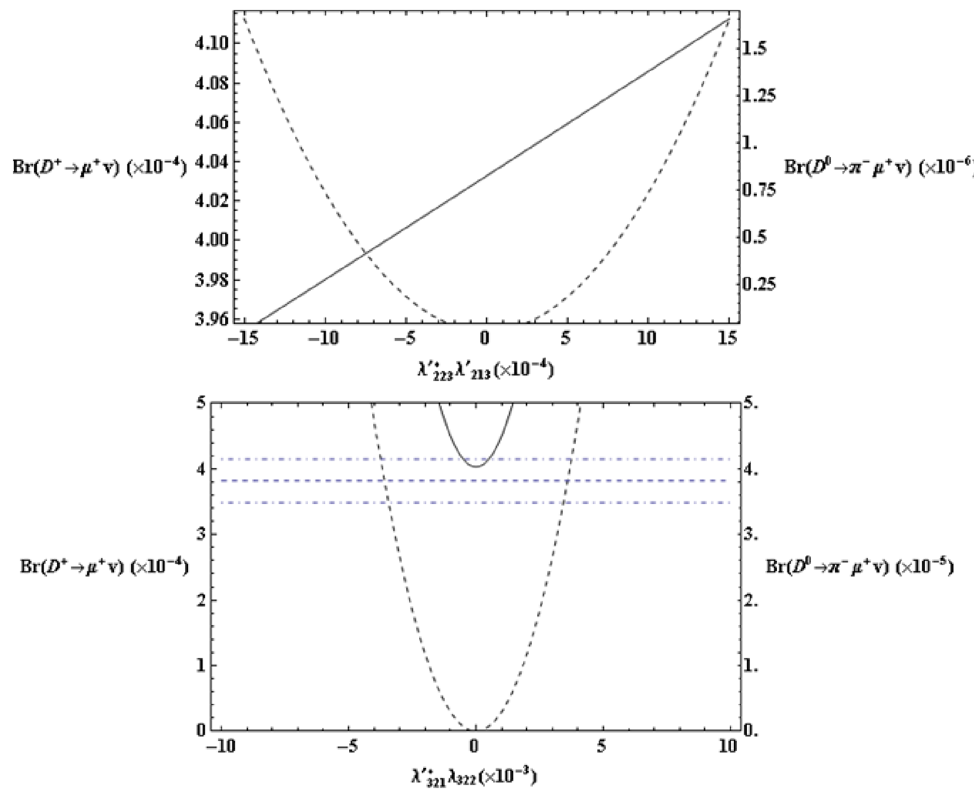
The dotted line shows the experimental bound on $D \to e^{+}\nu_{e}$. $\lambda^{'}_{113}\lambda^{'*}_{123}$ is expressed as $\frac{1}{(m_{\tilde{d}_L}/100GeV)^2}$.

Figure 6.8.4: Graphs showing relation between branching fraction of leptonic and semileptonic decay of charm meson.



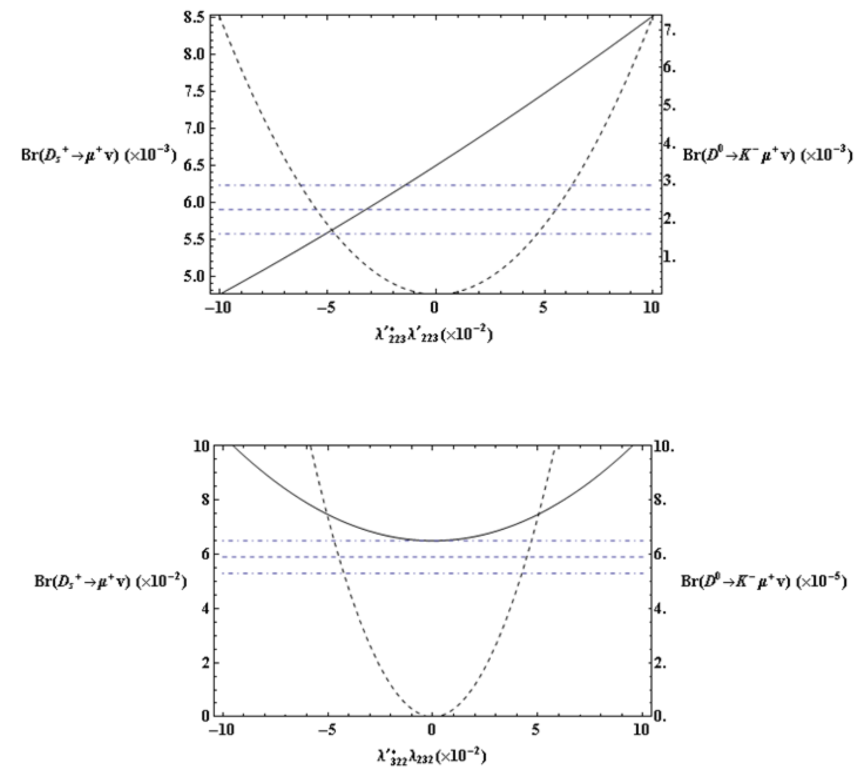
The dotted line shows the experimental bound on $D \to e^+\nu_e$. $\lambda'_{311}\lambda'^{*}_{321}$ is expressed as $\frac{1}{(m_{\tilde{l}_L}/100GeV)^2}$.

