

DRELL-YAN PROCESSES IN NUCLEAR TARGETS

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Corrections to cross sections for the Drell-Yan process on a nuclear target due to the change of quark distributions inside the nucleus are estimated. It is shown that these corrections should in principle be observable, especially for pion beams.

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1. Introduction

Recent EMC data have shown that the electromagnetic structure function measured on iron F_2^{Fe} differs from that measured on deuterium F_2^{D} (so-called EMC effect) [1]. Corrections for neutron excess and Fermi motion [2] turn out not to reproduce the data. This effect shows that a change of shape of quark densities inside nuclear targets (as compared to free nucleon) takes place [3]. Several explanations of this phenomenon were recently proposed [4-8]. Here we investigate how the difference in the distribution of the quarks can influence the Drell-Yan process. In the first part of our paper we try to do this in a model-independent way. Later on we use a specific model [6] in order to obtain quark densities in the nuclear target.

2. General (model-independent) considerations

The structure function of an isoscalar target can be written as

$$\frac{1}{x} F_2^{\text{T}}(x) = \frac{4}{9} u^{\text{T}}(x) + \frac{1}{9} d^{\text{T}}(x) + \frac{4}{3} s^{\text{T}}(x) = \frac{1}{2} \frac{5}{9} (u^{\text{p}}(x) + d^{\text{p}}(x)) + \frac{4}{3} s^{\text{p}}(x), \quad (1)$$

where u, d, s denote quark densities respectively of up, down and sea quark, indices T and p denote target and proton inside nucleus respectively.

The above formula is valid for any kind of isoscalar target, although quark densities can be different for different nuclei.

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We are considering massive muon pair production by quark-antiquark annihilation (i.e. the Drell-Yan process [9]). Using standard model [10] we obtain the following formula for cross sections:

$$\frac{d\sigma}{dx_1 dx_2} = K \frac{\sigma_0}{x_1 x_2} N_{q\bar{q}}(x_1, x_2) \quad (2)$$

where $N_{q\bar{q}} = \frac{1}{3} \sum_{l=u,d,s} e_l^2 [q_l(x_1) \bar{q}_l(x_2) + q_l(x_2) \bar{q}_l(x_1)]$; $\sigma_0 = 4\pi\alpha^2/3s$, e_l denotes the quark charge, $\bar{q}_l$, q_l are the antiquark and quark density respectively, s is CM energy squared and K is a multiplicative constant called K -factor. $N_{q\bar{q}}$ can be understood as the probability of finding a quark-antiquark pair with momentum fractions x_1, x_2 respectively.

For pion beams equation (2) implies

$$\begin{aligned} \frac{1}{A} \left(\frac{d\sigma}{dx_1 dx_2} \right)^{\pi^-} &= K \frac{\sigma}{x_1 x_2} \bar{u}^{\pi^-}(x_1) \left(\frac{4}{9} u^T(x_2) + \frac{5}{9} s^T(x_2) \right), \\ \frac{1}{A} \left(\frac{d\sigma}{dx_1 dx_2} \right)^{\pi^+} &= K \frac{\sigma}{x_1 x_2} \bar{d}^{\pi^+}(x_1) \left(\frac{1}{9} d^T(x_2) + \frac{5}{9} s^T(x_2) \right), \end{aligned} \quad (3)$$

if we neglect the meson sea contribution. This approximation is a good one because the process is dominated by annihilation of valence antiquark from the pion.

