PERFORMANCES OF THE R&D SUPER-ATTENUATOR (SA) CHAIN OF THE VIRGO EXPERIMENT

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A complete long SA chain has been recently put in operation at I.N.F.N. Pisa. The Inverted Pendulum (IP), the Seismic Filters (SF), the Suspension Wires (SW) and any other detail of the apparatus are identical to the final ones which are being installed at the Virgo site in Cascina. The Vertical Transfer Function (VTF) and the Horizontal Transfer Function (HTF) have been measured adopting a simple method based on the product of the single stages attenuation coefficient. A description of the used technique as well as the excellent results obtained by measuring the attenuation coefficient as function of the frequency for the complete chain will be presented.

1 Introduction

The detection of Gravitational Waves is a big challenge to confirm their existence predicted by the theory of General Relativity. Moreover, the direct observation of Gravitational Waves represents an important test of the Einstein's theory and, at the same time, it will open a fascinating field on the Universe investigation. For these reasons many laser interferometer antennas are presently under construction all over the world with the aim to detect gravitational radiation from different sources in the frequency range starting from a few Hz to a few kHz [1, 2, 3].

The detection of these weak signals is related to the sensitivity of the experimental apparatus and its capability to isolate the real signals from the fake ones, usually named "seismic noise". The Virgo Collaboration has developed and built in Pisa a sophisticated suspension system, called Super-Attenuator (SA), which will isolate the interferometer optical components from seismic noise. In this way it is possible either to avoid local effects which can mask gravitational waves detection or to obtain a mirror displacement sensitivity better than $3 \times 10^{-18}\ m/\sqrt{Hz}$ starting from 4 Hz. This corresponds to an obtainable strain sensitivity of $\tilde{h} = 10^{-21} Hz^{-1/2}$ in the low frequency region, where gravitational waves emitted by pulsars and coalescing binaries are expected [4].

The solution adopted to filter seismic noise transmitted to the optical components, is based on a multistage pendulum which is able to drastically reduce local disturbances on the mirror. The Super-Attenuator is, indeed, a cascade of low pass mechanical filters connected each other by means of a metallic suspension wire. In this paper we describe the working principle of an attenuation system based on a multistage pendulum and in the second paragraph we shortly present the Super-Attenuator studied for the Virgo experiment. In the last two paragraphs a simple method to measure the vertical and horizontal transfer functions as well as the very encouraging results obtained with the R&D chain will be presented.

2 The working principle of a multistage pendulum

The general approach used in the design of our attenuation system, is based on a simple idea. Considering a n-stages pendulum it is easy to prove that an horizontal motion of its suspension point, at a frequency f higher than the frequencies of the normal modes ($f > f_0 > f_1 > ... > f_n$), is transmitted to the suspended mass with an attenuation proportional to f^{-2n}. In particular, the ratio between the linear spectral density of the last mass displacement (the optical component) and the linear spectral density of the suspension point displacement, decreases as A/f^{2n} where $A = f_0^2 \cdot f_1^2 \cdot ... \cdot f_n^2$ and n is the number of stages.

With this system a very large attenuation of seismic noise horizontal component can be obtained at a frequency above the highest pendulum resonance. It is important to underline that lowering the pendulum resonant frequency, the attenuation capability of the system at a given frequency can be increased. The pendulum resonance frequency, indeed, depends on its length only and it can be changed by confining the peak in a low frequency region. This means that to reduce as much as possible the transmission of the seismic noise on the last stage of the chain, it would be better to have a cascade of pendula with long connection between two consecutive stages. Moreover, increasing the number of stages with the fixed total length of the chain, the attenuation response of the system in the region above the resonant frequencies can be improved. As a negative consequence of that the normal mode frequencies and then the detector threshold will be increased.

