Construction of higher-derivative supergravity models via superconformal formulation

February 2016

Yusuke YAMADA

Doctoral Thesis

Construction of higher-derivative supergravity models via superconformal formulation

February 2016

Waseda University

Graduate School of Advanced Science and Engineering
Department of Pure and Applied Physics, Research on Theoretical Particle Physics

Yusuke YAMADA

Abstract

Supergravity is one of the theories beyond the standard model of particle physics and solves some theoretical and phenomenological problems in the standard model. It is also regarded as an effective theory of superstring, which is a possible candidate for quantum gravity. It is known that, in effective theories of superstring, higher order derivative couplings generically appear. Such couplings may be important for physics at very high energy scale, especially in the early universe. From such perspectives, it is important to understand higher-derivative terms in supergravity. In this thesis, we construct the supergravity action including such terms via the superconformal tensor calculus, which reduces the complexity of calculations and unifies the three formulations of Poincaré supergravity. In particular, we construct supersymmetric higher-derivative terms of chiral and vector multiplets. We also discuss the effects of higher-derivative terms on inflation models in supergravity. As we will find, the supersymmetric higher-derivative terms are in general associated with nontrivial lower-(no-)derivative terms as a consequence of supersymmetry. We show that such nontrivial terms can play important roles during inflation.

Contents

Chapter 1

Introduction

1.1 Introduction and outline

The standard model of particle physics (SM) has been a successful model consistent with the collider experiments. In particular, in 2013, the last piece of SM, Higgs particle, was discovered and it also strongly supports the validity of SM. However, once we take our attention to cosmology, we notice that this is not the final theory explaining our universe completely. In other words, it is inconsistent with the standard cosmological model called the Λ-CDM model. The problems are obvious: in SM, both Λ and CDM are absent. Λ is the so-called cosmological constant, which is the most consistent source of the present accelerated expansion of the universe. In usual, although SM predicts the cosmological constant due to quantum corrections, it can be removed in the calculations. Therefore, it is essentially absent in SM. CDM means Cold Dark Matter, which is a neutral and non-relativistic massive particle. Such a particle is absent in SM, and this fact suggests that we need to extend SM for explaining them. An interesting model of CDM is the weakly interacting massive particle (WIMP) scenario, where CDM has a very weak coupling with particles in SM.

In the above argument, we have missed a more important point: before discussing the Λ and CDM, in the first place, *SM does not include gravity.* That is due to the non-renormalizable nature of quantum gravity. One of the promising candidates for the solution is provided by the superstring theory, where the fundamental material is a string. From the string theoretical viewpoint, the "particle" physics can be seen as the low energy effective theory.

The effective particle theory of superstring has greatly interesting features. For the consistency of the superstring theory, the number of the spacetime dimension is determined to be 10 (one for the time and the others for the space). Then, the effective particle theory should also be defined as 10 dimensional (10D) one. The other important feature is supersymmetry (SUSY) in spacetime. SUSY is one of the solutions for the stability of the electroweak scale. In superstring, SUSY is a local symmetry, and then, the effective theory becomes supergravity (SUGRA).

These facts imply that it is important to construct phenomenological and cosmological models within SUGRA. Such models of various kinds have been investigated in the literature so far. One way is to construct the model directly from the string theoretical side, that is, the top-down approach. Such an approach is better to clarify how the models can be realized in superstring. However, some assumptions are required and it is sometimes difficult to discuss the phenomenological and the cosmological consequences. In this thesis, we take the other approach, the bottom-up one, where we investigate the phenomenological and the cosmological models in SUGRA particle theory. In particular, we will construct and discuss the SUGRA models with some extensions motivated by the superstring theory.

In this thesis, we especially focus on the higher-derivative terms in the action, which contain spacetime derivatives more than two, in 4D SUGRA. It is known that such terms appear e.g. in the effective action of D-branes in type II superstring theory. One of the problems associated with higher-derivative terms is the so-called Ostrogradski instability [1, 2], which is equivalent to the appearance of ghost modes. The simplest way to avoid ghosts is to impose a requirement that the equation of motion (E.O.M) of a system should be the second order differential equation with respect to time. In non-SUSY theories, the most general scalar-tensor system satisfying the requirement is known as the Horndeski action [3, 4]. On the other hand, in SUSY case, such a general system has never been known so far, and furthermore, ghost-free higher-derivative terms of matter multiplets are less known [5, 6]. For example, known ghost-free SUSY higher-derivative terms of chiral multiplets are classified into two types. We will focus on one of them and also develop a special class of higher-derivative terms of a gauge multiplet called the Dirac-Born-Infeld (DBI) action [7, 8].

We will construct such models via the superconformal formulation [9, 10, 11, 12], where we consider a theory with superconformal symmetry. The symmetry includes the physical Poincaré SUSY as its subgroup, and this means that there are some unphysical symmetries in the superconformal theory. As we will see in some parts of this thesis, such unphysical gauge degrees of freedom can be used as a tool for reducing some complexities of calculations. The other utility of superconformal formulation is the unification of the Poincaré SUGRA formulations. Three different off-shell formulations of the Poincaré SUGRA have been known, in which the sets of auxiliary fields in the gravity multiplet are different from each other. It is known that the relation between one and the other formulations can be understood from the superconformal viewpoint [12]. Our construction of models with SUSY higher-derivative terms will be done in the superconformal formulation, and therefore, we can completely understand how such models are realized in all the formulations. This is one of the achievements of this thesis.

We will also discuss the cosmological consequences of our SUGRA models. Since the higher-derivative terms are expected to be suppressed by some scale much larger than one seeable in the collider experiments so far and near future, the effects on the phenomenology seem less important. However, in the early universe where the typical energy scale is much larger than that of the experiments today, effects of such terms may affect the dynamics of the universe. The main focus of our discussion is on *cosmic inflation* [13, 14, 15, 16, 17].

Inflation was first proposed as solutions for the horizon, the flatness, the monopole problems and so on. It solves such problems simultaneously, and so, it became one of a paradigm of the early universe. A class of inflation called the *slow-roll inflation* [18] predicts the primordial fluctuation of gravity field, whose almost scale independent spectra are consistent with the cosmic microwave background (CMB) observation so far. The primordial curvature fluctuation is the origin of the large-scale structure of our universe, and therefore, an inflationary era in the early universe seems necessary for explaining the universe observed today. The best way to explain the latest CMB data is a slow-roll inflation driven by a single scalar field called an *inflaton*.

Models of inflation in 4D SUGRA have been studied so far and numbers of models have been proposed. The models we will discuss are based on three classes among them called the F-term chaotic inflation [19, 20], the old and the new minimal Starobinsky inflation [21, 22, 23], and the massive vector multiplet inflation [24, 25]. We study the effects of the higher-derivative extension on those models.

The novel features of SUSY higher-derivative terms, which we will discuss, come from the nature of SUSY. Even if we try to construct only a specific interaction term, it is, in general, impossible in SUGRA because of the strong requirement of SUSY. The specific interaction always brings another unexpected interaction to the system. Such things do not occur in a non-SUSY system and the investigation of such a nontrivial interaction is more important to understand the SUSY/SUGRA system. As we will see below, SUSY higher-derivative terms lead to some nontrivial interactions other than higher-derivative ones. Those nontrivial interactions show some interesting and cosmologically favored features in each model.

This thesis consists of five chapters and appendices. In Chapter 2, we review the construction of SUGRA action in the superconformal formulation, which makes the construction rather simple and universal than the formulations based on Poincaré SUSY. Certain detailed advantages of the superconformal formulation are summarized in Sec. 2.1. Sections. 2.2, 2.3 and 2.4 are devoted to a review of some details about its structure and its basic application.

Models of inflation in SUGRA related to this thesis are summarized and reviewed in Chapter 3. First, we overview structures of the scalar potential of chiral multiplets and its consequences in inflation. After that, we review three models in SUGRA, the F-term chaotic inflation, the Starobinsky inflation in the old and the new minimal SUGRA, and the massive vector multiplet inflation.

In Chapter 4, we discuss one of the ghost-free higher-derivative terms of chiral multiplets. In Sec. 4.1, we introduce such a term in global SUSY. Then, we embed it into the superconformal formulation and discuss some features in Sec. 4.2. Taking those features into account, we construct an F-term chaotic inflation model with the higher-derivative term in Sec. 4.3, and then, find that the model behaves in a drastically different way from the original one. The reason for the behavior is clarified by a simplified model in Sec. 4.4.

We focus on the DBI action in SUGRA and its application to inflation in Chapter 5. Our construction is based on Refs. [26, 27] in global SUSY, which is reviewed in Sec. 5.1. Then, we promote the global SUSY expression to the superconformal one in Sec. 5.2, which enables us to discuss e.g., the cosmological application. As an application to inflation, we consider the DBI extension of the massive vector multiplet inflation model in Sec. 5.4. The other application, the DBI-Starobinsky model in the new minimal SUGRA, is discussed in Sec. 5.5.

Finally, we conclude in Chapter 6. The notation in this thesis is summarized in Appendix A. We give transformation laws of a general superconformal multiplet in Appendix B, and a brief review of inflation is given in Appendix C. In Appendix D, we give futher details of discussion in Sec. 4.3.

Chapter 2

Review of conformal SUGRA

2.1 Why conformal SUGRA?

In this section, we briefly introduce conformal SUGRA and overview its advantages. In this thesis, we consider 4D SUSY with four supercharges, which is called $\mathcal{N} = 1$. In Poincaré SUGRA, the action of supermultiplets is invariant under the set of local symmetries: the general coordinate transformation, the local Lorentz symmetry, and local SUSY, collectively called Poincaré SUSY. Conformal SUGRA has a much larger set of local symmetries: in addition to the above symmetries, dilatation, chiral U(1) symmetry, special SUSY, and special conformal symmetry, collectively called superconformal symmetry. In such a theory, supermultiplets should also be the representations of such additional symmetries. Those additional symmetries should be broken in the physical system. By setting gauge fixing conditions on the additional symmetries, as we will review in Sec. 2.4, we can obtain the physical Poincaré SUGRA theory as the broken superconformal one.

We employ conformal SUGRA mainly by the following two reasons. One of the reasons is that the conformal SUGRA can produce the Poincaré SUGRA theory in various frames in a systematic and simple way. In most formalisms of Poincaré SUGRA, scalars ϕ^i and graviton $g_{\mu\nu}$ couple to each other through the non-minimal Ricci scalar term $\sim f(\phi^i)R$, which represents the kinetic mixings between ϕ^i and $g_{\mu\nu}$. To make graviton canonical, we have to redefine the metric as $g_{\mu\nu} \to \tilde{g}_{\mu\nu} = e^{-2\sigma(x)} g_{\mu\nu}$ with an appropriate real function $\sigma(x)$. We now discuss the SUSY theory, and therefore, the non-minimal kinetic mixing between the fermions χ^i and gravitino ψ_μ also appears as $\sum_j g_j(\phi^i)\chi^j\gamma^{\mu\nu}\mathcal{D}_\mu\psi_\nu$ where $g_j(\phi^i)$ is a function of scalars, and $\mathcal{D}_\mu$ denotes the covariant derivative defined later. In contrast to the case of the canonicalization of graviton, the procedure to make gravitino canonical is much complicated in many cases.[1] Such a complexity is relaxed in the superconformal

[1] For example, in Ref. [28], the redefinitions procedure for canonicalization of graviton and gravitino

formulation.

Here, we show how the procedure is simplified in conformal SUGRA. Although we will show the procedure precisely in the following sections, it is meaningful to demonstrate it in a simplified example for understanding. As we mentioned above, to produce the Poincaré SUGRA from conformal one, we need to set gauge fixing conditions on the symmetries other than Poincaré SUSY. We focus especially on the dilatation and the special SUSY gauge fixings. Dilatation and the special SUSY have a real bosonic and a fermionic generators respectively. Therefore, we can put conditions on a real bosonic and a fermionic quantities. Let us recall the mixing terms given by

$$\mathcal{L}_{\rm mix} = f(\phi^i)R + g_j(\phi^i)\chi^j\gamma^{\mu\nu}\mathcal{D}_\mu\psi_\nu. \tag{2.1}$$

To make the non-minimal Ricci scalar term canonical $\sim fR \to \frac{1}{2}R$, we put the dilatation gauge fixing condition $f(\phi^i) = \frac{1}{2}$. This condition eliminates one of the scalar degrees of freedom and we assume ϕ^0 is removed. We can solve the condition and then ϕ^0 becomes a function of the other scalars as $\phi^0 = \Phi^0(\phi^{i\neq 0})$. The effects of the dilatation fixing on the other terms appear through ϕ^0 substituted by Φ^0 in the action. As in the same way, the second term in Eq. (2.1) can be removed by using the special SUSY fixing condition $\sum_j g_j(\phi^i)\chi^j = 0$. The condition eliminates one of the fermionic degrees of freedom, which we choose as χ^0. Then, χ^0 becomes a linear combination of the other fermions like $\chi^0 = -g_0^{-1}\sum_{j\neq 0} g_j\chi^j$. Thus, we can obtain canonical kinetic terms of graviton and gravitino. As we find from the above discussion, the superconformal action should have some degrees of freedom other than ones remaining in the gauge-fixed system, that is, in the physical Poincaé SUSY theory. The fields such as ϕ^0 and χ^0 are called *compensator* fields and the simplest choice of them is so that they form a superconformal multiplet, called a *compensator multiplet*.

The second advantage of conformal SUGRA is that it unifies Poincaré SUGRA with different types of gravity multiplet. It has been known that there are some possible sets of auxiliary fields of gravity multiplet in Poincaré SUGRA. From the viewpoint of the Poincaré SUGRA, it is difficult to find the relation between theories with such different sets of auxiliary fields. However, in Ref. [12], it was found that those theories can be understood as conformal SUGRA models with *different compensator multiplets*. Three classes of different Poincaré SUGRA, which can be realized with irreducible superconformal compensator multiplets; a chiral, a real linear, and a complex linear multiplets, are known as the old minimal, the new minimal, and the non-minimal formulations, respectively. The relation between them was studied in Ref [29], and it was found that the new and the non-minimal formulations are equivalent to special classes of the old minimal formulation. It is also known, however, that the equivalence holds only if there are no

coupled to chiral multiplets are shown.

SUSY higher-derivative terms of a compensator. In this sense, if higher-derivative terms exist, the difference of formulations has physical meanings. Such examples include the Starobinsky model in the old and the new minimal formulations, which will be discussed in Sec. 3.3. In this thesis, we will discuss the action with higher-derivative terms in chapters 4 and 5. To understand whether the behavior of such terms depends on the choice of the formulations, the conformal SUGRA formalism is quite useful.

There are other utilities of conformal SUGRA for understanding phenomenological and cosmological aspects of SUGRA systems although we just comment on them briefly. The anomaly mediated SUSY breaking found in Refs. [30, 31] can be simply described in terms of the compensator multiplet [32]. In Ref. [33], using gauge fixing conditions alternative to the conventional one [34], the effective SUGRA action in a flat spacetime was simplified, and expressed in terms of the flat superspace. The conformal SUGRA approach was also applied to cosmology, especially for an understanding of inflation models [35]. In Ref. [36], the general Jordan frame SUGRA has been constructed by using conformal SUGRA, which is useful for constructing inflation models with non-minimal Ricci scalar terms and for clarifying the underlying conformal symmetry. The author showed how the SUSY breaking effects affects on SUGRA inflation models in view of the conformal SUGRA structure [37].

2.2 Basics of conformal SUGRA

We review the construction of conformal SUGRA in this section on the basis of Refs. [12, 38]. For the construction, three formalisms are known. The first is the *superconformal tensor calculus* which we will use below and is based on the gauge theory of superconformal symmetry on the spacetime manifold. The second is conformal superspace approach [39], which is very similar to the previous one, but the base space is superspace. The third one is the group manifold approach [40]. For a practical application, the first method has been used, and so we employ it.

In the superconformal tensor calculus, we consider the gauge theory of the superconformal symmetry. The strategy is quite simple: From the superconformal algebra, we can find representations of the algebra, which is a covariant quantity in the gauge theory. Then, we can construct the action of representations, which is invariant under the superconformal transformations.

First, we define the *covariant quantities* and the *covariant derivative*. Let us consider a set of transformations

$$\delta(\epsilon) \equiv \epsilon^A T_A, \tag{2.2}$$

where A is a label of generators, ϵ^A is a transformation parameter, and T_A is a generator. We assume the algebra formed by T_A as

$$[T_A, T_B] = f_{AB}^C T_C, \tag{2.3}$$

where [,] denotes the commutator, and f_{AB}^C is the structure constant of the algebra, which is antisymmetric with respect to the exchange of A and B.[2] The algebra can also be expressed with transformation parameters ϵ_1^A and ϵ_2^B as

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = (\epsilon_2^B \epsilon_1^A f_{AB}^C) T_C = \delta(\epsilon_3), \tag{2.4}$$

where $\epsilon_3^C \equiv \epsilon_2^B \epsilon_1^A f_{AB}^C$. Under the transformation, a *covariant quantity* Φ is defined so that it transforms as

$$\delta(\epsilon)\Phi = \epsilon^A K_A \tag{2.5}$$

where $K_A \equiv T_A \Phi$. The important point is that the transformation of the covariant quantity does not have the differentiated ϵ^A-terms like $\partial_\mu \epsilon^A$.