For $x_2 > 0.35$ we can neglect also the target sea and for the sum of π^+ and π^- induced cross sections (remembering that $\bar{u}^{\pi^-} = \bar{d}^{\pi^+}$) we obtain from (3):

$$\begin{aligned} \frac{1}{A} \left(\frac{d\sigma}{dx_1 dx_2} \right)^{\pi^+} + \frac{1}{A} \left(\frac{d\sigma}{dx_1 dx_2} \right)^{\pi^-} &= \frac{1}{A} \left(\frac{d\sigma}{dx_1 dx_2} \right)^{\pi^\pm} \\ &= K \frac{\sigma_0}{x_1 x_2} \bar{u}^{\pi^\pm}(x_1) \left(\frac{4}{9} u^T(x_2) + \frac{1}{9} d^T(x_2) \right). \end{aligned} \quad (4)$$

This leads to following expression for the ratio of Drell-Yan cross sections measured on different targets:

$$\frac{\frac{1}{A_1} \left(\frac{d\sigma}{dx_1 dx_2} \right)^{\pi^\pm}}{\frac{1}{A_2} \left(\frac{d\sigma}{dx_1 dx_2} \right)^{\pi^\pm}} = \frac{\frac{1}{9} d_{A_1}^T(x_2) + \frac{4}{9} u_{A_1}^T(x_2)}{\frac{1}{9} d_{A_2}^T(x_2) + \frac{4}{9} u_{A_2}^T(x_2)} = \frac{F_2^{A_1}}{F_2^{A_2}}. \quad (5)$$

Here we assumed that $x_2 > 0.35$ and that the K factor does not depend on the number of nucleons in target A_i [11].

From (5) it follows that the ratio of sums of π^+ and π^- induced cross sections measured in the Drell-Yan process on different targets for $x_2 > 0.35$ should behave like the ratio of structure functions measured in DIS. This conclusion does not depend upon any particular assumption about shape of quark distribution in the target.

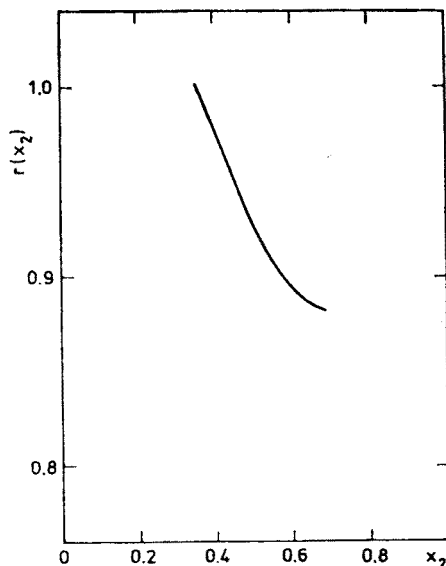


Fig. 1. Ratio $r(x_2)$ of Drell-Yan $\left(\frac{d\sigma}{dx_2}\right)$ pion induced cross sections measured on iron and deuterium vs x_2 , obtained in a model-independent way. The line is fitted to the data [1]

In Fig. 1 we have drawn the ratio of nuclear cross sections to those observed on deuterium (formula (4)). As we have already pointed out this ratio is equal to the ratio of structure functions, which in this kinematical region is smaller than unity.

If we however assume something about change of shape of quark distribution inside nuclei, then we can get some results separately for π^+ and π^- beams. The simplest assumption, again in isoscalar case, is that the shape of valence quark distributions change in the same way for up and down quark. In the region of $x > 0.35$ this implies:

$$\frac{q^T(x)}{q^f(x)} = r(x) = \frac{F_2^T(x)}{F_2^f(x)}, \quad x > 0.35, \quad (6)$$

where q is the quark density and the index f means "free", i.e. one without nuclear corrections (e.g. inside deuterium). $r(x)$ is already plotted in Fig. 1.

Equation (6) is valid under assumption that target is isoscalar, i.e. $u^T(x) = d^T(x) = q^T(x)$. Although the iron target is not isoscalar the discrepancy from isoscalarity is only about 4% ($Z/A = 0.46$); thus our analysis is valid within this limit of accuracy.