Figure 6.8.5: Graphs showing relation between branching fraction of leptonic and semileptonic decay of charm meson.



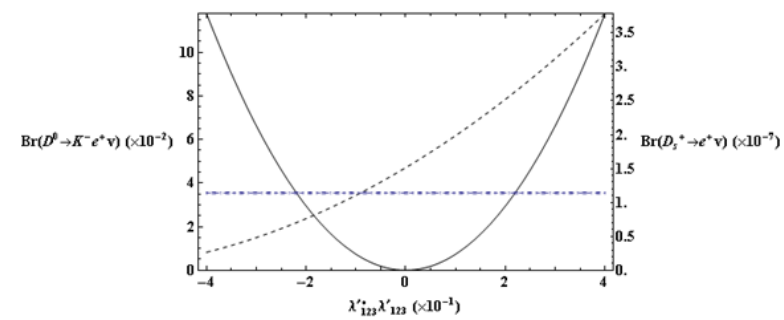
The dotted line shows the variation of $D^0 \to \pi^- \mu^+ \nu_\mu$. $\lambda_{322}\lambda^{'*}_{321}$ is expressed in the units of $\frac{1}{(m_{\tilde{l}_L}/100GeV)^2}$.$\lambda^{'}_{213}\lambda^{'*}_{223}$ is expressed in the units of $\frac{1}{(m_{\tilde{d}_L}/100GeV)^2}$

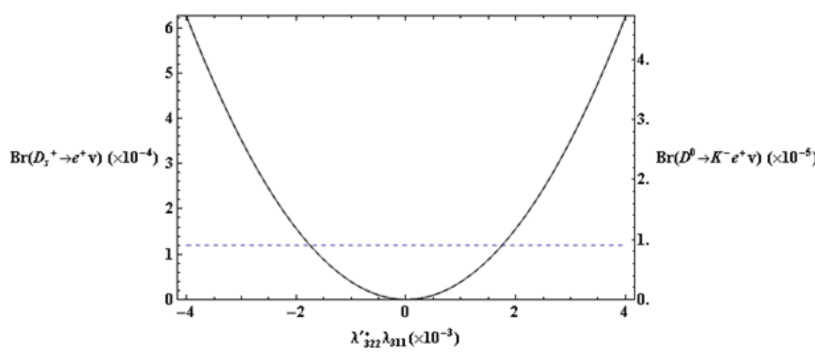
Figure 6.8.6: Graphs showing relation between branching fraction of leptonic and semi-leptonic decays of charm meson



The dotted line shows the variation of $D_s \rightarrow \mu^+ \nu_\mu . \lambda'_{223} \lambda'^{*}_{223}$ is expressed in the units of $\frac{1}{(m_{\tilde{d}_L}/100GeV)^2}$

Figure 6.8.7: Graphs showing relation between branching fraction of leptonic and semi-leptonic decays of charm meson





