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Unitarity with Closed Timelike Curves

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Abstract. We conjecture that, in certain cases, quantum dynamics is consistent in the presence of closed timelike curves. We consider time dependent orbifolds of three dimensional Minkowski space describing, in the limit of large AdS radius, BTZ black holes inside the horizon. Although perturbative unitarity fails, we show that, for discrete values of the gravitational coupling, particle propagation is consistent with unitarity. This quantization corresponds to the quantization of the black hole angular momentum. We perform the computation at very low energies, where string effects are irrelevant and interactions are dominated by graviton exchange in the eikonal regime.

One of the outstanding difficulties of the AdS/CFT correspondence [1] is to understand physics in the bulk of the AdS space in terms of CFT data. In particular, understanding the space–time causal structure of black holes is still a fundamental problem from the view point of the duality. The AdS_3/CFT_2 case is one of the best studied examples of the duality, with extremal black hole geometries given by the BTZ metric [2, 3]

$$ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2 \left(d\phi - N_\phi dt\right)^2 \,, \tag{1}$$

where

$$N = \frac{1}{\ell r}\left(r^2 - r_+^2\right) \,, \qquad N_\phi = \frac{1}{\ell}\frac{r_+^2}{r^2} \,.$$

The AdS_3 radius is given by ℓ, and r_+ is the position of the horizon determining the mass and the angular momentum of the black hole

$$M_{\rm bh} = \frac{\pi M}{4}\left(\frac{2r_+^2}{\ell^2} + 1\right) \,, \qquad J = \frac{\pi M}{2}\frac{r_+^2}{\ell} \,,$$

in terms of the three–dimensional Planck mass[1] M.

In the dual CFT_2 description, these black holes correspond to states with [4]

$$L_0 + \tilde{L}_0 = \ell M_{\rm bh}, \qquad L_0 - \tilde{L}_0 = J,$$

where $L_0, \tilde{L}_0$ are the Virasoro zero modes. For a supersymmetric theory the spin eigenvalue J is naturally quantized in half integral units

$$2J \in \mathbb{Z} \,. \tag{2}$$

[1] For notational convenience, we normalize the Planck mass in terms of the Newton constant G as $M^{-1} = 2\pi G$.

On the other hand, from a purely gravitational view point, the quantization of the angular momentum is rather mysterious. Classically, J is a continuous parameter, and the usual arguments leading to (2) rely on the asymptotic symmetries of quantum gravity on AdS_3 [5], and therefore implicitly on the existence of a dual CFT_2.

A basic property of the BTZ black holes is the existence of closed causal curves (CCC's) in the geometry. Therefore, if we ignore the dual CFT description, we naively expect that quantum gravity in the BTZ geometry violates unitarity. By studying quantum field theory in the flat space limit $\ell \to \infty$ of the BTZ geometry, we shall show that the quantization condition (2) can be obtained by demanding that quantum propagation of fields is consistent with unitarity, even in the presence of CCC's. More specifically, we will consider corrections to free propagation of scalar fields due to interactions with particles winding around the closed timelike direction, as shown in Figure 1.

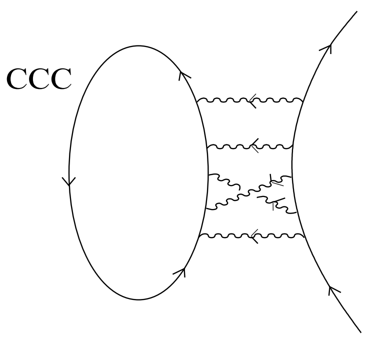


Figure 1. Leading correction to the free propagation of a scalar field due to gravitational interactions with virtual particles winding closed causal curves.

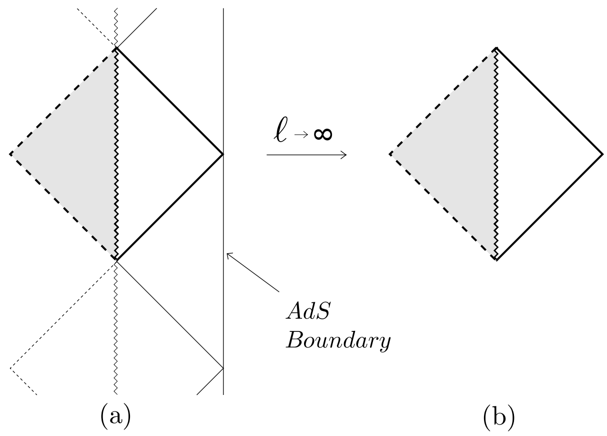


Figure 2. Penrose diagram of the extremal BTZ black hole (a). The shaded area represents the region behind the chronological singularity, where closed causal curves are present. In the limit $\ell \to \infty$, J fixed, one focuses on the region inside the black hole horizon and obtains a flat space orbifold with Penrose diagram (b).

