Chapter 18
String Theory

Paolo Di Vecchia

Abstract I start describing my interaction with Bruno during my thesis and then in his group in Frascati in connection with the calculation of the total cross-section of double bremsstrahlung that, at that time, was considered a good candidate as a monitor reaction for Adone. Then I discuss my transition to S-matrix theory and to the work that brought from the Dual Resonance Model to String Theory. I conclude describing the main results of String Theory in a way that could be followed by non-experts in the field.

18.1 Bruno and Me

In my third year of physics in 1964 I decided to follow the course on Statistical Mechanics held by Prof. Bruno Touschek and I was immediately fascinated by his personality. I liked a lot his informal personality, his way of doing and explaining physics and his free spirit. Therefore, at the beginning of 1965, I went to him asking for a thesis. He was very positive and told me to follow his course at the Scuola di Perfezionamento on QED where he was discussing first the Bloch-Nordsieck method and then he went on to discuss the quantisation of the electromagnetic field. When he finished with the Bloch-Nordsieck method [1, 2] he told me to use it for computing the cross-section of the double bremsstrahlung, corresponding to the process $e^+ + e^- \to e^+ + e^- + 2\gamma$, that, at that time, he was thinking to use as a monitor process for the luminosity of Adone. After his lectures it was easy to compute such a cross-section obtaining the leading soft term. Bruno helped me to write a letter on this result [3]. Adding to my thesis also the calculation of the forward amplitude I managed to get my Laurea in Physics in February 1966.

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Immediately after I got a fellowship to work in Bruno's group in the Laboratori Nazionali di Frascati (LNF). There I met Mario Greco who was also trying to compute the total cross-section for the double bremsstrahlung at high energy for any frequency of the two photons. We joined the forces and we published a paper after many months of work [4]. We computed the cross-section in two different ways finding agreement with [5] but not with [6]. After one year of fellowship I received a permanent position and continued to work in Bruno's group in Frascati.

After having finished the calculation of the total cross-section for double bremsstrahlung I did not want to go into the computation of loops and into the study of infrared divergences relevant for the experiments with Adone, as the rest of the group was doing. Instead I decided to move into S-matrix theory that was being developed in the sixties under the influence of the Berkeley school. For my work on the finite energy sum rules I got the possibility of visiting Caltech for three months where I discussed various issues of S-matrix theory with Frautschi and followed the wonderful lectures on particle physics given by Feynman. It was an incredible experience that made me to apply for a NATO fellowship to be able to go back to the US. In 1969 I got a NATO fellowship and I decided to use it at MIT to work with Sergio Fubini on the newly found Veneziano model. I had leave of absence from LNF for one year, but, when I asked to continue it for a second year, I received a negative answer from the Director of LNF. At the end of 1970 I resigned from LNF and stayed one more year at MIT.

This is the beginning of my peregrinations that brought me first to Cern for two years, then to Nordita in Copenhagen for four and half years, then to Cern again for one year. In 1979 I got a permanent position at the Freie Universität in Berlin and in 1980 I moved to a better position at the Bergische Universität Gesamthochschule Wuppertal where I stayed until the end of January 1986. From February 1986 I started to work again at Nordita and now I divide my time between the Niels Bohr Institute in Copenhagen and Nordita in Stockholm.

The Bloch-Nordsieck method and, more in general, the study of the infrared divergences in QED was the main activity in the theoretical group led by Bruno at the LNF and this activity has been very important for the experiments done with Adone. In the last couple of years I went back to these methods applying them to gravity [7] in connection with the experiments done at Ligo/Virgo where gravitational waves coming from the merging of two black holes have been observed.

18.2 From Dual Resonance Model to String Theory

The big successes obtained in the forties and fifties in the computation of the electromagnetic processes starting from the QED Lagrangian did not seem to be possible for strong interactions because of the large value of the pion-nucleon coupling constant $\frac{g^2_{\pi NN}}{4\pi} \sim 14$ that did not allow perturbative calculations.

Therefore, in the sixties, many people, led by Chew and Mandelstam in Berkeley, gave up the idea of writing a Lagrangian for strong interactions and pushed instead the idea of computing directly the S-matrix of the strong processes by implementing its basic properties as analyticity, Regge behaviour, crossing symmetry and unitarity by means of a not well specified bootstrap approach.

