Relation between mass splittings in isomultiplets of the baryon octet

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Abstract

The mass-difference relation is derived in the framework of the following principal assumptions: 1) mass splittings in isomultiplets are solely long-distance hadronic effects; 2) these effects are not purely electromagnetic in origin; and 3) the non-electromagnetic part of the mass differences in the isomultiplets of the baryon octet is due to the $\pi^0 - \eta$ mixing effects. The equation derived seems to be attractive. Firstly, in this equation there are no free parameters. Secondly, it reduces to the well-known Coleman-Glashow relation when $M_N = M_\Sigma = M_\Xi$. Thirdly, it is in good agreement with experimental data.

1 Introduction

So far there is no model explaining reasonably mass splittings in hadronic isomultiplets. The search for such a model is one the oldest problems in hadronic physics and its solution [1–8] has proved to be more difficult than it might have been expected.

The most popular approach to the problem is based on the study of dynamical quark models [1–4]. However, this way of calculating mass differences in isomultiplets has thus far yielded no unique results since such models contain experimentally uncertain parameters. Moreover, this drawback is characteristic of any model [1, 5–8].

We shall consider the mass differences in isomultiplets of the baryon octet within the framework of the approach which has been proposed in the papers [7,8]. However, in this paper we are going to concentrate on derivation of the mass-splitting relation without direct estimation of the mass differences.

2 The long-distance model

The mass-splitting relation is derived under the following principal assumptions: 1) mass splittings in isomultiplets are solely long-distance hadronic effects; 2) these effects are not purely electromagnetic in origin; and 3) the non-electromagnetic part of the mass differences in the isomultiplets of the baryon octet is due to the $\pi^0 - \eta$ mixing effect.

We consider a mass splitting as a self-energy shift of states within a given isomultiplet. The main contributions to the mass shift in isomultiplets of the baryon octet arise from the self-energy diagrams containing the first few low-lying virtual states: $B + \gamma$, $B + \pi^0$, and $B + \eta$.

By the Feynman rules, a meson exchange yields the mass shift

$$\delta M_B^\Pi = \frac{g_{BB\Pi}^2}{2M_B i} \int \frac{d^4 q}{(2\pi)^4} \bar{u}(p)\gamma_5 \frac{1}{\hat{p} - \hat{q} - M_B} \gamma_5 u(p) \frac{[F(q^2)]^2}{q^2 - \mu_\Pi^2}, \tag{1}$$

where $\hat{p} = \gamma_\mu p^\mu$, $\hat{q} = \gamma_\mu q^\mu$ and γ_5 are the Dirac operators; $u(p)$ is the baryon spinor; p is the momentum of the baryon in the initial and final states; q is the momentum of a meson in the intermediate state; M_B and μ_Π are, respectively, the baryon and meson masses; $F(q^2)$ is the hadronic form factor of a baryon; $g_{BB\Pi}$ is the Yukawa coupling constant.

We suppose the function $F(q^2)$ in Eq.(1) to be universal for the octet and quite similar to the hadronic form factor of a proton:

$$F(q^2) = (1 - q^2/m^2)^{-1}, \quad m = 324\, MeV. \tag{2}$$

This is the empirical form factor which has been proposed by Ferrari and Selleri [9, 10].

Another assumption is that the relations between the Yukawa coupling constants are defined by the standard $SU(3)$ invariant term [10, 11]:

$$L_{int} = \sqrt{2}g(M_B/M_N)(FTr[B,\bar{B}]\Pi + DTr\{B,\bar{B}\}\Pi), \tag{3}$$

where F and D are parameters of the F and D couplings.

Certainly, in the absence of the isospin symmetry breaking, the π^0 and η exchanges cannot split masses of particles in an isomultiplet because the shifts for all its members should be the same in this case. However, when breaking is turned on, the physical states are

$$\begin{aligned} |\pi^0>&=|\pi'>\cos\theta - |\eta'>\sin\theta, \\ |\eta>&=|\pi'>\sin\theta + |\eta'>\cos\theta, \end{aligned} \tag{4}$$

where $|\pi'>$ and $|\eta'>$ are the pure isospin states and θ is the $\pi^0-\eta$ mixing angle.