The dotted line shows the variation of $D^+ \to e^+\nu_e$. $\lambda_{311}\lambda_{322}^{'*}$ is expressed in the units of $\frac{1}{(m_{\tilde{l}_L}/100GeV)^2}$.$\lambda_{123}^{'}\lambda_{123}^{'*}$ is expressed in the units of $\frac{1}{(m_{\tilde{d}_L}/100GeV)^2}$

Figure 6.8.8: Graphs showing relation between branching fraction of leptonic and semi-leptonic decays of charm meson

Processes	Quark level Process	Branching Fraction	Branching Fraction	Branching Fraction
		(Experimental)	SM	($\not{R}_p$ contribution)
$D^0 \to \pi^- e^+ v_e$	$(c \to u\ e^+ v_e)$	$(2.89 \pm 0.08) \times 10^{-3}$		$< 2.0 \times 10^{-7}$
$D^+ \to e^+ v_e$		$< 8.8 \times 10^{-6}$	1.18×10^{-8}	$< 9.4 \times 10^{-9}$
$D^+ \to \pi^0 e^+ v_e$		$(4.05 \pm 0.18) \times 10^{-3}$		$< 5 \times 10^{-7}$
$D^0 \to \pi^- \mu^+ v_\mu$	$(c \to d\ \mu^+ v_\mu)$	$(2.37 \pm 0.24) \times 10^{-3}$		$< 1.5 \times 10^{-6}$
$D^+ \to \mu^+ v_\mu$		$(3.82 \pm 0.33) \times 10^{-4}$	5×10^{-4}	$< 3.96 \times 10^{-4}$
$D^0 \to K^- e^+ v_e$	$(c \to s\ e^+ v_e)$	$(3.55 \pm 0.04)\%$		$< 3.55\%$
$D_s^+ \to e^+ v_e$		$< 1.2 \times 10^{-4}$	1.5×10^{-7}	$< 3.8 \times 10^{-7}$
$D^0 \to K^- \mu^+ v_\mu$	$(c \to s\ \mu^+ v_\mu)$	$(3.30 \pm 0.13)\%$		$< 1.93 \times 10^{-3}$
$D_s^+ \to \mu^+ v_\mu$		$(5.90 \pm 0.33) \times 10^{-3}$	6.5×10^{-3}	$< 5.90 \times 10^{-3}$

Table 6.8.2: Comparison of Branching Ratios in SM and SUSY with experiments

$(D_s,\ D^0,\ D^\pm)$. Squark Yukawa couplings products are normalized as $1/(m_{\widetilde{d_3^c}}/100GeV)^2$. Bounds on $\lambda'_{123}\lambda'^{*}_{123}$ $(< 2.22 \times 10^{-1})$ have been calculated from $\mathrm{Br}(D^0 \to K^- e^+ v_e)$

Bounds on $\lambda'_{223}\lambda'^{*}_{223}$ $(< -1.45 \times 10^{-2})$ have been calculated from $\mathrm{Br}(D_s^+ \to \mu^+ v_\mu)$

Processes	*Subquark* Process	*Branching* Fraction	*Branching* Fraction	*Branching* Fraction
		(Experimental)	SM	($\not{R}_p$ contribution)
$D^0 \to \pi^- e^+ v_e$	$(c \to u\ e^+ v_e)$	$(2.89 \pm 0.08) \times 10^{-3}$		$< 1.51 \times 10^{-6}$
$D^+ \to e^+ v_e$		$< 8.8 \times 10^{-6}$	1.18×10^{-8}	$< 8.8 \times 10^{-6}$
$D^+ \to \pi^0 e^+ v_e$		$(4.05 \pm 0.18) \times 10^{-3}$		$< 1.42 \times 10^{-6}$
$D^0 \to \pi^- \mu^+ v_\mu$	$(c \to d\ \mu^+ v_\mu)$	$(2.37 \pm 0.24) \times 10^{-3}$		$< 7.41 \times 10^{-7}$
$D^+ \to \mu^+ v_\mu$		$(3.82 \pm 0.33) \times 10^{-4}$	5×10^{-4}	$< 3.82 \times 10^{-4}$
$D^0 \to K^- e^+ v_e$	$(c \to s\ e^+ v_e)$	$(3.55 \pm 0.04)\%$		$< 9.79 \times 10^{-6}$
$D_s^+ \to e^+ v_e$		$< 1.2 \times 10^{-4}$	1.5×10^{-7}	$< 1.2 \times 10^{-4}$
$D^0 \to K^- \mu^+ v_\mu$	$(c \to s\ \mu^+ v_\mu)$	$(3.30 \pm 0.13)\%$		$< 1.54 \times 10^{-4}$
$D_s^+ \to \mu^+ v_\mu$		$(5.90 \pm 0.33) \times 10^{-3}$	6.5×10^{-3}	$< 6.23 \times 10^{-3}$