Again restricting ourselves to the region $x_2 > 0.35$ we can write:

$$\frac{\left(\frac{d\sigma}{dx_2}\right)_A^{\pi^-}}{\left(\frac{d\sigma}{dx_2}\right)_f^{\pi^-}} = \frac{\left(\frac{d\sigma}{dx_2}\right)_A^{\pi^+}}{\left(\frac{d\sigma}{dx_2}\right)_f^{\pi^+}} = r(x_2). \quad (7)$$

Other cross sections, e.g. $m^3 d\sigma/dm$ as a function of $\sqrt{\tau}$ ($\tau = x_1 x_2$), can also be calculated in principle, but restriction to the region $x_1, x_2 > 0.35$ (where the sea contributions can be neglected) requires that $\sqrt{\tau} > 0.592$.

3. Δ isobar model

Somewhat more specific predictions can be made using a particular model to obtain the parametrization of quark densities inside the nuclear target. In this paper we use Δ isobar model proposed recently by Szwed [6]. In his model it is assumed that pions inside nucleus resonate with some nucleons and form Δ isobars [12]. This admixture of Δ isobars changes effectively quark distributions in the target, though remaining nucleons (i.e. those which do not resonate into Δ isobars) have their quark distributions unchanged.

Detailed shape of Δ isobar-caused corrections depends on a manner in which nucleons are replaced by Δ 's. As a first approximation we take into account only processes like $pn \rightarrow \Delta^+ \Delta^0$ (or $pp \rightarrow \Delta^{++} \Delta^0$ and $nn \rightarrow \Delta^+ \Delta^-$ with equal weights), i.e. we assume that Δ isobar-induced correction is isosinglet.

We can now write the target structure function as:

$$F_2^T = \left(\alpha - \frac{\beta}{2} \right) F_2^p + \left(1 - \alpha - \frac{\beta}{2} \right) F_2^n + \frac{1}{2} \beta (F_2^{\Delta^+} + F_2^{\Delta^0}), \quad (8)$$

where $F_2^p, F_2^n, F_2^{\Delta^+}, F_2^{\Delta^0}$ denote respectively proton, neutron, Δ^+, Δ^0 structure functions, $\alpha = Z/A$ where Z is the number of protons in the nucleus with A nucleons and β denotes the fraction of nucleons changed to Δ isobars.

If we assume that equation (8) is valid not only for the whole structure function but also for quark densities, we obtain:

$$\begin{aligned} u^T &= \left(\alpha - \frac{\beta}{2} \right) u^p + \left(1 - \alpha - \frac{\beta}{2} \right) d^p + \frac{\beta}{2} (u^{\Delta^+} + u^{\Delta^0}), \\ d^T &= \left(\alpha - \frac{\beta}{2} \right) d^p + \left(1 - \alpha - \frac{\beta}{2} \right) u^p + \frac{\beta}{2} (d^{\Delta^+} + d^{\Delta^0}), \\ s^T &= (1 - \beta) s^p + \beta s^{\Delta}. \end{aligned} \quad (9)$$

It is easily seen that quark densities in the target can be divided into two parts: "ordinary" quark densities $q(x)$ and corrections $\delta q(x)$. These corrections correspond to quark density corrections in [4].

Following [6] we assume for Δ quark densities:

$$\frac{1}{2} u^{\Delta^+} = \frac{1}{2} d^{\Delta^0} = d^{\Delta^+} = u^{\Delta^0} = d^p. \quad (10)$$

That comes from simple isospin symmetry arguments for Δ quadruplet.

Using (10) we can rewrite (9) extracting δq and obtaining:

$$\delta u^T = \frac{\beta}{2} (2d^p - u^p), \quad \delta d^T = \frac{\beta}{2} (2d^p - u^p), \quad \delta s^T = \beta (s^{\Delta} - s^p). \quad (11)$$

Again following [6] we assume that s^Δ distribution differs from s^p only in normalization, but not in shape. Normalization should be chosen in such a way that the total momentum fraction carried by valence and sea quarks is equal for proton and for Δ . It has been noticed [4], [6], however, that slight decrease of momentum carried by gluons (i.e. increase of momentum carried by sea) helps to achieve better agreement with the data.

