Triangle relations for XYZ states

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We propose novel triangle relations, not the well-known triangle singularity, for better understanding of the exotic XYZ states. Nine XYZ resonances, $X(3872)$, $Y(4230)$, $Z_c(3900)$, $X(4012)$, $Y(4360/4390)$, $Z_c(4020)$, $X(4274)$, $Y(4660)$, and $Z_{cs}(3985)$ have been classified into triples to construct three triangles based on the assumption that they are all tetra-quark states. Here $X(4012)$ has not been observed experimentally yet, and we predict its mass and roughly the width. We also suggest some channels deserving search for with high priority based on this hypothesis, as well as predictions of a few production/decay rates of these channels. We hope further experimental studies of the XYZ states will benefit from our results.

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Charmonium-like states, the so-called XYZ particles, have been hot topics in high energy physics community since $X(3872)$ was discovered in 2003 by Belle.[1] Their special characters make them good candidates of multiple-quark states[2] beyond the conventional mesons (quark–antiquark pair) and baryons (triple quarks). In addition to being a novel material form, they also provide idea labs for studying the nonperturbative QCD, the theory describing strong interaction. Studying their production and decay rates individually, and comparing them to that of the charmonium states, should shed light on the internal structures and interaction mechanism of these exotic states. Then it will lead to better understanding of the

strong interaction, such as color confinement, at the several GeV energy region. Various theoretical models have been proposed to interpret the experimental results and describe the nature of these XYZ states, including models, for example, tetra-quark, hybrid, conventional charmonium, molecular, and even cusp effect. For more details, our readers may read the fourth part of Ref. 2 and the references quoted in this review. However, in short, a global and satisfied frame, that can describe the whole XYZ states well, is still in absent. The situation of our knowledge on the multiple-quark states is somewhat similar to that on the glueballs, both of them are allowed by QCD but without experimentally conclusive evidence yet. Also, the mixing between these exotic states with normal mesons or baryons makes the situation much more complex.

Recently, three results reported by BESIII collaboration have attracted our attention. Two of them are the confirmation of the radiative transition from $Y(4230)$ to $X(3872)$ (Ref. 3) and pion transition from $Y(4230)$ to $Z_c(3900)$ (Ref. 4). The third one is the first observation of $Z_{cs}(3985)$,[5] it is based on the data samples of e^+e^- collision at the center-mass-energy close to $Y(4660)$. Even the statistic is low, it seems $Z_{cs}(3985)$ is produced by a kaon transition from $Y(4660)$. These new information and previous experimental results inspire us to investigate the relations between these XYZ states and suggest a general frame containing them. In this frame, XYZ states are classified into various sets, and each contains one X, one Y, and one Z states with similar quark components. So a single set can be illuminated by a triangle. Each point of a triangle is a state, while each side of the triangle is a hadronic or radiative transition. In this frame, nine XYZ states, $X(3872)$, $Y(4230)$, $Z_c(3900)$, $X(4012)$, $Y(4360/4390)$, $Z_c(4020)$, $X(4274)$, $Y(4660)$, and $Z_{cs}(3985)$ have been classified into three sets to construct triple triangles. We believe such a frame would provide insights on them, and can be a guide to future experimental searches and measurements.

The construction of a triangle start with a simple hypothesis, that the XYZ states are tetra-quark states and can be described universally in a similar quark components, i.e. $|h\rangle |c\bar{c}\rangle$. Here the $|h\rangle$ is a superimposition of light quark components, and same for X and Y states as

$$|h\rangle_{X/Y} = p_{h1}|u\bar{u}\rangle + p_{h2}|d\bar{d}\rangle + p_{h3}|s\bar{s}\rangle , \qquad (1)$$

where p_{h1}, p_{h2}, and p_{h3} are amplitude strengths that can be determined as parameters via experimental measurements. However, the form of $|h\rangle$ for Z states should be slightly different since their iso-spin is not zero, and can be written as

