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Assessing lepton-flavour non-universality from $B \to K^* \ell\ell$ angular analyses

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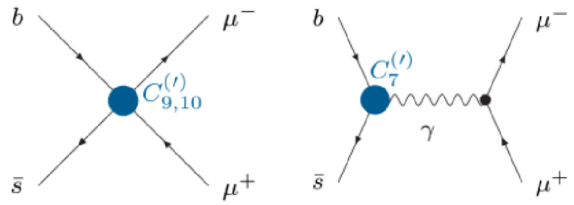
Abstract. The recent years of activity at the LHC have brought to light several anomalies in exclusive semileptonic $b \to s\ell^+\ell^-$ decays. Even though the latest model independent analyses are pointing to sizable New Physics contributions to different Wilson coefficients (especially C_9), the possibility of interpreting these results unambiguously in terms of New Physics is spoilt by non-perturbative QCD corrections entering at amplitude level. In this proceeding, we present a brief summary of the latest results of our global fits. Also, the origin of the above mentioned QCD corrections, plus their interplay with possible New Physics contributions, is shortly reviewed. A discussion about the properties of the observable R_K leads to the central part of this work: the proposal of a new set of observables able to test for lepton-flavour universality violation, where hadronic uncertainties cancel (in the Standard Model). To conclude, the properties of these observables are analysed within the Standard Model as well as within several New Physics benchmark scenarios.

1. Introduction

After the Higgs search era at the LHC, flavour physics has become one of the most promising windows for studying possible Beyond the Standard Model (BSM) effects. In particular, processes driven by the flavour-changing neutral current (FCNC) transition $b \to s\ell^+\ell^-$ have been providing the framework for the raise of several deviations from the Standard Model (SM) in various observables. In 2015, the LHCb analysis [1] of the 3 fb^{-1} data on $B \to K^*\mu^+\mu^-$ confirmed a $\sim 3\sigma$ anomaly in two large K^*-recoil bins of the angular observable P_5' [2, 3] that was already present in the 2013 results with 1 fb^{-1} [4]. The same experiment measured the observable $R_K = \mathcal{B}(B \to K\mu^+\mu^-)/\mathcal{B}(B \to Ke^+e^-)$ [5] in the dilepton mass range from 1 to 6 GeV^2 and found a 2.6σ tension with its SM value, predicted to be equal to 1 (to a high accuracy). Further deviations were observed by LHCb in the two large-recoil bins of the branching ratio of $B_s \to \phi\mu^+\mu^-$ [6]. And finally, a few months ago, the Belle experiment performed an independent measurement of P_5' [7] which confirmed the deviation w.r.t. SM observed by LHCb.

The relevance of the mentioned tensions (P_5', R_K, $\mathcal{B}(B_s \to \phi\mu^+\mu^-)$ and several low-recoil bins of $B \to V\ell^+\ell^-$ and $B \to P\ell^+\ell^-$ decays) is that all of then are sensitive to the same effective couplings, the Wilson coefficients $C_{7,9,10}^{(\prime)}$ induced by the four-fermion operators in the effective Hamiltonian approach (see Fig. 1):

$$\begin{aligned}
&\mathcal{O}_7^{(\prime)} = \frac{\alpha}{4\pi} m_b \left[\bar{s}\sigma_{\mu\nu} P_{R(L)} b\right] F^{\mu\nu}, && \mathcal{O}_9^{(\prime)} = \frac{\alpha}{4\pi} \left[\bar{s}\gamma^\mu P_{L(R)} b\right] \left[\bar{\ell}\gamma_\mu \ell\right], && \mathcal{O}_{10}^{(\prime)} = \frac{\alpha}{4\pi} \left[\bar{s}\gamma^\mu P_{L(R)} b\right] \left[\bar{\ell}\gamma_\mu\gamma_5 \ell\right], \\
&C_7^{\rm SM}(\mu_b) = -0.29, && C_9^{\rm SM}(\mu_b) = 4.07, && C_{10}^{\rm SM}(\mu_b) = -4.31, \qquad (1)
\end{aligned}$$



processes	$\mathscr{C}^{(\prime)}_7$	$\mathscr{C}^{(\prime)}_9$	$\mathscr{C}^{(\prime)}_{10}$
$B \to X_s\gamma,\ B \to K^*\gamma$	✓		
$B \to X_s\mu^+\mu^-$	✓	✓	✓
$B_s \to \mu^+\mu^-$			✓
$B_{(s)} \to (K^{(*)},\phi)\mu^+\mu^-$	✓	✓	✓

