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Nuclear isomers in neutron stars

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In fact, nuclear isomers are studied in rapid neutron capture process so-called r - process of the nucleosynthesis in neutron stars which occurs at the energy state about several MeV (dozens of GK) causing cooling of the star matter. Thus, isomers freeze out in thermal equilibrium due to cooling the neutron star matter by the r - process, i.e. they can immediately be populated in the star medium as it is in astrophysical isomers (astromers). There are two different main states of the astromers: ground state, where the isomer transition rates characterize; and thermalization temperatures, which describes the transition rates between pairs in the nuclear states. Studying unknown behaviour of astromers in models of neutron star matter may be helpful in understanding the cooling processes, in particular, in estimations of age of radio pulsars.

I. INTRODUCTION

A nucler isomer is a metastable state of an atomic nucleus, in which one or more nucleons occupy higher energy levels than in the ground state of the same nucleus. Metastable describle nuclei whose excited states have half-lives 100 to 1000 times longer than the half-lives of the excited nuclear states that decay with a "prompt" half life (ordinarily on the order of 10-12 seconds). The term "metastable" is usually restricted to isomers with half-lives of 10^{-9} seconds or longer. Some references recommend 510^{-9} seconds to distinguish the metastable half life from the normol "prompt" gamma-emission half-life [1].

Astrophysical nucleosynthesis calculations rely on nuclear reaction rates. Arguably, even a single reaction rate can profoundly affect the astrophysical evolution [2]. Therefore, researchers have been very busy for several years to calculate the rates of nuclear weak interactions [3], [4], [5], neutron capture cross sections [6] they did great things. Usually, only the ground-state rate is used to calculate nucleosynthesis rates, or the levels are calculated in a thermal-equilibrium probability distribution. When a nucleus is formed, it falls to a lower state (usually by gamma-ray emission). If it is held in an isomer until it reaches the ground state, it does not have to undergo further reactions at the rate of the ground state. Isomers can also cause the distribution of nuclear energy levels to fail to reach thermal equilibrium. The decay rates (eg, beta-decay) of the long-lived states (ground state and isomers) of a single nuclear species can differ dramatically; the most famous example in astrophysics is ^{26}Al, which is an isomer with a ground-state beta-decay half-life of 717 kyr but a beta-decay half-life of 6.346 s. Although the existence of isomeric states in nuclei has been known for about a century, much remains to be learned about their effect on the creation of elements in astrophysical nucleosynthesis, are expected to have significant effects due to their unique decay properties [7].

We see that some isomers can play an influential role in astrophysical nucleosynthesis. But most isomers are not. This distinction defines astrophysical isomers, or "astromers", they are nuclear isomers and have such effects in the astrophysical environment.

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We apply our methods to multiple nuclei in a range of astrophysical locations, including the analysis of uncertainty effects for isomers. We develop in Section 2 a very accurate means of calculating the effective transition rates between long-lived nuclear states in hot environments. In Section 3 , we apply our methods to several known and potential astromers. We provide concluding remarks in Section 4.

II. TRANSITION-RATE

In this article, we use the formulation of [8] Considering a connected system, we divide these states into two classes: endpoint E, in our application, they are the ground state and isomers in the atomic nucleus; and intermediate states I, which are nonisomeric states for us. This discussion specializes in two end-point cases (one isomer), but the arguments are easily generalized. Disruption is then introduced as a different set of rates that do not affect internal throughput. We will keep this section generic by using the symbols A and B for endpoint states and i, j, etc. for intermediate states. For concreteness, take A as the ground state and B as the nuclear isomer. Figure 1 provides a schematic of the thermally driven transitions from A to B through intermediate states i, j, and k. When a system goes from state s, it goes to state t with probability b_{st}.