$$|h\rangle_Z = p'_{h1}|u\bar{d}\rangle + p'_{h2}|u\bar{s}\rangle + p'_{h3}|d\bar{u}\rangle + p'_{h4}|s\bar{u}\rangle$$
$$+ p'_{h5}|d\bar{s}\rangle + p'_{h6}|s\bar{d}\rangle + p'_{h7}|u\bar{u}\rangle - p'_{h7}|d\bar{d}\rangle , \qquad (2)$$

where parameters p'_h's will take corresponding values for various states. Similarly $|c\bar{c}\rangle$ can be expanded into a superimposition of hidden-charm states as

$$|c\bar{c}\rangle = p_{c1}|\eta_c\rangle + p_{c2}|J/\psi\rangle + p_{c3}|\chi_{c0}\rangle + p_{c4}|\chi_{c1}\rangle$$
$$+ p_{c5}|\chi_{c2}\rangle + p_{c6}|h_c\rangle + p_{c7}|\eta'_c\rangle + p_{c8}|\psi'\rangle + \cdots , \qquad (3)$$

where only the chamonia below the open-charm threshold have been explicitly addressed, since we assume the contribution from the hidden-charm above the threshold is negligible in the cases we discussed. Notice our hypothesis is significantly different to the models with these XYZ states, which are pure charmonia or fake resonances from the cusp effect. From this hypothesis, a naive inference is that a mass of any XYZ state is mainly determined by the heavy quark component, and strongly affected by their couplings to open-charm channels. This inference would be used to determine the mass weights of these triangles. Since there are many different open-charm combinations, here we only consider the thresholds of the following S-wave open-charm channels, i.e. $DD_1(2420)$, $D^*D_1(2420)$, $D_sD_{s1}(2460)$, $D_s^*D_{s1}(2460)$, $D_s^*D_{s1}(2536)$, $D^*D_2(2460)$, and $D_s^*D_{s2}(2573)$. They are believed to provide the largest contributions. From a simple sum of the masses of the D mesons, we know the thresholds are roughly at 4.29 GeV, 4.43 GeV, 4.43 GeV, 4.57 GeV, 4.65 GeV, 4.47 GeV, and 4.68 GeV, respectively. It seems if there are seven triangles, two of them are according to the same 4.43 GeV would be considered degeneracy. However, after considering the quark component, it is more likely the two pairs $\left(D^*D_1, D^*D_2\right)$ and $\left(D_s^*D_{s1}, D_s^*D_{s2}\right)$ should be considered as degeneracies. So the reduced four thresholds are 4.29 GeV, 4.43(4.47) GeV, 4.57 GeV, 4.65(4.68) GeV. And the shifts between the excited ones to the fundamental one would be 140 MeV, 280 MeV, 360 MeV.

Due to the open-charm thresholds and the masses of the well-known XYZ states $X(3872)$, $Y(4230)$, and $Z_c(3900)$,[6] we assume that the three states construct the triple points of the fundamental triangle, and the radiative and hadronic transitions are illustrated as the triple sides as shown in the left-bottom of Fig. 1. So starting from the fundamental triangle, i.e. the $X(3872)$–$Y(4230)$–$Z_c(3900)$ triangle, we can predict an excited triangle with the mass shift between 4.29 GeV and
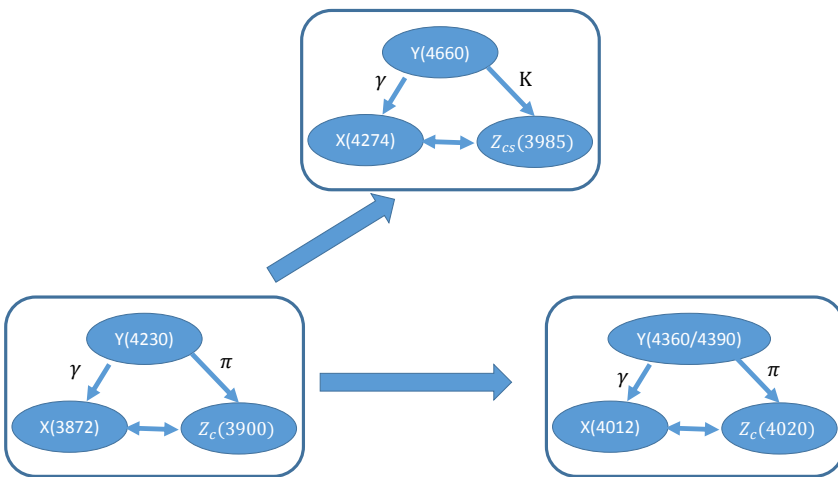