Table 6.8.3: Normalized Yukawa couplings

$(D_s,\ D^0,\ D^\pm)$. Slepton Yukawa couplings products are normalized as $1/(m_{\tilde{l}_3^c}/100GeV)^2$.

Bounds on $|\lambda'^*_{321}\lambda_{311}|$ $(< 4.33 \times 10^{-4})$ have been calculated from $\mathrm{Br}(D^+ \to e^+ v_e)$

Bounds on $|\lambda'^*_{3j1}\lambda_{322}|$ $(< 5.0 \times 10^{-4})$ have been calculated from $\mathrm{Br}(D^+ \to \mu^+ v_\mu)$

Bounds on $|\lambda'^*_{332}\lambda_{311}|$ $(< 1.82 \times 10^{-3})$ have been calculated from $\mathrm{Br}(D_s^+ \to e^+ v_e)$

Processes	Quark Process	Yukawa Couplings	Bounds	Bounds
			(New)	(Old)
$D_s^{\pm} \to K^{\pm} e^+ e^-$	$(c \to u\ e^+ e^-)$	$\lambda'^{*}_{113}\lambda'_{123}$	$< 5.61 \times 10^{-4}$	$< 8 \times 10^{-4}$
$D^+ \to e^+ v_e$	$(c \to u\ e^+ v_e)$	$\lambda'^{*}_{321}\lambda_{311}$	$< 8.8 \times 10^{-6}$	$< 2.8 \times 10^{-3}$
$D_s^{\pm} \to K^{\pm} e^+ \mu^-$	$(c \to u\ e^+ \mu^-)$	$\lambda'^{*}_{213}\lambda'_{123}$	$< 1.09 \times 10^{-3}$	
$D^0 \to K^- e^+ v_e$	$(c \to s\ e^+ v_e)$	$\lambda'^{*}_{123}\lambda'_{123}$	$< 2.22 \times 10^{-1}$	
$D_s^+ \to e^+ v_e$	$(c \to s\ e^+ v_e)$	$\lambda'^{*}_{332}\lambda_{311}$	$< 1.82 \times 10^{-3}$	
$D_s^{\pm} \to K^{\pm} \mu^+ \mu^-$	$(c \to u\ \mu^+ \mu^-)$	$\lambda'^{*}_{213}\lambda'_{223}$	$< 1.34 \times 10^{-3}$	$< 4.0 \times 10^{-3}$
$D^+ \to \mu^+ v_\mu$	$(c \to u\ \mu^+ v_\mu)$	$\lambda'^{*}_{321}\lambda_{322}$	$< 5.0 \times 10^{-4}$	$< 1.01 \times 10^{-2}$
$D_s \to K e^+ \mu^-$	$(c \to u\ e^+ \mu^-)$	$\lambda'^{*}_{213}\lambda'_{123}$	$< 1.09 \times 10^{-3}$	$< 9.0 \times 10^{-3}$
$D_s^+ \to \mu^+ v_\mu$	$(c \to s\ \mu^+ v_\mu)$	$\lambda'^{*}_{223}\lambda'_{223}$	$< 1.45 \times 10^{-2}$	$< 1.0 \times 10^{-2}$