According to the previous considerations, the optimization of an attenuation system, based on a multistage pendulum, passes through the difficult choice of many parameters which give opposite contributions. In the attenuation of vertical vibrations (vertical component of the seismic noise), for instance, a multistage pendulum does not have any effect in isolating the optical components. Any vertical vibration will be partially transmitted to the interferometer laser beam (horizontal direction) because of an unavoidable coupling among different degrees of freedom. A vertical attenuation, comparable with the horizontal one, is reachable by replacing each suspension wire with a spring. This spring should support a big load and, at the same time, it should be soft enough to exhibit a low resonant frequency. Using these cautions it is possible to confine the vertical resonance of the chain in a low frequency region obtaining a strong attenuation starting from a few Hz.

Changes in the phase beam can also be induced by horizontal displacements of the mirror due to the rotations of the pendulum chain around the vertical axis. To confine these rotational mode frequencies below the detection band of our ideal apparatus, each pendulum mass has to be replaced by a structure having a high momentum of inertia. In addition the diameter of the suspension wire, connecting two consecutive stages, has to be small enough to reduce its return torque which, opposing to the rotation of the chain, determines its rotational frequency. In order to decrease the coupling between linear and rotational modes, the suspension of the structure has to be done in a concentric way, so that a seismic vibration will induce a very small torque on the multistage pendulum. At the same time an interconnection of the stages at small distance and as close as possible to their centers of mass, will guarantee a negligible coupling effects on the horizontal motion of the suspended mass with the rotational and tilt modes (rotational mode around the horizontal axis) of the chain.

Assuming an ideal multistage pendulum as suspension system for our mirror, we can write the linear spectral density ($\tilde{X}_m(f)$) of its residual displacement in the horizontal direction as:

$$\tilde{X}_m(f) = \tilde{X}_s(f)\,[HTF(f) + \alpha \cdot VTF(f)]$$

where $\tilde{X}_s(f) \approx \delta/f^2 \; m/\sqrt{Hz}$, and the constant δ ranges from 10^{-9} to $10^{-6} mHz^{3/2}$ (depending on the geographical location of the measurement [5]) is the linear spectral density of the seismic noise acting on the suspension point (assumed isotropic), HTF(f) is the horizontal transfer function, α is the vertical-horizontal coupling factor ($\approx 10^{-2}$) and VTF(f) is the vertical transfer function.

The drawback of using purely mechanical oscillators as displacement attenuators is that they inhibit seismic noise transmission above their resonant frequency, but magnify the amplitude of any excitation at the resonant frequencies. At these frequencies the seismic motion is amplified by a factor proportional to the resonant frequency quality factor. Without low-frequency seismic motion pre-attenuation and proper damping of these modes, the payload is subject to a slow movement, called residual motion of the attenuator chain, that can have an amplitude of several microns in all the three degrees of freedom.

3 The Virgo suspension system

Keeping in mind the idea of a multistage pendulum and all the considerations underlined in the previous paragraph, the Virgo Collaboration has developed and built a long Super Attenuator chain [6]. The prototype, called R&D Chain, has been used to check the solution adopted in the final design as well as the performances of the apparatus. It consists of three fundamental parts: the Inverted Pendulum (IP), the Seismic Filters (SF) connected each other by metallic Suspension Wires and the Last Stage (LS) or payload equipped with a Marionette, a reference mass and a mirror. The only differences between the R&D Chain and the Virgo SA are:

- the length of the metallic wire connecting the suspension point to the first attenuating stage (about 1.5 m shorter), because the entire structure of the chain (about 7 m high) does not fit in the building;
- the cables used during the tests. They were not identical to the Virgo final ones which will be softer in order to avoid damping effects and to reduce internal friction of the cable itself;
- the mirror. It has been replaced with an aluminum disk having the same geometry and weight, but completely different optical characteristics (not relevant for our measurements);
- the attenuation system was not enclosed in a vacuum vessel.

Here below we shortly describe the apparatus used for the test. A more detailed description of each element can be found in [7].

3.1 The Inverted Pendulum (IP)

The IP is an elastic metallic structure on top of which a steel ring (called "Top Ring") rigidly connected to a mechanical filter (hereafter called Filter Zero) is suspended by three thin wires (31 mm long) accommodated on the legs (see fig. 1). The elasticity of the structure is due to three flexible metallic joints. They are screwed onto the legs to form an interconnection element between the upper part and the bottom one of the aluminum pipes. A bottom steel ring, on which the Inverted Pendulum is anchored, completes this pre-isolator stage.