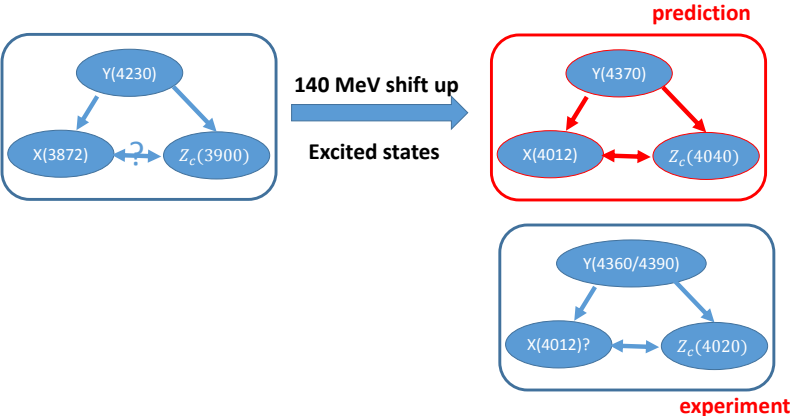Fig. 1.   Triple triangles for nine XYZ states.

Fig. 2. The prediction and experimental observation of the first excited triangle of XYZ states.
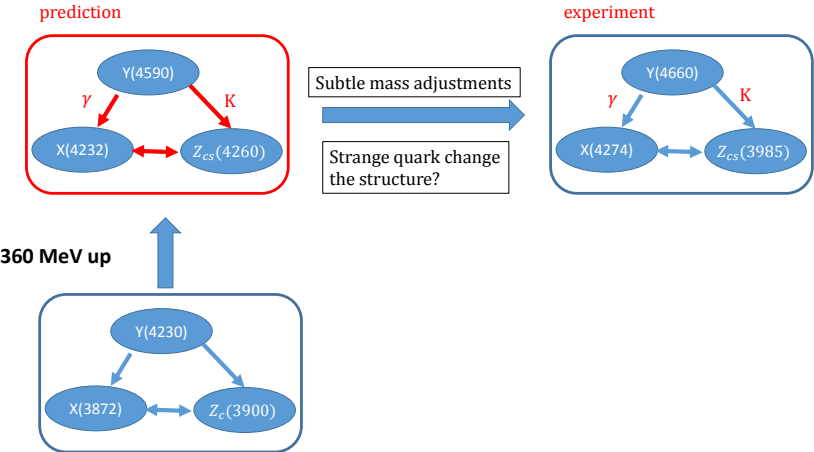

Fig. 3. The prediction and experimental observation of the second excited triangle of XYZ states involving with strange quark heavily.

4.43 GeV, i.e. 140 MeV. As shown in Fig. 2, it is obvious the Y and Z states belonging to the predicted triangle can be associated to the experimental observations, $Y(4360/4390)$ and $Z_c(4020)$, respectively. Here, we assume $Y(4360)$ and $Y(4390)$ are actually one vector state due to their similar masses. However, there is no such evidence of the X state close to $X(4012)$ experimentally. Only it is well matched with another theoretical prediction.[7] Based on the similar argument, it is also that another triangle can be obtained by shifting the masses up to 360 MeV, the difference between thresholds of 4.29 GeV and 4.65 GeV, as shown in Fig. 3. Here, the predicted X and Y states are perfectly matched with the experimental results $X(4274)$ and $Y(4660)$. While the mass of predicted Z state is obviously larger than the recently observed $Z_{cs}(3985)$, this discrepancy may be caused by the