Table 6.8.4: Comparison of Yukawa couplings

Chapter 7

Summary and Discussion

Currents, connecting fermions of the same electric charge, but with different flavours, are called flavour changing neutral current *(FCNC)*. It is widely believed that *FCNC* processes are very rare in the standard model *(SM)* due to *GIM* suppression. *FCNC* performs dual fuction: on the one hand, it is an important and critical test of the radiative structure of the SM, and, on the other hand, it acts as a sensitive and effective probe of physics beyond the *SM*: *"new physics"*. This belief becomes more firm when we concentrate on those *FCNC* processes, which have two neutrinos in their final state. Neutrino is the only matter content, which is treated massless in *SM*, while the results from all existing neutrino experiments clearly indicate the fact that neutrinos are massive.This implies that the present *SM* of particle physics is not the whole story. Hence, it needs to be revised or extended in order to accommodate neutrino masses, mixing and other properties related to it. In general, there are many extensions of *SM* which not only include masses and mixing, but also generate new kind of interactions known as non-standard interaction *(NSI)*. *NSI* could establish the *SM* gauge principle at energies near electroweak breaking, including new nonstandard bosons, induced at eight dimensional operators. Regardless of the origin, quantifying the strength of a new interaction is very important, which may appear in the form of unknown couplings. this is usually referred to as non-standard non-universal couplings. These new interactions do not spoil the several *SM* predictions, but improve the theoretical calculations, which are consistent with the present experimental picture; and hence provide the additional information in terms of known physical phenomena, justifying more precise measurements. In this scenario, for a detailed illustration, we use the pure and semileptonic rare decays of pseudoscalar mesons with missing energy, i.e. ($M^0 \to \nu_\alpha \overline{\nu}_\beta, M'^{\pm,0} \to M'^{\pm,0}\nu_\alpha \overline{\nu}_\beta$ and $M_X^{\pm,0} \to M'^{\pm,0}\nu_\alpha \overline{\nu}_\beta$; notice that $M > M'$, where $M = B, D, K$ and $M' = \pi, K, D$, here subscript $X = S, C$). At the quark level, all $M_X^{\pm,0} \to M^{\pm,0}\nu_\alpha \overline{\nu}_\beta$ decays are represented by $q \to q'\ \nu_\alpha \overline{\nu}_\beta$ ($q = b, c, s, d$), and all these processes can be divided into two categories on the bases of lepton flavours, i.e.,

1. lepton *flavour conserving* ($\alpha = \beta$), and
2. lepton flavour violating ($\alpha \neq \beta$) decays.

The *first type* of decays $q \to q'\ \nu_\alpha \overline{\nu}_\alpha$ ($\alpha = e, \mu, \tau$) is absent in the SM at tree level, however it is induced by GIM mechanism at the quantum loop level, which makes their effective strength very small. Further suppression is caused by the weak mixing angles of the quark flavor rotation matrix, called Cabibbo-Kobayashi-Maskawa *(CKM)* matrix. These two types of suppressions make *FCNC* decays very rare. Furthermore, these processes provide indirect test of high energy scales through a low energy process. Such type of processes (if $q = b, c, s$) have only short distance dominant contribution, whereas long distance contribution is subleading. As we analyze pure and semileptonic decays, which can be accurately predicted in the *SM* due to the fact that the only relevant hadronic operators are just the current operators, whose matrix elements can be extracted from their respective leading decays.

The second type of decays $q \to q'\ \nu_\alpha \overline{\nu}_\beta$ (($\alpha \neq \beta$; $\alpha, \beta = e, \mu, \tau$) is strictly forbidden to all orders in *SM* due to lepton flavour violation, and so the only possible explanation for these type of processes are non standard/new interactions. Hence, one can say that these are the "golden channels" for the study of new physics.

In this thesis, we have analyzed the above mentioned decays in *SM* (for $\alpha = \beta$) and in none standard model (for $\alpha = \beta$ and $\alpha \neq \beta$), by using model independent and model dependent (R parity violating Supersymmetric Model) approaches. Our aim is to predict the branching fraction (in some cases) and limits on *NSI* parameters.