By maintaining the horizontal resonant frequency below 100 mHz, this element performs a pre-isolation of the seismic noise horizontal component while the Filter Zero will reduce the noise transmission in vertical direction. Moreover, because of its soft structure, the IP requires very low forces to move the chain suspension point. This guarantees an easy control of the chain in compensating for the horizontal slow ground motion. Finally, the predicted performances of the IP and the top stage, allowed to design a SA with five filters instead of seven as in the former design. For safety reasons a metallic frame (the Inner Structure) ground connected,

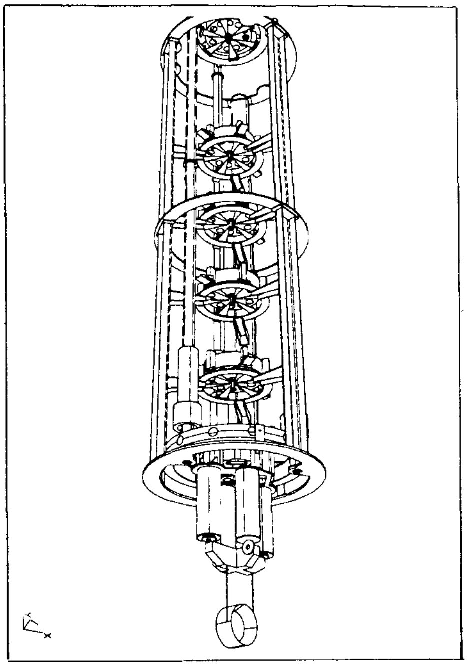

Figure 1: View of an assembled long Super-Attenuator chain.

surrounds the IP and supports a bracket underneath each stage to stop the fall of the filter chain if a suspension wire breaks. An experimental study of the mechanical design as well as the performances of the Inverted Pendulum can be found in [8, 9].

3.2 The Seismic Filters (SF)

In our SA system each pendulum mass has been replaced by a rigid metallic structure drum shaped acting as an oscillator in vertical direction too. A sequence of five mechanical filters guarantees a good seismic noise isolation of the optical components according to the working principle of a multistage pendulum. A detailed description of the design and performances of the single filter can be found in [10].

It is a rigid steel cylinder (70 cm diameter, 18.5 cm high for a total weight of about 100 kg) suspended as close as possible to its center of mass. On the outer circumference of the filter body bottom part, a set of triangular cantilever spring blades are clamped (fig. 2). Each blade is bent at a constant curvature radius and with different base width according to the load to be supported.

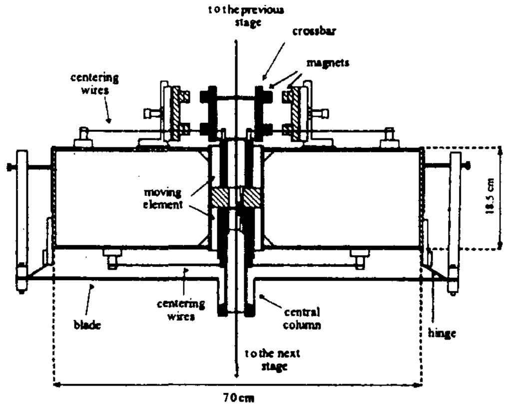

Figure 2: Drawing of a mechanical seismic filter. All the components are indicated

A nominal load ranging between 48 kg and 96 kg hung on the blade tip will force it in flat and horizontal position. The blade tip is connected by a 1mm diameter wire to a central column. inserted through a hole in the center of the filter body. Any movement of the central column, apart in the vertical direction, has been prevented by two sets of four centering wires accommodated on the top and on the bottom of the filter body. A crossbar bolted on the upper part of the central column has been used as a mechanical support for the magnetic anti-spring system described in the next sub-paragraph. In working conditions the central column, the crossbar and the anti-spring system represent the moving part of the mechanical filter from which the load of the lower stages is suspended by a steel suspension wire. Connecting each filter to the other, we can obtain a chain of mechanical oscillator in the vertical direction.