Figure 1. Effective couplings $C^{(\prime)}_{7,9,10}$ contributing to $b \to s\ell^+\ell^-$ transitions and sensitivity of the various radiative and (semi-)leptonic $B_{(s)}$ decays modes to them.

where $P_{L,R} = (1 \mp \gamma_5)/2$, m_b stands for the mass of the b quark and $\mu_b = 4.8$ GeV denotes the energy scale. The right-handed (primed) Wilson coefficients are not given since they either vanish or can be neglected in the SM.

The next point that needs to be addressed is whether a New Physics (NP) contribution to the coefficients $C^{(\prime)}_{7,9,10}$ could simultaneously account for the various tensions in data. Plausible BSM scenarios with the required characteristics to generate this type of contributions include SM extensions with Z' bosons or lepto-quarks. The couplings $C^{(\prime)}_{7,9,10}$ can be constrained through various observables in radiative and (semi-)leptonic $B_{(s)}$ decays, each of them sensitive to a different subset of coefficients (see Fig. 1). Therefore, in order to assess the extend of potential NP effects it is required to perform a combined study of these observables, including their correlations. Several analysis of this fashion can be found in the literature, see Refs. [8, 9, 10, 11, 12, 13, 14]. Differences among them include the use of different data sets, different form factor inputs and different/additional observables.

In order to be able to interpret the observed deviations in the (semi-)leptonic $B_{(s)}$ decays in terms of NP effects a detailed analysis of all possible hadronic uncertainties has to be done (see, for instance, [15]). The theoretical predictions are plagued by perturbative and non-perturbative QCD effects, and it can be argued that some of the non-perturbative effects may mimic a NP signal. In a very recent paper [16], some of the authors carefully discuss the different sources of hadronic uncertainties and provide robust arguments disfavouring the possibility that these non-perturbative effects are the origin of the observed anomalies.

Another tentative path for assessing possible NP effects goes along the line pointed by the deviation observed in R_K. The tension in this observable can be understood if NP would affect predominantly $b \to s\mu^+\mu^-$ compared to $b \to se^+e^-$ [5] and thus violate lepton-flavour universality. Also the fact that no deviation was observed in $B \to K^*e^+e^-$ data at very large K^* recoil [17] supports the hints suggested by R_K. In Ref. [18] we assume lepton-flavour universality is maximally violated between the electronic and muonic modes and exploit the resulting new paradigm to build observables where the presence of NP in $b \to s\mu^+\mu^-$ can be proved in a clean way, via comparison of $b \to s\mu\mu$ and $b \to see$ observables.

Last December, Belle presented a separate measurement [19] of P_5' in the muon and electron channels, and hence of the observable $Q_5 = P_5^{\mu\prime} - P_5^{e\prime}$ proposed in Ref. [18]. Though not yet statistically significant, the result points to a violation of lepton-flavour universality in P_5' in compliance with R_K.

The structure of this proceeding is the following: in Sec. 2 we report the features and most important results of the analysis in Ref. [11]. The origin of the hadronic uncertainties and how they can interfere with a NP signal is sketched in Sec. 3. Finally, in Sec. 4 we present the observables postulated in [18] and review their characteristics.

