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Testing relativistic gravity with binary and millisecond pulsars

J H Taylor

Joseph Henry Laboratories and Physics Department
Princeton University, Princeton, New Jersey 08544, USA

Abstract. Binary and millisecond pulsars offer unique opportunities for high precision experiments in relativistic gravity, probing well beyond the weak-field, slow-motion limit of all previous experimental tests. They also provide the means for accurate measurements of neutron star masses, placing rigorous constraints on the energy density of low-frequency gravitational radiation in the universe, and a number of other significant results. The first known binary pulsar, PSR B1913+16, has now been observed for more than 18 years. Its timing measurements have conclusively established the existence, quadrupolar nature, and propagation speed of gravitational waves; the results are presently in accord with general relativity at the 0.4% level. A more recently discovered binary pulsar, PSR B1534+12, has provided clean access to a test of gravity under strong-field conditions, independent of gravitational radiation effects. In this paper I summarize and update the status of experiments involving these two pulsars, and provide references to other related work.

1. Introduction

The discovery of pulsars 25 years ago introduced radio astronomers to a new type of natural clock, one that has proven remarkably useful for high-precision experiments in relativity. When a pulsar was found in a gravitationally bound orbiting system (Hulse & Taylor 1975), the way was opened for making explicit comparison of terrestrial atomic time with the time kept by a spinning macroscopic object located in a strong gravitational field and moving with mildly relativistic velocity. Measurements of this orbiting pulsar have been pursued intensively and have borne excellent fruit, as predicted. Meanwhile, a number of other pulsars have been found with characteristics and circumstances that make them interesting for applications in gravitational physics. Interesting results have now been obtained in areas as diverse as (1) determining neutron star masses, (2) placing limits on the cosmic gravitational wave background, (3) testing the constancy of the gravitational coupling parameter G, (4) establishing the existence and quadrupolar nature of gravitational waves, and (5) establishing constraints on possible departures of the "correct" theory of gravity from general relativity, in the strong-field regime. A number of papers have been published on these topics in the last few years. I shall summarize and update a few of the results here, and provide references to more complete work published elsewhere.

2. Overview of experimental details

The first binary pulsar to be discovered, named PSR B1913+16 according to its position in the B1950 system of celestial coordinates, turned out to be the harbinger of a new subclass of pulsars whose evolution is substantially modified by the proximity of an orbiting companion star. More than 40 of these "recycled" pulsars have now been found, about half of them since 1990. They are believed to have passed through an evolutionary phase in which mass and angular momentum were accreted from an evolving companion after it left the hydrogen-burning main sequence and began shedding its outer layers. The recycled pulsars have much shorter rotation periods and weaker magnetic fields than other pulsars. Moreover, they exhibit even more remarkable long-term timing stabilities — approaching and possibly surpassing those of the best atomic clocks.

Pulsar timing experiments are conceptually straightforward, even though one of the "clocks" being compared may be located several kiloparsecs from Earth. The observations are usually signal-to-noise limited, so they are best carried out with the largest available radio telescopes, such as the 305 m spherical reflector of the Arecibo Observatory in Puerto Rico. Interstellar propagation effects and galactic background noise compromise the measurements below about 0.3 GHz, while steep radio frequency spectra make most pulsars hard to observe at frequencies much above 3 GHz. Therefore, timing observations are usually carried out somewhere within this one-decade frequency interval.

In the pulsar timing measurements that my colleagues and I make regularly at Arecibo, radio-frequency signals induced in the feed antennas are amplified, converted to intermediate frequency, and passed through a multi-channel spectrometer. Digital signal averagers accumulate the pulsar's periodic intensity waveform in each frequency channel, using circuitry under computer control and accurately synchronized with the observatory's time and frequency standard. A programmable synthesizer, whose output frequency is adjusted once a second in a phase-continuous manner, compensates for changing Doppler shifts caused by known accelerations of the pulsar and the observatory. Average pulse profiles are recorded every few minutes together with appropriate time tags. Phase offsets of individual profiles are measured relative to a high signal-to-noise standard profile, converted to time delays, and added to the recorded start times of the integrations, thereby yielding topocentric times of arrival (TOAs) according to the observatory's master clock. TOAs obtained for different spectral channels are combined after correcting for dispersive delays caused by the ionized interstellar medium, and clock offsets measured by means of Earth-orbiting GPS satellites (Lewandowski & Thomas 1991) are applied to correct all TOAs to the best available standard of terrestrial atomic time.

