What is the Radius of a Nucleus?

The radius of a nucleus is an interesting quantity in itself, but it has recently gained further interest in particle physics. A nucleus has a higher density of nucleons than any other material; thus unstable particles can pass through much material before decaying, and it is possible to measure the cross section in nuclear matter; we then hope to interpret this as a cross section of the unstable particle on a free nucleon, a quantity we cannot otherwise measure. Production from different parts of a nucleus can be coherent. Whether or not coherence exists can be an indication of selection rules. The size and density of the nucleus enters into all of these calculations in a sensitive way.

The distribution of charge inside a nucleus is accurately measured either by electron scattering or by a study of μ-mesic x-rays. Both methods agree to about 1%. If we make the usual approximation that the nucleus consists of Z protons and $(A - Z)$ neutrons, with defined locations, we obtain the distribution of protons by unfolding the proton charge distribution. The distribution of the neutrons is harder to measure. Fortunately at high energies, where we wish to study these processes, further approximations can be made which make calculations possible.

If the wavelength of the incident particle is much smaller than the radius ($\lambdabar \ll R$) the particle can be considered as travelling along a straight line through the nucleus (the "eikonal" approximation); it is usual, though not necessary, to neglect internucleon correlations. Then it is possible to describe the motion by the high-energy optical model; this was first written down by Fernbach, Serber, and Taylor,[1] but since then the nature of the approximation has been made clearer by the work of many others—of whom Glauber[2] is the best known.

For example, the scattering amplitude, at small angles, for a neutron of momentum k on a nucleus, neglecting spin, becomes

$$f(\theta) = ik\int b\,db\{1 - \exp[-\tfrac{1}{2}\bar{\sigma}T(b)(1 - i\beta)]\}\, J_0(kb\theta). \qquad (1)$$

This formula may be visualized as follows: b is the impact parameter at which the neutron hits the nucleus,

$$T(b) = \int_{-\infty}^{+\infty} \rho(b, z)\, dz$$

is the integral over the nucleon density along the trajectory of the neutron, $\bar{\sigma}$ is the average nucleon–nucleon cross section,

$$A\bar{\sigma} = Z\sigma_p + (A - Z)\sigma_n ,$$

and β is the ratio of the real to imaginary part of the forward amplitude; $\exp[-\frac{1}{2}\bar{\sigma}T(b)]$ shows the attenuation of the wave passing through the nucleus; $i\beta\frac{1}{2}\bar{\sigma}T(b)$ is its phase shift. In the integrand we take the difference between the wave going through the nucleus and that without the nucleus present. Finally, a wave front at the rear of the nucleus emits wavelets (according to Huyghen's principle) which add up with differences in phase according to $J_0(kb\theta)$. This formula displays the forward diffraction peak and the diffraction minima.

A variant of this formula has been used to describe the forward ρ photoproduction amplitude:

$$f = 2\pi f_0 \int^{\infty} b\, db \int_{-\infty}^{+\infty} dz\, J_0(b\sqrt{|t_\perp|}) \exp(iz\sqrt{|t_\parallel|})\, \rho(z, b)$$

$$\times \exp\left[-\tfrac{1}{2}\bar{\sigma}(1 - i\beta) \int_z^{\infty} \rho(z', b)\, dz'\right]. \qquad (2)$$

Here the photoproduction of a ρ meson by a nucleon is assumed to proceed without spin flip with an amplitude f_0, at a position z in the nucleus. Subsequently it can be absorbed ($\bar{\sigma}$) or have its phase changed ($i\beta\bar{\sigma}$). An extra kinematic phase change is included. This case, $\exp(iz\sqrt{|t_\parallel|})$, where $t_\parallel$ is the longitudinal momentum transfer, is due to the fact that the photon and vector meson proceed with different momenta. Applications of these formulas always lead to radii appreciably larger than the proton radius.

These formulas preceded the first electron-scattering measurements, and this difference in derived radii has always been present. Johnson and Teller[3] at once suggested that the Coulomb barrier could make the radii of the protons smaller than the radii of the neutrons. On the other hand the protons repel each other and go to the surface. Clearly the theoretical situation is complex, but interesting.