Coefficient	Best fit	1σ	3σ	Pull_{SM}	p-value (%)
$C_7^{\rm NP}$	-0.02	$[-0.04, -0.00]$	$[-0.07, 0.04]$	1.1	13.0
$C_9^{\rm NP}$	-1.05	$[-1.25, -0.85]$	$[-1.62, -0.40]$	**4.7**	61.0
$C_{10}^{\rm NP}$	0.55	$[0.34, 0.77]$	$[-0.05, 1.24]$	2.8	24.0
$C_{7'}^{\rm NP}$	0.02	$[-0.00, 0.04]$	$[-0.05, 0.09]$	0.9	13.0
$C_{9'}^{\rm NP}$	0.06	$[-0.18, 0.30]$	$[-0.65, 0.78]$	0.3	12.0
$C_{10'}^{\rm NP}$	-0.03	$[-0.20, 0.14]$	$[-0.55, 0.48]$	0.2	12.0
$C_9^{\rm NP} = C_{10}^{\rm NP}$	-0.18	$[-0.36, 0.02]$	$[-0.68, 0.50]$	0.9	13.0
$C_9^{\rm NP} = -C_{10}^{\rm NP}$	-0.59	$[-0.74, -0.44]$	$[-1.06, -0.17]$	**4.3**	51.0
$C_{9'}^{\rm NP} = C_{10'}^{\rm NP}$	0.00	$[-0.24, 0.24]$	$[-0.74, 0.71]$	0.0	12.0
$C_{9'}^{\rm NP} = -C_{10'}^{\rm NP}$	0.03	$[-0.08, 0.13]$	$[-0.30, 0.35]$	0.2	12.0
$C_9^{\rm NP} = -C_{9'}^{\rm NP}$	-1.00	$[-1.20, -0.78]$	$[-1.55, -0.32]$	**4.4**	54.0
$C_9^{\rm NP} = -C_{10}^{\rm NP}$ $= -C_{9'}^{\rm NP} = -C_{10'}^{\rm NP}$	-0.61	$[-0.45, -0.45]$	$[-1.17, -0.17]$	**4.3**	50.0
$C_9^{\rm NP} = -C_{10}^{\rm NP}$ $= C_{9'}^{\rm NP} = -C_{10'}^{\rm NP}$	-0.24	$[-0.33, -0.15]$	$[-0.53, 0.03]$	2.7	23.0

Table 1. Best-fit point, confidence intervals, pulls for the SM hypothesis and p-value for different 1D NP scenarios, including $b \to see$ data but assuming NP only in $b \to s\mu\mu$.

2. A short review of the global fits

The reference fits in [11] are obtained using the branching ratios and angular observables for $B \to K^*\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$, the branching ratios of the charged and neutral modes $B \to K\mu^+\mu^-$, the branching ratios of $B \to X_s\mu^+\mu^-$, $B_s \to \mu^+\mu^-$ and $B \to X_s\gamma$, as well as the isospin asymmetry A_I and the time-dependent CP asymmetry $S_{K^*\gamma}$ of $B \to K^*\gamma$. For the theoretical predictions, lattice form factors from Refs. [20, 21] are used in the low-recoil region, while we resort to light-cone sum rule (LCSR) form factors from Ref. [22], with their correlations assessed from the large-recoil symmetries, in the low-q^2 region. The only exception being $B_s \to \phi$, which requires the use of the form factors in Ref. [23].

The hypothesis tested in our analysis is modeled by treating the NP contributions to the Wilson coefficients $\{C_i^{\rm NP}\}$ as parameters allowed to vary freely. We estimate the value of these parameters by performing a frequentist fit including experimental and theoretical correlation matrices. In Tab. **??** we present our most updated 1D fit results, where the new $B_s \to \phi$ form factors [23] and the new experimental results on $\mathcal{B}(B^0 \to K^*(892)^0\mu^+\mu^-)$ [24] are included. We report no significant changes in the fit results from the ones in Ref. [11]. In the last two columns we provide information about the goodness-of-fit by displaying the SM-pull, i.e. the number of standard deviations by how much the best fit point is preferred over the SM point $\{C_i^{\rm NP}\} = 0$, and the p-value for each scenario. A scenario with a large SM-pull, or equivalently a large p-value, indicates a parameter selection that allows for a better description of the data. Therefore, results in Tab. **??** establish the hypothesis of having a contribution to the C_9 coefficient of $\sim$

