Attractors in higher-order viscous hydrodynamics for Bjorken flow

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We consider causal higher order theories of relativistic viscous hydrodynamics in the limit of one-dimensional boost-invariant expansion and study the associated dynamical attractor. We obtain evolution equations for the inverse Reynolds number as a function of inverse Knudsen number [1]. The solutions of these equations exhibit attractor behavior. We compare the attractors of the second-order Müller-Israel-Stewart (MIS) [2], transient Denicol-Niemi-Molnar-Rischke (DNMR) [3], and third-order (TO) [4] theories with the exact solution of the Boltzmann equation in the relaxation-time approximation (RTA).

Relativistic dissipative hydrodynamics has been applied quite successfully to explain the collective behaviour observed in high energy heavy-ion collisions. The simplest relativistic dissipative theory, relativistic Navier-Stokes theory, imposes instantaneous constitutive relations between the dissipative flows and their generating forces, expressed through first-order gradients of equilibrium quantities. This approach was found to be plagued by acausality and instability which was rectified in the second-order MIS theory by introducing a relaxation type equation for the dissipative flows and thus turning them into independent dynamical degrees of freedom of the system. As discussed in [5], even the minimal causal conformal theory given by MIS introduces new modes called non-hydrodynamic modes that were absent in Navier-Stokes theory. These non-hydrodynamic modes are now known to play an important role in determining the regime of applicability of hydrodynamics, also known as the "hydrodynamization" process [6]. In the present study, we will focus on yet another interesting feature that appears in a causal theory of relativistic hydrodynamics, "the hydrodynamic attractor" [7].

	β_π	a	λ	χ	γ
MIS	$4P/5$	4/15	0	0	4/3
DNMR	$4P/5$	4/15	10/21	0	4/3
TO	$4P/5$	4/15	10/21	72/245	412/147

TABLE I: Coefficients for causal viscous hydrodynamic evolution of shear stress tensor in Bjorken flow for the three theories considered here.

In the case of longitudinal Bjorken flow expressed in Milne coordinates $x^\mu = (\tau, x, y, \eta_s)$ [with $\tau = \sqrt{t^2 - z^2}$ and $\eta_s = \tanh^{-1}(z/t)$], the evolution equation for non-vanishing shear stress tensor component, $\pi \equiv -\tau^2 \pi^{\eta_s \eta_s}$, can be written in the following generic form [1]:

$$\frac{d\epsilon}{d\tau} = -\frac{1}{\tau}\left(\frac{4}{3}\epsilon - \pi\right), \tag{1}$$

$$\frac{d\pi}{d\tau} = -\frac{\pi}{\tau_\pi} + \frac{1}{\tau}\left[\frac{4}{3}\beta_\pi - \left(\lambda + \frac{4}{3}\right)\pi - \chi\frac{\pi^2}{\beta_\pi}\right]. \tag{2}$$

The coefficients β_π, a, λ, χ, and γ appearing in Eq. (2) and Eq. (4) are tabulated in Table I for the three theories studied in this work.

In terms of the normalized shear stress (inverse Reynolds number) $\bar{\pi} \equiv \pi/(\epsilon + P)$ and the rescaled time variable $\bar{\tau} \equiv \tau/\tau_\pi$ (which is the inverse Knudsen number for Bjorken flow),

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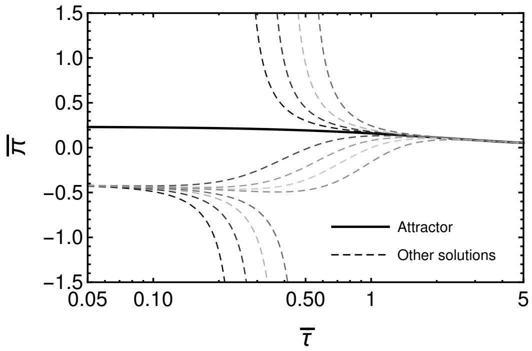

FIG. 1: Attractor behavior of the solutions for the third-order theory. The two (attracting and repulsing) fixed points at $\bar{\tau} \to 0$ are clearly visible.

Eq. (1) can be used to obtain the relation

$$\bar{\pi} = 3\left(\frac{\tau}{\bar{\tau}}\right)\frac{d\bar{\tau}}{d\tau} - 2. \tag{3}$$

Here we also used that for a conformal system $\epsilon \propto T^4$ and $T\tau_\pi = 5\bar{\eta} =$ const. where $\bar{\eta} \equiv \eta/s$ is the specific shear viscosity. Eqs. (2) and (3) can be combined to obtain a first-order nonlinear ordinary differential equation for $\bar{\pi}$ that is completely decoupled from the evolution of the energy density [1]:

$$\left(\frac{\bar{\pi}+2}{3}\right)\frac{d\bar{\pi}}{d\bar{\tau}} = -\bar{\pi} + \frac{1}{\bar{\tau}}\left(a - \lambda\,\bar{\pi} - \gamma\,\bar{\pi}^2\right). \tag{4}$$

The solutions of Eq. (4) posses an interesting feature called the "hydrodynamic attractors" as shown in Fig. 1. The initial condition for the attractor solution is obtained by imposing the boundary conditions that both $\bar{\pi}$ and $d\bar{\pi}/d\bar{\tau}$ remain finite as $\bar{\tau} \to 0$ which results in the quadratic equation $\gamma\,\bar{\pi}^2 + \lambda\,\bar{\pi} - a = 0$ for the initial value of $\bar{\pi}$. One finds two solutions, one of which (the positive root) is stable and corresponds to the unique attractor solution whereas the negative root corresponds to a repulsor. An illustration of this generic behavior is shown in Fig. 1 for the case of the attractor and other solutions of Eq. (4) corresponding to the third-order theory [1].

In Fig. 2, we show the attractor solutions for $\bar{\pi}$ for the MIS, DNMR, and third-order theories, as well as for the exact solution of Boltzmann equation (BE) in the relaxation-time approximation (RTA) [8] and the Navier-Stokes solution. We see that of these the MIS attractor approaches the exact attractor most slowly, while the attractor of the third-order theory exhibits the best agreement with the exact RTA BE attractor [1]. This adds to the evidence of the superior performance of the third-order theory over different variants of second-order theories based on expansions around a locally equilibrated isotropic momentum distribution.

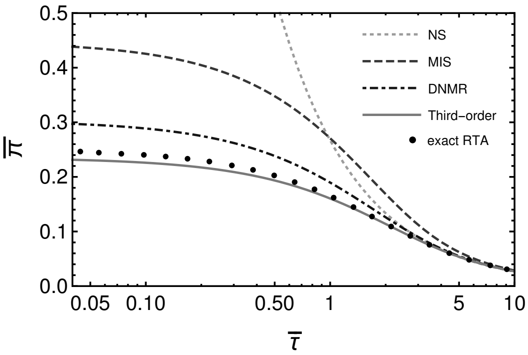

FIG. 2: Attractors for the MIS, DNMR, and third-order theories, compared with the exact numerical attractor of the RTA Boltzmann equation and Navier-Stokes solution.

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