Corrections in quark densities lead to changes in Drell-Yan cross sections. Using densities given by (9), one obtains from equation (2) for proton and pion beams:

$$\begin{aligned}
 N_{q\bar{q}}^p &= \frac{1}{3} \left(\frac{4}{9} u^p(x_1) + \frac{1}{9} d^p(x_1) + \frac{4}{3} s^p(x_1) \right) [s^p(x_2) + \beta(s^\Delta(x_2) - s^p(x_2))] \\
 &\quad + s^p(x_1) \left[\left(\frac{1}{9} + \frac{\alpha}{3} - \frac{5}{18} \beta \right) u^p(x_2) + \left(\frac{4}{9} - \frac{\alpha}{3} + \frac{5}{9} \beta \right) d^p(x_2) \right], \\
 N_{q\bar{q}}^{\pi^-} &= \frac{1}{3} (\bar{u}^{\pi^-}(x_1) + s^{\pi^-}(x_1)) \left[\frac{4}{9} \left(\alpha - \frac{\beta}{2} \right) u^p(x_2) + \frac{4}{9} (1 - \alpha + \beta) d^p(x_2) \right. \\
 &\quad \left. + \frac{5}{9} (s^p(x_2) + \beta(s^\Delta(x_2) - s^p(x_2))) \right] + s^{\pi^-}(x_1) \left[\frac{1}{9} (\alpha + \beta) d^p(x_2) \right. \\
 &\quad \left. + \frac{1}{9} \left(1 - \alpha - \frac{\beta}{2} \right) u^p(x_2) + \frac{7}{9} (s^p(x_2) + \beta(s^\Delta(x_2) - s^p(x_2))) \right], \\
 N_{q\bar{q}}^{\pi^+} &= \frac{1}{3} (\bar{u}^{\pi^-}(x_1) + s^{\pi^-}(x_1)) \left[\frac{1}{9} \left(\alpha + \frac{\beta}{2} \right) d^p(x_2) + \frac{1}{9} \left(1 - \alpha - \frac{\beta}{2} \right) u^p(x_2) \right. \\
 &\quad \left. + \frac{5}{9} (s^p(x_2) + \beta(s^\Delta(x_2) - s^p(x_2))) \right] + s^{\pi^-}(x_1) \left[\frac{4}{9} \left(\alpha - \frac{\beta}{2} \right) u^p(x_2) \right. \\
 &\quad \left. + \frac{4}{9} (1 - \alpha + \beta) d^p(x_2) + \frac{7}{9} (s^p(x_2) + \beta(s^\Delta(x_2) - s^p(x_2))) \right]. \tag{12}
 \end{aligned}$$

Extracting nuclear corrections we get:

$$\begin{aligned}
 \Delta N_{q\bar{q}}^p &= \frac{1}{3} \beta \left(\frac{4}{9} u^p(x_1) + \frac{1}{9} d^p(x_1) + \frac{4}{3} s^p(x_1) \right) (s^\Delta(x_2) - s^p(x_2)) \\
 &\quad + \frac{5}{18} \beta s^p(x_1) (2d^p(x_2) - u^p(x_2)) \\
 \Delta N_{q\bar{q}}^{\pi^-} &= \frac{1}{3} \beta (\bar{u}^{\pi^-}(x_1) + s^{\pi^-}(x_1)) \left[\frac{2}{9} (2d^p(x_2) - u^p(x_2)) + \frac{5}{9} (s^\Delta(x_2) - s^p(x_2)) \right] \\
 &\quad + \frac{1}{18} \beta s^{\pi^-}(x_1) [(2d^p(x_2) - u^p(x_2)) + 14(s^\Delta(x_2) - s^p(x_2))], \\
 \Delta N_{q\bar{q}}^{\pi^+} &= \frac{1}{3} \beta (\bar{u}^{\pi^-}(x_1) + s^{\pi^-}(x_1)) \left[\frac{1}{18} (2d^p(x_2) - u^p(x_2)) + \frac{5}{9} (s^\Delta(x_2) - s^p(x_2)) \right] \\
 &\quad + \frac{1}{9} \beta s^{\pi^-}(x_1) [2(2d^p(x_2) - u^p(x_2)) + 7(s^\Delta(x_2) - s^p(x_2))], \tag{13}
 \end{aligned}$$