The most successful result of these new ideas was the Veneziano model [8], originally constructed for the process $\pi\pi \to \pi\omega$ and immediately after extended to the scattering of four scalar particles:

$$B_4 \sim \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} \sim \Gamma(-\alpha(t))(-\alpha' s)^{\alpha(t)} \tag{18.1}$$

It contains an infinite number of narrow width resonances lying on a linearly rising Regge trajectory $\alpha(t) = \alpha_0 + \alpha' t$ and on its daughter trajectories spaced by integers and, for $s >> |t|$, has Regge behaviour as shown in the right-hand-side of the previous equation. Also the external scalar particle lies on the leading Regge trajectory and has a mass given by $\alpha_0 + \alpha' m^2 = 0$ in terms of the intercept α_0 and the slope α' of the leading Regge trajectory.

Shortly after the Veneziano model the previous amplitude has been extended to the scattering of N scalar particles [9]

$$B_N \sim \int_{-\infty}^{\infty} \frac{\prod_1^N dz_i \theta(z_i - z_{i+1})}{dV_{abc}} \prod_{i=1}^{N} \left[(z_i - z_{i+1})^{\alpha_0 - 1} \right] \prod_{i<j} (z_i - z_j)^{2\alpha' p_i \cdot p_j} \tag{18.2}$$

and this N-point amplitude was called Dual Resonance Model (DRM). It satisfies all the axioms of S-matrix theory, except unitarity, with an infinite number of zero-width resonances lying on linearly rising Regge trajectories.

Having found the S-matrix the next questions were: is it a consistent S-matrix? What is the underlying theory?

The first step in this direction was taken by Fubini, Gordon and Veneziano [10] and by Nambu [11] and Susskind [12] who rewrote it in terms of an infinite number of harmonic oscillators spaced by integers satisfying the commutation relation: $[a_{n\mu}, a^{\dagger}_{m\nu}] = \delta_{nm}\eta_{\mu\nu}$ with $n, m = 1, 2\ldots$ and with the Lorentz metric given by $\eta_{\mu\nu} = (-1, 1, 1, 1)$. In particular, in [10] it was written as follows:

$$B_N = \int \frac{\prod_{i=1}^N dz_i}{dV_{abc}} \langle 0| \prod_{i=1}^{N} V(z_i, p_i) |0\rangle \tag{18.3}$$

in terms of the vertex operator of the external scalar particle $V(z, p)$ and $Q^{\mu}(z)$:

$$V(z,p) =: e^{ipQ(z)} :$$
$$Q^{\mu}(z) = \hat{q}^{\mu} - 2i\alpha' \hat{p}^{\mu} \log z + i\sqrt{2\alpha'} \sum_{n=1}^{\infty} \sqrt{n} \left(a_n^{\mu} z^{-n} - a_n^{\dagger\mu} z^n \right) \tag{18.4}$$

where $\hat{q}_0$ and $\hat{p}_0$ satisfy the commutation relation $[\hat{q}_0, \hat{p}_0] = i\eta^{\mu\nu}$.

Factorising the amplitude at the pole for $s_{ij} \sim M^2 = \frac{N-1}{\alpha'}$ (for simplicity in the case with $\alpha_0 = 1$) for $N = 0, 1 \ldots$ one gets the states $|\lambda\rangle$ with mass M^2 that contribute to its residue:

$$N|\lambda\rangle = \sum_{n=1}^{\infty} n a^{\dagger}_{n\mu} a_{n\nu} \eta^{\mu\nu} |\lambda\rangle \;\; ; \;\; \eta_{\mu\nu} = (-1, 1, 1, 1) \tag{18.5}$$

For $N = 0$ the previous equation is satisfied by the vacuum $|0\rangle$, for $N = 1$ by the vector state $a_1^{\dagger\mu}|0\rangle$ and so on.

For a generic value of α_0 it turns out that the DRM contains states with negative norm [13, 14] violating tree-level unitarity.

Virasoro [15] found that for $\alpha_0 = 1$ there are extra conditions (Virasoro conditions) that possibly eliminate ghosts.