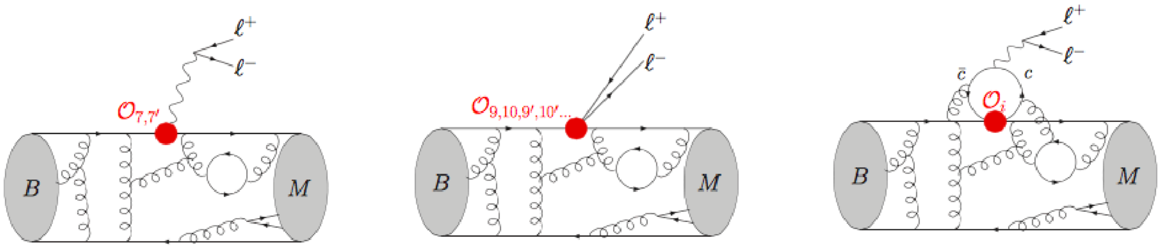


Figure 2. Illustration of factorisable (first two diagrams) and non-factorisable (third diagram) QCD corrections to exclusive $B \to M\ell^+\ell^-$ matrix elements.

-25 % w.r.t. its SM value as the most favored one.

3. Some comments about hadronic uncertainties

Theoretical computations in the framework of semi-leptonic B decays necessarily require accounting for contributions coming from QCD effects both of perturbative and non-perturbative nature. From the amplitude level perspective, predictions involve tree-level diagrams with insertions of the operators $\mathcal{O}_{7,9,10}$ (generated at loop level in the SM), as well as one-loop diagrams with an insertion of the charged-current operator

$$\mathcal{O}_2 = [\bar{s}\gamma^\mu P_L c]\,[\bar{c}\gamma_\mu P_L b] \tag{2}$$

(generated at tree level in the SM). In contributions of the first type, the leptonic and the hadronic currents factorise, and QCD corrections are restricted to the hadronic $B \to M$ current (first two diagrams in Fig. 2). This class of *factorisable QCD corrections* thus forms part of the hadronic form factors parametrizing the $B \to M$ transition. On the other hand, contributions of the second type receive *non-factorisable QCD corrections* (third diagram in Fig. 2) that cannot be absorbed into form factors.

3.1. Form factor uncertainties and power corrections

In order to be protected from uncertainties stemming from factorisable QCD corrections enclosed in the form factors, one can exploit the large-recoil symmetries of QCD to build observables such that their form factor sensitivity is minimised [2, 3, 25, 26, 27, 28]. Following this line of thought, a set of observables that only exhibit a mild form factor dependence (suppressed by powers of α_s and Λ/m_b) was proposed, the so-called optimised observables P_i.

It is essential for the analysis of the observables to control the correlations of the errors of the different form factors. For that purpose, our approach relies on the assessment of the correlations by means of the large-recoil symmetry relations. This method guarantees a model independent determination of the correlations from first principles but, as a drawback, the result is only valid up to order Λ/m_b corrections that need to be estimated. To estimate these *factorisable power corrections* we follow the technique first introduced in Ref. [29], which was improved shortly after in Ref. [15]. Numerically, we allow for a generic size of 10% power corrections to the form factor, as suggested by fitting to the particular LCSR form factors from Refs. [22, 23].

The size of the factorisable power corrections has been under intense debate recently due to attemps to explain the LHCb anomalies because of their impact. This possibility has been dismissed by the arguments provided in Ref. [16].

3.2. Impact of long-distance $c\bar{c}$ loops

Contributions to the amplitude coming from insertions of the $\mathcal{O}_2$ effective operator are of non-factorisable type, meaning that cannot be encoded into form factors. These contributions, commonly referred as long-distance charm-loop effects, can mimic a shift in the Wilson coefficient C_9 and thus have been suggested as a solution of the anomaly in $B \to K^*\mu^+\mu^-$ [30, 31]. Unlike a NP contribution, due to the non-local structure of the mentioned corrections, they are expected to show a non-constant q^2-dependence (q^2 stands for the dilepton invariant mass). This contribution always accompanies the perturbative SM part of C_9, and enter in the structure of the effective Wilson coefficient like

$$C_9^{\mathrm{eff}\,i}(q^2) = C_{9\,\mathrm{pert}}^{\mathrm{eff\,SM}}(q^2) + C_9^{\mathrm{NP}} + C_9^{c\bar{c}\,i}(q^2) \tag{3}$$

Charm-loop effects are also expected to depend on the helicity of the final state. For this reason we allow for three different $C_9^{c\bar{c}\,i}$ contributions, and thus three different $C_9^{\mathrm{eff}\,i}$, depending on the transversity amplitude under consideration $i = 0, \|, \perp$.