In gauge theory, the transformation parameter depends on the spacetime; $\epsilon^A(x)$. Therefore, the spacetime derivative of Φ does not become a covariant quantity, which transform as

$$\delta(\epsilon)\partial_\mu \Phi = \partial_\mu(\epsilon^A K_A) = \epsilon^A \partial_\mu K_A + \partial_\mu \epsilon^A K_A. \tag{2.6}$$

To construct the *covariant derivative*, *gauge fields* $\mathcal{B}_\mu^A$ are required, which transform under the symmetry transformation as

$$\delta(\epsilon)\mathcal{B}_\mu = \partial_\mu \epsilon^A + \epsilon_C \mathcal{B}_\mu^B f_{BC}^A. \tag{2.7}$$

Then we can define a *covariant derivative* $\mathcal{D}_\mu$ on Φ by

$$\mathcal{D}_\mu \Phi = (\partial_\mu - \mathcal{B}_\mu^A T_A)\Phi = (\partial_\mu - \delta(\mathcal{B}_\mu))\Phi. \tag{2.8}$$

Indeed, this is a covariant quantity, which transforms as

$$\begin{aligned}
\delta(\epsilon)\mathcal{D}_\mu \Phi =& \epsilon^A \partial_\mu K_A + (\partial_\mu \epsilon) K_A - (\delta(\epsilon)\mathcal{B}_\mu^A) K_A - \mathcal{B}_\mu^A \delta(\epsilon) K_A \\
=& \epsilon^A \partial_\mu K_A - \epsilon_C \mathcal{B}_\mu^B f_{BC}^A K_A - \mathcal{B}_\mu^A \delta(\epsilon) K_A \\
=& \epsilon^A \partial_\mu K_A - \epsilon^A \mathcal{B}_\mu^B [T_B, T_A]\Phi - \mathcal{B}_\mu^B \epsilon^A T_A T_B \Phi \\
=& \epsilon^A (\partial_\mu K_A - \mathcal{B}_\mu^B T_B K_A) \\
=& \epsilon^A \mathcal{D}_\mu K_A,
\end{aligned} \tag{2.9}$$

[2]Here, we assume the generators T^A are bosonic quantities. Even if the generators are fermionic, we can use the final form of each expression given in the following discussion.

where we have used the algebra (2.3) in the third equality.

From the gauge fields, we can construct the other covariant quantity called *curvature*. The curvature $R^A_{\mu\nu}$ is defined by

$$R^A_{\mu\nu} = 2\partial_{[\mu}\mathcal{B}^A_{\nu]} + \mathcal{B}^C_\nu \mathcal{B}^B_\mu f^A_{BC}, \tag{2.10}$$

where $[\cdots]$ is the antisymmetrization defined below Eq. (A.1). We can derive the transformation rule of curvatures as

$$\begin{aligned}\delta(\epsilon)R^A_{\mu\nu} =&2\partial_{[\mu}\partial_{\nu]}\epsilon^A + 2\partial_{[\mu}(\epsilon^C\mathcal{B}^B_{\nu]})f^A_{BC} + \left((\partial_\nu\epsilon^C + \epsilon^D\mathcal{B}^E_\nu f^C_{ED})\mathcal{B}^B_\mu f^A_{BC} - (\mu\leftrightarrow\nu)\right)\\ =&\epsilon^C(2\partial_{[\mu}\mathcal{B}^B_{\nu]}f^A_{BC}) + \epsilon^D\mathcal{B}^E_\nu\mathcal{B}^B_\mu f^C_{ED}f^A_{BC} + \epsilon^D\mathcal{B}^C_\nu\mathcal{B}^E_\mu f^B_{ED}f^A_{BC}\\ =&\epsilon^C(2\partial_{[\mu}\mathcal{B}^B_{\nu]}f^A_{BC}) - \epsilon^C\mathcal{B}^E_\nu\mathcal{B}^D_\mu f^B_{ED}f^A_{BC}\\ =&\epsilon^C R^B_{\mu\nu}f^A_{BC},\end{aligned} \tag{2.11}$$

where we have used a Jacobi identity $\sum_B f^B_{CE}f^A_{BD} + (\text{cyclic with respect to C, D, E}) = 0$ in the third equality.

So far, we have discussed a general gauge theory. Next, let us focus on the superconformal symmetry. We define the superconformal transformation δ_{sc} as

$$\delta_{sc} = \xi^a P_a + \epsilon Q + \frac{1}{2}\lambda^{ab}M_{ab} + \lambda_D D + \theta A + \eta S + \lambda^a_K K_a, \tag{2.12}$$

In Eq. (2.12), P_a, Q, M_{ab}, D, A, S, and K_a denote the generators of translation, SUSY, Lorentz transformation, dilatation, chiral U(1) symmertry, conformal SUSY (S-SUSY), and special conformal transformation, respectively. The subscripts a, b denote the local Lorentz indices. The coefficients ξ^a, ϵ, λ^{ab}, λ_D, θ, η, and λ^a_K are corresponding transformation parameters, respectively. These generators form the superconformal algebra as (other combinations are commutative)

$$\begin{aligned}[P_a, M_{bc}] =&2\eta_{a[b}P_{c]}, \quad [P_a, D] = -P_a, \quad [P_a, S_\alpha] = (\gamma_a Q)_\alpha,\\ \{Q_\alpha, Q^\beta\} =& -\frac{1}{2}(\gamma^a)_\alpha^\beta P_a, \quad [Q_\alpha, M_{ab}] = \frac{1}{2}(\gamma_{ab}Q)_\alpha, \quad [Q_\alpha, D] = -\frac{1}{2}Q_\alpha,\\ [Q_\alpha, A] =&\frac{3i}{2}(\gamma_* Q)_\alpha, \quad \{Q_\alpha, S_\beta\} = -\frac{1}{2}\mathcal{C}_{\alpha\beta}D - \frac{1}{4}(\gamma^{ab})_{\alpha\beta}M_{ab} + \frac{i}{2}(\gamma_*)_{\alpha\beta}A,\\ [Q_\alpha, K_a] =& -(\gamma_a S)_\alpha, \quad [M_{ab}, M_{cd}] = 4\eta_{[a[c}M_{d]b]}, \quad [M_{ab}, S_\alpha] = -\frac{1}{2}(\gamma_{ab}S)_\alpha,\\ [K_a, M_{bc}] =&2\eta_{a[b}K_{c]}, \quad [D, S_\alpha] = -\frac{1}{2}S_\alpha, \quad [D, K_a] = -K_a, \quad [A, S_\alpha] = \frac{3i}{2}(\gamma_* S)_\alpha\\ \{S_\alpha, S^\beta\} =& -\frac{1}{2}(\gamma^a)_\alpha^\beta K_a, \quad [P_a, K_b] = 2(\eta_{ab}D + M_{ab}),\end{aligned} \tag{2.13}$$

where the subscripts α and β denote the spinor indices, and $\mathcal{C}_{\alpha\beta}$ is the charge conjugation matrix defined below Eq. (A.4). $\{\cdots\}$ denotes the anti-commutator, and γ_a, γ_{ab}, and γ_* are defined in Appendix A. We also define the set of gauge fields as

$$\mathcal{B}_\mu^A T_A = e_\mu^a P_a + \bar{\psi}_\mu Q + \frac{1}{2}\omega_\mu^{ab} M_{ab} + b_\mu D + A_\mu A + \bar{\phi}_\mu S + f_\mu^a K_a. \tag{2.14}$$

From the algebra, we can read off the structure constants, and obtain the following set of curvatures (2.10) for each generator,

$$R_{\mu\nu}^a(P) =2(\partial_{[\mu} + b_{[\mu})e_{\nu]}^a + 2\omega_{[\mu}^{ab} e_{\nu]b} - \frac{1}{2}\bar{\psi}_\mu \gamma^a \psi_\nu \tag{2.15}$$

$$R_{\mu\nu}(Q) =2\left(\partial_{[\mu} + \frac{1}{2}b_{[\mu} - \frac{3i}{2}A_{[\mu}\gamma_* + \frac{1}{4}\omega_{[\mu}^{ab}\gamma_{ab})\right)\psi_{\nu]} - 2\gamma_{[\mu}\phi_{\nu]}, \tag{2.16}$$

$$R_{\mu\nu}^{ab}(M) =2\partial_{[\mu}\omega_{\nu]}^{ab} - 2\omega_{[\mu c}^{a}\omega_{\nu]}^{cb} + 8f_{[\mu}^{[a}e_{\nu]}^{b]} - \bar{\psi}_{[\mu}\gamma^{ab}\phi_{\nu]}, \tag{2.17}$$

$$R_{\mu\nu}(D) =2\partial_{[\mu}b_{\nu]} - 4f_{[\mu}^a e_{\nu]a} - \bar{\psi}_{[\mu}\phi_{\nu]}, \tag{2.18}$$

$$R_{\mu\nu}(A) =2\partial_{[\mu}A_{\nu]} + i\bar{\psi}_{[\mu}\gamma_*\phi_{\nu]}, \tag{2.19}$$

$$R_{\mu\nu}(S) =2\left(\partial_{[\mu} - \frac{1}{2}b_{[\mu} + \frac{3i}{2}A_{[\mu}\gamma_* + \frac{1}{4}\omega_{[\mu}^{ab}\gamma_{ab}\right)\phi_{\nu]} - 2\gamma_a f_{[\mu}^a\psi_{\nu]}, \tag{2.20}$$

$$R_{\mu\nu}^a(K) =2(\partial_{[\mu} - b_{[\mu})f_{\nu]}^a + 2\omega_{[\mu}^{ab}f_{\nu]b} - \frac{1}{2}\bar{\phi}_\mu\gamma^a\phi_\nu. \tag{2.21}$$

In the above construction, we have defined the symmetry algebra and corresponding gauge fields, as in the case of the gauge theory of "internal" symmetries. However, the superconformal symmetry would be a "spacetime" symmetry, and therefore, we have to relate the symmetry to the general coordinate transformation (GCT) of spacetime. For such a purpose, we need to change the meaning of "translation" as follows: We define translation $\tilde{P}_a$, whose transformation is defined as

$$\begin{aligned}\xi^a\tilde{P}_a \equiv&\delta_{\rm GC}(\xi^\mu) - \sum_{A\neq P}\xi^\mu\mathcal{B}_\mu^A T_A\\ =&\xi^\mu(\nabla_\mu - \sum_{A\neq P}\mathcal{B}_\mu^A T_A)\\ \equiv&\xi^\mu D_\mu,\end{aligned} \tag{2.22}$$

where $\delta_{\rm GC}(\xi^\mu)$ denotes GCT with a parameter $\xi^\mu(x)$, $\xi^a \equiv e_\mu^a\xi^\mu$, and ∇_μ is a covariant derivative with respect to GCT. Now the translation becomes the covariantized GCT with respect to all the symmetries. This modification requires the replacement of P^a to $\tilde{P}^a$ in the algebra. Therefore, the covariant derivative $\mathcal{D}_\mu$ in Eq. (2.8) does not appear in the following discussion but the deformed covariant derivative D_μ does. Then, we have to know whether the deformed algebra requires some conditions or not for the closure of the algebra. Indeed, it is known that three conditions on curvatures and the modification of the transformation laws of three gauge fields are required. We do not discuss them in detail, but show how those conditions and transformations are derived in the following.

In the algebra (2.13), we find that only the equation $\{Q_\alpha, Q^\beta\} = \frac{1}{2}(\gamma^a)_\alpha^\beta P_a$ contains P_a on the right-hand side without including it on the left-hand side. The modification $P_a \to \tilde{P}_a$ especially affects such a relation. Let us discuss the original anti-commutation relation of Q on e_μ^a,

$$[\bar{\epsilon}_1 Q, \bar{\epsilon}_2 Q] e_\mu^a = \xi^b (P_b e_\mu^a), \tag{2.23}$$

where $\xi^b \equiv \frac{1}{2}\bar{\epsilon}_2 \gamma^b \epsilon_1$. Before starting deformation, we note the following relation,

$$\xi^a P_a \mathcal{B}_\mu^A = \xi^a \tilde{P}_a \mathcal{B}_\mu^A + \xi^\nu R_{\nu\mu}^A, \tag{2.24}$$

which can be derived from the definition of $\tilde{P}_a$ (2.22) straightforwardly. With this identity, we obtain

$$[\bar{\epsilon}_1 Q, \bar{\epsilon}_2 Q] e_\mu^a = \xi^b (\tilde{P}_b e_\mu^a) + \xi^\nu R_{\nu\mu}^a(P). \tag{2.25}$$

To deform the algebra so that $P_a \to \tilde{P}_a$,

$$R_{\nu\mu}^a(P) = 0 \tag{2.26}$$

is required. This condition can be solved with respect to ω_μ^{ab}. Thus, ω_μ^{ab} becomes a dependent field given as a function of e_μ^a, b_μ, ψ_μ (see the definition of $R_{\mu\nu}(P^a)$ in Eq. (2.15)).

We also notice that the condition $R_{\mu\nu}^a(P) = 0$ is not SUSY invariant. Indeed, $\bar{\epsilon}Q(R_{\mu\nu}^a(P)) = \frac{1}{2}\bar{\epsilon}\gamma^a R_{\mu\nu}(Q) \neq 0$ if we follow the original transformation rule. Such a contradiction can be solved by taking into account the modification of the transformation law of ω_μ^{ab}. Now, it is a dependent field determined by Eq. (2.26), and so the SUSY transformation of ω_μ^{ab} should be consistent with the constraint. Therefore, the transformation law of ω_μ^{ab} should be modified so that

$$\bar{\epsilon}Q(R_{\mu\nu}^a(P)) = \frac{1}{2}\bar{\epsilon}\gamma^a R_{\mu\nu}(Q) + 2\delta_Q'(\epsilon)\omega_{[\mu}^{ab} e_{\nu]b} = 0, \tag{2.27}$$

where δ_Q' is the additional SUSY transformation other the original one. Equation (2.27) can be solved, and we obtain

$$\delta_Q'(\epsilon)\omega_\mu^{ab} = -\frac{1}{2}\bar{\epsilon}\gamma^{[a} R_\mu^{b]}(Q) - \frac{1}{4}\bar{\epsilon}\gamma_\mu R^{ab}(Q). \tag{2.28}$$

We need to perform the same procedure for ψ_μ, b_μ, and A_μ, and find that the modification can be completed with the following conditions,

$$\gamma^\mu R_{\mu\nu}(Q) = 0, \tag{2.29}$$
$$R^{\rm cov}_{\mu\nu}(M) + i\tilde{R}_{\mu\nu}(A) = 0, \tag{2.30}$$

where $R^{\rm cov}_{\mu\nu}(M) = R^{{\rm cov}ab}_{\mu\rho}(M) e^\rho_a e_{b\nu}$, and $R^{{\rm cov}ab}_{\mu\rho}(M) = R^{ab}_{\mu\rho}(M) - \frac{1}{2}\bar{\psi}_\rho \gamma_\mu R^{ab}(Q)$. These conditions can be solved with respect to ϕ_μ and f^a_μ, respectively. Then those fields become dependent fields and their SUSY transformations have additional terms as ω^{ab}_μ:

$$\delta'_Q(\epsilon)\phi_\mu = \frac{i}{2}\gamma^\nu(\gamma_* R_{\mu\nu}(A) + \tilde{R}_{\mu\nu}(A))\epsilon, \tag{2.31}$$
$$\delta'_Q(\epsilon) f^a_\mu = \frac{1}{4}\bar{\epsilon}(\gamma^{ab} R^{\rm cov}_{b\mu}(S) + \gamma_* \tilde{R}^{{\rm cov}a}_{\mu}(S)), \tag{2.32}$$

where $R^{\rm cov}_{\mu\nu}(S) = R_{\mu\nu}(S) + \frac{i}{2}\gamma^\rho(\gamma_* R_{\mu\rho}(A) + \tilde{R}_{\mu\rho}(A))\psi_\nu$. Note that all the other transformations are the same with that determined by the original algebra.

Thus, the deformation of the algebra is completed, and the superconformal symmetry becomes a "spacetime" symmetry.

2.3 Superconformal multiplet

2.3.1 General multiplet

In the previous section, we have discussed the basic structure of the superconformal tensor calculus and seen the deformed superconformal algebra. Here, let us construct the representations of the algebra. From a representation Φ, we can construct finite numbers of its SUSY descendants $\sim Q\Phi$, $\bar{Q}\Phi, \cdots$, $QQ\bar{Q}\bar{Q}\Phi$. We call such a set of fields a *supermultiplet*. However, a supermultiplet may not be a *superconformal multiplet*, which is a set of fields transformed under the superconformal transformations. The reason can be understood by the following example: Let us consider the S-transformation of $Q\Phi$. From the algebra (2.13), $S(Q\Phi) = \{S, Q\}\Phi - Q(S\Phi) \sim (M + D + A)\Phi - Q(S\Phi)$. We find that the transformation of $Q\Phi$ depends on the Q-transformation of a new field $S\Phi$. This means that $S\Phi$ is also a "superpartner" of Φ. This implies that a supermultiplet itself can not determine the transformation law under the superconformal symmetry.

From the above argument, we define a superconformal multiplet as a supermultiplet which satisfies the following condition: $S\Phi = 0, K\Phi = 0$. Let us construct a superconformal multiplet, more concretely.

Let us consider a superconformal multiplet $\mathcal{C}$, which has the lowest component C and transforms as

$$\delta_Q C = \frac{i}{2}\bar{\epsilon}\gamma_*\zeta, \quad \delta_M C = 0, \quad \delta_D C = w\lambda_D C, \quad \delta_A C = in\theta C$$
$$\delta_S C = 0, \quad \delta_K C = 0, \tag{2.33}$$

where we have used the notation $\delta_I = \epsilon^I T_I$ (without a summation with respect to I). In the D- and A-transformations of C, we have introduced two real parameter w and n called the *Weyl* and the *chiral* weights, respectively. These weights are the most important quantities for characterizing the superconformal multiplet $\mathcal{C}$. ζ is an arbitrary spinor whose transformation law is uniquely determined by the algebra. Let us demonstrate the procedure to determine the transformation law of ζ. As an example, we consider the S-transformation of ζ. From the algebra (2.13), we find

$$[\delta_S(\eta), \delta_Q(\epsilon)] = \delta_D\left(\frac{1}{2}\bar{\eta}\epsilon\right) + \delta_M\left(\frac{1}{2}\bar{\epsilon}\gamma\eta\right) + \delta_A\left(\frac{i}{2}\bar{\epsilon}\gamma_*\eta\right). \tag{2.34}$$

We also know the transformation law of C, and then obtain

$$\begin{aligned}\delta_S(\eta)\delta_Q(\epsilon)C =& [\delta_S(\eta), \delta_Q(\epsilon)]C + \delta_Q(\epsilon)\delta_S(\eta)C \\ =& \left(\delta_D\left(\frac{1}{2}\bar{\eta}\epsilon\right) + \delta_M\left(\frac{1}{2}\bar{\epsilon}\gamma\eta\right) + \delta_A\left(\frac{i}{2}\bar{\epsilon}\gamma_*\eta\right)\right)C + 0 \\ =& \frac{1}{2}\bar{\eta}\epsilon w C + 0 + in\left(\frac{i}{2}\bar{\epsilon}\gamma_*\eta\right)C \\ =& \frac{i}{2}\bar{\epsilon}\gamma_*(iw\gamma_* + in)\eta C.\end{aligned} \tag{2.35}$$

The left-hand side can be rewritten as

$$\delta_S(\eta)\delta_Q(\epsilon)C = \frac{i}{2}\bar{\epsilon}\gamma_*\delta_S(\eta)\zeta, \tag{2.36}$$

and then, combining these equations, we obtain

$$\delta_S(\eta)\zeta = (iw\gamma_* + in)\eta C. \tag{2.37}$$

From the above example, we can confirm that the transformation law of a descendant field can be uniquely determined by that of the lower (ascendant) components.