Let us recall the Penrose diagram of the extremal BTZ black hole given in Figure 2a. In the flat space $\ell \to \infty$ limit, keeping the energy scale

$$E = \frac{\ell}{(2\pi r_+)^2}$$

fixed, the region inside the black hole horizon becomes an orbifold of flat Minkowski space $\mathbb{M}^3/e^{\kappa}$, introduced in [7]. Choosing coordinates $x^{\pm}, x$ on $\mathbb{M}^3$, such that the metric is

$$ds^2 = -2dx^+dx^- + dx^2 \,,$$

the orbifold generator κ is the Killing vector

$$\kappa = i\left(L_{+x} + E^{-1}K_-\right) = -\left(x^-\partial_x + x\partial_+\right) + E^{-1}\partial_- \,,$$

where L_{ab}, K_a are, respectively, the generators of Lorentz transformations and translations, and where E parameterizes inequivalent orbifolds.

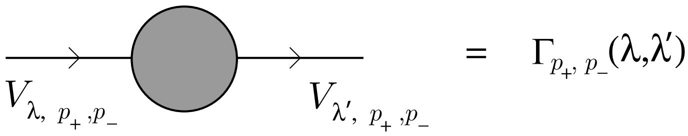


Figure 3. Correction to the free scalar propagator due to interactions. The conserved momenta $p_\pm$ flow through the diagram, whereas λ, λ' are the off–shell mass squared of the external legs.

Under a change of coordinates the metric can be written as

$$ds^2 = -2dy^+dy^- + 2Ey\left(dy^-\right)^2 + dy^2 \tag{3}$$

and the Killing vector as

$$\kappa = \frac{1}{E}\frac{\partial}{\partial y^-}\,.$$

The direction y^- is therefore compact with period

$$y^- \sim y^- + \frac{1}{E}\,.$$

This geometry focuses on the region inside the horizon of the extremal BTZ black hole as seen in figure 2b. The quantization of the BTZ black hole angular momentum (2) becomes, in the flat space limit, the condition

$$2J = \frac{M}{4\pi E} \in \mathbb{Z}\,. \tag{4}$$

In this case, on the other hand, one cannot justify this quantization condition with arguments relying on asymptotic symmetries and on the existence of a dual CFT. In fact, the Minkowski space orbifold just described focuses on the region inside the horizons, and the asymptotic AdS boundary is no longer part of the geometry.

We will derive the quantization condition (4) purely within the framework of quantum field theory in the presence of gravitational interactions. From this perspective, (4) is obtained by requiring unitarity in the space $\mathbb{M}^3/e^\kappa$, which possess CCC's. Hence we see that unitarity in the presence of CCC's is related to charge quantization in dual descriptions of the system. The mechanics that protects chronology is rather different than that proposed by Hawking [8], which is based on a large backreaction due to UV effects.

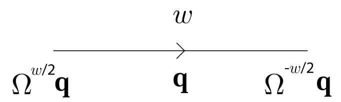
To investigate the possible restoration of unitarity, we shall study the two–point function $\Gamma_{p_+,p_-}(\lambda,\lambda')$ of a scalar field as represented in Figure 3. The external states V_{λ,p_+,p_-}, which are invariant under the action of the orbifold group $\Omega = e^\kappa$, are labelled by the mass squared λ and by the conserved momenta $p_\pm$, conjugate to the Killing vectors $\partial_{y^\pm}$. They are given explicitly by the wave functions

$$V_{\lambda,p_+,p_-}(\mathbf{x}) = \frac{1}{|p_+|}\int dk\, e^{i\left(p_+x^+ + k_-x^- + kx\right)} \times \tag{5}$$

$$\times \exp\frac{i}{2Ep_+^2}\left[\left(2p_+p_- - \lambda\right)k - \frac{k^3}{3}\right], \tag{6}$$

where $k_- = (k^2+\lambda)/(2p_+)$. The full propagator becomes

$$16\pi^2 E\,(\lambda + i\epsilon)\,\delta\left(\lambda-\lambda'\right) + \Gamma_{p_+,p_-}\left(\lambda,\lambda'\right)\,. \tag{7}$$



$$\longrightarrow \qquad \frac{-i}{\mathbf{q}^2+m^2-i\epsilon}\; e^{\frac{i}{E}\left(wq_-+\frac{w^3}{24}\,q_+\right)}$$

Figure 4. Scalar propagator for a particle winding w times the compact y^- direction. The incoming and outgoing momenta are related by the action of the orbifold generator and the usual propagator is multiplied by a momentum dependent phase.