Following the description of our filtering system, is clear that the total load of the chain is hung to the triangular steel blades 3.5 mm thick and 385.5 mm long. The base width of the triangle ranges from 180 mm to 110 mm according to the load to be supported. For the same reason the top filter is equipped with twelve blades and the last one with four.

As calculated, once properly loaded, the main vertical resonance frequency of this system is about 1.5 Hz. In order to reduce the peak amplitude of the blade first flexural mode (about 100 Hz) in the mechanical transfer function, a damper is mounted in the middle of each blade. Another special damper, placed on the crossbar, will suppress a mode at about 50 Hz due to the suspension wire connecting the crossbar to the next filter which acts as a rigid spring.

The material used for the blades construction is a Maraging steel instead of a standard steel to minimize micro-creep effects [11] due to the high load applied. The same material has been used to machine the suspension wires with a nail-head at both ends. Since the suspended load decreases going from the top to the bottom of the chain, the wire and nail-head diameters change along the chain in the range 4-1.85 mm and 8-6 mm respectively [12]. This allows to confine the violin mode in the high frequency band and, reducing as much as possible the diameter of the wire, to control its angular stiffness which determines the rotational frequency around the vertical axis.

The two nail-heads of the wires connecting a filter on the chain to the previous and to the next one, are screwed in the central part of its body at a relative distance of 5 mm, very close to the filter center of mass. This guarantees a small return torque to the rotations of the filter around the horizontal axis and thus a low tilt frequency.

3.3 The Magnetic Anti-springs

The suspension wire length (l) of 1.15 m in our SA, sets the pendulum resonant frequency (f_H) of each stage at:

$$f_H = \frac{1}{2\pi} \cdot \sqrt{\frac{g}{l}} \approx 0.5 \text{ Hz}$$

In vertical direction the stiffness of the triangular blade springs fixes the natural resonant frequency of the filter at about 1.5 Hz. As consequence of this the highest mode of the chain should be at 7.5 Hz, above the designed Virgo detection frequency threshold (4 Hz).

To reduce the vertical stiffness of the blades, and then to confine the main vertical resonant frequency of the filter below the pendulum one (f_H), a system of magnetic anti-springs has been conceived. It consists of two sets of permanent magnets (the first assembled on the crossbar and the second one on the filter body) facing each other (see fig. 3) with opposite horizontal magnetic moment (namely in a repulsive configurations). In this way the two matrices screwed on the crossbar are forced to move in vertical direction only.

The working principle of this system is very simple. When the magnets are perfectly faced the repulsive force has a null vertical component, but as soon as a matrix is moved in vertical

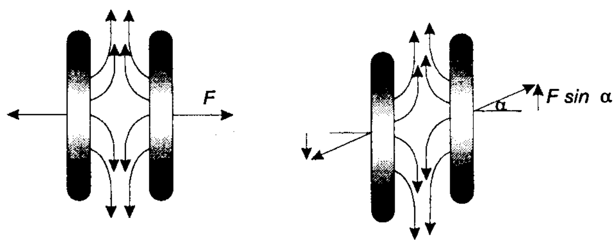

Figure 3: The working principle of the magnetic anti-spring system: when the magnets are displaced in the vertical direction a vertical component of the repulsive force appears.

direction, a vertical component of the magnetic force appears. Considering a small relative displacement (Δy), compared with the distance (d) between two matrices of magnets and with their transverse dimension, the vertical component of the repulsive force (F_y) is proportional to Δy:

$$F_y \approx F_0 \cdot \frac{\Delta y}{d}$$

where F_0 is the module of the repulsive force. A similar device is equivalent to a vertical spring with a negative elastic constant (anti-spring) whose module is F_0/d. Its rest position is thus the position for which the two couple of matrices are perfectly faced.

On a seismic filter the magnetic anti-springs act on the crossbar, the moving part of the filter connected to the next stage, in parallel with the blade springs. So that the vertical modes frequency of the chain are confined below the highest frequency of the horizontal modes. For this reason the detection band will be limited by the horizontal resonant peaks only.