from (2) it follows that:

$$\Delta r = \frac{\Delta \left(\frac{d\sigma}{dx_1 dx_2} \right)}{\left(\frac{d\sigma}{dx_1 dx_2} \right)_f} = \frac{\Delta N_{q\bar{q}}(x_1, x_2)}{N_{q\bar{q}}^f(x_1, x_2)}, \tag{14}$$

because kinematic terms are equal for $\Delta(d\sigma/dx_1 dx_2)$ and $(d\sigma/dx_1 dx_2)_f$. Here $\Delta(d\sigma/dx_1 dx_2)$ denotes correction to the Drell-Yan cross section caused by presence of Δ isobars inside nucleus, $(d\sigma/dx_1 dx_2)_f$ is this cross section without corrections.

The formula for the ratio of this correction to the value of uncorrected cross section as a function of x_2 reads:

$$\Delta r(x_2) = \frac{\int_{x_1 \text{ low}}^{x_1 \text{ high}} dx_1 \Delta N_{q\bar{q}}(x_1, x_2)}{\int_{x_1 \text{ low}}^{x_1 \text{ high}} dx_1 N_{q\bar{q}}^f(x_1, x_2)}. \quad (15)$$

We analysed also changes in $m^3 d\sigma/dm$ cross section. For this cross section we get the expression:

$$m^3 \frac{d\sigma}{dm} = \frac{8\pi\alpha^2}{9} \int_{\sqrt{\tau}}^1 \frac{dx_1}{x_1} \tau N_{q\bar{q}} \left(x_1, \frac{\tau}{x_1} \right); \quad x_F > 0, \quad (16)$$

where $x_F = x_1 - x_2$, $\tau = x_1 x_2$.

4. Results

We performed numerical calculations according to the formulae (15) and (16) using the quark structure functions measured in NA3 experiment [13]. All calculations have been made for the iron target with values for sea normalization constant and percentage of Δ isobars inside nucleus fitted to the data [1].

The ratio Δr from (15) is plotted against x_2 in Fig. 2. This ratio falls down with x_2 . Because the target is not isoscalar a small difference between π^+ and π^- beam can be seen.

Analogous ratio but for $m^3 d\sigma/dm$ (equation (16)) for all kinds of beams is plotted versus $\sqrt{\tau}$ in Fig. 3. It is seen that for pions the same pattern is repeated as in Fig. 2. It can be noticed that the corrections for proton beam are almost evenly distributed over whole range of $\sqrt{\tau}$. This implies only multiplicative correction to the cross section which can be difficult to observe due to the presence of the fitted K -factor. The shape of corrections to the proton cross section can be understood by observation that proton beam does not contain valence antiquark, thus all contributions to the Drell-Yan process arise from sea antiquarks. Corrections to the cross section itself can be hardly seen in experimental data because of errors and logarithmic scaling on y axis. The best test would be the measurement of ratios of cross sections on different targets, preferably comparison of nuclear cross sections with the ones measured on deuterium, where the nuclear corrections should be negligible:

$$R_c = \frac{\frac{1}{A} \left(\frac{d\sigma}{dx_2} \right)_A}{\frac{1}{2} \left(\frac{d\sigma}{dx_2} \right)_{D_2}}. \quad (17)$$

Such a measurement eliminates K -factor from the formulas.