An extensive theoretical framework for analyzing pulsar timing data has been developed over the years (e.g., Manchester & Taylor 1977, Damour & Deruelle 1986, Taylor & Weisberg 1989, Ryba & Taylor 1991, Damour & Taylor 1992). Each observatory-centered TOA, say $t_{\rm obs}$, is first transformed to the reference frame of the solar system barycenter, and then, for binary pulsars, to the pulsar co-moving frame (modulo an unknown constant velocity offset). In this frame the TOAs should be spaced so that pulse number $\mathcal{N}$ (an integer) is emitted at time $T_{\mathcal{N}}$, given implicitly by

$$\mathcal{N} = \nu T_{\mathcal{N}} + \frac{1}{2}\dot{\nu}T_{\mathcal{N}}^2 + \ldots , \tag{1}$$

where $\nu = 1/P$ is the neutron star's rotation frequency and time derivatives higher than the first are generally found to be negligible. The transformation from terrestrial time to pulsar proper time is carried out in the weak-field, slow-motion limit of general relativity; to sufficient accuracy, other viable theories of gravity would yield identical results. The necessary equations include terms related to the positions, velocities, and masses of objects within the solar system and to frequency-dependent propagation effects in the interstellar medium. For binary pulsars there are also terms representing the consequences of orbital motion. The orbital effects have been worked out and parametrized in a general phenomenological way by Damour and Deruelle (1985, 1986).

To an accuracy consistent with the experimental state of the art, all significant terms appearing in the time transformation can be summarized in the single equation

$$\begin{aligned} T \;=\;& t_{\mathrm{obs}} - t_0 + \Delta_{\mathrm{C}} - D/f^2 + \Delta_{\mathrm{R}\odot}(\alpha, \delta, \mu_\alpha, \mu_\delta, \pi) + \Delta_{\mathrm{E}\odot} - \Delta_{\mathrm{S}\odot}(\alpha, \delta) \\ & - \Delta_{\mathrm{R}}(x, e, P_b, T_0, \omega, \dot{\omega}, \dot{P}_b, \dot{x}, \dot{e}, \delta_\theta) - \Delta_{\mathrm{E}}(\gamma) - \Delta_{\mathrm{S}}(r, s) - \Delta_{\mathrm{A}} \,. \end{aligned} \tag{2}$$

Here t_0 is a nominal equivalent TOA at the solar system barycenter; Δ_{C} represents the measured offset between the observatory reference clock and the best terrestrial standard of time; D/f^2 is the dispersive delay for propagation at frequency f over the path from pulsar to Earth; $\Delta_{\mathrm{R}\odot}$, $\Delta_{\mathrm{E}\odot}$, and $\Delta_{\mathrm{S}\odot}$ are propagation delays and relativistic time adjustments within the solar system; and Δ_{R}, Δ_{E}, Δ_{S}, and Δ_{A} are similar terms for a binary pulsar's orbit. Subscripts on the various Δ's indicate the nature of the delays, which include "Roemer," "Einstein," and "Shapiro" effects within the solar system, and these as well as "Aberration" effects in the pulsar orbit. Note that the Roemer terms have approximate amplitudes given by the orbital periods times v/c, where v is a speed characteristic of orbital motion and c the speed of light. The Einstein terms are proportional to (v^2/c^2), multiplied by the orbital eccentricities. The Shapiro delay (Shapiro 1964) in the solar system has a maximum value of $\approx 120\,\mu\mathrm{s}$ when the line of sight grazes the limb of the Sun, and depends logarithmically on the impact parameter. The corresponding delay within the binary orbit depends on the companion star's mass, the orbital phase, and the inclination i between the plane of the orbit and the plane of the sky. Full details on all of the terms in Eq. (2) can be found in the references quoted earlier.