For $\alpha_0 = 1$, together with Del Giudice [16], we found that the on-shell physical states are characterised by the following conditions

$$L_n|Phys.\rangle = (L_0 - 1)|Phys.\rangle = 0 \;\; ; \;\; n > 0 \tag{18.6}$$

that generalise the Fermi condition in QED: $\partial^{\mu} A_{\mu}^{(+)}|Phys.\rangle = 0$.

Fubini and Veneziano [17] showed that the Virasoro generators L_n satisfy the conformal algebra in two space-time dimensions called Virasoro algebra:

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{D}{24} n(n^2 - 1)\delta_{n+m,0} \tag{18.7}$$

where the central charge was obtained by Weis [18], using the expression of L_n in terms of the oscillators.

Campagna et al. [19] generalised the amplitude to include any physical state (not just the ground state). For $\alpha_0 = 1$ the lowest state is a tachyon with mass $m^2 = -p^2 = -\frac{1}{\alpha'}$ and the next state is a massless photon with vertex operators [19, 20]

$$V(p, z) =: e^{ipQ(z)} : \;\; ; \;\; V_i(k, z) = \left(\epsilon_i \frac{dQ(z)}{dz}\right) e^{ikQ(z)} \tag{18.8}$$

Those vertex operators and, more in general, the vertex operators of any physical state are conformal fields with dimension $\Delta = 1$ that satisfy the following commutation relation with the conformal generators L_n:

$$[L_n, V_{\alpha}(z, p)] = \frac{d}{dz}\left(z^{n+1} V_{\alpha}(z, p)\right) \tag{18.9}$$

The N-point amplitude involving N physical states can be written in terms of their vertex operators [19]:

$$A_N = \int \frac{\prod_{i=1}^{N} dz_i}{dV_{abc}} \langle 0| \prod_{i=1}^{N} V_{\alpha_i}(z_i, p_i)|0\rangle \tag{18.10}$$

realising the idea [21] that there is a complete democracy among the physical states and, as a consequence, there is no physical state more fundamental than the others. All of them lie on Regge trajectories.

The photon vertex operator was then used to construct the $(D-2)$-dimensional DDF operators [22]:

$$A_{n,i} = \frac{i}{\sqrt{2\alpha'}} \oint_0 dz(\epsilon_i \frac{dQ(z)}{dz})e^{ikQ(z)} \;;\; k^2 = \epsilon k = 0 \;;\; [L_m, A_{n,i}] = 0 \tag{18.11}$$

They satisfy the algebra of the harmonic oscillators:

$$[A_{n,i}, A_{m,j}] = n\delta_{ij}\delta_{n+m;0} \;\;;\;\; A_{-m,i} \equiv A^{\dagger}_{m,i} \;\;;\;\; n > 0 \tag{18.12}$$

and generate an infinite number of physical states with positive norm (no ghosts): but not all of them for arbitrary D (only for $D = 26$).

Already in 1969 Nambu [11, 23], Nielsen [24] and Susskind [12] suggested that the structure underlying the DRM was that of a string theory. In particular, in the Nambu formulation, the Virasoro algebra appeared to be a classification algebra as in Conformal Field Theory, while in the DRM was a gauge algebra needed to eliminate ghost states. It took a while to understand how to eliminate this discrepancy and this delayed the connection of the DRM with string theory.

The Nambu-Goto [23, 25] action was written down in 1970 as a generalisation of the one for a point particle

$$L_{part.} = -m \int \sqrt{-dx_\mu dx^\mu} \;\;;\;\; L_{string} = -T \int \sqrt{-d\sigma_{\mu\nu} d\sigma^{\mu\nu}} \tag{18.13}$$

where m is the mass of the particle and $T = \frac{1}{2\pi\alpha'}$ is the string tension. But it took a while to understand how to use it. In terms of the world-sheet coordinates σ and τ one gets

$$L_{String} = -T \int_{\tau_i}^{\tau_f} d\tau \int_0^{\pi} d\sigma \sqrt{(\dot{x}x')^2 - \dot{x}^2(x')^2} \tag{18.14}$$

where $x^\mu(\tau, \sigma)$ is the string coordinate, $\dot{x}^\mu = \frac{\partial x^\mu}{\partial \tau}$ and $(x')^\mu = \frac{\partial x^\mu}{\partial \sigma}$. It is invariant under any choice of coordinates σ and τ.