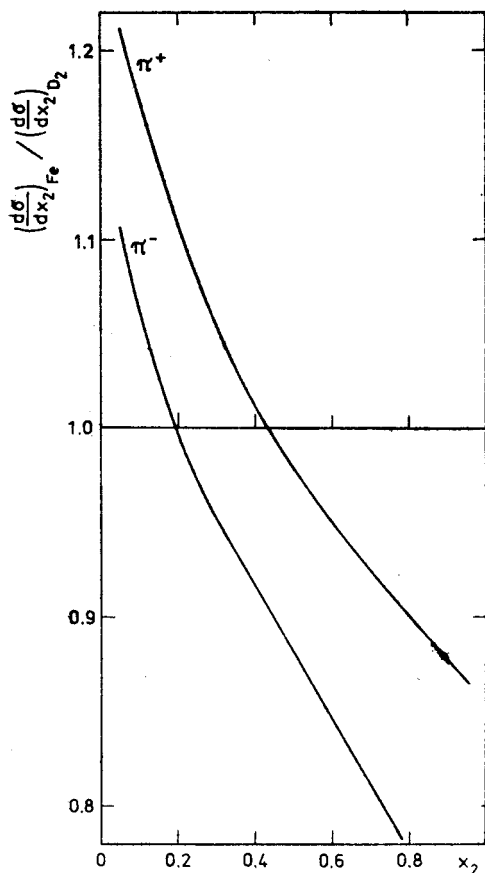


Fig. 2

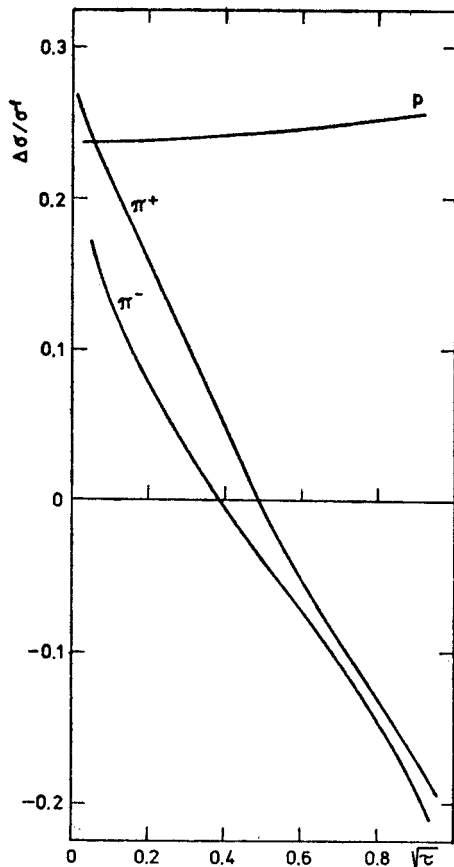


Fig. 3

Fig. 2. Ratio $\Delta r = \Delta \left(\frac{d\sigma}{dx_2} \right)_T / \left(\frac{d\sigma}{dx_2} \right)_f$ for π^+ , π^- beam vs x_2 (cf. (14)). In Figs. 2-6 we used following parameters for the model: $\alpha = 26/56$, $\beta = 0.23$, $c_s = 2.2$ ($s^\Delta = c_s \cdot s^p$)

Fig. 3. Analogous ratio as in Fig. 2 but for $\left(\frac{d\sigma}{dm} \right)$ cross section vs $\sqrt{\tau}$

Predictions for such ratios in the case of iron are drawn in Fig. 4. Unfortunately the largest nuclear effects are in the region of large x_2 where statistics is small, thus experimental errors are large.

Ratios $\frac{1}{A_{Fe}} \left(\frac{d\sigma}{dm} \right)_{Fe} / \frac{1}{2} \left(\frac{d\sigma}{dm} \right)_{D_2}$ versus $\sqrt{\tau}$ are plotted in Fig. 5.