replacement of the strange quark with up/down quark, that will change the internal structure of the multiple quark state then decrease the mass of Z state. Notice the subtle shift is about 300 MeV that is just the difference between the masses of kaon meson and pion meson. Another possibility is the $Z_{cs}(3985)$ is not the Z state that should be filled in this triangle, so another heavier Z state deserves searching for with different channels. Till now, there is no evidence of the $X(4274)$, discovered in $B \to J/\psi\phi K$ process,[8] is observed via $Y(4660) \to \gamma X(4274)$, $X(4274) \to \phi J/\psi$. So, a search on this radiative transition channel is deserving. Additionally, because mass estimation of $X(4274)$ is based on a very rough way, other X structures seen in $B \to J/\psi\phi K$ cannot be excluded such as $X(4140)$, and so $X(4274)$ is the only one of the possible candidates.

The previous predictions on the two excited triangles have implicitly applied part of the interaction mechanism proposed in this paper. The complete form of the interaction hypothesis can be summarized as the following two rules:

- *Shifts between triangles*: The radial position or orbital momentum between $|h\rangle$ and $|c\bar{c}\rangle$ for excited triangles keeps similar to the fundamental triangle, but the $|c\bar{c}\rangle$ changes between the fundamental and excited ones according to the corresponding open-charm thresholds.
- *Transitions within a triangle*: The $|c\bar{c}\rangle$ keeps same, only the radial position or orbital momentum between $|h\rangle$ and $|c\bar{c}\rangle$ should change.

Due to the second rule mentioned above and the observed channels, we can see there are more channels deserving search for. Some of them with higher priority are listed in Table 1. Here almost the observed or searched results are from PDG,[6] while the others are from recent published results.[4,9–14]

Table 1. The observed/searched and proposed channels for XYZ states. Here $X\eta_c$ represents $\eta\pi^+\pi^-\eta_c$, $3\pi\eta_c$, $\pi^+\pi^-\eta_c$, and $\gamma\pi^0\eta_c$; ψ_2 is $\psi_2(3823)$; D_1 is $D_1(2420)$.

States	Observed/searched	Proposed
$X(3872)$	$\rho J/\psi$, $\omega J/\psi$, $D^0\bar{D}^{*0}$, $\pi^0\chi_{c1}$, $\gamma J/\psi$, $\gamma\psi'$, $\gamma D\bar{D}$, $\pi^0 D^0\bar{D}^0$	$\pi\pi\eta_c$, $\pi\pi\chi_{cJ}$
$Y(4230)$	$\omega\chi_{c0}$, $\pi\pi J/\psi$, $\pi\pi h_c$, $\pi\pi\psi'$, $\pi^+ D^0 D^{*-}$, $\eta J/\psi$, $\eta' J/\psi$, $\pi^+\pi^-\chi_{cJ}$, $X\eta_c$	$\pi^0\psi'$, $\gamma\chi_{cJ}$, $D^*\bar{D}^*$
$Z_c(3900)$	$\pi J/\psi$, $\rho\eta_c$, $D\bar{D}^*$, $\pi\eta_c$	πh_c, $\pi\psi'$, $\pi\pi\chi_{c0}$
$X(4012)$		$\pi\pi\psi'$, $\pi\pi\psi_2$, $\pi\pi h_c$, $\pi\pi D\bar{D}$
$Y(4360/4390)$	$\pi\pi\psi'$, $\pi\pi\psi_2$, $D_1\bar{D}$, $\pi\pi h_c$, $\pi\pi\psi''$, $\eta J/\psi$	$\pi D^*\bar{D}^*$
$Z_c(4020)$	πh_c, $D\bar{D}^*$	$\pi\psi'$, $\pi\psi_2$, $D\bar{D}^*$, $\pi D\bar{D}$
$X(4274)$	$\phi J/\psi$	$D_s^{(*)}\bar{D}_s^{(*)}$, $\eta\chi_{cJ}$, $\eta'\chi_{cJ}$
$Y(4660)$	KKJ/ψ, KKh_c, $KK\psi'$, $KD^{(*)}D_s^{(*)}$	$\phi\chi_{cJ}$, $\eta J/\psi$, $\eta' J/\psi$, $\eta\psi'$, $\eta'\psi'$
$Z_{cs}(3985)$	DD_s^*, D^*D_s	KJ/ψ, $K\eta_c$, Kh_c, $K\psi'$, $K\chi_{c0}$