3.4. The Last Stage (LS)

The Last Stage of the SA chain is hung to a mechanical filter historically called "filter 7". It consists of a special element anvil shaped with four wings named "marionette", a reference mass and a mirror. The marionette has been designed with four wings on which four small permanent magnets are accommodated. In front of these magnets four coils attached at the end of four aluminum pipes screwed on the bottom part of the filter 7, are placed. In this way the magnet-coil system and the marionette allow the control of the interferometer optical component in three degrees of freedom: the displacement along the beam direction, the rotation around the vertical axis and the rotation around the horizontal axis perpendicular to the beam. From the marionette four thin wires start. The first couple, 1.9 m long, supports the mirror in a cradle and the second one, with the same length and with the same technique, supports the reference mass.

The final control of the mirror displacement in the beam direction, will be performed by four coils acting on the magnets glued on the back side of the mirror. In order to have the best efficiency, the coils have been mounted on the reference mass, so that, in working conditions, it recoils against the mirror. As final effect the common center of mass will be at rest.

4 A simple method to measure the Transfer Function

The Transfer Function (TF) of a linear mechanical system can be defined as a linear operator connecting the Fourier transform of a displacement X_{in} (in all 6 degree of freedom), considered as an input point, to the Fourier transform of the displacement X_{out} (in all 6 degree of freedom), considered as the output point of the system.

In general the linear operator will be represented by 6×6 matrix and then the TF measurement will consist in determining the 36 elements.

Supposing that our system interacts with the external environment through the input point only, the TF does not depend on the force applied, but on the mechanical elements only. In this case we can write:

$$\begin{aligned} X_{in} &= T_1 \cdot F_{in} \\ X_{out} &= T_2 \cdot F_{in} \end{aligned}$$

and then

$$X_{out} = T_2 \cdot T_1^{-1} \cdot X_{in}$$

and the matrices T_1 and T_2 do not depend on the applied force, because the system is linear. In other words the total TF of our multistage pendulum does not depend on the elements and external forces which are upstream of the input point (X_{in}).

According to these considerations we conclude that the total TF of a multistage attenuation system, can be obtained multiplying the TF of each stage. This is a very important result, because it gives the possibility to measure the ratio between the Fourier transform of the output point displacement and the Fourier transform of an input point displacement (where the force is applied), exciting each stage of our SA chain.

In addition, adopting this simple method, the measurement of the total TF is performed in a wide frequency band. The attenuation capability of our suspension system, indeed, is so strong that after a few Hz it is impossible to accomplish a direct measurement of the 36 elements of the matrix. This is due to the fact that commercial accelerometers are not so sensitive to detect the small mirror residual displacement after five attenuation stages.

5 The Transfer Function Measurements with the R&D chain

In order to evaluate the attenuation efficiency of our suspension system as a function of the frequency, a long SA chain (the R&D chain) has been used to measure the vertical and horizontal TF with the method described above.

During the test period the set-up used was identical to the final one apart a few small details (not relevant for this kind of measurement) as reported in a previous paragraph. For the "stage by stage" measurement the accelerometers used are commercial ones, because they are sensitive enough to detect a typical output signal (one attenuation stage only). A total number of 12 accelerometers (6 for the input point and 6 for the output one) connected to a spectrum analyzer with the same number of channels, would be necessary to evaluate each matrix element of the TF measurement.

In vertical direction each filter, as well as the entire SA, have a weak coupling with the other degrees of freedom. This means that the 6×6 matrix is quasi-diagonal and two accelerometers (one in each filter) are sufficient in determining the diagonal element of the TF between two consecutive stages. Moreover the diagonal term in the final product of matrices (the total VTF) is well represented by the product of their single diagonal terms.

The fig. 4 shows the vertical TF measurement between filter 2 and filter 3. It should be noted that the attenuation factor is about 10^{-2} per stage in the region 10-40 Hz. In fig. 5 the total VTF for the SA chain obtained as product of single stages is showed. After five attenuation stages the ratio between the Fourier transform of the output point displacement and the Fourier transform of an input point displacement, ranges in the interval 10^{-9} - 10^{-13} for the frequency band 0-100 Hz.