It is more convenient to use a simpler and classically equivalent action:

$$S(x, g) = -\frac{T}{2} \int_{\tau_i}^{\tau_f} d\tau \int_0^{\pi} d\sigma \sqrt{-g} g^{ab} \partial_a x^\mu \partial_b x^\nu \eta_{\mu\nu} \tag{18.15}$$

where g^{ab} is the metric of the two-dim world-sheet (σ, τ). The equation of motion for g_{ab} implies the vanishing of the world-sheet energy-momentum tensor:

$$T_{ab} = \partial_a x \partial_b x - \frac{1}{2} g_{ab} g^{cd} \partial_c x \partial_d x = 0 \Longrightarrow \dot{x}^2 + (x')^2 = \dot{x} x' = 0 \quad (18.16)$$

Choosing the conformal gauge

$$g_{ab} = \rho(\sigma, \tau) \eta_{ab} \quad (18.17)$$

we get a free action with the constraint of the vanishing of T_{ab}. The gauge is not completely fixed because we can still perform conformal transformations remaining in this gauge. To fix the gauge completely we can go to the light-cone gauge imposing an extra condition:

$$x^+ = 2\alpha' p^+ \tau \ ; \ x^\pm = \frac{x^0 \pm x^{D-1}}{\sqrt{2}} \quad (18.18)$$

Using the light-cone gauge condition and the vanishing of the two-dimensional energy momentum tensor, one can determine x^- in terms of the components $x^\perp$ (orthogonal to $x^\pm$). The only independent components are the $D-2$ transverse $x^\perp$. This analysis was performed by Goddard et al. [26]. They checked that the D-dimensional Lorentz generators, written only in the terms of the $D-2$ transverse $x^\perp$, satisfy the Lorentz algebra only if

$$\alpha_0 = 1 \ ; \ D = 26 \quad (18.19)$$

For $D = 26$ the DDF operators generate a complete set of physical states implying that the bosonic string is ghost free. This finally shows that the spectrum of physical states of the DRM for $D = 26$ is identical to the spectrum of string theory described by the Nambu-Goto action. Concerning the interaction Cremmer and Gervais [27] and Mandelstam [28] showed that the on shell three-point amplitude computed in string theory was identical to that of three arbitrary DDF states [29]. The equivalence of the DRM and string theory for higher N-point amplitudes was shown in [28, 30].

Even before finding its connection with string theory, the DRM, with the infinite set of zero width resonances, was considered a tree diagram of a unitary theory. At tree level, unitarity requires absence of ghosts and this property was satisfied for $\alpha_0 = 1$. On the other hand, loop diagrams were necessary in order to have the total widths of the resonances Γ_T to be equal to the sum of all partial widths $\sum_i \Gamma_i$. In order to implement this property one-loop and even multiloop amplitudes were constructed [31–33].[1]

[1] A complete expression for the multiloop amplitude in the bosonic string was only possible in the eigthies [35, 36] after the discovery of BRST symmetry.

Lovelace [34] showed that the non-planar loop had cuts violating unitarity unless $D = 26$. If $D = 26$ those cuts become poles that later turned out to be the states of a closed string that also lie on linearly rising Regge trajectories:

$$\alpha_{open}(s) = 1 + \alpha' s \;\; ; \;\; \alpha_{closed}(s) = 2 + \frac{\alpha'}{2} s \tag{18.20}$$

This means that unitarity requires that open strings always include closed strings. Closed strings require open strings but only at non-perturbative level as we will see later. As a consequence, Gauge Theories always include Gravity and vice-versa.