The theoretical computation of this-long distance contribution is technically very challenging. At the moment, there is only a partial computation in the literature [22], which resorts on light-cone sum rules combined with a dispersion relation.

4. Observables testing lepton-flavour universality violation

Looking only to $B \to K^*\mu^+\mu^-$ data, one might be tempted to explain the anomaly in terms of long-distance charm, even despite all the arguments that support the contrary [16]. However, this temptation could just be the result of a excessively narrow point view. Actually, just by broadening the frame to include $B \to K^*e^+e^-$ data, the hypothesis of long-distance charm effects as the responsible for the observed tensions starts to loose weight, because of R_K. The deviation observed in this observable cannot be explained by long-distance charm, while it adds coherently with the pattern of deviations observed in $B \to K^*\mu^+\mu^-$, being possible to explain both tensions by introducing a signal of lepton-flavour universality violation (LFUV) that couples only to the muonic channel and not the the electronic one. In other words, both the tensions in $B \to K^*\mu^+\mu^-$ and in R_K are alleviated by adding a constant contribution to $C_{9\mu}$ but leaving C_{9e} SM-like.

R_K alone, though, is not enough, since it has very little discrimination power among different generic NP scenarios. It is thus necessary to complement R_K with more observables able to test for LFUV, we proposed in [18] several of them: Q_i, B_i and M.

4.1. A new basis of observables: Q_i

A particularly interesting set of observables with the desired properties comes from the direct comparison of $P_i^{(\prime)}(B \to K^*\mu\mu)$ and $P_i^{(\prime)}(B \to K^*ee)$. These observables are the so-called $Q_i = P_i^{(\prime)\,\mu} - P_i^{(\prime)\,e}$. Being defined in terms of optimised observables, Q_i inherit their properties and show a reduced sensitivity to hadronic uncertainties. In particular, these observables are very protected against long-distance charm-loop contributions in the SM, since the effective operator $\mathcal{O}_2$ couples identically to muons and electrons. A measurement of Q_i different from zero would point to NP in an unambigous way, confirming the violation of lepton flavour universality observed in R_K. Obviously, in presence of NP the problem of hadronic uncertainties reemerges, but then we enter in a completely different field, that of a NP Discovery. To ilustrate this crucial property of Q_i, in Fig. 3 we portray $P_5^{\prime\,\mu}$ and Q_5 both in the SM and in a NP scenario with $C_{9\mu}^{\mathrm{NP}} = -1.1$ (taking $C_{9e}^{\mathrm{NP}} = 0$). Notice how tiny are the uncertainties in the SM predictions (grey boxes) for Q_5 compared with $P_5^{\prime\,\mu}$. On the other hand, when we allow for a NP contribution, the theoretical uncertainties in Q_5 grow, as expected, but are more limited than the ones of $P_5^\prime$.

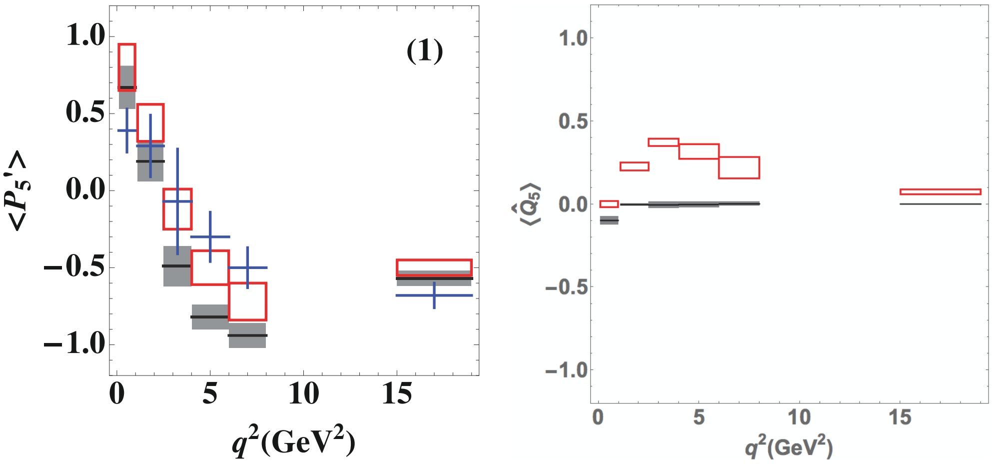


Figure 3. Predictions for P_5' and Q_5 in the SM (grey boxes) and in presence of NP (red boxes) in $C_{9\mu}^{\rm NP} = -1.1$ (with $C_{9e}^{\rm NP} = 0$).