Another important example is the Q-transformation of ζ. Following the (deformed) algebra, we have

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)]C = -\frac{1}{2}(\bar{\epsilon}_1\gamma^a\epsilon_2)D_a C. \tag{2.38}$$

To compute the left-hand side of this equation, we need to know the form of $\delta_Q(\epsilon)\zeta$, the general form of which is

$$\delta_Q(\epsilon)\zeta = (\Phi^0 + \Phi^1_a\gamma^a + \Phi^2_{ab}\gamma^{ab} + \gamma^a\gamma_*\Phi^3_a + \gamma_*\Phi^4)\epsilon, \tag{2.39}$$

where Φ^I $(I = 0, \cdots, 4)$ are bosonic fields with (or without) Lorentz indices. This expression contains all possible fields as the SUSY descendants of ζ associated with the complete set of γ-matrices in 4D. Then, we can compute the left-hand side of Eq. (2.38) as

$$\begin{aligned}[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)]C =& \frac{i}{2}\bar{\epsilon}_2\gamma_*\delta_Q(\epsilon_1)\zeta - (\epsilon_1 \leftrightarrow \epsilon_2)\\ =& \frac{i}{2}(\bar{\epsilon}_2\gamma_*\epsilon_1\Phi^0 + \bar{\epsilon}_2\gamma_*\gamma^a\epsilon_1\Phi^1_a + \bar{\epsilon}_2\gamma_*\gamma^{ab}\epsilon_1\Phi^2_{ab})\\ &+ \frac{i}{2}(\bar{\epsilon}_2\gamma_*\gamma^a\gamma_*\epsilon_1\Phi^3_a + \bar{\epsilon}_2\epsilon_1\Phi^4) - (\epsilon_1 \leftrightarrow \epsilon_2)\\ =& 0 + 0 + i\bar{\epsilon}_2\gamma_*\gamma^{ab}\epsilon_1\Phi^2_{ab} + i\bar{\epsilon}_1\gamma^a\epsilon_2\Phi^3_a,\end{aligned} \tag{2.40}$$

where we have used the Majorana flip identities (A.6). Comparing this and the right-hand side of Eq. (2.38), we find

$$\Phi^2_{ab} = 0, \tag{2.41}$$

$$\Phi^3_a = \frac{i}{2}D_aC. \tag{2.42}$$

The undetermined components Φ^0, Φ^1_a and Φ^4 should be understood as SUSY descendants of ζ. We conventionally define the new fields H, K, and B_a as $\Phi^0 = -\frac{1}{2}K$, $\Phi^1_a = -\frac{1}{2}B_a$, and $\Phi^4 = \frac{i}{2}H$. Thus we have determined the Q-transformation as

$$\delta_Q(\epsilon)\zeta = \frac{1}{2}(iH\gamma_* - K - \not{B} - i\gamma_*\not{D}C)\epsilon. \tag{2.43}$$

In this way, we can determine all the components of $\mathcal{C}$ and their transformation laws. The components are summarized as

$$\mathcal{C} = [C, \zeta, H, K, B_a, \lambda, D], \tag{2.44}$$

and we will use this notation to express superconformal multiplets. The transformation law of each component is summarized in Appendix B. Note that the Weyl and chiral weights of each component of $\mathcal{C}$ are uniquely determined by those of the lowest component C. Therefore, we caracterize the Weyl and the chiral weights of a superconformal multiplet by those of its lowest component.

In this subsection, we have discussed a general multiplet whose lowest component is a scalar. However, we can further consider a general multiplet with external Lorentz indices defined in Ref. [41]. We do not use it directly in this thesis, and therefore, we do not discuss it here.

2.3.2 Irreducible multiplets

We have constructed a general multiplet in the previous subsection. Here, we introduce some special multiplets, which are constrained by specific conditions.

First, we define a *chiral multiplet*. A chiral multiplet Φ consists of three independent fields:

$$\Phi = [\phi, P_L\chi, F], \tag{2.45}$$

where $P_L = \frac{1+\gamma_*}{2}$ is a left projection. Embedding a chiral multiplet into a general multiplet (2.44), we can express Φ as

$$\mathcal{C}(\Phi) = [\phi, -\sqrt{2}iP_L\chi, -F, iF, iD_a\phi, 0, 0], \tag{2.46}$$

where we have assumed that ϕ does not have any gauge charges of internal symmetries. This multiplet cannot have arbitrary Weyl and chiral weights because of the following consistency condition: We assume Φ has its Weyl and chiral weights (w, n). Then, the S-SUSY transformation of $P_L\chi$ gives

$$\delta_S(\eta)P_L\chi = \frac{1}{\sqrt{2}}(w\gamma_* + n)\eta\phi. \tag{2.47}$$

The left-hand side of this equation has the definite chirality projected by $P_L = \frac{1}{2}(1+\gamma_*)$. However, the right-hand side can have a different one in general. Only the solution for this contradiction is the choice $w = n$. Therefore, chiral multiplets should have the Weyl and the chiral weights satisfying $\boxed{w = n}$.[3] Chiral multiplets are important to describe chiral fermions such as those in SM. Although a chiral multiplet here is a singlet for any internal symmetries, gauged chiral multiplets, which are important for describing SUSY SM, will be shown later.

From a specific multiplet which is not a chiral one, we can construct a chiral multiplet by using the *chiral projection*. Let us consider a general multiplet (2.44) with its Weyl and chiral weights $(w, n) = (w_C, n_C)$, which satisfy $w_C - n_C = 2$. Then, the following combination is S-inert:

$$\phi_{\mathcal{C}} = \frac{1}{2}(H - iK). \tag{2.48}$$

Indeed,

$$\begin{aligned}\delta_S\phi_{\mathcal{C}} =& \frac{i}{4}\bar{\eta}(w_C - n_C - 2)(\gamma_* - 1)\zeta \\ =& 0,\end{aligned} \tag{2.49}$$

[3] We will give another definition of a chiral multiplet in Sec. 4.2.

where we have used the transformation laws of H and K in Eqs. (B.3) and (B.4). The Q-transformation of $\phi_{\mathcal{C}}$ is

$$\delta_Q \phi_{\mathcal{C}} = \frac{i}{2}\bar{\epsilon} P_L(\lambda + \not{D}\zeta), \tag{2.50}$$

which is the same as that of a chiral multiplet (2.46) by the following identifications: $\phi = \phi_{\mathcal{C}}$ and $P_L\chi = \frac{i}{\sqrt{2}} P_L(\lambda + \not{D}\zeta)$. The Weyl and the chiral weights of ϕ_C is determined as $(w, n) = (w_C+1, w_C+1)$ from Eqs. (B.3) and (B.4). Therefore, the multiplet $\Phi_{\mathcal{C}}$, whose lowest component is $\phi_{\mathcal{C}}$, is a chiral multiplet. Therefore, we can define the projection Σ on a superconformal multiplet $\mathcal{C}$ with $w_C - n_C = 2$ as

$$\Sigma(\mathcal{C}) = \Phi_{\mathcal{C}} = \left[\frac{1}{2}(H - iK), \frac{i}{\sqrt{2}} P_L(\lambda + \not{D}\zeta), -\frac{1}{2}(D + \Box C + iD^a B_a)\right]. \tag{2.51}$$

We call Σ the *chiral projection.*

We can define an irreducible superconformal multiplet $\mathbf{L}$, whose Weyl and chiral weights $w_{\mathbf{L}}$ and $n_{\mathbf{L}}$ satisfy $w_{\mathbf{L}} - n_{\mathbf{L}} = 2$, so that

$$\Sigma(\mathbf{L}) = 0. \tag{2.52}$$

Then, $\mathbf{L}$ is called a (complex) *linear multiplet.* In Sec. 3.3.2, we will focus on a *real linear multiplet* L with $(w, n) = (2, 0)$. The components of L are given by

$$\mathcal{C}(L) = [C^L, \zeta^L, 0, 0, B^L_a, -\not{D}\zeta^L, -\Box C^L], \tag{2.53}$$

where C^L is a real scalar, ζ^L is a Majorana spinor, and B^L_a is a real vector, which satisfies $D^a B^L_a = 0$.

The other special multiplet is a *gauge multiplet.* For a real general multiplet (2.44) with $(w, n) = (0, 0)$, let us consider the following combination:

$$\hat{B}_\mu \equiv e^a_\mu B_a - \frac{1}{2}\bar{\psi}_\mu \zeta. \tag{2.54}$$

Its Q-transformation is given by

$$\delta_Q \hat{B}_\mu = -\frac{1}{2}\bar{\epsilon}\gamma_\mu \lambda - \frac{1}{2}\partial_\mu(\bar{\epsilon}\zeta). \tag{2.55}$$

The last term looks like a U(1) transformation of a gauge field $\hat{B}_\mu \to \hat{B}_\mu - \partial_\mu \sigma$ where σ is a real scalar field. If we regard the last term as a U(1) transformation, the appearance of ζ is not physical. Then, we can construct the following superconformal multiplet:

$$\mathbf{V}^A \equiv [(\hat{B}_\mu)^A, (\lambda^G)^A, (D^G)^A], \tag{2.56}$$

where we have extended the U(1) gauge multiplet to that of general gauge symmetries, A denotes the index of internal gauge symmetries, $(\lambda^G)^A$ is a Majorana spinor, and $(D^G)^A$ is a real scalar. The transformation laws of $\mathbf{V}^A$ are as follows:

$$\begin{aligned}
(\hat{B}_\mu)^A : \ &\delta_Q(\hat{B}_\mu)^A = -\frac{1}{2}\bar{\epsilon}\gamma_\mu(\lambda^G)^A, \quad \delta_M(\hat{B}_\mu)^A = 0, \\
&\delta_D(\hat{B}_\mu)^A = 0, \quad \delta_A(\hat{B}_\mu)^A = 0, \\
&\delta_S(B_\mu)^A = 0, \quad \delta_K(\hat{B}_\mu)^A = 0, \qquad (2.57)\\
(\lambda^G)^A : \ &\delta_Q(\lambda^G)^A = \left(\frac{i}{2}\gamma_*(D^G)^A + \frac{1}{4}\gamma^{ab}(\hat{F}^G_{ab})^A\right)\epsilon, \quad \delta_M(\lambda^G)^A = -\frac{1}{4}\lambda^{ab}\gamma_{ab}(\lambda^G)^A, \\
&\delta_D(\lambda^G)^A = \frac{3}{2}\lambda_D(\lambda^G)^A, \quad \delta_A(\lambda^G)^A = \frac{3i}{2}\theta\gamma_*(\lambda^G)^A, \\
&\delta_S(\lambda^G)^A = 0, \quad \delta_K(\lambda^G)^A = 0, \qquad (2.58)\\
(D^G)^A : \ &\delta_Q(D^G)^A = \frac{i}{2}\bar{\epsilon}\gamma_*(\not{D}\lambda)^A, \quad \delta_M(D^G)^A = 0, \\
&\delta_D(D^G)^A = 2\lambda_D(D^G)^A, \quad \delta_A(D^G)^A = 0, \\
&\delta_S(D^G)^A = 0, \quad \delta_K(D^G)^A = 0, \qquad (2.59)
\end{aligned}$$

up to the gauge transformation of $(\hat{B}_\mu)^A$, where

$$(\hat{F}^G_{ab})^A \equiv e^\mu_a e^\nu_b \left(2\partial_{[\mu}(\hat{B}_{\nu]})^A + f^A_{BC}(\hat{B}_\mu)^B(\hat{B}_\nu)^C + \bar{\psi}_{[\mu}\gamma_{\nu]}(\lambda^G)^A\right), \qquad (2.60)$$

and f^A_{BC} is the structure constant of gauge symmetries.

Now we have a gauge multiplet, and therefore, we can discuss a general multiplet which has a U(1) gauge charge. In such a case, the Q-transformation law of a general multiplet (2.44) changes by the following reason: If the lowest component of a multiplet transforms under internal gauge symmetries, its covariant derivative should include $(\hat{B}_\mu)^A$. Then, the Q-transformation of such a term has gaugino $(\lambda^G)^A$ terms. More concretely, we assume that C^I, where I is a label of general multiplets, transforms as $C^I \to k^I_A(C)$ under the internal gauge transformation, where $k^I_A(C)$ is the Killing vector of C^I. Then, the covariant derivative of C^I includes a term like $-(\hat{B}_\mu)^A k^I_A$, which transforms under Q as

$$\delta_Q\left((\hat{B}_\mu)^A k^I_A\right) \sim (\hat{B}_\mu)^A\partial_J k^I_A\zeta^J + \gamma_\mu(\lambda^G)^A k^I_A, \qquad (2.61)$$

where ∂_J denotes the derivative with respect to C^J. The first term in the right-hand side is a part of the covariant derivative of ζ^I but the second term is a new contribution due to the internal gauge symmetry. Therefore, the transformation law of superconformal multiplets are different if they have charges under internal gauge symmetries. Note that the transformation laws of symmetries other than Q are the same as the original ones because $\hat{B}_\mu^A$ is inert under the other superconformal transformations. The extra terms in Q-transformations of each component are summarized in Appendix B.

It is worth noting that the gauged chiral multiplet consists not only of the original components but also of those of gauge multiplets. We show its components:

$$\mathcal{C}(\Phi^{\text{gauged}}) = [\phi, -\sqrt{2}iP_L\chi, -F, iF, iD_a\phi, -2iP_R(\lambda^G)^A k_A(\phi), -ik_A(D^G)^A], \tag{2.62}$$

where $k_A(\phi)$ is the Killing vector for gauge symmetries.

2.3.3 Multiplication law of supermultiplets

We have discussed irreducible superconformal multiplets, which satisfy some conditions. Next, let us construct a multiplet from multiplications of superconformal multiplets. For general multiplets $\mathcal{C}_1$ and $\mathcal{C}_2$ with (w_1, n_1) and (w_2, n_2), the multiplication of their lowest components C_1C_2 is obviously S- and K-inert and has $(w, n) = (w_1 + w_2, n_1 + n_2)$. Therefore, that can be the lowest component of a superconformal multiplet denoted by $\mathcal{C}_1\mathcal{C}_2$. From this observation, we find that the function of the lowest components forms a superconformal multiplet, as long as all the terms in it have the same Weyl and chiral weights.

It is important to stress that the Weyl and the chiral weights are additive with respect to the multiplication of superconformal multiplet. This rule is important to construct the invariant action of superconformal multiplets.

With general multiplets $\mathcal{C}^I$, the function of them $f(\mathcal{C}^I)$ is given by

$$\begin{aligned} f(\mathcal{C}^I) =& \Bigg[f(C^I),\ f_I\zeta^I,\ f_I H^I - \frac{1}{4}\bar{\zeta}^I\zeta^J f_{IJ},\ f_I K^I + \frac{i}{4}\bar{\zeta}^I\gamma_*\zeta^J f_{IJ}, \\ & f_I B_a^I + \frac{i}{4}\bar{\zeta}^I\gamma_*\gamma_\zeta^J f_{IJ},\ \lambda',\ D' \Bigg], \end{aligned} \tag{2.63}$$

where

$$\lambda' = f_I\lambda^I + \frac{1}{2}(H^I - i\gamma_*K^I + i\gamma_*\not{B}^I - \not{D}C^I)\zeta^J f_{IJ} - \frac{1}{4}(\bar{\zeta}^I\zeta^J)\zeta^K f_{IJK}, \tag{2.64}$$

$$\begin{aligned} D' =& f_I D^I + \frac{1}{2}f_{IJ}(H^IH^J + K^IK^J - B_a^I B^{Ja} - D_aC^I D^aC^J) - \bar{\zeta}^I\lambda^J f_{IJ} \\ & - \frac{1}{4}\bar{\zeta}^K(H^I - i\gamma_*K^I + i\gamma_*\not{B}^I)\zeta^J f_{IJK} + \frac{1}{16}f_{IJKL}(\bar{\zeta}^I\zeta^J)(\bar{\zeta}^K\zeta^L), \end{aligned} \tag{2.65}$$

and the subscripts on $f = f(C^I)$ denote the derivatives with respect to C^Is (e.g., $f_{IJ} = \partial_I\partial_J f$).

2.4 Invariant action and superconformal gauge fixing

In this section, we show invariant action formulae in conformal SUGRA and discuss the superconformal gauge fixing to derive a physical action.

2.4.1 F- and D-term density formulae

With a chiral multiplet (2.45) with $(w, n) = (3, 3)$, which is a singlet of any internal gauge symmetry, the following action is invariant under the superconformal transformation:

$$[\phi]_F \equiv \int d^4xe \left[F + \frac{1}{\sqrt{2}}\bar{\psi}_\mu \gamma^\mu P_L \chi + \frac{1}{2}\phi\bar{\psi}_\mu \gamma^{\mu\nu} P_R \psi_\nu + \text{h.c.} \right], \tag{2.66}$$

where $e \equiv \det(e_\mu^a)$. This is the so-called *F-term density formula*. Although we do not show the invariance of this action explicitly, we note that the only nontrivial part of the proof is the Q-invariance.

We can construct the other useful action formula called *D-term density formula*. From a real general multiplet $\mathcal{C}$ (2.44) with $(w, n) = (2, 0)$, we can construct a chiral multiplet $\Sigma(\mathcal{C})$ (2.51). As we mentioned, $\Sigma(\mathcal{C})$ has $(w, n) = (3, 3)$, which can be applied to the F-term density formula. By substituting it into the formula (2.66), we obtain the following action after some partial integrals,

$$\begin{aligned}[C]_D \equiv \int d^4xe \Bigg[& D - \frac{1}{3}C\hat{R} + \frac{1}{6}(C\bar{\psi}_\mu \gamma^{\mu\rho\sigma} - i\bar{\zeta}\gamma_{\rho\sigma}\gamma_*)R'_{\rho\sigma}(Q) - \frac{i}{2}\bar{\psi}_\mu\gamma^\mu\gamma_*\lambda \\ & + \frac{1}{4}\epsilon^{abcd}\bar{\psi}_a\gamma_b\psi_c(B_d - \frac{1}{2}\bar{\psi}_d\zeta) \Bigg], \end{aligned} \tag{2.67}$$

where $\hat{R} \equiv R^{\text{cov}}_{\mu\nu}(M)g^{\mu\nu}$, and $R'_{\mu\nu}(Q) \equiv 2(\partial_{[\mu} + \frac{1}{4}\omega^{ab}_{[\mu}\gamma_{ab} + \frac{1}{2}b_{[\mu} - \frac{3i}{2}A_{[\mu}\gamma_*)\psi_{\nu]}$. This action is also superconformal invariant because it is an alternative form of the F-term formula.