In Fig. 6 experimental results for the ratio $\left(\frac{d\sigma}{dx_2} \right)_{H_2} / \frac{1}{A_{Pt}} \left(\frac{d\sigma}{dx_2} \right)_{Pt}$ versus x_2 are plotted. Theoretical curves with corrections due to Δ isobars are plotted with solid line (while

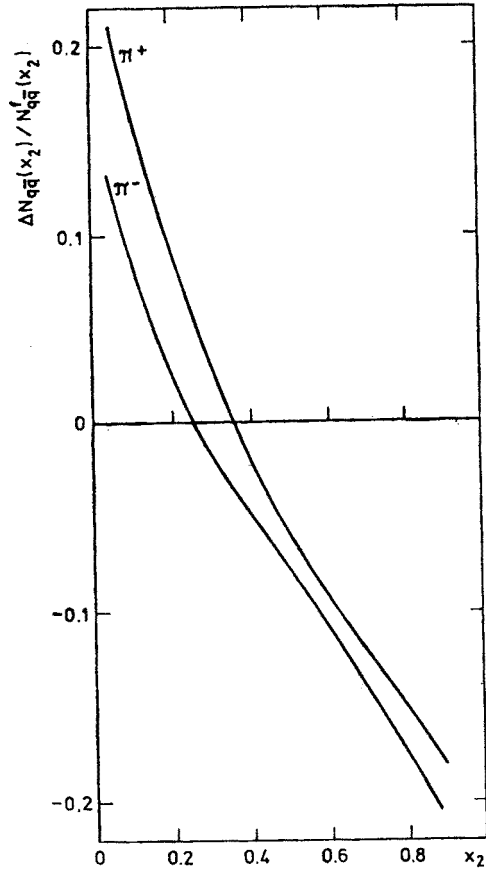


Fig. 4

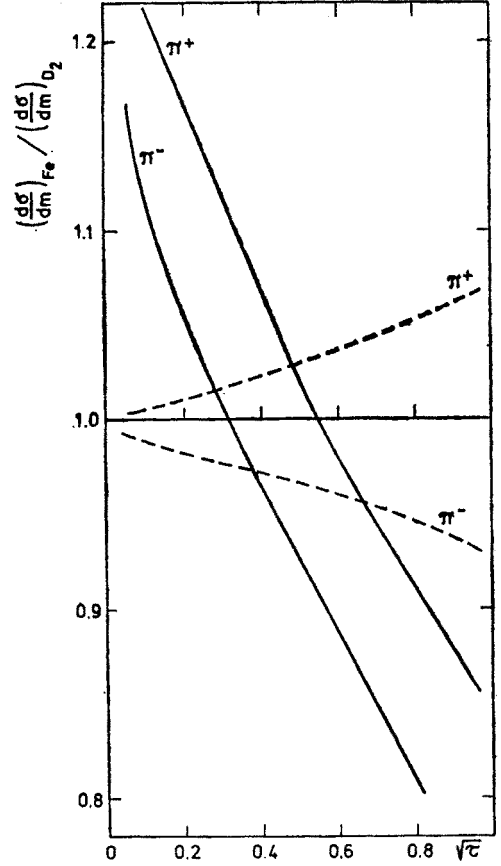


Fig. 5

Fig. 4. Ratio of cross sections $\left(\frac{d\sigma}{dx_2}\right)$ measured on iron and deuterium vs x_2

Fig. 5. Analogous ratio as in Fig. 4 but for $\left(m^3 \frac{d\sigma}{dm}\right)$ cross sections vs $\sqrt{\tau}$. Dashed line denotes uncorrected cross section (i.e. without taking Δ isobars into account), continuous line denotes corrected one

$F_2^{\text{Pt}}/F_2^{\text{D}_2}$ ratio is unmeasured we simply assume for Pt the same β and s^A parameters as for Fe), whereas uncorrected results are plotted with dashed line. Because of large experimental errors it is impossible to rule out any of the curves.