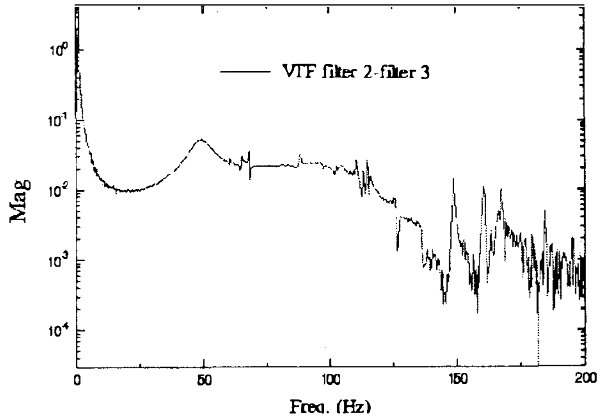

Figure 4: The vertical transfer function between filter 2 and filter 3

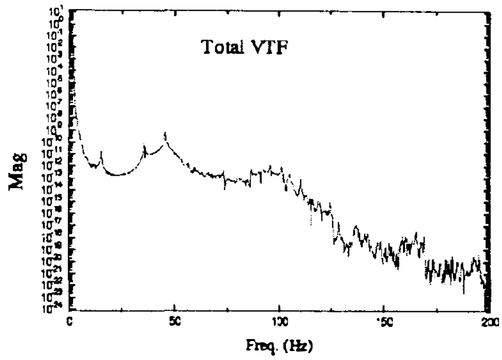

Figure 5: The total VTF obtained by the "stage by stage" method.

In horizontal direction the situation is much more complicated. Any horizontal vibration has a coupling with the other degrees of freedom not negligible. For this reason we carefully excited the filter body in the horizontal direction, so that to minimize the coupling with the rotational degrees of freedom. As an example, in fig. 6 a comparison between vertical and horizontal transfer function for the stage filter 1-filter 2 is reported. From this comparison it is possible to conclude that the total horizontal transfer function will be many orders of magnitude smaller than the vertical one thanks to the better performances of the filters in that direction.

6 Conclusions

The first long SA chain for the VIRGO experiment has been successfully built and tested at I.N.F.N. Pisa. All the construction details are identical to the final ones.

The test results have demonstrated that the system conceived is able to isolate the optical components of the VIRGO apparatus from the seismic noise. In particular, for the vertical component of the seismic noise an attenuation factor of about 10^{-2} per stage, has been measured in the frequency region 10-40 Hz. The horizontal component is attenuated of about 10^{-3} in the same frequency region.

Adopting the "stage by stage" method to measure the total Transfer Function, an attenuation factor ranging between 10^{-9} and 10^{-13}, has been obtained for the vertical component of the seismic noise in the frequency band 0-100 Hz.

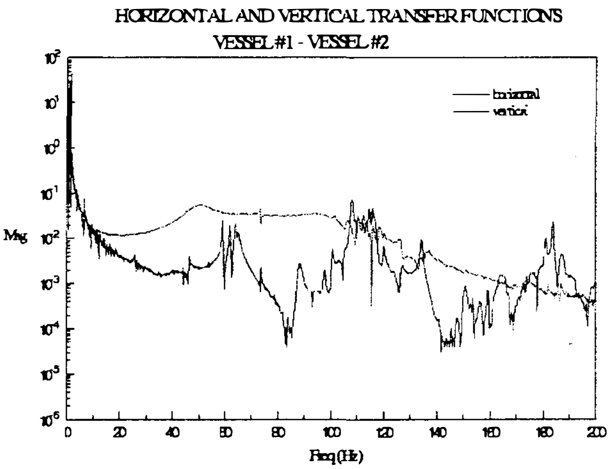

Figure 6: Comparison between vertical and horizontal transfer function for filter 1-filter 2

Acknowledgments

We would like to thank the Virgo Pisa Electronics Group for their fundamental support during the electronics installation and cabling of the chain. We are also grateful to the Virgo Roma Group responsible for the marionette construction.

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