2.4.2 Superconformal gauge fixing and compensators

Now we can construct the superconformal action and make it the Poincaré one by the superconformal gauge fixing. Before discussing the procedure in a concrete example, we have to know the general feature of the superconformal gauge fixing.

To obtain the Poincaré SUGRA action from conformal one, we have to break the full superconformal symmetry to the Poincaré SUSY. To do so, we fix gauge degrees of freedom other than that of Poincaré SUSY. As an exception of this argument, A can remain because P, Q, M and A can form a subalgebra. Then, the number of conditions we have to set equals to the number of generators corresponding to the "would-be" broken symmetries, D, (A), S, and K. The total number of bosonic constraints is five (six) and that of fermionic one is one. If we require that graviton and gravitino remain as the physical degrees of freedom, we can not impose gauge conditions on them. The remaining degrees of freedom are b_μ and A_μ because other fields are dependent fields. Therefore, we need, at least, one superconformal multiplet other than the superconformal gauge fields. Such an additional multiplet is the compensator.

The minimal choice of a compensator is to choose one irreducible multiplet. Note that it cannot be a real multiplet with $(w, n) = (2, 0)$ as discussed in Ref. [42]. Therefore, possible candidates are a chiral, a real linear, and a complex linear multiplets, and indeed, they can be compensator multiplets. After superconformal gauge fixing, some of the components in a compensator multiplet remain in the physical theory. With these three types of compensators, we find that the remaining components of them do not have kinetic terms, as long as we do not consider the higher-derivative couplings of compensator multiplet, which are equivalent to the higher-derivative gravitational couplings. Therefore, in the physical theory, the remaining compensator components behave as auxiliary fields.

As we discussed in the previous section, all the irreducible multiplets have different components, which means that, with the different compensator, the auxiliary fields in Poincaré SUGRA are different from each other. Historically, Poincaré SUGRA formulations with different sets of auxiliary fields were known. The relation between them had never been known before the appearance of Ref. [12], in which it was first clarified from the conformal SUGRA viewpoint. As shown in Ref. [12], the old minimal formulation can be realized with a chiral compensator, the new minimal one with a real linear compensator, and the non-minimal one with a complex linear compensator.

In the most of remaining parts, we focus on the old minimal formulation, that is, the superconformal action with a chiral compensator. That is because this formulation can realize the broadest class of models as shown in Ref. [29]. However, as mentioned in the above, the higher-derivative terms in conformal SUGRA may induce the kinetic terms of compensator components. We will briefly discuss SUGRA with a real or a complex linear compensator later.

2.4.3 Action of chiral multiplets

Here, we construct a simple action of a chiral multiplet to show the procedure of the superconformal gauge fixing. We consider the following action,

$$S = \left[\frac{1}{2}\mathcal{N}\right]_D, \tag{2.68}$$

where

$$\mathcal{N} \equiv -3S_0\bar{S}_0 e^{-\frac{|\phi|^2}{3}}. \tag{2.69}$$

S_0, ϕ are a chiral compensator with $(w,n) = (1,1)$ and a matter chiral multiplet with $(w,n) = (0,0)$, respectively.[4] $\mathcal{N}$ is a real multiplet with $(w,n) = (2,0)$, and its components are summarized as follows:

$$\mathcal{C}(\mathcal{N}) = [\mathcal{N}, (-\sqrt{2}i\mathcal{N}_I P_L\chi^I + \text{h.c.}),\ ...,\ ..., B_a^{\mathcal{N}}, \lambda^{\mathcal{N}}, D^{\mathcal{N}}], \tag{2.70}$$

where

$$B_a^{\mathcal{N}} =(i\mathcal{N}_I D_a\phi^I + \text{h.c.}) + i\bar{\chi}^I P_L\gamma_*\gamma_a P_R\chi^{\bar{J}}\mathcal{N}_{I\bar{J}}, \tag{2.71}$$

$$\lambda^{\mathcal{N}} = \left(\sqrt{2}i(\bar{F}^{\bar{J}} + \not{D}\bar{\phi}^{\bar{J}})P_L\chi^I\mathcal{N}_{I\bar{J}} + \frac{i}{\sqrt{2}}\mathcal{N}_{IJ\bar{K}}(\bar{\chi}^I P_L\chi^J)P_R\chi^{\bar{K}} + \text{h.c.}\right), \tag{2.72}$$

$$\begin{aligned} D^{\mathcal{N}} =&2(F^I\bar{F}^{\bar{J}} - D_aX^ID^a\bar{X}^{\bar{J}} - \bar{\chi}^I P_L\not{D}\chi^{\bar{J}})\mathcal{N}_{I\bar{J}} \\ &+\left(-\bar{\chi}^I P_L\chi^J\bar{F}^{\bar{K}}\mathcal{N}_{IJ\bar{K}} - \bar{\chi}^{\bar{K}}\not{D}\phi^I\chi^J\mathcal{N}_{IJ\bar{K}} + \text{h.c.}\right), \end{aligned} \tag{2.73}$$

and I is the index of chiral multiplets. Substituting this into the D-term formula (2.67), we obtain a corresponding action of chiral multiplets. After some simplification, the action becomes

$$\begin{aligned} S = \int d^4xe\Bigg[&\frac{1}{6}\mathcal{N}(-R(e,b) + \bar{\psi}_\mu\mathcal{R}^\mu + e^{-1}\partial_\mu(e\bar{\psi}_\nu\gamma^\nu\psi_\mu) - \mathcal{L}_{\text{SGT}}) \\ &+\mathcal{N}_{I\bar{J}}\left(F^I\bar{F}^{\bar{J}} - \hat{D}_a\phi^I\hat{D}^a\bar{\phi}^{\bar{J}} - \frac{1}{2}\bar{\chi}^I P_L\hat{\not{D}}\chi^{\bar{J}} - \frac{1}{2}\bar{\chi}^{\bar{J}} P_R\hat{\not{D}}\chi^I\right) \\ &\frac{1}{2}(\mathcal{N}_{IJ\bar{K}}(-\bar{\chi}^J P_L\chi^I\bar{F}^{\bar{K}} + \bar{\chi}^J P_L\hat{\not{D}}\phi^I\chi^{\bar{K}}) + \text{h.c.}) \\ &+\frac{1}{4}\mathcal{N}_{IJ\bar{K}\bar{L}}\bar{\chi}^I P_L\chi^J\bar{\chi}^{\bar{K}} P_R\chi^{\bar{L}} + \mathcal{L}_{3/2}^{\text{int}}\Bigg], \end{aligned} \tag{2.74}$$

[4]Hereafter, we refer to a superconformal multiplet by its lowest component.

where

$$R(e,b) = \hat{R}|_{\psi_\mu=0}, \tag{2.75}$$

$$\mathcal{R}^\mu = \gamma^{\mu\nu\rho}\left(\partial_\nu - \frac{3i}{2}\gamma_* A_\nu + \frac{1}{4}\omega_\nu^{ab}\gamma_{ab}\right)\psi_\rho, \tag{2.76}$$

$$\mathcal{L}_{SGT} = \frac{1}{16}\left[-(\bar{\psi}_\mu\gamma_\nu\psi_\rho)(\bar{\psi}^\mu\gamma^\nu\psi^\rho) - 2(\bar{\psi}_\mu\gamma_\nu\psi_\rho)(\bar{\psi}^\mu\gamma^\rho\psi^\nu) + 4(\bar{\psi}_\mu\gamma^\mu\psi_\nu)^2\right], \tag{2.77}$$

$$\begin{aligned}\mathcal{L}_{3/2}^{\rm int} =& \Bigg(-\frac{\sqrt{2}}{3}\mathcal{N}_I\bar{\chi}^I P_L\gamma^{ab}D_a\psi_b + \frac{1}{\sqrt{2}}\mathcal{N}_{I\bar{J}}\bar{\psi}_a\hat{\not{D}}\bar{\phi}^{\bar{J}}\gamma^a P_L\chi^I \\ &+ \frac{i\epsilon^{abcd}}{8}(\bar{\psi}_a\gamma_b\psi_c)\mathcal{N}_I\hat{D}_d\phi^I + \text{h.c.}\Bigg) - \frac{1}{2}\mathcal{N}_{I\bar{J}}(\bar{\psi}_a P_L\chi^I)(\bar{\psi}^a P_R\chi^{\bar{J}}) \\ &+ \frac{i\epsilon^{abcd}}{16}\bar{\psi}_a\gamma_b\psi_c\mathcal{N}_{I\bar{J}}(\bar{\chi}^{\bar{J}}P_R\gamma_d\chi^I).\end{aligned} \tag{2.78}$$

I is the index of the multiplets (S_0, ϕ), subscripts of $\mathcal{N}$ denote the derivative with respect to $I, \bar{J}$, and $\hat{D}_\mu = D_\mu|_{\psi_\mu=0}$.

On the first line of Eq. (2.74), we find a non-minimal coupling between the "Ricci scalar" term and scalar fields,

$$-\frac{1}{6}\mathcal{N}R(e,b) = \frac{1}{2}S_0\bar{S}_0 e^{-\frac{|\phi|^2}{3}}R(e,b). \tag{2.79}$$

Let us construct SUGRA action in Einstein frame, where the coefficient of Ricci scalar is $\frac{1}{2}$.[5] To obtain the action in that frame, we use the D-gauge fixing condition given by

$$S_0\bar{S}_0 e^{-\frac{|\phi|^2}{3}} = 1. \tag{2.80}$$

We also use the A-gauge fixing condition,

$$S_0 = \bar{S}_0. \tag{2.81}$$

Combining these conditions, we can solve them in terms of S_0 and $\bar{S}_0$ as

$$S_0 = \bar{S}_0 = e^{\frac{|\phi|^2}{6}}. \tag{2.82}$$

As the K-gauge fixing condition, we require

$$b_\mu = 0. \tag{2.83}$$

[5]Throughout this thesis, we will use the Planck unit convention $M_{\rm Pl} = 1$ where $M_{\rm Pl}$ is the Planck mass ($\sim 2.4 \times 10^{18}$ GeV).

By the set of conditions, the "Ricci scalar" term takes the standard form $\frac{1}{2}R$. It is worth noting that this procedure does not require any complicated Weyl rescaling processes mentioned before. This is a benefit of the superconformal formulation.

We also have the S-gauge condition, which is useful to eliminate the following kinetic mixing between matter and gravitino,

$$-\frac{\sqrt{2}}{3}\mathcal{N}_I\bar{\chi}^I P_L\gamma^{ab}D_a\psi_b, \tag{2.84}$$

the first term in $\mathcal{L}^{\text{int}}_{3/2}$ (2.78). To eliminate this term, we set the following S-gauge condition,

$$\mathcal{N}_I P_L\chi^I = 0. \tag{2.85}$$

This condition can be solved with respect to $P_L\chi^{S_0}$ and we obtain

$$P_L\chi^{S_0} = \frac{1}{3}e^{\frac{|\phi|^2}{6}}\bar{\phi}P_L\chi^\phi. \tag{2.86}$$

We note that all the gauge fixing conditions should invariant under the remaining symmetries, that is, Poincaré SUSY. Obviously, they are invariant under the GCT and the local Lorentz transformation but not under the Q-transformation. This implies that SUSY transformation in a gauge-fixed system should be deformed so that all the conditions are invariant in the sense of the deformed SUSY. We do not discuss it since it is not so important for our later discussion.[6]

The remaining procedure to complete the construction is quite simple: We just substitute the solutions (2.82), (2.83) and (2.86) into the action (2.74). Then we obtain

$$\begin{aligned}
S = \int d^4x e\Bigg[&\frac{1}{2}(R - \bar{\psi}_\mu\hat{\mathcal{R}}_\mu + \mathcal{L}_{SGT}) - (1-\frac{1}{3}|\phi|^2)\partial_\mu\phi\partial^\mu\bar{\phi} - \frac{1}{12}(\bar{\phi}\partial_\mu\phi + \phi\partial_\mu\phi)^2 \\
&- \bar{\chi}P_L\not{D}_{(P)}\chi + \left(-\frac{1}{6}\bar{\phi}\bar{\chi}P_L\not{\partial}\phi\chi + \text{h.c.}\right) - \frac{1}{6}(\bar{\chi}P_L\chi)(\bar{\chi}P_R\chi)\left(1+\frac{|\phi|^4}{18}\right) \\
&+ \frac{1}{\sqrt{2}}\left(\bar{\psi}_a\not{\partial}\bar{\phi}\gamma^a P_L\chi + \text{h.c.}\right) - \frac{1}{2}(\bar{\psi}_a P_L\chi)(\bar{\psi}^a P_R\chi) \\
&+ \frac{i}{16}\epsilon^{abcd}(\bar{\psi}_a\gamma_b\psi_c)\bar{\chi}P_R\gamma^\chi_d + \frac{i}{8}\epsilon^{abcd}\bar{\psi}_a\gamma_b\psi_c(\bar{\phi}\partial_d\phi - \phi\partial_d\bar{\phi}) + \mathcal{L}_{\text{aux}}\Bigg],
\end{aligned} \tag{2.87}$$

[6]If one is interested in the deformation, see Ref. [12] for the review.

where

$$\hat{\mathcal{R}}_\mu = \mathcal{R}_\mu|_{A_\mu=0}, \tag{2.88}$$

$$\begin{aligned}\mathcal{L}_{\text{aux}} =& -3e^{-\frac{|\phi|^2}{3}}|F^{S_0}|^2 + 3A_\mu A^\mu + e^{-\frac{|\phi|^2}{6}}(\bar{\phi}F^\phi \bar{F}^{S_0} + \text{h.c.}) - iA_\mu(\bar{\phi}\partial^\mu\phi - \phi\partial^\mu\bar{\phi}) \\ &+ \left(1 - \frac{1}{3}|\phi|^2\right)|F^\phi|^2 + \frac{i}{2}(\bar{\chi}P_L\gamma^a\chi)A_a + \frac{1}{18}(\bar{\phi}|\phi|^2\bar{\chi}P_L\chi\bar{F}^{\bar{\phi}} + \text{h.c.}).\end{aligned} \tag{2.89}$$

We find that A_a, F^ϕ, and F^{S_0} do not have the kinetic terms, and therefore, these are the auxiliary fields. The E.O.M of them are easily solved, and then, we finally obtain the following on-shell action,

$$\begin{aligned}S = \int d^4xe\Bigg[&\frac{1}{2}(R - \bar{\psi}_\mu\hat{\mathcal{R}}_\mu + \mathcal{L}_{SGT}) - \partial_\mu\phi\partial^\mu\bar{\phi} - \bar{\chi}P_L\not{D}_{(P)}\chi \\ &+ \left(-\frac{1}{4}\bar{\phi}\bar{\chi}P_L\not{\partial}\phi\chi + \text{h.c.}\right) - \frac{1}{8}(\bar{\chi}P_L\chi)(\bar{\chi}P_R\chi) \\ &+ \frac{1}{\sqrt{2}}\left(\bar{\psi}_a\not{\partial}\bar{\phi}\gamma^a P_L\chi + \text{h.c.}\right) - \frac{1}{2}(\bar{\psi}_a P_L\chi)(\bar{\psi}^a P_R\chi) \\ &+ \frac{i}{16}\epsilon^{abcd}(\bar{\psi}_a\gamma_b\psi_c)\bar{\chi}P_R\gamma_d\chi + \frac{i}{8}\epsilon^{abcd}\bar{\psi}_a\gamma_b\psi_c(\bar{\phi}\partial_d\phi - \phi\partial_d\bar{\phi})\Bigg].\end{aligned} \tag{2.90}$$

As we expected, in the final expression (2.90), the physical degrees of freedom are ϕ, $P_L\chi$, their complex conjugates, graviton e^a_μ, and ψ_μ. The compensator components are completely eliminated. In such a way, we can construct the Poincaré SUGRA action from superconformal one even in a more general situation.

Chapter 3

Review of SUGRA inflation models

3.1 General properties of SUGRA inflation models

In this chapter, we review some SUGRA inflation models related to the models in Chapters 4 and 5. Before discussing concrete models, we first overview difficulties and features of SUGRA inflation models in this section. The brief review of inflation is given in Appendix C.

We will focus on the inflation models in which inflation is driven by a single scalar field. In particular, we discuss the case that the scalar potential dominates the energy of the universe during inflation. In such a case, it is important to understand the structure of the scalar potential in SUGRA.

The generic SUGRA action of chiral and gauge multiplets,ϕ^I and V^A, is given by

$$S = \frac{1}{2}[S_0 \bar{S}_0 \Omega]_D + [S_0^3 W]_F - \frac{1}{4}[f_{AB} \mathcal{W}^{A\alpha} \mathcal{W}_\alpha^B]_F, \tag{3.1}$$

where Ω is a real arbitrary function of ϕ^I and $\bar{\phi}^{\bar{J}}$, W and f are holomorphic functions of ϕ^I, $f_{AB}\mathcal{W}^{A\alpha}\mathcal{W}_\alpha^B$ is a chiral multiplet whose lowest component is $f_{AB}(\phi^I)\bar{\lambda}^A P_L \lambda^B$, and λ^A is the fermionic component of a gauge multiplet (2.56). $I(\bar{J})$ and A denote the (anti-)chiral multiplets' and gauge indices, respectively. Here we have chosen the weights of ϕ^I as $(w, n) = (0, 0)$ and assumed the absence of any higher-derivative terms. Note that the action corresponds to the one in the old minimal SUGRA formulation, which is the most general in all the SUGRA formulations as long as higher-derivative terms are absent.