The formulas written above have been used in a very detailed study of ρ photoproduction in nuclei.[4] The authors determine radii by studying the forward diffraction peak, and they call these "strong-interaction" radii without explaining the words, but with an implication of generality. One important correction should be, but has not been, made to these formulas before they are applied. An assumption is made in deriving the formula that the ρ–nucleon cross section has a zero range. This approximation may be relaxed by replacing $(1 - i\beta)\frac{1}{2}\bar{\sigma}T(b)$ by

$$\int \frac{2}{ik} \bar{M}(q)\, S(q)\, e^{i\mathbf{q}\cdot\mathbf{r}}\, d^2q, \tag{3}$$

where $\bar{M}(q)$ is the average particle–nucleon amplitude at a momentum transfer q and $S(q)$ is the nuclear form factor

$$S(q) = \rho(r)\, e^{i\mathbf{q}\cdot\mathbf{r}}\, d^3q.$$

These formulas are equal in the limit $\bar{M}(q) = \text{constant} = M(0)$.

In the limit of a transparent nucleus, the corrected formula leads to $f(\theta) = \bar{M}(q)\, S(q)$ which is clearly correct.

At small angles, we expand the terms in powers of q^2. The coefficients of q^2 are clearly the mean-square radii, and we find

$$\langle r^2 \rangle_{\text{eff}} = \langle r^2 \rangle_{\text{nucleons}} + \langle r^2 \rangle_{\rho N}. \tag{4}$$

The correction clearly depends on the process—and the spin. For nucleon–nucleon scattering at 150 MeV, this correction has been confirmed[5] and found to have three values, for real, imaginary, and spin-orbit parts of the nucleon nuclear amplitude. This correction is clearly an extension of the radius due to the range of nuclear forces. If nuclear forces have zero range, $M(q) = \text{constant}$.

For the ρ photoproduction case (which is similar to ρ nuclear scattering), $\langle r^2 \rangle_{\rho N}$ can be derived from a fit to ρ photoproduction from a nucleon. At 6 BeV, $\langle r^2 \rangle \approx 1.2\ \text{Fm}^2$. The correction to the radii becomes about 10% for light nuclei and 2% for heavy nuclei. The rms radius of the lead nucleus becomes 5.56 Fm, 0.2 Fm *larger* than the radius of the protons (5.34 Fm) in the lead nucleus (the μ-mesic x-ray radius corrected for the proton size).

A similar result is obtained from an examination of K_0^2 regeneration in nuclei[6] at a few GeV, or from an examination of proton interactions with nuclei at 20 GeV. No other correction of this magnitude has been suggested at these energies.

The suggestion that the radius of the neutron distribution is greater than that of the proton distribution has some support from a study of x-rays from the cascading of K mesons in a K^--mesic atom.[7] However, K-mesic x-rays measure the extreme tails of the neutron distribution and the proton distribution is itself badly known in the tails.

It is not clear whether the model, even with the correction here noted, is accurate enough so that the authors of Refs. 4 and 5 have *not* claimed definitively that the radii of neutrons and protons are different; however at energy high enough, it is generally believed that the model should be accurate.

There are some processes which selectively pick out the protons or neutrons in the nucleus. Thus at 150 MeV $\sigma_{pn} \approx 2\sigma_{nn}$ and a comparison of neutron and proton absorption cross sections can give results.[8] Unfortunately $\lambda\!\!\!^{-}$ is too large for the results to be believable. At 700 MeV $\sigma_{\pi-p} \approx 2\frac{1}{2}\sigma_{\pi+p}$ and $\sigma_{\pi-p} = \sigma_{\pi+n}$, so a comparison of π^+ and π^- cross sections will give a difference in radii. The experiment[9] yields $r_n - r_p = -0.2 \pm 0.3$ Fm, an effect which is not significant but in the opposite direction to the previous experiment.

Other experiments have been suggested[10]; $K^{\pm}$ charge exchange producing K_2^0 mesons and $\pi^{\pm}$ charge exchange producing π^0 mesons. The positive particles interact *only* with neutrons and the negative particles *only* with protons. This is a bigger factor than the factor of two in the previous experiments.

Until this problem is solved, the analysis of the experiments remains in doubt. We hope to gain interesting information therefrom, a value of $\bar{\sigma}$. This we should be able to interpret as ρ scattering off individual nucleons (with a 5% correction for correlations). However, if we put in the electron scattering radius, inconsistent values of $\bar{\sigma}$ appear.

This procedure for obtaining cross sections for unstable particles on nucleons has been extended to ϕN and ωN by photoproduction on complex nuclei and can be applied to many other unstable particles. The procedure is also important for extracting the phase of the *CP*-violating amplitudes from interference with K_2^0 regeneration. It is therefore a technological problem of great import to particle physicists.

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