Spectrum and Decay Constants of B and B_s Mesons

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Introduction

It has long been pointed out that light-heavy mesons act as the hydrogenic atoms of QCD and represent a unique laboratory to test our understanding of QCD[1]. Theoretically, our knowledge of hadron physics is mainly based on phenomenological quark confinement models. Though some of the successful models could reproduce the masses of the low-lying states, their predictions for the excited states were not satisfactory with respect to the experimental values. The pseudoscalar decay constants of the light-heavy mesons have also been estimated in the context of many QCD-motivated approximations. The predictions of each of these constants cover a wide range of values from one model to another. Phenomenologically, it is important to have reliable estimates of these decay constants as they are useful in many weak processes such as quark mixing, CP violation etc.

Theoretical Formulation

For the study of the light-heavy bound state system(B and B_s meson) we are treating both the quark and the antiquark relativistically. The Hamiltonian for the case can be written as[2]

$$H = \sqrt{\mathbf{p}^2 + m_Q^2} + \sqrt{\mathbf{p}^2 + m_{\bar{q}}^2} + V(\mathbf{r}) \quad (1)$$

where $\mathbf{p}$ is the relative momentum of the quark-antiquark and m_Q is the heavy quark mass and $m_{\bar{q}}$ is the light quark mass and $V(\mathbf{r})$ is the quark-antiquark potential [3],

$$V(r) = -\frac{\alpha_c}{r} + Ar^\nu \quad (2)$$

where A is the potential parameter and ν is a general power index. $\alpha_c = (4/3)\alpha_S$, α_S is the strong running coupling constant.

We assume the trial wave function for the (n, l) state to be the form given by the hydrogenic radial wave function,

$$R_{nl}(r) = \left(\frac{\mu^3(n-l-1)!}{2n(n+l)!}\right)^{1/2} (\mu r)^l \, e^{-\mu r/2} L_{n-l-1}^{2l+1}(\mu r) \quad (3)$$

Here, μ is the variational parameter and $L_{n-l-1}^{2l+1}(\mu r)$ is Laguerre polynomial. For a chosen value of ν, the variational parameter, μ is determined for each state using the virial theorem

$$\left\langle \frac{P^2}{2m} \right\rangle = \frac{1}{2} \left\langle \frac{rdV}{dr} \right\rangle \quad (4)$$

We find the expectation value of H(Eqn. (1)) using the equation

$$H\psi = E\psi \quad (5)$$

This gives the spin average masses of the system in terms of the power index ν. We fit the value of A for the ground state spin averaged mass for the light heavy mesons (B and B_s) to their experimental values using the expression

$$M_{SA} = M_P + \frac{3}{4}(M_v - M_p) \quad (6)$$

where M_v and M_p are the vector and pseudoscalar meson masses. With these values of μ and A in the wavefunction and potential respectiverly, we calculate the excited states of B and B_s mesons with different choices of ν. The calculated values are listed for a particular value of ν in table I.

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TABLE I: Pseudoscalar and vector masses of mesons (in GeV)

System	State	ν	M_P			M_V		
			Present	Expt.[4]	DI[5]	Present	Expt.[4]	DI[5]
	1S	0.9	5.329	5.325	5.324	5.269	5.279	5.279
$b\bar{u}$	2S	1.7	5.870		5.886	5.915		5.920
(B^*, B)	3S	1.5	6.324		6.320	6.350		6.347
	4S	1.5	6.820			6.839		
	1S	0.9	5.363	5.366	5.373	5.414	5.413	5.421
$b\bar{s}$	2S	2.0	5.969		5.985	6.017		6.019
(B_s^*, B_s)	3S	1.7	6.442		6.421	6.470		6.449
	4S	1.7	6.988			7.009		

TABLE II: Pseudoscalar and vector decay constants of mesons (in GeV)

System	State	ν	f_P			f_V		
			Present	[6]	[7]	Present	[8]	[9]
	1S	0.9	0.213	0.203±0.023	0.196	0.214	194 ± 0.018	0.190
$b\bar{u}$	2S	0.9	0.118			0.118		
(B^*, B)	3S	0.9	0.088			0.088		
	4S	0.9	0.072			0.073		
	1S	0.9	0.242	0.236 ± 0.030	0.216	0.243	0.272 ± 0.020	0.217
$b\bar{s}$	2S	0.9	0.135			0.136		
(B_s^*, B_s)	3S	0.9	0.102			0.106		
	4S	0.9	0.084			0.084		

Decay Constants

We compute the decay constants using the Van Royen-Weisskopff formula[10],

$$f_{P/V}^2 = \frac{12\left|\psi_{P/V}(0)\right|^2}{M_{P/V}} \qquad (7)$$

where, $\psi_{P/V}(0)$ is the wavefunction at the origin of the pseudoscalar or vector meson. Results are tabulated in table II.

Results

We have calculated the S wave masses and decay constants of B and B_s mesons. The ground state masses and decay constants are in good agreement with other theoretical models at $\nu = 0.9$ but excited states masses are coming between $\nu 1.5$ and $\nu = 2$.

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