In the following, we focus on the bosonic part of the action, which is given by

$$S_B = \int d^4xe\Bigg[-\frac{1}{6}|S_0|^2\Omega R(e,b) + \Omega|F^{S_0}|^2 + (S_0\Omega_I F^I \bar{F}^{S_0} + \text{h.c.}) + |S_0|^2\Omega_{I\bar{J}}F^I\bar{F}^{\bar{J}}$$
$$- \Omega|D_\mu S_0|^2 - (S_0\Omega_I D_\mu\phi^I D^\mu\bar{S}_0 + \text{h.c.}) - |S_0|^2\Omega_{I\bar{J}}D_\mu\phi^I D^\mu\bar{\phi}^{\bar{J}}$$
$$- i|S_0|^2\Omega_I k_A^I D^A + \big(3S_0^2 W F^{S_0} + S_0^3 W_I F^I + \text{h.c.}\big)$$
$$- \frac{1}{4}(\text{Re}f_{AB})(F_{ab}^A F^{Bab} - 2D^A D^B) - \frac{i}{4}(\text{Im}f_{AB})F_{ab}^A\tilde{F}^{Bab}\Bigg], \tag{3.2}$$

where $D_\mu S_0 = \partial_\mu S_0 - iA_\mu S_0$, $D_\mu\phi^I = \partial_\mu\phi^I - k_A^I\hat{B}_\mu^A$ and $\tilde{F}_A^{ab} \equiv -\frac{i}{2}\epsilon^{abcd}F_{Acd}$.

This superconformal action includes a compensator S_0, which should be eliminated by appropriate gauge fixing conditions. Even in the presence of S_0, we can read off a property of SUGRA inflation models from this expression. Let us focus on the coefficient of the Ricci scalar $R(e,b)$. Since it should be positive, the function Ω should be negative definite. Then, the scalar potential contribution from F^{S_0} is also negative definite. It is a nontrivial task to realize the "positive" energy during inflation because the F-term of the compensator produces an opposite contribution. Indeed, the terms including F^{S_0} can be rewritten as

$$\begin{aligned}\mathcal{L}_{F_0} =& \Omega|F^{S_0}|^2 + (S_0\Omega_I F^I\bar{F}^{S_0} + 3S_0^2WF^{S_0} + \text{h.c.})\\ =& \Omega|F^{S_0} + S_0\Omega_I F^I + 3\bar{S}_0^2\bar{W}|^2 - \Omega|S_0\Omega_I F^I + 3\bar{S}_0^2\bar{W}|^2.\end{aligned} \tag{3.3}$$

The first term vanishes after solving the E.O.M of F^{S_0} and the second term gives the negative definite scalar potential term for the negative definite Ω. Thus, we find that, for realizing inflation, it is important to pay attention to the contributions (3.3) from F^{S_0}.

Next, we consider the action in the Einstein frame, which can be realized with the following set of superconformal gauge conditions [34],

$$S_0 = \bar{S}_0 = \sqrt{-\frac{3}{\Omega}},\ b_\mu = 0. \tag{3.4}$$

In such a frame, it is useful to define the following quantity, which is the so-called Kähler potential,

$$K \equiv -3\log\left(-\frac{\Omega}{3}\right). \tag{3.5}$$

With this quantity, the action can be rewritten as[1]

$$S_B|_E = \int d^4xe\left[\frac{1}{2}R - K_{I\bar{J}}D_\mu\phi^I D^\mu\bar{\phi}^{\bar{J}} - V_F - V_D\right.$$
$$\left. - \frac{1}{4}(\mathrm{Re}f_{AB})F^A_{ab}F^{Bab} - \frac{i}{4}(\mathrm{Im}f_{AB})F^A_{ab}\tilde{F}^{Bab}\right], \tag{3.6}$$

where we have integrated out auxiliary fields; F^{S_0}, F^I, their complex conjugates, A_μ, and D^A. The potential terms V_F and V_D, called F- and D-term potentials, respectively, are given by

$$V_F \equiv e^K(K^{I\bar{J}}D_IWD_{\bar{J}}\bar{W} - 3|W|^2), \tag{3.7}$$
$$V_D \equiv -\frac{1}{2}(\mathrm{Re}f_{AB})^{-1}(k^I_AK_I)(k^J_BK_J), \tag{3.8}$$

where $D_IW \equiv W_I + K_IW$. Note that $k^I_AK_I$ should be pure imaginary from the gauge invariance and the vacuum expectation value (VEV) of the gauge kinetic function f_{AB} gives the inverse square of the gauge coupling constant. Therefore V_D is positive definite since $\mathrm{Re}f_{AB}$ should be positive definite.

For the F-term potential (3.7), we notice that it depends on the superpotential W, which can contain mass parameters smaller than the Planck scale. Inflation scale is typically required to be smaller than the Planck scale, and therefore, such a structure of F-term potential is important.

We also notice that there is an overall factor e^K in V_F. Naively, due to this factor, all the scalar fields obtain their mass term because, for a scalar ϕ, the mass is given by $m^2_{\phi\bar{\phi}} \sim \partial_\phi\partial_{\bar{\phi}}V_F \sim K_{\phi\bar{\phi}}V_F + \cdots \sim 3K_{\phi\bar{\phi}}H^2 + \cdots$. The ellipses denote mass terms coming from the other parts and H is the Hubble parameter during inflation. This mass contribution is the so-called *Hubble induced mass*. Although the total effective mass also depends on the ellipses parts, without any assumption, such a mass contribution appears in the universal way. This is on the one side a good feature to realize the effectively single field inflation model, because the Hubble induced masses stabilize all the scalar fields during inflation.

On the other hand, although the F-term potential seems to be appropriate to inflation models from the above aspect, there is a serious problem called the *η-problem*, which is caused by the Hubble induced mass itself. As we find, all the scalar fields, including the inflaton field, obtain masses of the order of H^2. However, to realize the slow-roll inflation, the mass of the inflaton should be much smaller than H. Therefore, the inflaton mass

[1] Detailed derivation of the F-term potential can be found, e.g., in Ref. [37].

should be protected from the Hubble induced mass by some mechanisms. Although such a situation may happen accidentally,[2] a reasonable mechanism is provided by a symmetry for the inflaton multiplet. We will discuss it in the following sections.

Next, let us focus on the D-term potential (3.8). As mentioned above, VEV of gauge kinetic function f_{AB} gives the inverse square of the gauge coupling constant. Therefore, we can effectively rewrite the D-term potential as

$$V_D \sim -\frac{(g^2)^{AB}}{2}(k_A^I K_I)(k_B^J K_J). \tag{3.9}$$

As we see, this potential only depends on the Kähler potential, and therefore, the mass scale of the potential typically becomes the Planck scale (~ 1). More precisely, the potential scale is given by $g^2(\sim g^2 M_{\rm Pl}^4)$, that is, we have to require a sufficiently small gauge coupling to realize the inflation scale smaller than $\mathcal{O}(1)$.

The other feature of the D-term potential is the absence of the exponential factor e^K in contrast to the F-term one. This is a good feature to achieve successful inflation because the potential can have a plateau efficient to continue the inflationary era. On the other hand, it may imply the absence of the Hubble induced mass during inflation. If the inflaton couples to the other multiplets in the Kähler potential, their mass terms can appear as in the case of the F-term potential. However, if it is absent, some directions may also be flat, and then, such directions also produce the quantum fluctuation, which can lead to the scalar curvature perturbation with a non-Gaussian spectrum. The situation is constrained by the result from CMB observations. However, it is a highly model dependent argument, and we need to investigate the detailed thermal history of the universe in each model.

As a concluding remark of this section, we briefly comment on the SUSY breaking and the late time universe. In inflation models with both the F- and the D-term potentials, it is important to note that the inflation can happen if the positive energy is realized, that is, if SUSY is broken. In models we will discuss, the inflaton (or the other field) breaks SUSY only during inflation, and SUSY is restored at the vacuum. However, in realistic models of our universe, SUSY should be broken also at the vacuum to explain the dark energy and unobserved SUSY partners of the standard model particles. We will not discuss in detail about thermal histories after inflation in this thesis, but in general we have to take into account the decay of the inflaton after inflation. Especially, in models with broken SUSY, the gravitino production by the thermal [52, 53, 54] and non-thermal processes [55, 56, 57] is important since the light gravitino may become a dark matter candidate, or decay at the time of the nucleosynthesis, which prevents the successful BBN

[2]Such an accidental inflation[43, 44, 45, 46] may be important to realize the TeV scale SUSY compatible with the moduli stabilization in superstring theory [47, 48, 49, 50, 51].

scenario [58]. It has been known that even if the inflaton does not break SUSY at the vacuum, it can decay into gravitino due to the kinetic or mass mixings between SUSY breaking sector and the inflaton [59, 60]. Also, the oscillation of the SUSY breaking sector occurs after inflation which decays into gravitino. Such a problem was raised earlier in Refs. [61, 62, 63] and reinvestigated in Refs. [64, 65, 66]. The simplest way to avoid such a problem is to introduce a non-minimal Kähler potential term [67, 68], and cosmological constraints on such a model were discussed in Ref. [69].

3.2 Chaotic inflation with F-term potential

In this section, we review the chaotic inflation models with the F-term potential (3.7). The chaotic inflation [18] is a class of inflation models which is free from the initial condition problem that we will discuss below. The simplest version of it is realized in the following system,

$$S = \int d^4xe \left[\frac{1}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2\right], \tag{3.10}$$

where ϕ is a real scalar field which is the inflaton and m is a real parameter corresponding to the mass of ϕ. During inflation, the spacetime becomes sufficiently homogeneous, and then, $\phi = \phi(t)$. The slow-role parameters ϵ (C.7) and η (C.10) in this model are given by

$$\epsilon = \eta = \frac{2}{\phi^2}. \tag{3.11}$$

From this expression, we notice that ϕ should be much larger than $1(= M_{\rm Pl})$ to realize the slow-roll, $\epsilon, \eta \ll 1$. In other words, inflation happens in a very wide range of the field space because the required condition is only $\phi > 1$. That is an important feature of this model by the following reason: Although we do not know the details of the very early universe before inflation, we can estimate that the universe was so small and the size of the universe was of the order of the Planck length. In such a case, the typical energy scale is expected to be very high, and then, the potential energy of the inflaton is negligibly small as long as $\phi < m^{-1}$ in this case. Then, it is reasonable to assume that there are in some particuler patches of the universe with different values of ϕ due to the quantum fluctuation. If the inflaton potential is flat only around a specific value ϕ_0, inflation only happens some patches where $\phi \sim \phi_0$. This is the initial condition problem in inflation models. For the sake of simplicity of the requirement $\phi \gg 1$, in the chaotic inflation scenario, inflation can occur in many patches satisfying it. Therefore, the initial condition problems are absent in chaotic inflation models. Here, we have shown the simplest version of the chaotic inflation, but we can also use other forms of the potential. As long as inflation can occur with a chaotic initial condition, we call the model the chaotic inflation as in Ref. [70].

Let us consider the chaotic inflation in SUGRA with the F-term potential. It is known that, in general, SUGRA realization of chaotic inflation models are quite difficult because of two problems which we will discuss later. The chaotic inflation model in SUGRA was first proposed in Ref. [71] with specific choices of Kähler and super-potentials. However, a simple way to avoid these two problems had not been known until Ref. [19] was proposed.

For the realization of the chaotic inflation, one of the requirement is the flatness of the potential in the broad range of the field value of the inflaton. However, as we discussed in the previous section, the η-problem in SUGRA exists. The solution proposed in Ref. [19] is to impose a shift symmetry of the inflaton on the Kähler potential. The simplest example is the following term,

$$K = \frac{1}{2}(\Phi + \bar{\Phi})^2, \tag{3.12}$$

where Φ is the inflaton chiral multiplet. This Kähler potential is invariant under the transformation $\Phi \to \Phi + iC$, where C is a real constant. Then, ImΦ direction is free from the η-problem in SUGRA, because of the absence of the inflaton field in the Kähler potential term. Therefore, Im$\Phi \equiv \phi$ can be a candidate for the inflaton. Recently, the other type of the Kähler potential was proposed, which realizes the so-called α-attractor model [72]. The Kähler potential is given by

$$K = -\frac{3\alpha}{2}\log\left(\frac{(\Phi + \bar{\Phi})^2}{4\Phi\bar{\Phi}}\right), \tag{3.13}$$

where α is a real constant. This is invariant under the following transformations, $\Phi \to a\Phi$, $\Phi \to b\Phi^{-1}$ where a and b are real constants [73]. The former corresponds to the shift symmetry of the real scalar ϕ defined by $\Phi = \exp\left(\sqrt{\frac{2}{3\alpha}}\phi\right) + i\chi$, where χ is a real scalar field. Note that ϕ is a canonically normalized field. In Ref. [74], it was found that the α-attractor type Kähler potential (3.13) becomes a simple shift symmetric one (3.12) in the limit $\alpha \to \infty$.

In both cases, Kähler potential is flat with respect to the inflaton. However, superpotential terms, in general, are not invariant under the shift of the inflaton, which is necessary to produce the inflaton potential. As we mentioned in the previous section, the superpotential term has parameters determining the scale of the inflaton potential, and they should be smaller than 1 to realize the inflation scale consistent with the observation. In the 't Hooft's sense, the requirement of the small parameter is technically natural because the superpotential terms break the shift symmetry, which can be restored when the parameters become zero.

The other problem is the difficulty of the positive energy. To realize the single field inflation, ReΦ should be stabilized at a specific point, which we assume as its origin. With a simple Kähler potential $K = \frac{1}{2}(\Phi + \bar{\Phi})^2$ and a superpotential term $W = W(\Phi)$, the scalar potential at $\Phi = 0 + i\phi$ becomes

$$V_F = |W'(i\phi)|^2 - 3|W(i\phi)|^2, \tag{3.14}$$

where prime denotes the derivative with respect to ϕ. As we find from this expression, the scalar potential with a power type superpotential $W = \sum_n a_n \Phi^n$ becomes negative if ϕ takes a sufficiently large value. For simplicity, we show the case where ReΦ is stabilized at its origin but, even if we assume different VEVs of ReΦ, we also come across the similar problem. Therefore, we can not realize the simple chaotic inflation with this kind of simple setup.

Two solutions for these problems have been known. One of the solutions is introducing an additional multiplet called the *stabilizer*, first proposed in Ref. [19]. Before discussing models with the stabilizer, we briefly comment on the second solution: That solution is provided by introducing the additional Kähler potential term, which stabilizes the direction orthogonal to the inflaton, and also eliminates the negative contribution in the F-term potential. Such an extension was pioneered in Refs [75, 76]. Recently, it was found that such a situation can also be realized in the model with an α-attractor type Kähler potential [74, 77]. In all the cases, the inflation can be realized with a single chiral multiplet, which is good from the minimalistic viewpoint.[3]

Let us review the first solution, in which the additional multiplet called a stabilizer couples to the inflaton multiplet. In Ref. [19], the following set of a Kähler and a superpotential is assumed,

$$K = \frac{1}{2}(\Phi + \bar{\Phi})^2 + |S|^2, \tag{3.15}$$

$$W = m\Phi S, \tag{3.16}$$

where $\Phi = \frac{1}{\sqrt{2}}(\varphi + i\phi)$, S is the stabilizer multiplet, m is a real parameter. With this setup, S has a mass term $\sim m^2|S|^2$, and $\langle S \rangle = 0$ at the classical level. Then, the negative contribution in the F-term potential $-3|W|^2$ vanishes. We also find that the mass of φ is of the order of $H^2 \sim m^2\phi^2$, which stabilize φ at its origin during inflation. Then, the

[3]Recently, a new model with this kind of mechanisms was proposed in Ref. [78], where the inflaton is the phase of a complex scalar field in the multiplet.

inflaton potential becomes

$$V_F = \frac{1}{2}m^2\phi^2, \tag{3.17}$$

which is the same as in Eq. (3.10). Thus, the simplest chaotic inflation is effectively realized by introducing a *shift symmetric* Kähler potential and a stabilizer multiplet S.

It is worth noting that we can understand why the negative contribution vanishes from the conformal SUGRA viewpoint as follows: We mentioned that the F-term of the compensator produces a negative definite contribution to the F-term potential. This is the origin of $-3|W|^2$. In the above model, the F-term of the compensator is given by

$$\begin{aligned} F^{S_0} &= -\left(e^{-\frac{K}{6}}K_I F^I + 3e^{\frac{K}{3}}\bar{W}\right) \\ &= -\left(e^{-\frac{K}{6}}\varphi F^{\Phi} + e^{-\frac{K}{6}}\bar{S}F^S + 3e^{\frac{K}{3}}\bar{m}\bar{\Phi}\bar{S}\right), \end{aligned} \tag{3.18}$$

where we have solved the E.O.M of F^{S_0} derived from Eq. (3.3). This vanishes on the inflationary trajectory $\varphi = S = 0$. Therefore, the vanishing of F^{S_0} is a solution to avoid a negative potential. The vanishing of F^{S_0} also occurs with different mechanisms. In Refs. [76, 79], it occurs because of the so-called no-scale structure of the Kähler potential.

The stabilizer can solve the problem because it is stabilized around its origin. However, does it generically occur? In the model discussed above, the mass of S is almost the same with one of the inflaton. So, S is also light during inflation, and then, the quantum fluctuation of it is produced during inflation. This becomes an isocurvature mode of the scalar curvature perturbation, which becomes the adiabatic mode after S decays. If S dominates the universe after inflation, the model may predict a large non-Gaussianity of the scalar perturbation spectrum [80], which is constrained by the observation. More dangerous case is that the stabilizer obtains a tachyonic mass during inflation. If it is the case, the inflationary trajectory becomes unstable. Such situations can be circumvented if the stabilizer S is strongly stabilized during inflation. It can be realized by introducing a quartic Kähler potential term $-\zeta|S|^4$, where ζ is a sufficiently large positive constant [19, 20, 81, 82]. In Ref. [20], a SUGRA inflation model with an arbitrary scalar potential was constructed, in which the problems discussed so far are absent.

More recently, the nilpotent chiral multiplet [83, 84, 85, 86, 87, 88], which we denote as $\hat{X}$, has received much interests. It satisfies a superconformal condition $\hat{X}^2 = 0$, which is solved as

$$X = \frac{\bar{G}G}{2F^X}, \tag{3.19}$$

where X, G, F^X are the scalar, the fermion, and the auxiliary components of $\hat{X}$, respectively. Then, $\hat{X}$ does not have a scalar degree of freedom, and then, it plays the role of the stabilizer without the above problem [89, 90, 91, 92, 93, 94]. Technically, to obtain the bosonic action, we calculate the SUGRA action in the standard way, and just set $X = 0$ in the action.