The above considerations depend strongly upon assumptions concerning shape and normalization of s^A , which are not very well motivated. Also the coefficient β as estimated from the data seems to be too large. However we would like to point out that we treat Δ isobar model mostly as a convenient way of parametrising quark density functions and not as a full dynamical explanation of EMC effect.

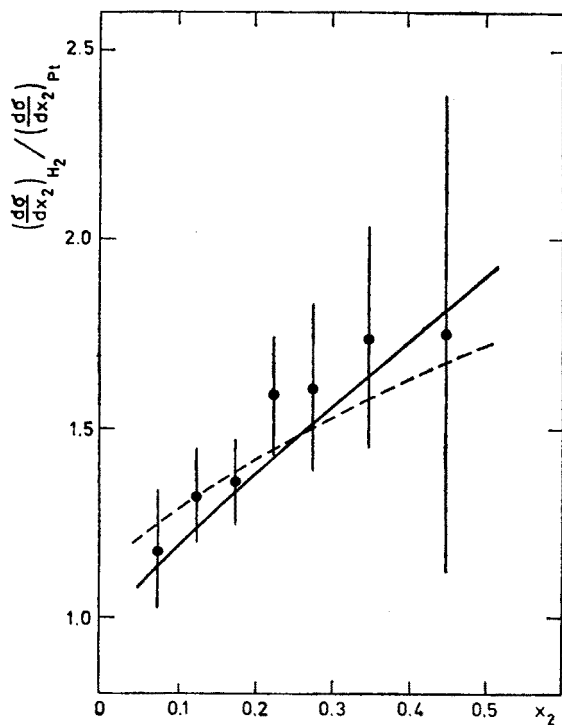


Fig. 6. Ratio $\left(\frac{d\sigma}{dx_2}\right)_{H_2} / \left(\frac{d\sigma}{dx_2}\right)_{Pt \text{ nucl}}$ vs x_2 . Experimental data is taken from NA3 experiment [11]. Dashed line denotes uncorrected (as in Fig. 5), continuous line corrected cross section

5. Conclusions

Drell-Yan cross section is usually assumed to be linear in A , i.e.

$$\left(\frac{d\sigma}{dx_2}\right)_A = A \left(\frac{d\sigma}{dx_2}\right)_{\text{nucl}}, \quad (18)$$

where $\left(\frac{d\sigma}{dx_2}\right)_A$ is the cross section measured on the target A and $\left(\frac{d\sigma}{dx_2}\right)_{\text{nucl}}$ is the cross section per one nucleon. That comes from parton model hypothesis of additive contribution from each nucleon. Changes in shape of quark distributions in nuclear target (in comparison to free nucleons) violate this simple relation, thus instead of (18) we can write:

$$\left(\frac{d\sigma}{dx_2}\right)_A = AR_c(x_2, A) \left(\frac{d\sigma}{dx_2}\right)_{\text{nucl}}, \quad (19)$$

or similarly for distribution in mass (for the definition of R_c cf. (17)).

R_c for $A = 56$ is plotted in Fig. 4 and Fig. 5. The shape of R_c is crucial. In the case of proton $R_c(m)$ does not depend strongly on kinematical variables such as m (cf. Fig. 5).

This leads only to multiplicative correction to the cross section which is hard to extract from the data. Much stronger dependence of R_c on x_2 can be seen in the case of pion beams, so nuclear effects are easier to observe there. Unfortunately corrections due to nuclear effects are of the same order as experimental errors in the available data [11]. A -dependence of nuclear effects can be extracted from β , but at present, having β as a fitted parameter and the data only for iron target it is impossible to tell anything about this dependence.

Summing up, we would like to emphasize that changes in the shape of quark distributions inside nucleus, which explain the behaviour of the $F_2^F/F_2^{D_2}$ ratio, lead to the corrections in the Drell-Yan cross section which in principle can be observed, especially for pion beams. Present lack of experimental evidence for this effect can be attributed to large experimental errors in measured cross sections.

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