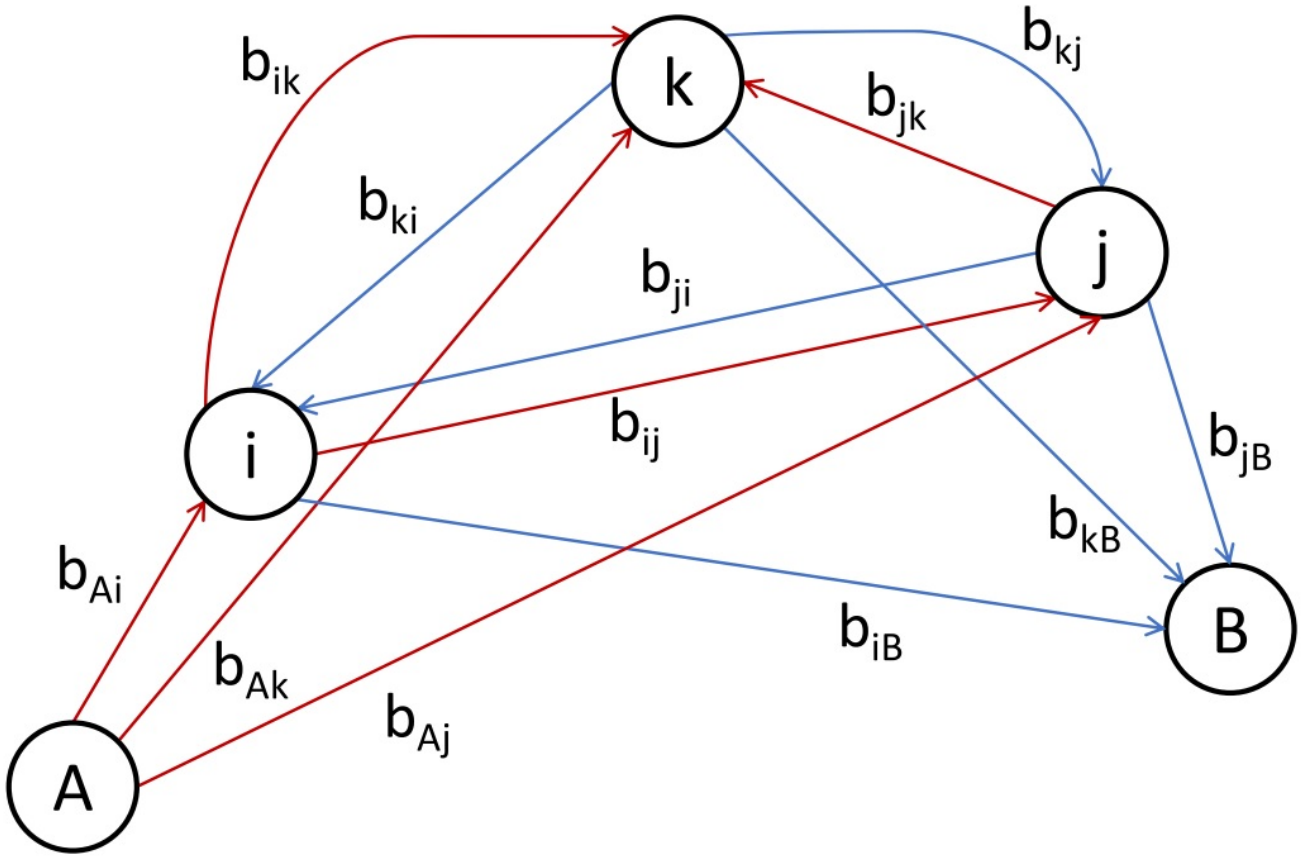

FIG. 1: Direct connection of A and B, scheme of transition from state A to state B. If the energy is increasing upward, the red arrows indicate the transitions and the blue arrows indicate the downward transitions. b_{st} indicates the probability of going directly to t when the system leaves s.

The transition probability b_{st} is related to the transition rate λ_{st}.

$$b_{\mathrm{st}} = \frac{\lambda_{\mathrm{st}}}{\lambda_{\mathrm{s}} = \sum_{\mathrm{f}} \lambda_{\mathrm{sf}}} \tag{1}$$

By checking, $\sum_I b_{\mathrm{st}} = 1$, ensure that there are no sources in the system. We calculate λ_{st} using the spontaneous γ-decay rate $\lambda^{s}_{\mathrm{hl}}$ from the higher h state to the lower l state.

$$\lambda_{hl} = \lambda^{s}_{\mathrm{hl}}(1+u), \tag{2}$$

$$\lambda_{hl} = \frac{2J_{\mathrm{h}}+1}{2J_{\mathrm{l}}+1}\lambda^{s}_{\mathrm{hl}}u, \tag{3}$$

$$u = \frac{1}{e^{E_{\mathrm{h}}-E_{\mathrm{l}}}/T-1}. \tag{4}$$

The factor u is a thermal photon Bose–Einstein distribution, reflecting that it stimulates the h to l transition and induces the l to h transition. As noted by [9], there may be other thermal interactions affecting λ_{st}. Equations (2)–(4) can be extended to include other interactions as needed. With the individual transition probabilities, we are ready to calculate the effective thermal transition rates between the end point states.