Under the assumption of maximal LFUV between the muonic and electronic modes, the chronic dichotomy of NP or charm is traded by the scenario of a Discovery, where charm-loop uncertainties only enter into the picture when discussing the type of NP. Precisely, for the purpose of distinguishing between potential NP scenarios, the angular analysis of a subset of Q_i observables $(i = 1, 2, 4, 5)$ provides precious information. The observables[1] $\hat{Q}_1$ and $\hat{Q}_4$ offer excellent tests for the presence of right-handed currents in $C_{9\mu}'$ and $C_{10\mu}'$, as it can be seen by the very distinctive signature of $\hat{Q}_1$ and the position of the bins in each end of the large-recoil region of $\hat{Q}_4$ (Fig. 4). Additional tests for distinguishing scenarios with NP contributions only in $C_{9\mu}$ from scenarios that allow for NP both in $C_{9\mu}$ and $C_{10\mu}$ can be found in the last two bins of the large recoil region of $\hat{Q}_2$ and $\hat{Q}_5$ and the first two bins of $\hat{Q}_4$ (see Fig. 5).

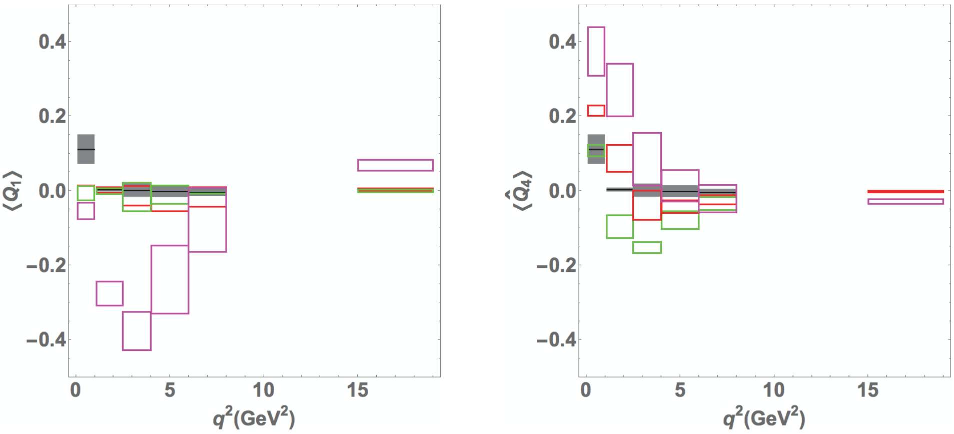


Figure 4. Predictions for $\hat{Q}_1$ and $\hat{Q}_4$ in the SM (grey boxes) and in presence of NP (red boxes) in $C_{9\mu}^{\rm NP} = -1.1$, $C_{9\mu}^{\rm NP} = -C_{10\mu}^{\rm NP} = -0.65$ and $C_{9\mu}^{\rm NP} = -C_{9\mu}' = -1.18$ & $C_{10\mu}^{\rm NP} = C_{10\mu}' = 0.38$ (with $C_{ie}^{(\prime)\rm NP} = 0$, $i = 9, 10$).

Belle has been the first experiment to measure the observable Q_5 [19], finding a 1.2σ tension

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[1] The hat notation specifies that F_L is assumed to be the one measured by LHCb (see [18] for definitions).

w.r.t. the Standard Model prediction in the relevant bin [4, 8] GeV^2 (this tension is reduced to 0.6σ in the presence of a NP contribution $C_{9\mu}^{\rm NP} = -1.1$). The low statistical significance of this result makes impossible to draw any conclusion, although it is interesting to notice its connection with the deviation in R_K.