Stable particle states will exist provided the reality condition

$$\Gamma^\star_{p_+,p_-}\left(\lambda,\lambda'\right) = \Gamma_{p_+,p_-}\left(\lambda',\lambda\right) \tag{8}$$

is satisfied. In particular we shall consider the specific kinematics with $p_- \in 2\pi E\mathbb{Z}$ fixed, $\lambda = \lambda' = 0$ and $p_+ \to 0$. The computation that follows is largely independent on the details of the underlying theory because gravitational interactions will be dominant in the specific kinematic regime just described [9].

A key ingredient in the computation is the propagator, which is simply given by the method of images. Denoting the Feynman propagator in the covering space by $\Delta\left(\mathbf{x},\mathbf{x}'\right)$, we can write the full propagator as a sum

$$\left\langle \Phi\left(\mathbf{x}\right)\Phi\left(\mathbf{x}'\right)\right\rangle = \sum_{w\in\mathbb{Z}} \Delta\left(\Omega^w\mathbf{x},\mathbf{x}'\right)\ .$$

The summand $\Delta\left(\Omega^w\mathbf{x},\mathbf{x}'\right)$ can then be written symmetrically as

$$\int \frac{d^3\mathbf{q}}{\left(2\pi\right)^3}\,\frac{-i}{\mathbf{q}^2+m^2-i\epsilon}\; e^{\frac{i}{E}\left(wq_-+\frac{w^3}{24}\,q_+\right)}\;\phi_{\Omega^{-w/2}\mathbf{q}}\left(\mathbf{x}\right)\phi^\star_{\Omega^{w/2}\mathbf{q}}\left(\mathbf{x}'\right)\ ,$$

so that, in Fourier space, a scalar propagator is labeled by a momentum $\mathbf{q}$ and a winding number w. The propagator itself is given by

$$\frac{-i}{\mathbf{q}^2+m^2-i\epsilon}\; e^{\frac{i}{E}\left(wq_-+\frac{w^3}{24}\,q_+\right)}\ .$$

Moreover, as we move along the propagator, the momentum gets transformed under the action of the orbifold group element Ω^{-w}. Therefore, the incoming momentum along the line is $\Omega^{w/2}\mathbf{q}$ and the outgoing one is $\Omega^{-w/2}\mathbf{q}$, as shown in Figure 4.

We will compute the first non–trivial contribution to $\Gamma_{p_+,p_-}\left(\lambda',\lambda\right)$ arising from the graph 5. The only propagator with non–vanishing winding number w is the loop propagator, which probes the non–causal structure of space–time. The bubble in the graph represents the four–point interaction in the parent theory on $\mathbb{M}^3$ to all orders in the couplings. In the limit of $p_+ \to 0$, we will only need control over the parent four–point amplitude in the eikonal kinematical regime, where resummation techniques are known and where general arguments indicate that interactions are dominated by graviton exchange. The full eikonal amplitude for spin–2 exchange reads [10, 11, 12]

$$1 + i\mathcal{A} \simeq -4iM\,\frac{s^2}{tM^2+s^2-i\epsilon}\,,$$

with poles in the physical region placed at

$$s,u = \pm\left(M\sqrt{-t}+i\epsilon\right)\ . \tag{9}$$

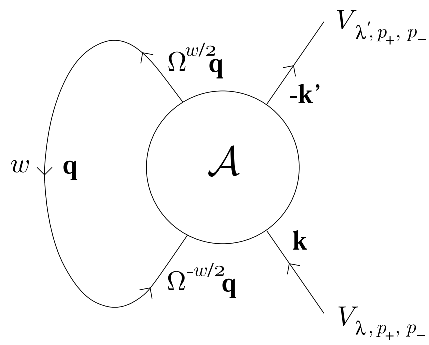


Figure 5. Leading contribution to the two–point function $\Gamma_{p_+,p_-}(\lambda, \lambda')$. The loop momentum has non–vanishing winding number w, whereas the blob $\mathcal{A}$ represents the four–point amplitude in the parent theory to all orders in the coupling constants.

We shall assume that the amplitude $\mathcal{A}$ has poles given by (9) also in the off–shell regime needed for the computation.