Interestingly, it was found that, such a nilpotent multiplet appears as the effective theory of a D-brane in superstring theory [95, 96, 97]. Under the constraint, SUSY is non-linearly realized, which can be considered as a specific limit of linearly realized SUSY [88, 98]. The full-component action of models with the nilpotent multiplet has been investigated in Refs. [99, 100, 101, 102].

It is also important to note that, to describe the universe with the nilpotent multiplet, $\langle F^X \rangle$ should take a non-zero value, during and *after* inflation. This means that the nilpotent multiplet should break SUSY otherwise it is ill-defined as found from Eq. (3.19). Then, the SUSY breaking in the present universe can also be described by $\hat{X}$, which is interesting from the minimalistic viewpoint. Such an aspect of the nilpotent multiplet is discussed in Ref. [90, 91, 94]

3.3 Starobinsky inflation in old and new minimal SUGRA

In this section, we review the SUSY version of the Starobinsky model [14] in which the higher-derivative term of the gravity $\sim R^2$ exists. As we mentioned in Sec. 2.1, the system including higher-derivative terms typically depends on the formulation of SUGRA, that is, the choice of the compensator multiplet. In the following subsections, we will review the old and the new minimal Starobinsky models, where any ghost modes do not appear. The Starobinsky model in non-minimal SUGRA has also been studied at the linearized level in Ref. [103]. It was found that such an extension leads to the appearance of the ghost modes.

3.3.1 Old minimal SUGRA

To construct a higher curvature action in SUGRA, we need a multiplet containing curvature terms in its components. In particular, the multiplet including R is important to realize the R^2 term. Such a superconformal chiral multiplet denoted $\mathcal{R}$ is

$$\mathcal{R} \equiv \frac{\Sigma(\bar{S}_0)}{S_0} \tag{3.20}$$

where S_0 is the chiral compensator with $(w, n) = (1, 1)$ [41]. Note that $\mathcal{R}$ has weights $(w, n) = (1, 1)$. With this multiplet, the SUGRA Starobinsky model can be described by the following action,

$$S = \left[-\frac{3}{2} S_0 \bar{S}_0 + \frac{\alpha}{2} \mathcal{R} \bar{\mathcal{R}} \right]_D , \tag{3.21}$$

where α is a positive constant. The components of the chiral multiplet $\mathcal{R}$ can be expressed as

$$\mathcal{R} = \left[-\frac{\bar{F}^{S_0}}{S_0}, \cdots, \frac{2|F^{S_0}|^2}{S_0^2} - \frac{\Box \bar{S}_0}{S_0} + \cdots \right], \tag{3.22}$$

where ellipses denote the fermionic parts. With this expression, the bosonic part of the action (3.21) can be written as

$$\begin{aligned} S|_B = \int d^4 x e \Bigg[& - 3|F^{S_0}|^2 + 3 D_a S_0 D^a \bar{S}_0 + \alpha |F^{\mathcal{R}}|^2 - D_a X^{\mathcal{R}} D^a \bar{X}^{\mathcal{R}} \\ & + \frac{1}{2} \left(S_0 \bar{S}_0 - \frac{\alpha}{3} |X^{\mathcal{R}}|^2 \right) R(e, b) \Bigg], \end{aligned} \tag{3.23}$$

where

$$F^{\mathcal{R}} = \frac{2|F^{S_0}|^2}{S_0^2} - \frac{\Box \bar{S}_0}{S_0}, \tag{3.24}$$

$$X^{\mathcal{R}} = - \frac{\bar{F}^{S_0}}{S_0}. \tag{3.25}$$

We take simple gauge fixing conditions $S_0 = \bar{S}_0 = 1$, and $b_\mu = 0$, and obtain the following action,

$$\begin{aligned} S|_B^{\text{g.f}} = \int d^4 x e \Bigg[& - 3|X^{\mathcal{R}}|^2 + 3 A_a A^a - D_a X^{\mathcal{R}} D^a \bar{X}^{\mathcal{R}} + \frac{1}{2} \left(1 - \frac{\alpha}{3} |X^{\mathcal{R}}|^2 \right) R \\ & + \alpha \left\{ \left(2|X^{\mathcal{R}}|^2 + \frac{1}{6} R - A_a A^a \right)^2 + (D_a A^a)^2 \right\} \Bigg]. \end{aligned} \tag{3.26}$$

From the last term of the action, we find not only the higher curvature term R^2 but also some other couplings between $X^{\mathcal{R}}$, A_a and graviton. In contrast to the case without higher curvature terms, the fields A_a and F^{S_0} are no longer the auxiliary ones because they have kinetic terms and (or) couplings to the Ricci scalar. Although there are some additional degrees of freedom, we can conclude that this is the SUGRA extension of the Starobinsky model in the old minimal formulation.

It is known that the original (non-SUSY) Starobinsky model has a dual picture, where the system consists of a scalar field coupled with Einstein gravity [104]. The scalar field is sometimes referred to as a scalaron. Such a dual picture can be obtained via conformal transformations, and the dual picture is useful to discuss e.g. the inflationary attractor in the Starobinsky model [105]. More generally, the system including an arbitrary coupling $F(\phi, R)$ can be rewritten as Einstein gravity with two scalar fields [106].

The superconformal version of such a transformation was shown by Ceccoti in Ref. [21]. It can be done as follows: First, with a Lagrange multiplier chiral multiplet T with $(w, n) = (0, 0)$, we rewrite the action (3.21) as

$$S^{\rm dual} = \left[-\frac{3}{2}S_0\bar{S}_0\left(1 - \frac{\alpha}{3}S\bar{S}\right)\right]_D + \left[\frac{3}{2}S_0^3 T\left(S - \frac{\mathcal{R}}{S_0}\right)\right]_F, \tag{3.27}$$

where S is a chiral multiplet with $(w, n) = (0, 0)$. The E.O.M of T gives a superconformal constraint $S = \frac{\mathcal{R}}{S_0}$ which reproduces the original action (3.21). On the other hand, we can perform the following transformation,

$$\begin{aligned}\left[-\frac{3}{2}S_0^3 T\left(\frac{\mathcal{R}}{S_0}\right)\right]_F &= \left[-\frac{3}{2}S_0 T\Sigma(\bar{S}_0)\right]_F \\ &= \left[\frac{3}{2}S_0\bar{S}_0(T + \bar{T})\right]_D + (\text{tot.div}),\end{aligned} \tag{3.28}$$

where in the last equality, we have used the following identity[4]

$$[\Lambda\Phi\Sigma(\bar{\Phi})]_F = [-(\Lambda + \bar{\Lambda})\Phi\bar{\Phi}]_D + (\text{tot.div}). \tag{3.29}$$

Using this transformation, we obtain the dual action

$$S^{\rm dual} = \left[-\frac{3}{2}S_0\bar{S}_0(1 + T + \bar{T} - \frac{1}{3}S\bar{S})\right]_D + \left[S_0^3 MTS\right]_F, \tag{3.30}$$

where we redefine multiplets as $S \to -\frac{1}{\sqrt{\alpha}}S$ and $T \to -T$, and $M \equiv \frac{3}{2\sqrt{\alpha}}$. This is the standard SUGRA action (3.6) with

$$K = -3\log\left(1 + T + \bar{T} - \frac{1}{3}S\bar{S}\right), \tag{3.31}$$

$$W = MTS. \tag{3.32}$$

[4]This identity is almost trivial if one recalls the construction of the D-term action from the F-term one.

From the above procedure, we can identify S as $X^{\mathcal{R}}$ and T as the scalaron multiplet.

The scalaron plays the role of the inflaton in the non-SUSY case, and then, we analogously expect that T plays the role of the inflaton multiplet in this case. Indeed, the potential of ReT is the same as the one in non-SUSY Starobinsky model if $S = \text{Im}T = 0$. However, once we calculate the scalar potential of this system, we notice that, on the trajectory $S = \text{Im}T = 0$, the mass of S during inflation (Re$T \gg 1$) becomes $m_{S\bar{S}} \equiv K^{S\bar{S}} V_{S\bar{S}} \sim -\frac{2}{3}M^2$. Therefore, such an inflationary trajectory is unstable. This problematic situation can be circumvented if one notices that S can play a role of the stabilizer. As we discussed in Sec. 3.2, the instability of the stabilizer can be relaxed by adding a quartic term like $\zeta|S|^4$, and then, the mass of the stabilizer can be made positive and heavier than the Hubble scale. Note that such a modification is possible if one adds a term $\tilde{\zeta}(S_0\bar{S}_0)^{-3}|\mathcal{R}|^4$ to the original action (3.21). The term does not produce higher order terms of the Ricci scalar like R^4 but yields terms like $\zeta|X^{\mathcal{R}}|^2 R^2$, which is the addtional term of S in the dual Kähler potential.

A simpler way to stabilize S is the nilpotency condition on $\mathcal{R}$ [89], which is equivalent to a nilpotency of S, as in the case of $\hat{X}$ discussed in Sec. 3.2. In Ref. [89], the following action is discussed;

$$S = \left[-\frac{3}{2} S_0 \bar{S}_0 + \frac{\alpha}{2} \mathcal{R}\bar{\mathcal{R}} \right]_D + [S_0 \Lambda \mathcal{R}^2]_F, \tag{3.33}$$

where Λ is a chiral multiplet with $(w, n) = (0, 0)$. The E.O.M of Λ gives a constraint $\mathcal{R}^2 = 0$, which is the nilpotency condition. Through the similar procedure, we can obtain the dual action of this one as,

$$S^{\text{dual}} = \left[-\frac{3}{2} S_0 \bar{S}_0 (1 + T + \bar{T} - \frac{1}{3} S\bar{S}) \right]_D + \left[S_0^3 M T S \right]_F + [S_0^3 \Lambda S^2]_F. \tag{3.34}$$

Thus the nilpotency condition on $\mathcal{R}$ becomes exactly that of S in the dual action, by which the instability problem can be solved. However, in this case, the VEV of F^S vanishes, which is inconsistent with the nilpotency condition (see Eq. (3.19)) [93]. One of the possibilities of changing the situation is to use a different type of constraint. In Ref. [107], the Starobinsky model with the following condition is discussed,

$$\left(\frac{\mathcal{R}}{S_0} - \mu \right)^2 = 0, \tag{3.35}$$

where μ is a real constant. Under this condition, F^S becomes non-zero at the vacuum, which is consistent with the solution (3.19). One can find that in such a model, SUSY breaking is dominantly caused by T, and $|F^T| \gg |F^S|$ in the case with $\mu \ll 1$. Interestingly, when matters are included, the SUSY breaking effect is only mediated by S to such as the MSSM sector, which leads to the hierarchical mass splitting between the gravitino and the MSSM superparticles. It is worth noting that under the constraint (3.35), the pure SUGRA action $[-S_0\bar{S}_0]_D$ becomes equivalent to the SUGRA coupled to a nilpotent multiplet X, as shown in Refs. [108, 109].

So far, we have discussed the SUSY Starobinsky model, which includes only R^2 term as a higher-derivative term. However, more generically, one can consider the system with R^n terms. In Ref. [21], such a system is also considered. Following the work, we can construct a multiplet like $\Sigma(\bar{\mathcal{R}})$ whose lowest component includes R. Then, with the multiplet, we can construct models with R^n. However, it is also shown that such a model generically has ghost modes whose kinetic coefficients are opposite to the ordinary matter fields. Therefore, the system including R^n terms is generically unstable. Recently, in Ref. [110], the authors discuss the situation where such ghost modes decouple.

A feature of the old minimal SUGRA Starobinsky model is the couplings of T, which are completely determined by its origin. For example, the allowed superpotential term of T is only $W = MTS$, because T is introduced as a Lagrange multiplier. Can we modify the terms of T as a modification of SUGRA action (3.21)? The answer to such a question has been given in Ref. [111]. It is shown that, if T has a superpotential $W = f(T)S$, the model is not a dual of the pure SUGRA action (3.21) unless $f(T)$ is a linear function of T. Such a model is dual to the higher curvature action with an additional chiral multiplet $\tilde{T}$ other than the curvature multiplet.

3.3.2 New minimal SUGRA

Next, we discuss the Starobinsky model in the new minimal SUGRA. The structure of the Starobinsky type action in new minimal SUGRA was studied in Ref. [112], and its application to inflation was discussed in Ref. [23]. Let us review the model on the basis of them.

As we mentioned, the new minimal SUGRA is the conformal SUGRA with a real linear compensator L_0. In this case, the pure SUGRA action is given by

$$S_{\text{pure}} = \left[\frac{3}{2}L_0 \ln\left(\frac{L_0}{\mathcal{S}\bar{\mathcal{S}}}\right)\right]_D, \tag{3.36}$$

where $\mathcal{S}$ is a chiral mutiplet with $(w, n) = (1, 1)$. Although one may think that there is an extra degree of freedom $\mathcal{S}$, it is not physical because of the following reason: The action (3.36) is invariant under the transformation $\mathcal{S} \to \mathcal{S}e^{\Lambda}$ where Λ is a chiral multiplet with $(w, n) = (0, 0)$. This is because $[L_0(\Lambda + \bar{\Lambda})]_D = -[\Sigma(L_0\Lambda)]_F = -[\Lambda\Sigma(L_0)]_F = 0$, where we have used the identity (3.29), and the last equality is due to the definition of

L_0.[5] Thus, $\mathcal{S}$ can be fixed as an extra gauge degree of freedom. Then, by using the extra gauge and superconformal gauge fixing conditions, we can eliminate L_0 and $\mathcal{S}$ (other than L_0's auxiliary field) from the physical theory.

For later discussions, it is useful to see that the pure SUGRA action (3.36) is equivalent to the old minimal one unless higher curvature terms exist. Let us review it on the basis of Ref. [29]. The action (3.36) can be rewritten as

$$S = \left[\frac{3}{2}U \ln\left(\frac{U}{\mathcal{S}\bar{\mathcal{S}}}\right)\right]_D + \left[\frac{3}{2}U(\Phi+\bar{\Phi})\right]_D, \tag{3.37}$$

where U is a real general multiplet with $(w,n)=(2,0)$ and Φ is a chiral multiplet with $(w,n)=(0,0)$. Note that U becomes a real linear multiplet by the E.O.M of Φ, because $[U(\Phi+\bar{\Phi})]_D = -[\Phi\Sigma(U)]_F$ and Φ's E.O.M gives $\Sigma(U)=0$. On the other hand, if we first vary a general multiplet U, we obtain the following equation,

$$\ln\left(\frac{U}{\mathcal{S}\bar{\mathcal{S}}}\right) + 1 + \Phi + \bar{\Phi} = 0. \tag{3.38}$$

We can solve this equation as

$$U = |e^{-\frac{1}{2}-\Phi}S|^2 = |S_0|^2, \tag{3.39}$$

where we have defined $S_0 = e^{-\frac{1}{2}-\Phi}S$. Then, the action can be rewritten as

$$S = \left[-\frac{3}{2}S_0\bar{S}_0\right]_D, \tag{3.40}$$

which is the same as the pure SUGRA action in the old minimal formulation. This is the essence to prove the equivalence between SUGRA actions with different compensators, shown in Ref. [29]. Even if the matter and other gauge multiplets are contained, this duality transformation can be applied. The important point of this duality transformation is the absence of the derivative terms of compensators. In other words, SUGRA with different compensators are, in general, not the same with each other in the presence of derivative operators on compensators.

Next, let us consider the action including the R^2 term in new minimal SUGRA. Before that, we focus on an interesting feature of the combination $V_R \equiv \log\left(\frac{L_0}{\mathcal{S}\bar{\mathcal{S}}}\right)$. Under the transformation $\mathcal{S}\to\mathcal{S}e^{\Lambda}$, V_R transforms as $V_R \to V_R - \Lambda - \bar{\Lambda}$, which is the same with the transformation of a gauge superfield in superspace. Indeed, V_R behaves as if it is a

[5] We can understand why the factor $\ln\left(\frac{L_0}{\mathcal{S}\bar{\mathcal{S}}}\right)$ is required from this discussion. In the absence of it, the action $\sim [L_0]_D$ vanishes.

gauge multiplet because V_R has $(w,n) = (0,0)$ and the gauge transformation under a U(1) symmetry. It is also useful to consider the Ricci scalar term in the action (3.36) as the Fayet-Iliopoulos (FI) term of a vector multiplet V_R.[6] Then, one can find that the field strength superfield of V_R like $[\mathcal{W}^2(V_R)]_F$ gives the square of the FI term, that is, the R^2 term. Thus, the Starobinsky model in new minimal SUGRA is described by

$$S = \left[\frac{3}{2}L_0 V_R\right]_D + [-h\mathcal{W}^2(V_R)]_F, \tag{3.41}$$

where h is a real constant, $\mathcal{W}^2(V_R)$ is a chiral multiplet whose lowest component is $\bar{\lambda}_R P_L \lambda_R$, and λ_R is the λ-component of V_R. After fixing D- and K-gauges by $L_0 = 1$ and $b_\mu = 0$, respectively, the bosonic part of the action becomes

$$\begin{aligned} S|_B = \int d^4x e \Bigg[&\frac{1}{2}R + \frac{2h}{9}R^2 - hF^R_{\mu\nu}F^{R\mu\nu} - \frac{3}{2}A^R_\mu B^\mu \\ &+ \left(\frac{3}{4} + \frac{2h}{3}R\right) B_\mu B^\mu + h(B_\mu B^\mu)^2 \Bigg], \end{aligned} \tag{3.42}$$

where A^R_μ and B_μ are vector components of V_R and L_0, respectively, and $F^R_{\mu\nu} = 2\partial_{[\mu}A^R_{\nu]}$. As we expected, the R^2 term appears, and there is a non-minimal coupling between R and $B_\mu B^\mu$ as in the old minimal case, which implies the appearance of new degrees of freedom other than the scalaron.

Let us discuss the dual action to (3.41) [23]. The procedure is a little bit different from that for the old minimal one. By using a real linear multiplier L, we can rewrite the action (3.41) as

$$S = \left[\frac{3}{2}L_0 V_R\right]_D + [-h\mathcal{W}^2(V)]_F + [L(V - V_R)]_D, \tag{3.43}$$

where V is a real multiplet with $(w,n) = (0,0)$. The E.O.M of L gives $V_R = V + \Phi + \bar{\Phi}$ where Φ is a chiral multiplet.[7] Obviously, we can reproduce the action (3.41) by substituting the solution V. Instead, we can solve the equation $V_R = V + \Phi + \bar{\Phi}$ with respect to L_0 as

$$L_0 = \mathcal{S}\bar{\mathcal{S}}e^{\Phi+\bar{\Phi}+V}, \tag{3.44}$$

[6]This understanding is useful for the following discussion, but more correctly saying, the Ricci scalar term also comes from the D-term of L_0.