We started our venture by developing our understanding of the SM, its limitations and phenomenological implications. Our focus is to analyze the $K^+ \longrightarrow \pi^+ e^- e^+$, $D^+ \longrightarrow \pi^+ \mu^+ \mu^-$, $D^0 \longrightarrow K^- \mu^+ \upsilon_\mu$, $B^+ \longrightarrow \overline{D}^0 l^+ \upsilon_l$, $K^+ \longrightarrow \mu^+ \upsilon_\mu$ and $B^+ \longrightarrow \tau^+ \upsilon_\tau$ and other processes. As the outcome of this analysis, we have learned that most of the pure and semileptonic two and three body decays of pseudoscalar mesons that proceed through tree level Feynman diagram (flavour conserving charge and neutral current processes), can be explained very well within the framework of the SM*. Therefore, these processes put stringent constraint (because of high theory-experiment compatibility) on physics beyond the SM. Contrary to that, all the *FCNC* processes are suppressed when we have di-charge lepton of same flavour, and are highly suppressed when we have di-neutral lepton of the same flavour due to GIM cancellation and chiral suppression factor, within the framework of SM. This type of processes can be explained well beyond the SM.

In support of our argument, we have investigated the $D_s^+ \to K^+ \upsilon\overline{\upsilon}$, $D^0 \to \pi^0 \upsilon\overline{\upsilon}$ decays. These are long distance dominated processes and are model dependent. In this case, we have found that the contribution from NSIs is very large as compared to the SM, so the SM contribution can easily be ignored, as depicted in table 4.4.1. Whereas $D_s^+ \to D^+ \upsilon\overline{\upsilon}$ is short distance (SD) dominant process, here SM contribution cannot be ignored, but NSIs can improve SD dominated contribution (as it appears as an additive term, evident from table 4.5.1). Fortunately, this fact is depicted by the analysis of the only one experimentally measured process, i.e. $K^+ \to \pi^+ \upsilon\overline{\upsilon}$, as given in table 4.5.1 and fig 4.5.1. The information (value of NSI) obtained from this process can be used in $D_s^+ \to D^+ \upsilon\overline{\upsilon}$ to get the contribution of NSI in total branching ratios. Thus the branching ratios of $D_s^+ \to K^+ \upsilon\overline{\upsilon}$, $D^0 \to \pi^0 \upsilon\overline{\upsilon}$ and $D_s^+ \to D^+ \upsilon\overline{\upsilon}$ decays are 2.23×10^{-8}, $3.21 \times 10^{-8}$and 2.33×10^{-15} respectively, in the framework of NSIs. The values of non-standard parameters are: $\epsilon_{\tau\tau}^{uL}$ and $\epsilon_{\tau\tau}^{dL}$ are $O(10^{-2})$ and ~ 1 for $\alpha = \beta = \tau$, and $\epsilon_{\alpha\beta}^{dL} < 1$ for $\alpha = \beta = e$ or μ. Hence, we can conclude that, in the rare decays of charm mesons, the long distance dominated processes are dominated by NSIs, whereas there is a considerable enhancement in the *Br* of short distance processes due to NSIs (see tables 4.4.1 and 4.5.1). The details of this work are provided in chapter 4.

The afore-mentioned analyzes of NSIs is extended by incorporating second and third generations of quarks. The reason for doing so is to investigate why the only available non-standard parameter constraints in the literature are $\epsilon_{\alpha\beta}^{uL}$ and $\epsilon_{\alpha\beta}^{dL}$†; and why we are unable to find bounds on non-standard parameters, pertaining to second and third generation, i.e., $\epsilon_{\alpha\beta}^{bL}$, $\epsilon_{\alpha\beta}^{sL}$, $\epsilon_{\alpha\beta}^{cL}$ and

*Except those mension in Table [6.12.2]

†As the term NSI is coined in neutrino interaction, and it is assumed that neutrino interact with matter and normal matter contains only up and down qauak as a stable matter content. This picture is true for the study of scattering not for decays.