The $\pi^0-\eta$ mixing generates contribution to isovector mass differences while similar contributions to isotensor differences cancel themselves completely. A set of Eqs. (1)–(4) leads to the following expressions for contributions from the $\pi^0-\eta$ mixing:

$$\begin{aligned} \Delta^{\pi-\eta}_{n-p} &= (1/8\sqrt{3}\pi^2)g^2 m\sin 2\theta\,(3F^2-D^2+2FD)I_N, \\ \Delta^{\pi-\eta}_{\Xi^--\Xi^0} &= (1/8\sqrt{3}\pi^2)g^2 m\sin 2\theta\,(-3F^2+D^2+2FD)(M^2_\Xi/M^2_N)I_\Xi, \\ \Delta^{\pi-\eta}_{\Sigma^--\Sigma^+} &= (1/8\sqrt{3}\pi^2)g^2 m\sin 2\theta\,4FD(M^2_\Sigma/M^2_N)I_\Sigma, \\ \Delta^{\pi-\eta}_{\Sigma^++\Sigma^--2\Sigma^0} &= 0. \end{aligned} \tag{5}$$

The numerical estimate [8] yields for integral I_B the values:

$$I_N = 0.498, \qquad I_\Sigma = 0.511, \qquad I_\Xi = 0.516. \tag{6}$$

As to the electromagnetic splittings, they have been thoroughly studied. According to [1], it should be expected that the magnitudes of the contributions from the γ exchanges are

$$\begin{aligned} \Delta^\gamma_{p-n} = 0.79\,MeV, &\qquad \Delta^\gamma_{\Xi^--\Xi^0} = 1.1\,MeV, \\ \Delta^\gamma_{\Sigma^--\Sigma^+} = 0.22\,MeV, &\qquad \Delta^\gamma_{\Sigma^++\Sigma^--2\Sigma^0} = 1.54\,MeV. \end{aligned} \tag{7}$$

Taking into account values (6) for I_B and values (7) for Δ^γ_B we assume approximately

$$\begin{aligned} I_N &= I_\Sigma = I_\Xi; \\ \Delta^\gamma_{p-n} &= \Delta^\gamma_{\Xi^--\Xi^0} = (1/2)\Delta^\gamma_{\Sigma^++\Sigma^--2\Sigma^0}, \qquad \Delta^\gamma_{\Sigma^+-\Sigma^-} = 0. \end{aligned}$$

3 The mass-difference relation

With these assumptions and the use of (5) the summation of the contributions from the $\pi^0-\eta$ mixing and the γ^0 exchanges gives for the mass differences the following expressions:

$$\begin{aligned} M_n - M_p &= a\,(3F^2-D^2+2FD) - \Delta^\gamma, \\ M_{\Xi^-} - M_{\Xi^0} &= a\,(-3F^2+D^2+2FD)(M^2_\Xi/M^2_N) + \Delta^\gamma, \\ M_{\Sigma^-} - M_{\Sigma^+} &= a4FD(M^2_\Sigma/M^2_N), \\ M_{\Sigma^+} + M_{\Sigma^-} - 2M_{\Sigma^0} &= 2\Delta^\gamma, \end{aligned} \tag{8}$$

where $a = (1/8\sqrt{3}\pi^2)g^2 m\sin 2\theta\, I$.

¿From a set of equations (8) one finds

$$\begin{aligned} \frac{M_{\Sigma^-} - M_{\Sigma^+}}{M^2_\Sigma} &= \frac{M_n - M_p}{M^2_N} + \frac{M_{\Xi^-} - M_{\Xi^0}}{M^2_\Xi} \\ &+ \frac{1}{2}\left(\frac{1}{M^2_N} - \frac{1}{M^2_\Xi}\right)(M_{\Sigma^+} + M_{\Sigma^-} - 2M_{\Sigma^0}). \end{aligned} \tag{9}$$

4 Conclusion

The mass-splitting relation derived seems to be attractive. Firstly, in this equation there are no free parameters. Secondly, it reduces to the well-known Coleman-Glashow relation [12] if one sets $M_\Xi = M_\Sigma = M_N$:

$$M_{\Sigma^-} - M_{\Sigma^+} = M_n - M_p + M_{\Xi^-} - M_{\Xi^0}$$

The substitution of the experimental values [13] for the particle masses and mass differences in Eq. (9) gives the following findings: value of its left-hand side is $(5.68 \pm 0.05)\, TeV^{-1}$ and the value of its right-hand side is $(5.61 \pm 0.34)\, TeV^{-1}$. Thus formula (9) is in good agreement with experimental data.

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