Eqs. (1) and (2) have been written to show explicitly the nature of the most significant dependences of pulsar TOAs on the set of potentially measurable parameters. In addition to the pulsar rotation frequency ν and its first time derivative $\dot{\nu}$, these parameters include the reference arrival time t_0, dispersion constant D, celestial coordinates α and δ, proper motion terms μ_α and μ_δ, and annual parallax π. For binary pulsars, as many as 13 orbital parameters are also measurable, at least in principle. These include five that appear even in a purely Keplerian analysis of orbital motion: the projected semi-major axis $x \equiv a_1 \sin i/c$, eccentricity e, binary period P_b, longitude of periastron ω, and time of periastron T_0. If the experimental timing precision is high enough, relativistic effects give access to as many as eight "post-Keplerian" (PK) measurables: the secular derivatives $\dot{\omega}$, $\dot{P}_b$, $\dot{x}$, and $\dot{e}$, the Einstein parameter γ, the "range" and "shape" of the orbital Shapiro delay, called r and s, and an orbital shape correction, δ_θ (see Damour & Taylor 1992, and references therein). Because seven quantities are required to fully specify the dynamics of a two-body orbiting system (up to theoretically uninteresting rotations about the line of sight), the measurement of any two PK parameters, in addition to the five readily measured Keplerian ones, provides a full description of the system, including predictions for the values of the remaining parameters. Thus, if a total of N post-Keplerian parameters are measurable one has immediate access to $N-2$ distinct tests of relativistic gravity.

In practice, parameter values are extracted from a set of TOAs by calculating the expected time of emission $T_{\mathcal{N}_i}$ for each observed pulse number $\mathcal{N}_i$ and then minimizing the weighted sum of squared residuals,

$$\chi^2 = \sum_i \left(\frac{T_i - T_{\mathcal{N}_i}}{\sigma_i}\right)^2 , \tag{3}$$

with respect to all phenomenological parameters to be determined. (Here T_i is the value of T corresponding to the i'th measured TOA, and σ_i is the measurement uncertainty. Note that with sufficiently frequent observations, the integers $\mathcal{N}_i$ are know exactly, so there is no ambiguity in determining $T_{\mathcal{N}_i}$.) In the analysis of a given set of data, some parameters will be more readily measurable than others. When TOAs are available for many observing dates distributed over a year or more, a pulsar's celestial coordinates, spin parameters, and Keplerian orbital elements are often measurable to accuracies of six or more significant digits. The PK parameters measure smaller effects, and are therefore more difficult to quantify. Extensive analyses of their measurabilities in practical circumstances have been carried out recently by Damour & Taylor (1992) and Taylor (1992).

As I have mentioned earlier, high-precision timing observations of pulsars have been put to many diverse uses. Even pulsars with no more than two measurable PK parameters can yield fundamentally important information. For example, the best available measurements of neutron star masses are derived from binary pulsar timing observations (Thorsett *et al* 1992, and references therein). Observations extending over many years have placed tight upper limits on the energy density of gravitational waves in the universe (Stinebring *et al* 1990) and on the constancy of the gravitational coupling parameter, G (Damour, Gibbons & Taylor 1988; Taylor 1992). Pulsar timing data are even proving useful as practical diagnostic tools, helping to establish time and frequency scales with the best possible long-term stabilities (Taylor 1991).

3. Binary pulsar PSR B1913+16

The first binary pulsar was found some 18 years ago, and its importance as a testbed for relativistic gravitation theories was recognized almost immediately (Hulse & Taylor 1975, Damour & Ruffini 1974, Brumberg *et al* 1975, Esposito & Harrison 1975, Wagoner 1975). In subsequent years, much effort has been put into making increasingly accurate measurements of its pulse arrival times and comparing the results with parametrized models such as the one summarized above. More than 4500 TOAs for PSR B1913+16 have now been recorded at the Arecibo Observatory (Taylor *et al* 1976; Taylor & Weisberg 1982, 1989, and unpublished work). These data determine the five Keplerian parameters and $\dot{\omega}$, the largest PK parameter, to a few parts per million or better (see Table 1). Two further PK parameters, γ and $\dot{P}_b$, have been determined with fractional accuracies better than 0.3%.

Table 1. Measured orbital parameters and derived masses for two binary pulsar systems. Figures in parentheses represent uncertainties in the last quoted digit; those in square brackets represent expected values of unmeasured quantities, according to general relativity.