$\epsilon^{tL}_{\alpha\beta}$ ‡. Contrary to quark sector, in charged lepton sector, non-standard parameters $\epsilon^{eL}_{\alpha\beta}$, $\epsilon^{\mu L}_{\alpha\beta}$and $\epsilon^{\tau L}_{\alpha\beta}$ relevant to second and third generations are good constraints [56]

In order to say something concrete about the sensitivity of different generations/flavours, we have investigated two sets of processes; one, in which FCNC involve only up type quarks, i.e. $c \longrightarrow u\upsilon\overline{\upsilon}$ as an external particles and down type (d, s, b) quarks propagating in the loop. The example of such processes are $D^+_s \to K^+\upsilon\overline{\upsilon}$, $D^0 \to \pi^0\upsilon\overline{\upsilon}$ and $D^+ \to \pi^+\upsilon\overline{\upsilon}$ decays (see table 5.5.2), while the other FCNC involves down type quarks, i.e. $s \longrightarrow d\upsilon\overline{\upsilon}$ as an external lines and up type (u, c, t) quark propagating inside the loop. Such processes, $K^+ \to \pi^+\upsilon\overline{\upsilon}, D^+_s \to D^+\upsilon\overline{\upsilon}$ and $B^0_s \longrightarrow B^0\upsilon\overline{\upsilon}$, are presented in table 5.5.1. The comparative study of the aforesaid processes indicates that the NSI parameters are highly generation sensitive. In the first case, we observe that the dominant and comparable contribution of NSI is due to the d and s quark, while b is highly suppressed at radiative level. The same is true for the second class of processes, where dominant and comparable contribution of NSI is due to the u and c quark, while t is highly suppressed at radiative level. Hence, we can conclude that the contribution due to third generation of quarks is highly suppressed in NSIs, contrary to SM. Whereas, the contribution from the first and second generations are comparable in size and can not be ignored. This is discussed in details in Chapter 5. In this chapter, we perform only model independent analysis by exploiting the facts of massive neutrinos and non-universal coupling. But, as it is known, we have at least one model in which neutrinos can acquire Majorana type masses via the mixing with gauginos and higgsinos at weak scale§, known as the SUSY Model. The R-parity violating part of this model is unique in the sense that it provides the potential to study FCNC at tree level. In this model, the SM particles decay through sparticle (as a resonance state) into ordinary SM particles. So the R-parity violating (Yukawa) coupling can be detected by using the usual particle detector. Therefore, it is really important to know what kinds of couplings are severely constrained by the present experimental data to provide the evidence for existence of supersymmetric particles. Keeping this in mind, we have analyzed the whole spectrum of pseudoscalar charm meson $D_s, D^{\pm}$ decays

‡In principle in the framework of the SM FCNC decays occur at the loop level, where heavy quarks (c, t) exchanged contribution maximally. Therefore the same type of contribution is expected in NSI at the vertex where quarks are involved.

§This mechanism does not involve physics at large energy scale $(M_{int}10^{12}GeV)$ in contrast with see-saw mechanism, hence makes it accessible for experimental searches.

$\Big(D \to l_\alpha \nu_\alpha, D \to l_\alpha \bar{l}_\beta, D_s^\pm \to K^\pm l_\alpha^+ l_\beta^- (v_\alpha)$ and $D \to M l_\alpha \bar{l}_\beta;\ M = \pi, K$ and $\alpha, \beta = e, \mu\Big)$. The technique that we have adopted is to make a comparison between those processes, represented by the same Feynman diagram in the R-parity violating SUSY Model, and hence, having a common set of combination $(\lambda\lambda')$ and product $(\lambda'\lambda')$ Yukawa couplings. The comparison shows that the contribution of the combination and product couplings $(\lambda_{\beta i\alpha}\lambda^{*\prime}_{ijq},\ \lambda'_{\beta qk}\lambda^{*\prime}_{\alpha jq})$ to the branching fractions of the above processes is consistent with or comparable to the experimental measurements in most of the cases. However, there exist some cases, where these contributions are highly suppressed (e.g., $D^0 \to \pi^- e^+ v_e, D^+ \to \pi^0 e^+ v_e$ etc.). This is evident in tables 6.8.1-6.8.4 and figs.6.8.1-6.8.8. We identify such cases in our analyzes and single out the important ones, suitable for exploring in the current and future experiments (e.g., $D^+ \to \mu^+ v_\mu, D_s^+ \to \mu^+ v_\mu$ etc.). So we conclude that, it is important to improve the precision of the measurements, to see whether or not a signal for new physics can be found. If not, tighter limits on the coupling products can be achieved.

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