[7]The reason why Φ appears is that the combination $[L(V - V_R)]_D$ is invariant under the shift $V(V_R) \to V(V_R) + \Phi + \bar{\Phi}$ due to a nature of L discussed below Eq. (3.36).

where we have used $V_R = \log\left(\frac{L_0}{\mathcal{S}\bar{\mathcal{S}}}\right)$. Then, by substituting it into Eq. (3.43), we obtain the following dual action,

$$\begin{aligned} S_{\text{dual}} &= \left[\frac{3}{2}\mathcal{S}\bar{\mathcal{S}}(\Phi+\bar{\Phi}+V)e^{\Phi+\bar{\Phi}+V}\right]_D + [-h\mathcal{W}^2(V)]_F \\ &= \left[\frac{1}{2}S_0\bar{S}_0\left(\frac{1}{2}(\Phi+\bar{\Phi}+gV)\right)e^{\frac{1}{2}(\Phi+\bar{\Phi}+gV)}\right]_D + \left[-\frac{g^2h}{4}\mathcal{W}^2(V)\right]_F, \end{aligned} \tag{3.45}$$

where in the last equality we have redefined fields as $\Phi \to \frac{1}{2}\Phi$, $V \to \frac{gV}{2}$, and $\mathcal{S} \to \frac{S_0}{\sqrt{3}}$. The scalar field Φ should be understood as a nonlinear realization of the $U(1)$ symmetry, which transforms $\Phi \to \Phi + ig$ under the symmetry. In terms of superfields, the combination $\Phi + \bar{\Phi} + gV$ is invariant under a set of gauge transformations $\Phi \to \Phi + g\Lambda$, and $V \to V - \Lambda - \bar{\Lambda}$, where Λ is a chiral multiplet. From this viewpoint, one can regard the action (3.45) as the one in Eq. (3.1) with

$$\Omega = \left(\frac{1}{2}(\Phi+\bar{\Phi}+gV)\right)e^{\frac{1}{2}(\Phi+\bar{\Phi}+gV)}, \tag{3.46}$$

and $W = 0$. The bosonic action of this system is given by

$$\begin{aligned} S|_B = \int d^4xe\Bigg[&\frac{1}{2}R - \frac{3}{4C^2}\partial_\mu C\partial^\mu C - \frac{9g^2}{8}\left(1+\frac{1}{C^2}\right)^2 \\ &- \frac{3g^2}{4C^2}\left(A_\mu - \frac{1}{g}\partial_\mu\theta\right)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\Bigg], \end{aligned} \tag{3.47}$$

where we have taken $h = g^{-2}$, $C \equiv \text{Re}\Phi$, $\theta \equiv \text{Im}\Phi$, A_μ is the vector component of V, and $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$. The first term on the second line of this equation is the kinetic term of θ but also looks like a mass term of A_μ. Indeed, by the gauge condition on the U(1) symmetry, we can set $\theta = 0$, and then, the term becomes exactly the mass of A_μ. This is because Φ is a nonlinear realization and θ corresponds to the so-called Stueckelberg field. For $\theta = 0$, the canonically normalized inflaton is $\phi = \sqrt{\frac{3}{2}}\log(-C)$ and the potential becomes $V = \frac{9g^2}{8}\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2$, which is the same as the one in the Starobinsky model.

We notice that, in contrast to the old minimal case, there is no stabilizer and massless fields since a massless field θ can be removed by A_μ. Therefore, the inflation is exactly driven by a single inflaton field in this model, and the instability problem is absent.[8]

[8]The energy of A_μ typically does not contribute to inflation.

3.4 Massive vector multiplet inflation

As we discussed in the previous section, the Starobinsky model in new minimal SUGRA is dual to the system with a chiral multiplet Φ coupled to a gauge multiplet V. Taking into account the gauge transformation $\Phi \to \Phi + g\Lambda$, we can rewrite the action (3.45), in terms of a gauge invariant combination $\hat{V} = V + \frac{1}{g}(\Phi + \bar{\Phi})$, as

$$S = \left[\frac{1}{2}S_0\bar{S}_0\left(\frac{1}{2}(g\hat{V})\right)e^{\frac{1}{2}(g\hat{V})}\right]_D + \left[-\frac{g^2 f}{4}\mathcal{W}^2(\hat{V})\right]_F, \tag{3.48}$$

where in the second term we formally change $V \to \hat{V}$ because $\mathcal{W}^2$ is a gauge invariant quantity. We find that the action can be written in terms of $\hat{V}$. As we see in the previous section, the vector field A_μ becomes massive. Therefore this combination is called a massive vector multiplet [113, 114], which is equivalent to a real multiplet with $(w, n) = (0, 0)$ without the gauge degree of freedom.

What we found in the previous section is that the massive vector multiplet has only a single scalar component and the stabilizer field is not required. That is because the inflaton potential comes from the D-term potential (3.8), which is definitely positive unlike the F-term potential. We note that the D-term potential is free also from the η problem appearing in the F-term inflation models.

From these observations, one expects that the massive vector multiplet can be a promising candidate for an inflaton multiplet. Such a model was considered in Refs. [24, 25, 115]. We review it on the basis of Ref. [25]. The master action of the massive vector multiplet in old minimal SUGRA is

$$S = \left[-\frac{3}{2}S_0\bar{S}_0\exp\left(-\frac{2}{3}J\right)\right]_D + \left[-\frac{1}{4}\mathcal{W}^2(V)\right]_F, \tag{3.49}$$

where $J = J\left(\frac{1}{2}(\Phi + \bar{\Phi} + gV)\right) = J(g\hat{V})$ is an arbitrary real function of the argument. The corresponding bosonic action is given by

$$\begin{aligned} S|_B = \int d^4x e\Bigg[&\frac{1}{2}R - \frac{1}{2}J''(C)\partial_\mu C\partial^\mu C - \frac{g^2}{2}(J'(C))^2 \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g^2}{2}J''(C)\left(A_\mu - \frac{1}{g}\partial_\mu\theta\right)^2\Bigg], \end{aligned} \tag{3.50}$$

where primes on J denote derivatives with respect to C. The scalar potential of the inflaton C is given by

$$V = \frac{g^2}{2}(J'(C))^2. \tag{3.51}$$

We can choose an arbitrary form of J, which means that the inflaton potential can take almost arbitrary forms.

The minimum of the potential is given by the condition $V'(C) = 0$, i.e., $J'(C) = 0$ or $J''(C) = 0$, and the former leads to the vanishing cosmological constant. The latter is singular because the kinetic coefficient of C vanishes, and therefore, we always obtain the former solution, which means that SUSY at the vacuum is not broken by this model itself unless the other terms of C come from the F-term potential. Indeed, because of SUSY at the vacuum $J' = 0$, the mass of the inflaton becomes $m_C^2 = V'' = g^2(J'')^2$, which is the same as that of the superpartner A_μ.

Interestingly, such a nonlinear realization may appear in an effective theory of superstring. In many superstring models, it is known that some anomalous U(1) symmetries appear in the effective theory. Such anomalies can be cancelled by the so-called Green-Schwarz mechanism [116]. In that case, the Green-Schwarz fields, which are nonlinear realization of each anomalous U(1) symmetry, appear. The inflaton multiplet Φ in the massive vector multiplet inflation model may appear as a Green-Schwarz multiplet. We note that, if it is the case, the Kähler potential term of Φ, that is, the function J is not arbitrary and determined by the string theoretical setup. This is a possibility of UV completion of the massive vector multiplet inflation. Although it may be an interesting scenario, the smallness of the gauge coupling g should also be explained by some mechanisms, in order the inflation scale characterized by g to be much smaller than $1(= M_{\rm pl})$ in the typical situation.

Chapter 4

Higher derivative terms of chiral multiplets

This chapter is based on Ref. [117].

4.1 Ghost free higher-derivative term of chiral multiplets

In this section, we discuss the higher-derivative couplings of chiral multiplets, which contain derivatives of more than second order. It is known that if the E.O.M of a bosonic field is a differential equation of more than third order with respect to time, the so-called Ostrogradski instability appears [1] in general.[1] For example, the following types of the higher-derivative terms of a scalar ϕ cause the instability,

$$\mathcal{L}_{\rm HD} = (\Box\phi)^2 + \partial_\mu\partial_\nu\partial_\rho\phi\partial^\mu\partial^\nu\partial^\rho\phi. \tag{4.1}$$

Each term gives a more than third-order time derivative term to the E.O.M of ϕ, and therefore, these terms make the system unstable. On the other hand, the following higher-derivative terms do not lead to the instability,

$$\mathcal{L} = (\partial_\mu\phi\partial^\mu\phi)^n, \tag{4.2}$$

where n is a positive integer. The most general scalar-tensor Lagrangian, which produce only the second order differential equations as E.O.M of a scalar and a graviton, has been investigated by Horndeski in Ref. [3]. Recently, the Lagrangian was reformulated in

[1]For a review, see, e.g., Ref. [2].

Ref. [4]. Such an E.O.M has been extended to the bi-scalar case in Ref. [118], although the corresponding action has not been known. Another recent development in this direction is given in Ref. [119], where an extension of Horndeski action, which leads to the third order equation but no ghost mode, is shown.

To construct the Horndeski action in SUGRA is interesting, but is difficult in the manifestly SUSY construction, that is, superspace and superconformal tensor calculus by the following reason: Horndeski constructed the most general ghost-free action in the way that, from all possible terms, he reduced terms by requiring some condition for realizing the second-order E.O.Ms of both graviton and a scalar [3]. Although, in principle, we can take such a procedure also in SUGRA, the construction of all the possible SUSY terms requires too many calculations. Even worse, a superfield contains at least 4 real scalar fields.[2] Also, as we will see, the form of higher derivative terms depend on formulations of SUGRA, of which we have three types. Therefore, the generic action has never been known, although some ghost-free higher-derivative terms of superfields have been studied and constructed.

We consider a specific SUSY higher-derivative term discussed in Ref. [5]. Before discussing it in SUGRA, let us consider it in global SUSY [120]. In the notation of Ref. [121], the term is given by

$$\mathcal{L} = \int d^4\theta \frac{1}{M^4} D\Phi D\Phi \bar{D}\bar{\Phi}\bar{D}\bar{\Phi}, \tag{4.3}$$

where M is a real mass parameter, Φ is a chiral multiplet, and D_α is the SUSY covariant derivative. The bosonic components of this term are given by

$$\mathcal{L}|_B = \frac{16}{M^4}(|\partial_\mu\Phi\partial^\mu\Phi|^2 - 2|F^\Phi|^2\partial_\mu\Phi\partial^\mu\bar{\Phi} + |F^\Phi|^4), \tag{4.4}$$

where F^Φ is the auxiliary component of Φ. The higher-derivative term $|\partial_\mu\Phi\partial^\mu\Phi|^2$ does not give the ghost instability obviously. The important point for later discussions is that, due to SUSY, some nontrivial terms $|F^\Phi|^2\partial_\mu\Phi\partial^\mu\bar{\Phi}$ and $|F^\Phi|^4$ appear. Such terms contribute to the kinetic and the scalar potential terms respectively. We expect that additional contributions may affect the inflaton dynamics if M is smaller than the Planck scale.

This action can be generalized to the following form

$$\mathcal{L} = \int d^4\theta T(\Phi, \partial_\mu\Phi, \bar{\Phi}, \partial_\mu\bar{\Phi}) D\Phi D\Phi \bar{D}\bar{\Phi}\bar{D}\bar{\Phi}, \tag{4.5}$$

[2] SUSY higher derivative terms often make "auxiliary fields" dynamical, and then, they are not "auxiliary" any more.

where T is a real function of arguments. The bosonic components of this are given by

$$\mathcal{L}|_B = 16T(\Phi, \partial_\mu\Phi, \bar{\Phi}, \partial_\mu\bar{\Phi})(|\partial_\mu\Phi\partial^\mu\Phi|^2 - 2|F^\Phi|^2\partial_\mu\Phi\partial^\mu\bar{\Phi} + |F^\Phi|^4). \tag{4.6}$$

Note that these terms do not lead to the ghost instability: All the terms consist of quantities containing at most the first derivative. The variation of differentiated parts give a derivative to other terms through integration by parts. However, a derived E.O.M of Φ contains at most the second-order derivative terms, since $\mathcal{L}$ originally consists of at most the first-order terms. In the next section, we consider an embedding of this class of term in SUGRA.

Interestingly, this class of higher-derivative terms may be related to superstring theory. In Refs. [27, 122, 123], it is pointed out that the effective action of the D3-brane in 6 dimensional spacetime is partly described by the following action,

$$\mathcal{L}_{\rm DBI} = \int d^4\theta \left(\Phi\bar{\Phi} + \frac{1}{8} f D\Phi D\Phi \bar{D}\bar{\Phi}\bar{D}\bar{\Phi}\right), \tag{4.7}$$

where

$$f^{-1} = 1 + \frac{1}{2}A + \sqrt{1+A+B}, \tag{4.8}$$

$$A = -4\partial_\mu\Phi\partial^\mu\bar{\Phi} - \frac{1}{4}D^2\Phi\bar{D}^2\bar{\Phi}, \tag{4.9}$$

$$B = 4(\partial_\mu\Phi\partial^\mu\bar{\Phi}) - 4|(\partial_\mu\Phi\partial^\mu\Phi)|^2. \tag{4.10}$$

Such a nontrivial action is derived as a dual action of the $\mathcal{N}=2$ Goldstino linear multiplet [27, 122, 123]. If this describes an effective action of D-branes, also from the string theoretical point of view, it may be important to understand the higher-derivative term (4.5).

Before we close this section, we comment on another known higher-derivative term in global SUSY. In Ref. [124], the following higher-derivative action was constructed,

$$\mathcal{L} = \int d^4\theta \left(\Phi\bar{\Phi} - \frac{1}{M^6}\Phi(\bar{D}_{\dot{\alpha}}\partial_\mu\bar{\Phi}\bar{\sigma}_\nu^{\dot{\alpha}\alpha}D_\alpha\partial_\rho\Phi)\epsilon^{\mu\nu\rho\sigma}\partial_\sigma\Phi\right), \tag{4.11}$$

where M is a real mass parameter. This Lagrangian is invariant under the following transformation of Φ,

$$\Phi \to \Phi + c + b_\mu y^\mu, \tag{4.12}$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$. This is the Galilean symmetry [125] in superspace, which contains the Galilean symmetry of the lowest component. The bosonic part of this Lagrangian is

$$\mathcal{L}|_B = -\partial_\mu\Phi\partial^\mu\bar{\Phi} - \frac{4}{M^6}\Phi\partial_{[\mu}\partial^\mu\bar{\Phi}\partial_\nu\partial^\nu\Phi\partial_{\rho]}\partial^\rho\bar{\Phi}. \tag{4.13}$$

In the second term, all the derivatives on each of Φ and $\bar{\Phi}$ are antisymmetrized, and therefore, the third order derivative term can not appear in the E.O.M of Φ, which ensures the absence of ghosts.

4.2 Superconformal realization of SUSY higher-derivative terms

In this section, we discuss the superconformal extension of the higher-derivative term (4.3). In the previous section, we have discussed the higher-derivative interactions in global SUSY. To promote them to that in SUGRA, we need the corresponding derivative operators in conformal SUGRA. Therefore, we first review the derivative operators on superconformal multiplets, on the basis of Ref. [41]. The operators which we need are D_α and ∂_μ on superconformal multiplets.

Let us recall the meaning of the operators. We consider a general superfield $\mathcal{C} = C + \theta\zeta + \cdots$, where C and ζ are a complex scalar and a Weyl spinor respectively, and the ellipses denote the other components. Then, the components of $D_\alpha \mathcal{C}$ are given by $\zeta_\alpha + \cdots$ where the ellipses denote higher order terms of θ and $\bar{\theta}$. In this respect, the operator D_α on $\mathcal{C}$ can be understood as an operation to construct a new superfield $D_\alpha \mathcal{C}$ whose lowest component is the θ-component of $\mathcal{C}$.

Taking these observations into account, let us consider the operator corresponding to D_α in conformal SUGRA, which is denoted by $\mathcal{D}$. In analogy with the global SUSY case, we assume that a supermultiplet $\mathcal{DC}$, where $\mathcal{C}$ is a general multiplet, has $P_L\zeta$ as the lowest component. However, such a multiplet can not be a superconformal one, in general, because the lowest component of a superconformal multiplet should be S- and K-inert. Indeed, although $\delta_K P_L \zeta = 0$, the S-transformation of $P_L\zeta$ is

$$\delta_S P_L \zeta = -i(w+n) P_L \eta C, \tag{4.14}$$

which vanishes if and only if $w + n = 0$. Therefore, we can define the spinor derivative operator in conformal SUGRA on the superconformal multiplet satisfying the constraint $w+n = 0$.[3] This is enough to construct the superconformal version of the term in Eq. (4.5) because the matter chiral multiplets have $(w, n) = (0, 0)$, which satisfies $w + n = 0$. Note that we can also define $\mathcal{D}$ on superconformal multiplet with Lorentz indices [41]. However, such multiplets are not required to construct the terms we will discuss below. We note that, although chiral multiplets Φ^I are defined so that their ζ-component is a Weyl fermion in Sec. 2.3.2, we can define it also in the algebraic way, by using $\bar{\mathcal{D}}_{\dot{\alpha}}$, as $\bar{\mathcal{D}}_{\dot{\alpha}} \Phi^I = 0$.

[3] More detailed analyses are given in Ref. [41].

Next, we consider the superconformal version of ∂_μ on superconformal multiplets. As in the case of $\mathcal{D}$, we assume that the superconformal derivative $\mathcal{D}_\mu$ is defined so that $\mathcal{D}_\mu\mathcal{C}$ has its lowest component $D_\mu C$. Then, we notice the same problem as the spinor derivative case, that is, $D_\mu C$ is not S- and K-inert. The S- and K-transformation of $D_\mu C$ are given by

$$\delta_{S,K} D_\mu C = -2w\lambda_{K\mu}C - \frac{i}{2}\bar{\eta}\gamma_\mu\gamma_*\zeta. \tag{4.15}$$

The first term can vanish if $w = 0$ but the second term can not in general. Therefore, it seems that the superconformal operator corresponding to ∂_μ in global SUSY can not be defined.