	PSR B1534+12	PSR B1913+16
Keplerian phenomenological parameters:		
Orbital period, P_b (s)	36351.70270(3)	27906.9807804(6)
Eccentricity, e	0.2736779(6)	0.6171308(4)
Projected semi-major axis, x (s)	3.729468(9)	2.3417592(19)
Time of Periastron, T_0 (MJD)	48262.8434966(2)	46443.99588319(3)
Longitude of periastron, ω (°)	264.9721(16)	226.57528(6)
Post-Keplerian phenomenological parameters:		
Advance of periastron, $\dot{\omega}$ (° yr^{-1})	1.7560(3)	4.226621(11)
Time dilation, γ (ms)	2.05(11)	4.295(2)
Orbital period derivative, $\dot{P}_b$ (10^{-12}) ...	−0.1(6)	−2.422(6)
Range of Shapiro delay, r (μs)	6.2(1.3)	[6.834]
Shape of Shapiro delay, $s \equiv \sin i$	0.986(7)	[0.734]
Derived masses:		
Pulsar mass ($M_\odot$)	1.34(7)	1.4410(7)
Companion mass ($M_\odot$)	1.34(7)	1.3874(7)

The PSR B1913+16 system thus has more measurable quantities than significant unknowns, and consequently provides a clean, accurate test of relativistic gravitation theories. It is this test that convincingly demonstrates the existence of quadrupolar gravitational radiation in general relativity. One uses the measured values of the five Keplerian parameters, $\dot{\omega}$, and γ to calculate the component masses and the expected rate of orbital period decay caused by gravitational radiation. This prediction is then compared with the observed value of $\dot{P}_b$. At present levels of accuracy it is necessary to include a small adjustment for the effect of galactic accelerations (Damour and Taylor 1991). An up-to-date error budget for the experiment is presented in Table 2, and Figure 1 illustrates the observed decay of the PSR B1913+16 orbit from September 1974 through April 1992. General relativity passes this "$\dot{\omega} - \gamma - \dot{P}_b$" test perfectly at the present level of accuracy, about 0.35% — an impressive confirmation of Einstein's theory in a regime where gravitation theories have not previously been testable.

Table 2. Error budget for the orbital period derivative of PSR B1913+16, and comparison of experimental result with general relativistic prediction.

Parameter	(10^{-12})
Observed value, $\dot{P}_b^{\rm obs}$	-2.4225 ± 0.0056
Galactic contribution, $\dot{P}_b^{\rm gal}$	-0.0124 ± 0.0064
Intrinsic orbital period decay, $\dot{P}_b^{\rm obs} - \dot{P}_b^{\rm gal}$	-2.4101 ± 0.0085
General relativistic prediction, $\dot{P}_b^{\rm GR}$	-2.4025 ± 0.0001
$\dot{P}_b^{\rm obs} - \dot{P}_b^{\rm gal} / \dot{P}_b^{\rm GR}$	1.0032 ± 0.0035

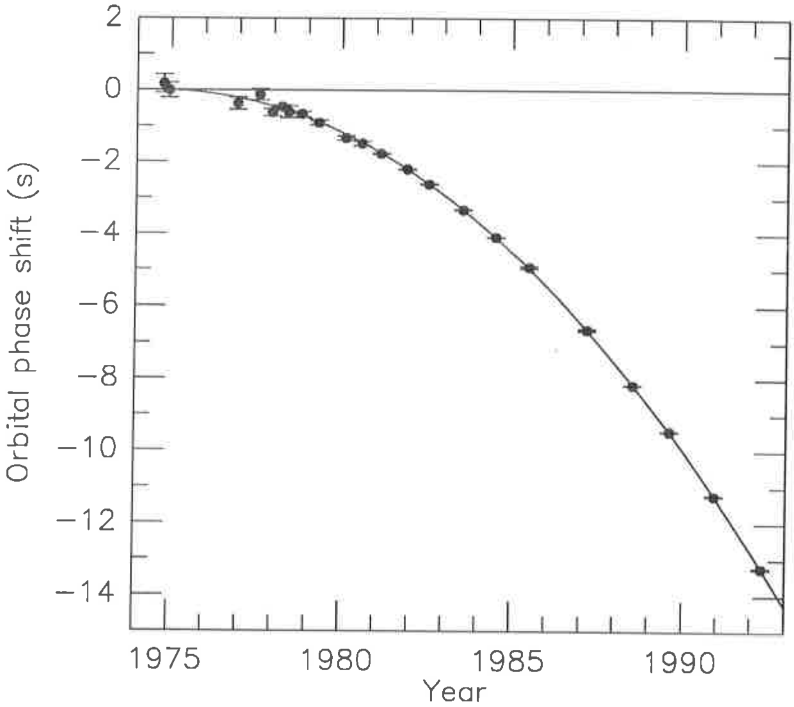

Figure 1. Filled circles represent the measured shifts of the times of periastron passage of PSR B1913+16, relative to a non-dissipative model in which the orbital period remains fixed at its 1974.78 value. The smooth curve illustrates the prediction of general relativity.