A. Transition rate effect

We express the effective transfer rate of AB from A to B as follows

$$\Lambda_{\mathrm{AB}} = \lambda_{\mathrm{AB}} + \sum_{i} \lambda_{\mathrm{Ai}} P_{\mathrm{IB}} \tag{5}$$

In the case of a system with more than two end point states, then equation (5) holds for all pairs of initial and end point states. Now we need to find P_{iB}. These can be calculated by considering the first step of all admissible routes from i to B. The step from i to s occurs with probability b_{is}. Now P_{iB} can be defined as the sum of the probabilities of the first possible steps from i multiplied by the probabilities P_{sB} of reaching the new state B. That is,

$$P_{\mathrm{iB}} = b_{\mathrm{iB}} + \sum_{\mathrm{j}} b_{\mathrm{ij}} P_{\mathrm{jB}} \tag{6}$$

From here we can express the problem in matrix form.

$$(1-b_{\mathrm{II}})\vec{P_{\mathrm{IB}}} = \vec{b_{\mathrm{IB}}} \tag{7}$$

III. APPLICATIONS

We now calculate the effective transition rates from isomer to ground and from ground to isomer. In addition, we calculate the rate of β-decay in several nuclei. Here, all calculations are based on the electron density $\rho Y_{\mathrm{e}} = 10^5 gsm^{-3}$. We use the method of Gupta & Meyer to calculate the effective decay rates for the states corresponding to the ground state and the isomer. The Gupta & Meyer method assumes that the intermediate states reach steady-state equilibrium instantaneously with the endpoint states They are the ratio of the intermediate state i to the endpoint state E occupations and the ratio of the direct transition rates between them noting that the steady-state occupation fractions are obtained by Y, and they call this the "inverse ratio" R_{Ei}

$$R_{\mathrm{Ei}} = \frac{\lambda_{\mathrm{Ei}}}{\lambda_{\mathrm{iE}}} = \frac{Y_{\mathrm{i}}}{Y_{\mathrm{E}}} \tag{8}$$

We calculate this ratio using the equation (2)-(4). This is undefined if the direct transfer rates are zero. But since this ratio does not depend on the size of the rates, it remains constant in the limit where the rates approach zero, we find that

$$R_{\mathrm{Ei}} = \frac{2J_{\mathrm{i}}+1}{2J_{\mathrm{E}}+1} e^{\frac{E_{\mathrm{E}}-E_{\mathrm{i}}}{T}} \tag{9}$$

Now, each intermediate state has a probability of going to the final state E, P_{iE}, before going to another final point. Gupta and Meyer (2001) interpret this as the fraction of the intermediate state i that is associated with the endpoint state E . This interpretation—in addition to assuming an instantaneous equilibrium of intermediate states—allows each state to be assigned and used WE weights. In fact, each state is assigned a modified Boltzmann factor for each ensemble:

$$W_{\mathrm{iE}} = P_{\mathrm{iE}} R_{\mathrm{Ei}} = P_{\mathrm{iE}} \frac{2J_{\mathrm{i}} + 1}{2J_{\mathrm{E}} + 1} e^{\frac{E_{\mathrm{E}} - E_{\mathrm{i}}}{T}} \tag{10}$$

We can now calculate any rate of effective β-decay for the ensemble associated with endpoint E with these weights,

$$\Lambda_{\mathrm{E}\beta} = \frac{\sum_{\mathrm{s}} W_{\mathrm{sE}} \lambda_{\mathrm{s}\beta}}{\sum_{\mathrm{s}} W_{\mathrm{sE}}} \tag{11}$$

In this expression, $\lambda_{\mathrm{s}\beta}$ is the individual β-decay rate of the s state. Now we are ready to calculate the rate of nuclear transition with isomers and the rate of β-decay.

IV. SUMMARY AND CONCLUSION

We have developed an efficient calculation of the thermal transition rates between ground-state and long-lived isomers in atomic nuclei. Our techniques do not rely on any physical assumptions. Although the technique is not complicated, we have rigorously proven its accuracy and developed detailed tools for analyzing which transitions are important at a given temperature.

Although our rates are similar to the estimates in these works by [10], [8]. Because we came to the same conclusions. Still, ours has its advantages. First, we do not rely on physical assumptions. Second, we do not use the Taylor expansion to solve equation (7). This means that there are sufficient expansion conditions for any nucleus at any temperature. However, we are greatly indebted to [10] and [8] for our calculation of the ensemble β-decay rate.

For our practical research, we can determine if there is a specific "thermalization temperature" for a given set of annihilation rates. Above this temperature, the core level can be in thermal equilibrium, and below it, the isotope must be considered as two independent species: the ground state and the astromer [10], [8].

We simply have to recognize that a fast annihilation rate increases the thermalization temperature. For example, an isomer that decays only through an internal transition to the ground state is converted to heat under most conditions. However, if an isomer has a neutron capture cross section that is very different from the ground state, it can become out of equilibrium in the r-process, thereby affecting the nuclear flow.

It should be noted that the nuclear isomer has astrophysical consequences and is therefore an "astromer" below its thermalization temperature. This temperature is sensitive to the different annihilation rates experienced by the nuclear species, increasing the temperature rapidly and thus extending the range of conditions under which the metastable state is an astromer. At sufficiently high temperatures, the transition rate dominates the decay rate, the astromeric property disappears, and the isotope can be considered a single species.

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