18.3 The Dual Pion Model

A N-point amplitude for pions was proposed by Neveu and Schwarz [37]. Unlike the one for the ground state particle of the bosonic string, it has the nice property that it vanishes for an odd number of pions, consistently with the fact that the pion has G-parity equal to -1. All previous analysis done for the DRM that turned out, for $\alpha_0 = 1$, to be equivalent to the bosonic string has been repeated for the NS model finding that it corresponds to a spinning string, i.e. a string with also spin degrees of freedom along it. It turned also out that it has no ghost if

$$\alpha_0 = 1 \;\; ; \;\; D = 10 \tag{18.21}$$

Actually, if $\alpha_0 = \frac{1}{2}$, as for the ρ Regge trajectory (for $m_\pi = 0$), one gets the four-point Lovelace-Shapiro model [38]:

$$A(s,t) \sim \frac{\Gamma(1-\alpha_\rho(s))\Gamma(1-\alpha_\rho(t))}{\Gamma(1-\alpha_\rho(s)-\alpha_\rho(t))} \;\; ; \;\; \alpha_\rho(s) = \frac{1}{2} + \alpha' s \; ; \; \alpha' = \frac{1}{2m_\rho^2} \tag{18.22}$$

with Adler zeroes as expected for pions (A(s = 0, t = 0) = 0).

The N-point generalisation of the LS model is discussed in a recent paper with Bianchi and Consoli [39]:

$$A_N = \int \frac{\prod_{1=1}^{N} d\theta_i dz_i}{dV_{abc}} \prod_{i<j} (Z_i - Z_j)^{2\alpha' k_i k_j} \prod_{i=1}^{N} (Z_i - Z_{i+1})^{-\frac{1}{2}-\alpha' m_\pi^2} \tag{18.23}$$

using a super-conformal formalism, where $Z_i = (z_i, \theta_i)$. It has the correct Adler zeroes. It reduces to the non-linear σ model when $\alpha' \to 0$. But it has negative norm states: ghosts.

The reason is that, while the NS model is super-conformal invariant, the integrand of the amplitude in (18.23) is only super-projective invariant. This means that there are not enough conditions to decouple the ghosts. The conclusion is that it seems

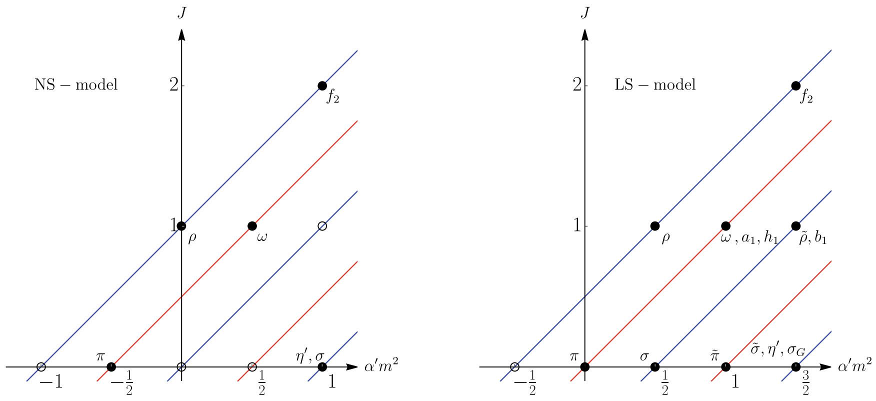

Fig. 18.1 Spectrum of NS (left) and LS (right) model in four dimensions, Regge trajectories in blue (red) have G-parity $+1$ (-1). Bullets represent 'physical' states, open circles represent 'missing' states

impossible to write a consistent N-pion amplitude with exact linearly rising Regge trajectories [40] (Fig. 18.1).

The NS model contains Regge trajectories with both integer and half-integer intercept. The particles lying on those with integer intercepts have G-parity $+1$, while the particles lying on those with half-integer intercepts have G-parity -1. As a consequence, the amplitudes with an odd number of particles lying on the Regge trajectories with half-integer intercepts are vanishing.

In order to extend the DRM to fermions Ramond [41] constructed the Ramond model and, later on, it turned out that the NS and the R model are part of the same model called RNS model. In addition to the bosonic oscillators of the bosonic string, it also contains an infinite set of fermionic oscillators with half-integer labels in the NS model and integer labels in the R model.

18.4 Unification of Gauge Theories and Gravity

In hadron physics only the pion is massless in the chiral limit. The consistent string theories that we have discussed do not allow for a massless pion, but contain instead massless gauge fields in the open string sector and a massless graviton in the closed string sector. This implies that the string theories that we have discussed cannot describe hadron physics as it was intended in the beginning with the Veneziano model.

In 1973 QCD was formulated and those interested in hadron physics left string theory and joined QCD.