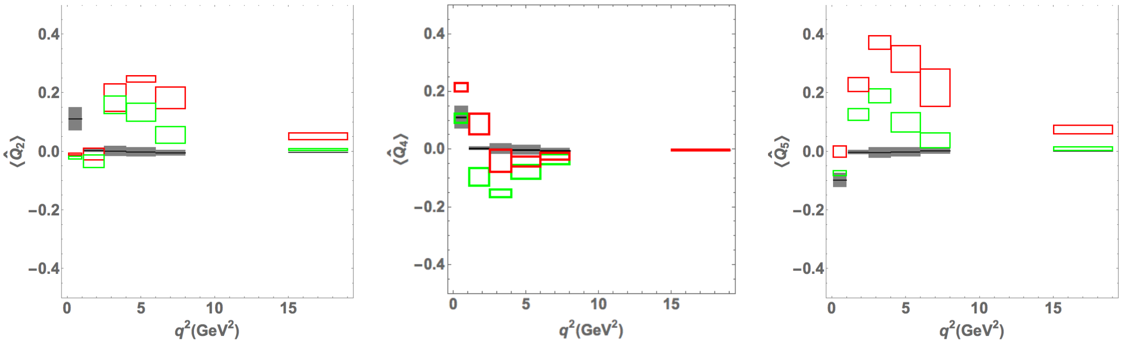

Figure 5. Predictions for $\hat{Q}_2$, $\hat{Q}_4$ and $\hat{Q}_5$ in the SM (grey boxes) and in presence of NP (red boxes) in $C_{9\mu}^{\rm NP} = -1.1$ and $C_{9\mu}^{\rm NP} = -C_{10\mu}^{\rm NP} = -0.65$ (with $C_{ie}^{(\prime)\rm NP} = 0$, $i = 9, 10$).

4.2. Additional LFUV observables

One can think of exploiting the angular coefficients in electron and muon modes, J_i^e and J_i^μ, in order to build observables sensitive to certain Wilson coefficients, and in some cases, insensitive to long-distance charm contributions.

Following this idea, in Ref. [18] we proposed

$$B_5 = \frac{J_5^\mu}{J_5^e} - 1 \qquad\qquad B_{6s} = \frac{J_{6s}^\mu}{J_{6s}^e} - 1 \qquad\qquad M = \frac{B_5 B_{6s}}{B_{6s} - B_5} \tag{4}$$

The observables B_5 and B_{6s} are form factor independent at all orders (up to corrections because of the different lepton masses) in the SM and share the charm-loop protection properties of the Q_i observables. In addition, B_5 and B_{6s}, assuming no right-handed currents, are proportional to the difference $(C_{10\mu} - C_{10e})$ which gives them unique capabilities for testing NP effects in C_{10}.

Finally, the last LFUV observable proposed in Ref. [18] is the M observable. This observable has the very singular property of being insensitive to contributions coming from long-distance charm-loops not only in the SM but also in presence of NP only in C_9, assuming *transversity-independent charm*[2]. If we consider *transversity-dependent charm* contributions or allow for the presence of NP in C_{10}, charm effects reemerge in M. Even though, its strong shielding from hadronic uncertainties makes M very sensitive to NP at low-q^2.

5. Conclusions

Recent experimental results provided by the LHCb and Belle experiments are showing a pattern of tensions with SM predictions in several $b \to s\ell^+\ell^-$ decay modes. Global analyses including all the available data show that scenarios with a large negative $C_9^{\rm NP}$ are preferred over the SM by typically more than 4σ. Explanations for the anomalies in terms of non-factorisable power corrections, although not fully excluded, are clearly disfavored by the theoretical arguments

[2] Charm-loop contributions that do not depend on the helicity of the final state, $C_9^{c\bar{c}} = C_9^{c\bar{c}\,\perp} = C_9^{c\bar{c}\,\parallel} = C_9^{c\bar{c}\,0}$.

provided in [16] and also by the appearance of tensions in the LFUV observables R_K and Q_5, measured by LHCb and Belle respectively. If confirmed, this tensions would unambiguously point to a NP scenario involving different couplings for muons and electrons. The current situation urges for the measurement of the newly proposed lepton flavour universality tests.

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