In the limit considered the two–point amplitude can be written as

$$\Gamma_{p_+,p_-}(0,0) \simeq \sum_{w \neq 0} \left(c_w \, \Gamma_w^+ + c_w^\star \, \Gamma_w^- \right) ,$$

where c_w is a constant and where

$$\Gamma_w^\pm = \int \frac{ds}{2\pi i} \, \frac{e^{\pm \frac{i}{4} \frac{w^2}{pE} s}}{|w| \, s - 4pp_- + i\epsilon} \, \mathcal{A}\left(s, t = -4p^2, u = s - \frac{8pp_-}{|w|} \right) . \tag{10}$$

The constant p is given by $p = \sqrt{2p_+p_-}$. The reality condition (8) is satisfied if

$$\left(\Gamma_w^+ \right)^\star = \Gamma_w^- \tag{11}$$

holds for each value of w. The amplitude $\Gamma_w^\pm$ is given by the contribution of the poles of the integrand in the upper (lower) complex s–plane, as shown in Figure 6. The reality condition

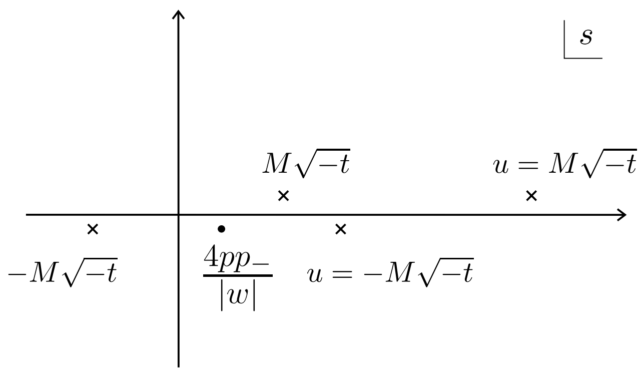


Figure 6. Poles of the integrand in equation (10) in the s–plane. The pole denoted with a dot comes from the winding propagator, whereas the poles marked with a cross come from the eikonal amplitude $\mathcal{A}$.

(11) becomes

$$-e^{i\frac{w^2M}{2E}}F_+ + e^{-i\frac{w^2M}{2E}}F_- = \mathcal{A}_{\text{on-shell}}\,, \tag{12}$$

where $F_\pm$ are real and are related to the residues of $\mathcal{A}$ at the eikonal poles, and where $\mathcal{A}_{\text{on-shell}}$ is the amplitude at the winding propagator pole. In order for (12) to be satisfied for all values of w, we must have that $e^{i\frac{M}{2E}} = 1$ and therefore that

$$\frac{M}{2E} \in 2\pi\,\mathbb{Z}\,,$$

which is the quantization condition (4). In this case, we have the additional requirement on the residues $F_- - F_+ = \mathcal{A}_{\text{on-shell}}$.

We have shown that, using limited information regarding the behavior of the gravitational interaction, we can recover the quantisation condition of the BTZ black hole angular momentum. Only for these discrete values is the theory unitary, even in the presence of closed causal curves.

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References

[1] J. M. Maldacena, *The large N limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys. **2** (1998) 231 [arXiv:hep-th/9711200].

[2] M. Banados, C. Teitelboim and J. Zanelli, *The Black hole in three-dimensional space-time*, Phys. Rev. Lett. **69** (1992) 1849 [arXiv:hep-th/9204099].

[3] M. Banados, M. Henneaux, C. Teitelboim and J. Zanelli, *Geometry of the (2+1) black hole*, Phys. Rev. D **48** (1993) 1506 [arXiv:gr-qc/9302012].

[4] A. Strominger, *Black hole entropy from near-horizon microstates*, JHEP **9802** (1998) 009 [arXiv:hep-th/9712251].

[5] J. D. Brown and M. Henneaux, *Central Charges In The Canonical Realization Of Asymptotic Symmetries: An Example From Three-Dimensional Gravity*, Commun. Math. Phys. **104**, 207 (1986).

[6] L. Cornalba and M. S. Costa, *Unitarity in the Presence of Closed Timelike Curves*, [arXiv:hep-th/0506104].

[7] L. Cornalba and M. S. Costa, *Time-dependent orbifolds and string cosmology*, Fortsch. Phys. **52** (2004) 145 [arXiv:hep-th/0310099].

[8] S. W. Hawking, *The Chronology protection conjecture*, Phys. Rev. D **46** (1992) 603.

[9] G. 't Hooft, *Graviton Dominance In Ultrahigh-Energy Scattering*, Phys. Lett. B **198** (1987) 61.

[10] S. Deser and R. Jackiw, *Classical And Quantum Scattering On A Cone*, Commun. Math. Phys. **118**, 495 (1988).

[11] G. 't Hooft, *Nonperturbative Two Particle Scattering Amplitudes In (2+1)-Dimensional Quantum Gravity*, Commun. Math. Phys. **117**, 685 (1988).

[12] S. Deser, J. G. McCarthy and A. R. Steif, *UltraPlanck scattering in D = 3 gravity theories*, Nucl. Phys. B **412**, 305 (1994) [arXiv:hep-th/9307092].