In 1974 Scherk and Schwarz [42] proposed to use string theory, not for hadrons, but as a theory consistently unifying gauge theories with gravity. A very important property of string theory is that Gauge Theories and GR are not put by hand, but emerge together as an unavoidable part of the theory, as gauge invariance and invariance under diffeomorphisms are necessary ingredients for a consistent description of massless spin 1 and 2, respectively.

Unlike in field theory, in string theory we have a single interaction: that among strings. The only free parameter is the Regge slope α' related to the string tension. The string coupling constant g_s that enters in the loop expansion is not a free parameter but is given by the vacuum expectation value of a particular state of the closed string, the dilaton $g_s \sim e^{\phi}$ and it is fixed by the minimum of the dilaton potential.

We must finally stress that String Theory is an extension of Field Theory: field theory amplitudes are recovered in the limit of $T \to \infty$ or $\alpha' \to 0$ [43, 44].[2]

In conclusion, the softness of string at high energy that was a problem for hadrons becomes now a virtue providing a finite theory of gravity.

18.5 From the RNS Model to Superstring

The RNS model contains two sectors: one with ten-dimensional fermions and another with ten-dimensional bosons. The spectrum of states in the bosonic sector is given by

$$\alpha' m_B^2 = N - \frac{1}{2} \;\; ; \;\; N|\lambda\rangle = \sum_{n=1}^{\infty} \left(n a_n^{\dagger} a_n + \left(n - \frac{1}{2} \right) \psi^{\dagger}_{n-\frac{1}{2}} \psi_{n-\frac{1}{2}} \right) |\lambda\rangle \qquad (18.24)$$

where N can be integer and half-integer. The lowest state is still a tachyon $|0\rangle$ and the next state is a massless gauge field $\psi^{\dagger}_{\frac{1}{2};\mu}|0\rangle$.

One can define a world-sheet fermion number:

$$(-1)^F \;\; ; \;\; F = \sum_{n=1}^{\infty} \psi^{\dagger}_{n-\frac{1}{2}} \psi_{n-\frac{1}{2}} - 1 \qquad (18.25)$$

The states with an even (odd) number of fermionic oscillators have eigenvalue -1 $(+1)$ under the action of $(-1)^F$. $(-1)^F$ corresponds to the G-parity that we discussed in the Dual Pion Model.

The spectrum of states in the fermionic sector is given by:

$$\alpha' m_F^2 = N \;\; ; \;\; N|\lambda\rangle = \sum_{n=1}^{\infty} \left(n a_n^{\dagger} a_n + n \psi_n^{\dagger} \psi_n \right) |\lambda\rangle \qquad (18.26)$$

[2] See also [45].

where N is an integer. The lowest state is a massless ten-dimensional fermion. There is a fermionic zero mode that satisfies the same algebra as that of the Dirac Γ-matrices:

$$\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu} \tag{18.27}$$

This means that the ground state $|0, A\rangle$ has a ten-dimensional Dirac spinor index A. Also in this case we have a fermion number operator:

$$(-1)^F = \Gamma_{11}(-1)^{F_R} \quad ; \quad F_R = \sum_{n=1}^{\infty} \psi_n^\dagger \psi_n \tag{18.28}$$

In 1976 Gliozzi et al. [46] proposed to truncate the spectrum of the RNS model keeping only the states that are even under the action of the fermion number operator:

$$(-1)^F |\psi\rangle = |\psi\rangle \tag{18.29}$$

It is called GSO projection.

The GSO projection eliminates the states in the NS sector that lie on half-integer Regge trajectories and in R sector imposes to the ground state to be a Weyl-Majorana fermion.

The two lowest states are a gauge field in the bosonic and a massless Weyl-Majorana fermion in the fermionic sector. They have the same number of physical degrees of freedom: 8 in $D = 10$. It turns out that, after the GSO projection, we get at each level of the spectrum the same number of bosons and fermions. One gets the spectrum of the open type I string theory that is supersymmetric in $D = 10$. Type I contains also a supersymmetric closed string sector with gravitons, gravitinos and other massless states. This is the first string theory without a tachyon in the spectrum that has been constructed.