Furthermore, based on Eqs. (1)–(3) and the following partial width formula

$$\Gamma(X/Y/Z \to f_h f_c) \propto |\langle f_h | h \rangle|^2 |\langle f_c | c\bar{c} \rangle|^2, \qquad (4)$$

where f_h and f_c indicate light hadrons and charmonia in the final states, we derive the formulae for relations of the partial widths of XYZ states decaying into light hadrons plus hidden-charm final states:

$$\frac{\Gamma(X \to f_{h1} f_{c1})}{\Gamma(X \to f_{h2} f_{c1})} = \frac{\Gamma(Y \to f_{h1} f_{c2})}{\Gamma(Y \to f_{h2} f_{c2})} \qquad (5)$$

and

$$\frac{\Gamma(Z \to f_{h1} f_{c1})}{\Gamma(Z \to f_{h1} f_{c2})} = \frac{\Gamma(Y \to f_{h2} f_{c1})}{\Gamma(Y \to f_{h2} f_{c2})}. \qquad (6)$$

Here the phase space effect has been ignored since the masses of XYZ states in one triangle are similar to each other. For example, from Eq. (5) and the assumption that $X(3872)$ and $Y(4230)$ are assigned in the same triangle, we can have

$$\frac{\Gamma(X(3872) \to \pi\pi J/\psi)}{\Gamma(X(3872) \to \omega J/\psi)} = \frac{\Gamma(Y(4230) \to \pi\pi \chi_{c0})}{\Gamma(Y(4230) \to \omega \chi_{c0})},$$

where the corresponding final light hadron or hidden-charm final states $\pi\pi$, J/ψ, ω, and χ_{c0} have replaced f_{h1}, f_{c1}, f_{h2}, and f_{c2}, respectively. However, because the available experimental measurements are very limited, only a few predictions are obtained. And we do not use the results from Ref. 4 since it only has a significance of 4.2 standard deviation, and its results are consistent with the charged mode,[15] which provides more precise measurements of $e^+e^- \to \pi\pi J/\psi$. And we do not use the results from Refs. 12–14, since unfortunately only upper limits of the Born cross sections are provided in these papers without the corresponding coupling strength that can be applied in our calculation directly. These predictions are listed in Table 2. All the obtained predictions are consistent with each other when different experimental inputs are used in the calculation. We notice both the values of $\Gamma(Z_c(3900) \to \pi h_c)/\Gamma(Z_c(3900) \to \pi J/\psi)$ and $\Gamma(Z_c(3900) \to \pi\psi')/\Gamma(Z_c(3900) \to \pi J/\psi)$ take large uncertainty. The reason is that the experimental result of $Y(4230) \to \pi\pi J/\psi$ is used, while in this measurement multiple solutions are found.[15] Considering no evidence has been observed in experiments till now, the destructive solution is more favored than the constructive

Table 2. The predictions of production or decay rates for XYZ states.

Items	Prediction	Comment
$B(Y(4230) \to \pi^+\pi^-\chi_{c0})\Gamma_{ee}^{Y(4230)}$ (in eV)	2.3 ± 0.9	with $Y(4230) \to \omega\chi_{c0}$
$B(Y(4230) \to \pi^+\pi^-\chi_{c0})\Gamma_{ee}^{Y(4230)}$ (in eV)	< 11.4	with $Y(4230) \to \gamma\chi_{c0}$
$\Gamma(Z_c(3900) \to \pi h_c)/\Gamma(Z_c(3900) \to \pi J/\psi)$	$0.4 \sim 3.0$	not seen in experiment
$\Gamma(Z_c(3900) \to \pi\psi')/\Gamma(Z_c(3900) \to \pi J/\psi)$	$0.1 \sim 1.0$	not seen in experiment

one for the multiple solutions. During our calculations we have also realized the negative results are worthy to be reported, since the upper limits sometimes will transform into bound limits or other constraints due to the relations.