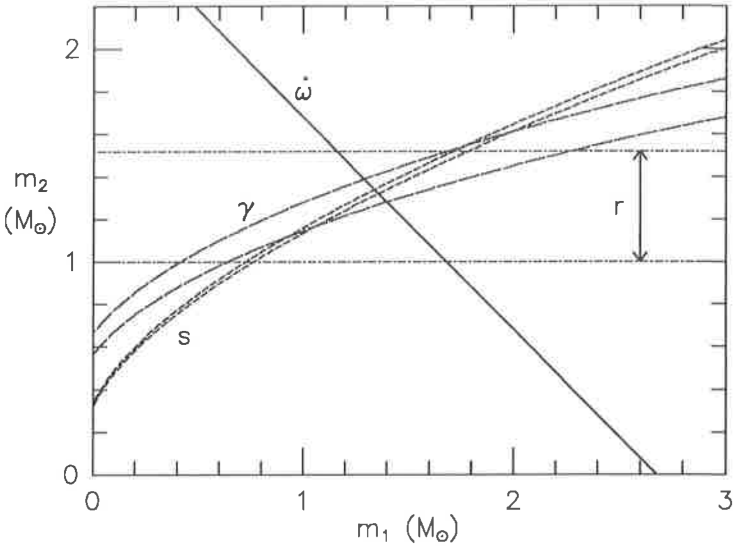

Figure 2. Parametric curves corresponding to experimental constraints on the post-Keplerian parameters $\dot{\omega}$, γ, r, and s for PSR B1534+12, according to general relativity. Pulsar and companion star masses $m_1 = m_2 = 1.34 \pm 0.07\,M_\odot$ are consistent with all of the measurements. (Observations carried out in collaboration with A. Wolszczan.)

4. Binary pulsar PSR B1534+12

About two years ago Wolszczan (1991) discovered a new binary pulsar system with relativistically interesting characteristics. Its orbit is less eccentric and slower than that of PSR B1913+16 (see Table 1), so most of its relativistic effects are smaller in magnitude. However, other circumstances contrive to make PSR B1534+12 an extremely attractive candidate for detailed study. The pulsar's radio signal is several times stronger than that of PSR B1913+16, and the pulse width is smaller. Consequently, in similar observations at Arecibo its TOAs can be measured about 5 times more accurately. In addition, the orbit of PSR B1534+12 is oriented more nearly edgewise to the line of sight ($\sin i > 0.97$, see Table 1), which greatly magnifies the Shapiro delay. PSR B1534+12 is also much closer to the Sun (approximately 0.7 kpc, compared with 7.1 kpc for PSR B1913+16; see Taylor & Cordes 1993), so contributions to measurable parameters from galactic kinematic effects are smaller and much easier to estimate.

The parameters of the PSR B1534+12 system measured by Taylor *et al* (1992) are listed in Table 1. For this pulsar, four PK parameters have already been measured with significance, and thus two distinct tests of relativistic gravity are available. The tests depend on the overall consistency of the parameter set, which is illustrated in Figure 2 by means of a plot of the constraints placed on the pulsar and companion masses, m_1 and m_2, by each of the measured PK parameters. The mass values $m_1 = 1.34 \pm 0.07$ and $m_2 = 1.34 \pm 0.07$ are consistent, within general relativity, with all of the directly measured parameters of PSR B1534+12 — and thus general relativity is found to be fully consistent with the data. It is noteworthy that unlike the $\dot{\omega} - \gamma - \dot{P}_b$ test for PSR B1913+16, which is a mixed test of the strong-field and radiative aspects of gravity, the $\dot{\omega} - \gamma - r - s$ test for PSR B1534+12 is purely a strong-field test (Damour & Taylor 1992, Taylor *et al* 1992).

Experimental results of the sort described here will almost certainly be improved and extended in the near future. In a recent paper (Taylor 1992) I have shown that on a 10-year time scale two more binary pulsar systems, PSR B1534+12 and PSR B2127+11C, should yield $\dot{\omega} - \gamma - \dot{P}_b$ tests of gravitational radiation damping at the 1% level or better. By that time PSR B1534+12 will have at least 5 measurable PK parameters, providing cleanly separated probes of the coupling of matter to gravitational radiation and the behavior of gravity in very strong fields. Moreover, combining the results of timing measurements of several binary pulsars can provide even tighter theoretical constraints than available from each pulsar separately (Taylor *et al* 1992).

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