18.6 Superstring Theories: Type IIA, Type IIB, Type I

After 1976 and before 1985 two closed superstring theories were constructed by Green and Schwarz [47]. They contain two bosonic sectors, called NS-NS and R-R, and two fermionic sectors, called R-NS and NS-R.

Type IIB theory is a chiral closed superstring theory that, in the massless NS-NS sector, contains a graviton, a dilaton and a Kalb-Ramond field described by an antisymmetric tensor $B_{\mu\nu}$ and in the R-R sector the potentials C_0, $C_{2\mu\nu}$, $C_{4\mu\nu\rho\sigma}$. The two fermionic sectors contain two gravitinos and two dilatinos with the same chirality.

Type IIA is instead a non-chiral closed superstring theory that has the same massless NS-NS sector as type IIB, while the R-R sector contains the potentials $C_{1\mu}$, $C_{3\mu\nu\rho}$.

The two fermionic sectors contain two gravitinos and two dilatinos with opposite chirality.

We conclude this section with Type I whose massless open string sector we have already discussed and we have seen that it contains a gauge boson and a gaugino. The closed string sector contains instead a graviton, a dilaton, a $C_{2\mu\nu}$ potential and one gravitino and one dilatino. Furthermore, in order to cancel gauge and gravitational anomalies the gauge group must be $SO(32)$.

Those are the three superstring theories that were constructed before string theory became popular again around 1985. The developments of string theory from the origin to 1985 are described in a book edited together with Cappelli et al. [48].

18.7 D(irichlet)p-Branes

In the previous section we have seen that the type I and type II theories contain potentials with more than one index. They are a generalisation of the electromagnetic potential A_μ and, as the electromagnetic potential is coupled to point-like particles, they are instead coupled to p-dimensional objects through the following generalisation of the electromagnetic coupling:

$$\int A_\mu dx^\mu \Longrightarrow \int A_{\mu_1\mu_2\ldots\mu_{p+1}} d\sigma^{\mu_1\mu_2\ldots\mu_{p+1}} \tag{18.30}$$

It turns out that there exist classical solutions of the low-energy string effective action that are coupled to the metric, the dilaton and are charged with respect to one of these RR fields [49]. For them we get the following asymptotic behaviour

$$C_{01\ldots p} \sim \frac{1}{r^{D-3-p}} \Longleftrightarrow C_0 \sim \frac{1}{r} \; if \;\; D=4, p=0 \tag{18.31}$$

that reduces to that of the electromagnetic vector potential for $p=0$. They correspond to non-perturbative states of string theory with tension and RR charge given by:

$$\tau_p = \frac{Mass}{p-volume} = \frac{(2\pi\sqrt{\alpha'})^{1-p}}{2\pi\alpha' g_s} \quad ; \quad \mu_p = \sqrt{2\pi}(2\pi\sqrt{\alpha'})^{3-p} \tag{18.32}$$

where g_s is the string coupling constant.

In 1994 Polchinski [50] showed that, in string theory, these objects are required by T-duality that, in the case of a closed string exchanges Kaluza-Klein modes with winding modes, while, in the case of an open string, changes Neumann with Dirichlet boundary conditions. For this reason they are called D(irichlet)p-branes. Open strings satisfy Neumann boundary conditions along the direction of the world-volume of the Dp-brane and Dirichlet boundary conditions along the directions orthogonal to the