In summary, we have proposed triple triangle relations based on the tetra-quark component hypothesis $|h\rangle|c\bar{c}\rangle$, with the S-wave open-charm thresholds as the weights, to classify the exotic XYZ states. Except $X(4012)$, that has not been observed experimentally yet, all the other predicted states can mostly be consistent with presently observed states. Notice this triangle relation proposed by us is not the well-known triangle singularity theory, and it is just experimental result-driven. Within this frame, we also propose some channels deserving search and predict a few production/decay rates of them. They are the main results of this paper and listed in Tables 1 and 2. We hope further experimental discoveries and measurements will benefit from our results.

It should also be mentioned that the original Z_{cs} states predicted by us are obviously heavier than the recently discovered $Z_{cs}(3985)$. Further searches in other channels such as KJ/ψ are deserving, even this mass difference may be explained by the mass difference between kaon and pion. We have not discussed the widths of the XYZ states, as well as the decay rates of open-charm channels since they should be significantly affected by the strong coupling to open-charm channel near thresholds. Till now, we do not have a convincing method to deal with it, but the widths should be similar to each other when the XYZ states are in one triangle due to their similar internal structure, except when one state is below the open-charm threshold then its width would be obviously narrower. It means the width of $X(4012)$ would be dozens of MeV or narrower since it seems just below the D^*D^* threshold. The number of S-wave open-charm thresholds is larger than the number of the triangles proposed in this paper due to limited experimental results, that implies there maybe undiscovered XYZ states and hence deserving more searches. And some thresholds are close to each other, that should make the identification and classification of the XYZ states difficult. We have not discussed other possible excited states, who will take different quantum numbers and larger radial positions or higher orbital momentum between $|h\rangle$ and $|c\bar{c}\rangle$, since the experimental results on the XYZ states are still too limited. These unknown states may expand the triangle relations to polygon relations in principle.

Acknowledgments

References

1. Belle Collab. (S. K. Choi *et al.*), *Phys. Rev. Lett.* **91**, 262001 (2003).
2. N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan, *Phys. Rep.* **873**, 1 (2020).

3. BESIII Collab. (M. Ablikim *et al.*), *Phys. Rev. Lett.* **122**, 232002 (2019).
4. BESIII Collab. (M. Ablikim *et al.*), *Phys. Rev. D* **102**, 012009 (2020).
5. BESIII Collab. (M. Ablikim *et al.*), arXiv:2011.07855 [hep-ex].
6. Particle Data Group (P. A. Zyla *et al.*), *Prog. Theor. Exp. Phys.* **2020**, 083C01 (2020).
7. J. Nieves and M. P. Valderrama, *Phys. Rev. D* **86**, 056004 (2012).
8. LHCb Collab. (R. Aaij *et al.*), *Phys. Rev. Lett.* **118**, 022003 (2017).
9. BESIII Collab. (M. Ablikim *et al.*), *Phys. Rev. D* **102**, 031101 (2020).
10. BESIII Collab. (M. Ablikim *et al.*), *Phys. Rev. Lett.* **124**, 242001 (2020).
11. BESIII Collab. (M. Ablikim *et al.*), *Phys. Rev. D* **101**, 012008 (2020).
12. BESIII Collab. (M. Ablikim *et al.*), arXiv:2012.02682 [hep-ex].
13. BESIII Collab. (M. Ablikim *et al.*), arXiv:2011.13850 [hep-ex].
14. BESIII Collab. (M. Ablikim *et al.*), arXiv:2010.14415 [hep-ex].
15. BESIII Collab. (M. Ablikim *et al.*), *Phys. Rev. Lett.* **118**, 092001 (2017).