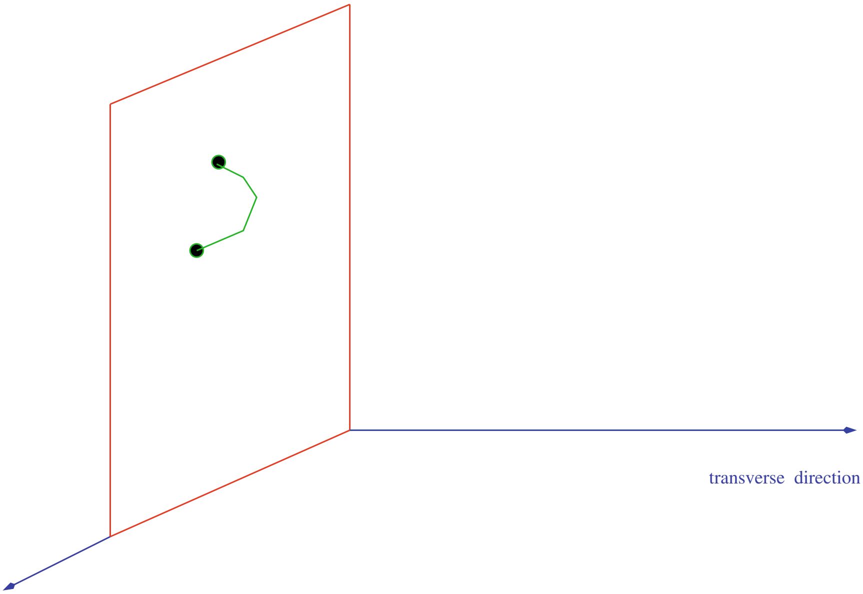

Fig. 18.2 The endpoints of an open string can move freely inside the world-volume of the Dp-brane, but they cannot move along the directions orthogonal to the world-volume of the Dp-brane

world-volume of a Dp-brane, as shown in Fig. 18.2. Besides the perturbative states, type I and II string theories contain also the Dp-branes that are non-perturbative states of type I and II string theories and are characterised by the fact of having open strings attached to their world-volume.

It follows that open strings and the corresponding gauge theories live in the (p+1)-dim. world-volume of a Dp-brane, while closed strings (gravity) live in the entire ten dimensional space.

If we have a stack of N parallel coincident Dp-branes, then we have N^2 open strings having their endpoints on the D branes, corresponding to the degrees of freedom of the adjoint representation of $U(N)$. The massless bosonic states correspond to the gauge fields of $U(N)$, while the massless fermionic states correspond to their supersymmetric partners, called gauginos.

The gauge theory living on N maximally supersymmetric D3-branes is the maximally supersymmetric $\mathcal{N} = 4$ super-Yang-Mills with $U(N)$ gauge group containing one gluon, 6 scalars and 4 Majorana fermions, all transforming according to the adjoint representation of the gauge group. It is conformal invariant with vanishing β-function. Maldacena [51] conjectured that this theory is equivalent to 10-dimensional string theory on $AdS_5 \otimes S_5$. By now there is a lot of evidence for it and a lot of applications have been made both for hadrons and condensed matter systems.

What remains to understand is how to extend the previous exact duality to non-conformal gauge theories as QCD and what is the string theory appearing in the 't Hooft large N expansion of QCD [52].

18.8 M-theory

Up to now we have discussed the three superstring theories that were constructed before 1984. After that, two more fully consistent string theories were constructed. They are the two heterotic strings. They are closed string theories, but, unlike type II theories, they contain a gauge theory: one with gauge group $SO(32)$ and the other with gauge group $E_8 \times E_8$. These two gauge groups are required in order not to have gauge and gravitational anomalies.

The five superstring theories that we have discussed are all consistent string theories in ten-dimensional Minkowski space-time and, at the perturbative level, they are all independent from each other.

This has generated a puzzle for many years: If string theory is a unique theory why do we have five theories instead of just one?

It turns out that, if we also include their non-perturbative behaviour, they are related to each other through a web of weak-strong dualities [53, 54] and they are all part of a unique 11-dimensional theory, called M-theory that, at low energy, reduces to the unique 11-dimensional supergravity. Starting from M-theory, in which two directions are compactified on $S^1 \times S^1$, one recovers type IIA and type IIB theories, in which one of the ten directions is compactified on S^1, that are T-dual to each other. If we instead compactify two directions of M-theory on $S^1 \times \frac{S^1}{Z_2}$ one recovers the two heterotic strings and type I theory [55]. In particular, type I and heterotic with gauge group $SO(32)$ are related by weak-strong duality [56].

The unification of all consistent string theories in ten dimensions in a unique 11-dimensional theory is a very beautiful result, but we should not forget that we live in four and not eleven dimensions. This means that eight directions of M-theory must live in a compact manifold that must be small enough in order not to contradict experiments. Unfortunately this compactification can be done in too many consistent ways and, at the moment, it seems impossible to use M-theory or string theory to predict the low energy physics that we see in experiments at present energies. This is the Landscape Problem that unfortunately is still with us at present.

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