This situation can be relaxed by introducing a superconformal multiplet with $w, n \neq 0$ as follows: Instead of $\mathcal{D}_\mu$, we define the **u**-associated derivative $\mathcal{D}_\mu^{(\mathbf{u})}$, where **u** is a superconformal multiplet with $(w, n) = (w_u, n_u)$. $D_\mu C$ can not be S- and K-inert by itself, and that is the reason why the derivative operator can not be a superconformal operator. Such a problem can be relaxed if the terms of S- and K-transformation (4.15) is subtracted. By using **u**, we find that the following combination is S- and K-inert,

$$D_\mu C - 2wV_\mu^K C + \frac{i}{2}\bar{\chi}^S\gamma_\mu\gamma_*\zeta - \frac{n}{4}\bar{\chi}^S\gamma_\mu\gamma_*\chi^S C, \tag{4.16}$$

where

$$\begin{aligned} V_\mu^K \equiv& \frac{1}{4w_u}(C_u^{-1}D_\mu C_u + C_u^{*-1}D_\mu C_u^{*-1}), \\ \chi_S \equiv& \frac{1}{2w_u} i\gamma_*(C_u^{-1}\zeta_u + C_u^{*-1}\zeta_u^C), \end{aligned} \tag{4.17}$$

C_u and ζ_u are C- and ζ-components of **u** respectively, and ζ^C is the charge conjugation of ζ. We have added V_μ^K and χ_S so that the terms in Eq. (4.15) are canceled. To confirm that the combination (4.16) is S- and K-inert, we need the S- and the K-transformations of V_μ, and χ^S given by

$$\begin{aligned} \delta_{S,K} V_\mu^K =& -\frac{1}{4}\bar{\eta}\gamma\chi^S - \lambda_{K\mu}, \\ \delta_{S,K}\chi^S =& \eta. \end{aligned} \tag{4.18}$$

Then a supermultiplet $\mathcal{D}_\mu^{\mathbf{u}}\mathcal{C}$, whose lowest component is given by Eq. (4.16), is a superconformal multiplet.

Thus, we find that the superconformal version of ∂_μ on superconformal multiplet can be defined as the so-called **u**-associated operation. Note that, unless $w_u \neq 0$, **u** is an arbitrary superconformal multiplet, and in an ordinary case, a compensator multiplet plays the role of $\mathbf{u}$. It is important to notice that we do not require any constraints on $\mathcal{C}$ in the above discussion, and therefore, the $\mathbf{u}$-associated derivative operation can be defined on any superconformal multiplets. We note that the superconformal derivative operators $\mathcal{D}_\alpha$ and $\mathcal{D}_\mu^{\mathbf{u}}$ make the superconformal multiplets with (w,n) that with $(w+\frac{1}{2}, n-\frac{3}{2})$ and $(w+1,n)$ respectively. In almost the same way, we can also define the $\mathbf{u}$-associated spinor derivative $\mathcal{D}_\alpha^{\mathbf{u}}$. However, it does not appear in the following discussion, and therefore, we omit it here.[4]

With these superconformal derivative operators, we can construct the higher-derivative term (4.5) in conformal SUGRA. In the old minimal SUGRA, that is, conformal SUGRA with a chiral compensator, such a higher-derivative term of Φ with $(w,n)=(0,0)$ can be written as

$$S_{\text{HD}} = [T(\Phi, \bar{\Phi}, |S_0|^{-1}\mathcal{D}_\mu^{S_0}\Phi, |S_0|^{-1}\mathcal{D}_\mu^{\bar{S}_0}\bar{\Phi})|\mathcal{D}\Phi\mathcal{D}\Phi|^2]_D, \tag{4.19}$$

where we have used S_0 as $\mathbf{u}$ and $|S_0| = \sqrt{S_0\bar{S}_0}$. T is a function of superconformal multiplets with $(w,n)=(0,0)$, and $\mathcal{D}\Phi\mathcal{D}\Phi$ should be understood as a multiplet of which the lowest component is $-2\bar{\chi}P_L\chi$.

Obviously, the term (4.19) depends on the choice of a compensator. Moreover, we can construct a similar action with $\mathbf{u} \neq S_0$.[5] Thus, we find that this kind of higher-derivative terms in different SUGRA formalism behaves in different ways. However, if T only depends on Φ and $\bar{\Phi}$, the term (4.19) becomes independent of the compensator. In the following, we focus on such a compensator-independent term given by

$$S_{\text{HD}} = [T(\Phi,\bar{\Phi})|\mathcal{D}\Phi\mathcal{D}\Phi|^2]_D. \tag{4.20}$$

As long as other higher-derivative terms including a compensator are absent, we can perform the duality transformation discussed in Sec. 3.3.2. For concreteness, let us consider the following system in new minimal SUGRA,

$$S = \left[\frac{3}{2}L_0 \ln\left(\frac{L_0\mathcal{F}(\Phi,\bar{\Phi})}{\mathcal{S}\bar{\mathcal{S}}}\right)\right]_D + [T(\Phi,\bar{\Phi})|\mathcal{D}\Phi\mathcal{D}\Phi|^2]_D, \tag{4.21}$$

where $\mathcal{F}$ is a real function. As in the same way with the case in Sec. 3.3.2, we can rewrite the action to that in the old minimal SUGRA as,

$$S_{\text{dual}} = \left[-\frac{3}{2}S_0\bar{S}_0\mathcal{F}^{-1}\right] + [T(\Phi,\bar{\Phi})|\mathcal{D}\Phi\mathcal{D}\Phi|^2]_D. \tag{4.22}$$

[4]For details of the $\mathbf{u}$-associated operators, see Ref. [41].

[5]For instance, we can construct L associate derivatives in the old minimal SUGRA, where L is a real linear multiplet. In this case, we regard L as not a compensator but a matter multiplet.

As we expected, under the duality transformation between the old and the new minimal SUGRA, the higher-derivative term is invariant.

It is also worth pointing out that the term (4.20) is manifestly invariant under the redefinition of the compensator. In the old minimal SUGRA, the action (3.1) has an ambiguity under the definition of the compensator. For example, under the change of the compensator given by $S_0 \to S_0 e^{\frac{1}{3}\Lambda(\Phi^I)}$, where Λ is a holomorphic function of Φ^I, Ω and W are changed as $\Omega \to \Omega e^{\frac{1}{3}(\Lambda+\bar{\Lambda})}$ and $W \to W e^{\Lambda}$. The change of Ω corresponds to the one of Kähler potential (3.5) as $K \to K - \Lambda - \bar{\Lambda}$. This is called *the Kähler transformation*. Because of the absence of S_0 in the term (4.20), it is invariant under the Kähler transformation.

The generalization of the term (4.20) to the multi-superfield case is straightforward. It is given by

$$S_{\text{HD}} = [T_{IJ\bar{K}\bar{L}} \mathcal{D}\Phi^I \mathcal{D}\Phi^J \bar{\mathcal{D}}\bar{\Phi}^{\bar{K}} \bar{\mathcal{D}}\bar{\Phi}^{\bar{L}}]_D, \tag{4.23}$$

where $T_{IJ\bar{K}\bar{L}}$ is a real function of Φ^I and $\bar{\Phi}^{\bar{J}}$, and its indices are symmetric under the exchanges $I \leftrightarrow J$ and $\bar{K} \leftrightarrow \bar{L}$. The properties discussed above are still true even for this case. The bosonic part of this action is given by

$$S_{\text{HD}}|_B = \int d^4x e \Bigg[32 T_{IJ\bar{K}\bar{L}} (F^I F^J \bar{F}^{\bar{K}} \bar{F}^{\bar{L}} - 2 F^I \bar{F}^{\bar{K}} \partial_\mu \Phi^J \partial^\mu \bar{\Phi}^{\bar{L}} + \partial_\mu \Phi^I \partial^\mu \Phi^J \partial_\nu \bar{\Phi}^{\bar{K}} \partial^\nu \bar{\Phi}^{\bar{L}}) \Bigg]. \tag{4.24}$$

It is worth noting that the action does not have the Ricci scalar term. This is because we only focus on the bosonic part, and actually, the higher-derivative term (4.23) includes the Ricci scalar term like $4T_{IJ\bar{K}\bar{L}} \bar{\chi}^I P_L \chi^J \bar{\chi}^{\bar{K}} P_R \chi^{\bar{L}} R$. Although such a term is omitted in the above, we have to take it into account if we construct the complete action including fermionic parts. In such a case with the standard action (3.1), to construct the action in the Einstein frame, we need to put the following D-gauge fixing condition

$$S_0 \bar{S}_0 \Omega - 4 T_{IJ\bar{K}\bar{L}} \bar{\chi}^I P_L \chi^J \bar{\chi}^{\bar{K}} P_R \chi^{\bar{L}} = 1. \tag{4.25}$$

Therefore, S_0 after gauge fixings becomes the function of not only scalars but also fermions. Such a structure may become important if the SUGRA corrections, that is, couplings between S_0 and matters are strong enough. In the low energy effective theory, such couplings seem typically not so strong because they are suppressed by mass parameters larger than the energy scale realized in collider experiments today.

We also notice that there is the higher order terms of the auxiliary fields F^I. In the ordinary case, the interactions of F-terms are at most quadratic order, and their E.O.Ms are linear equations which uniquely determine their solutions. However, in the presence of the higher order terms of F^I, the E.O.Ms of them become cubic equations, which are difficult to solve analytically in general and have three branches as solutions. As discussed in Refs. [5, 126, 127, 128], the solutions can be classified according to the dependence on the coupling $T_{IJ\bar{K}\bar{L}}$. One solution is regular in the limit of $T_{IJ\bar{K}\bar{L}} \to 0$, and the others are singular in the same limit. As shown in Ref. [127], the solutions singular at $T \to 0$ make the kinetic term of the scalar field non-canonical in the sense that the quadratic derivative terms of scalar vanish but quartic terms remain. Although that is an interesting solution of SUGRA, we focus on the regular solution in the following.

As a final remark of this section, we comment on the other ghost-free higher-derivative coupling discussed in Ref. [6]. The higher-derivative coupling $\sim [M^{-2}\Phi E^a D_a^{L_0}\bar{\Phi}]_D$[6] gives a non-minimal coupling between Φ and gravity like $M^{-2}G^{\mu\nu}\partial_\mu\Phi\partial_\nu\bar{\Phi}$ where $G_{\mu\nu}$ is the Einstein tensor. It is known that this interaction gives an interesting contribution to the inflaton dynamics owing to the gravitationally enhanced friction mechanism [130, 131, 132]. The essence of the mechanism is that such a coupling behaves as $\sim \frac{H^2}{M^2}\partial_\mu\Phi\partial^\mu\bar{\Phi}$ during inflation that makes slow the inflaton dynamics if $M \ll 1$. Interestingly, such a higher-derivative term is constructed only in the new minimal SUGRA. This term obviously depends on the choice of the compensator. Therefore, the form of these couplings must be varied under a duality transformation, even if it is possible. The construction of this kind of couplings in the old minimal SUGRA is interesting, although it has never been done so far.

4.3 Effects of SUSY higher-derivative terms on F-term inflation models

In this section, we discuss the effect of higher-derivative term (4.20) on the F-term inflation models based on Ref. [117]. The action we discuss here is

$$S = \left[-\frac{3}{2}S_0\bar{S}_0 e^{-\frac{K}{3}}\right]_D + [S_0^3 W]_F + [T_{IJ\bar{K}\bar{L}}\mathcal{D}\Phi^I\mathcal{D}\Phi^J\bar{\mathcal{D}}\bar{\Phi}^{\bar{K}}\bar{\mathcal{D}}\bar{\Phi}^{\bar{L}}]_D, \qquad (4.26)$$

[6] E_a is a real multiplet whose lowest component is $\sim C^{-1}B_a$ where C and B_a are components of a real linear compensator, and we have omitted the fermionic part. The full definition of it is given, e.g., in Ref. [129].

where

$$K = \hat{K}(\Phi,\bar{\Phi}) + |S|^2 - \zeta|S|^4, \tag{4.27}$$
$$W = f(\Phi)S, \tag{4.28}$$
$$\tag{4.29}$$

Φ and S are the inflaton and the stabilizer multiplet respectively, ζ is a positive constant, $f(\Phi)$ is a holomorphic function of Φ, and indices run over Φ and S. We have to note that the action (4.26) is not invariant under the duality transformation, nevertheless the higher-derivative term is so. The reason is as follows: In the presence of superpotential terms, we can not assume that all the matter multiplets have $(w,n)=(0,0)$ in the new and the non-minimal SUGRA. That is because, in those formalisms, compensators are a real or a complex linear multiplets, which can not compensate weights in the F-term formula. If one wants to maintain the duality invariance, some additional multiplet will be required, which compensates the weights of terms in the superpotential part. By assuming that such an additional multiplet is stabilized with its mass heavier than the inflation scale, the following discussion and result hold. If we do not add such a compensator like field, the following discussion is valid only in the old minimal SUGRA.

In the absence of the last term, the inflation is realized with the shift symmetric Kähler potential $\hat{K}(\Phi,\bar{\Phi}) = \tilde{K}(\Phi+\bar{\Phi})$, and the inflaton is $\mathrm{Im}\Phi$. The inflationary trajectory in such a case is summarized as follows.

- $f(\Phi)$ becomes non-zero, and then the masses of S and $\mathrm{Re}\Phi$ of the order of H^2 appear. Then, they are stabilized at the origin.
- Along $S=\mathrm{Re}\Phi=0$, $F^{\Phi} = -K^{\Phi\bar{\Phi}}D_{\bar{\Phi}}\bar{W}$ vanishes and $F^S = -K^{S\bar{S}}\bar{W}_{\bar{S}} = -K^{S\bar{S}}\bar{f}(\bar{\Phi}) \neq 0$.
- The effective potential along the inflationary trajectory is given by $V = K^{S\bar{S}}|f(i\phi)|^2$ where $\phi = \mathrm{Im}\Phi$.
- Inflation ends at $f'(i\phi)=0$ or $f(i\phi)=0$, and the former solution can realize the SUSY breaking by S at the vacuum.

In the presence of the higher-derivative term, the situation is quite different. With the higher-derivative term, the off-shell bosonic action in Eq. (4.26) is given by

$$S|_B = \int d^4xe\Bigg[\frac{1}{2}R - K_{I\bar{J}}\partial_\mu\phi^I\partial^\mu\bar{\phi}^{\bar{J}} + K_{I\bar{J}}F^I\bar{F}^{\bar{J}} + (e^{\frac{K}{2}}D_IWF^I + \text{h.c.}) + 3e^K|W|^2$$
$$+ 32T_{IJ\bar{K}\bar{L}}(F^IF^J\bar{F}^{\bar{K}}\bar{F}^{\bar{L}} - 2F^I\bar{F}^{\bar{K}}\partial_\mu\phi^J\partial^\mu\bar{\phi}^{\bar{L}} + \partial_\mu\phi^I\partial^\mu\phi^J\partial_\nu\bar{\phi}^{\bar{K}}\partial^\nu\bar{\phi}^{\bar{L}})\Bigg], \tag{4.30}$$

where we have used the D-, A- and K-gauge fixing conditions in Eq. (3.4).

As mentioned in the previous section, there are quartic couplings of F^I, by which the E.O.Ms of F-terms become cubic equations. Then, it is not easy to derive an on-shell action, and therefore, we assume a particular inflationary trajectory with which we can approximately solve the E.O.Ms of F-terms. What we assume is as follows:

- During inflation, as in the case of the ordinary chaotic inflation, S is stabilized at the origin due to the mass term $m_{S\bar{S}}^2 \sim \zeta H^2$. Then, $D_S W = f(\Phi)$, $W, D_\Phi W \propto S = 0$ and $\partial_\mu S = 0$.
- We *do not* assume the shift symmetry of Φ in the Kähler potential term, but assume that the kinetic coefficient from Kähler potential is not so large $K_{\Phi\bar{\Phi}} \sim \mathcal{O}(1)$ during inflation.
- For simplicity, we require $T_{\Phi S\bar{S}\bar{S}}$, $T_{S\Phi\bar{\Phi}\bar{\Phi}}$, and their conjugates are zero.[7]

Note that though we require these assumptions to obtain an on-shell action, the following discussion holds as long as $|D_S W| \gg |D_\Phi W|$. The assumption $S \sim 0$ and $\partial_\mu S = 0$ are consistent with the inflationary trajectory described below, and are automatically satisfied once inflation starts.

Here, we solve the E.O.M of F-terms in Eq. (4.30) under the assumptions. The E.O.M of $F^{\bar{S}}$ is given by

$$64\left(\frac{1}{64}K_{S\bar{S}} + 2T_{\Phi S\bar{\Phi}\bar{S}}|F^\Phi|^2 - T_{\Phi S\bar{\Phi}\bar{S}}|\partial_\mu\Phi|^2 + T_{SS\bar{S}\bar{S}}|F^S|^2\right)F^S + e^{\frac{K}{2}}D_{\bar{S}}\bar{W} = 0. \quad (4.31)$$

We can rewrite this equation as

$$64\left(\frac{1}{64}K_{S\bar{S}} + 2T_{\Phi S\bar{\Phi}\bar{S}}|F^\Phi|^2 - T_{\Phi S\bar{\Phi}\bar{S}}|\partial_\mu\Phi|^2 + T_{SS\bar{S}\bar{S}}|F^S|^2\right)A + e^{\frac{K}{2}}|D_{\bar{S}}\bar{W}|^2 = 0, \quad (4.32)$$

where $A = F^S D_S W$. We notice that all the quantities in Eq. (4.32) other than A are real, and therefore, A should also be real. In the parentheses of the above equation, we assume that the second term is much smaller than the forth one, and it will be confirmed below. By neglecting the former, the equation becomes a closed form with respect to A, which is given by

$$64\left(\frac{1}{64}K_{S\bar{S}} - T_{\Phi S\bar{\Phi}\bar{S}}|\partial_\mu\Phi|^2 + \frac{T_{SS\bar{S}\bar{S}}}{|D_S W|^2}A^2\right)A + e^{\frac{K}{2}}|D_{\bar{S}}\bar{W}|^2 = 0. \quad (4.33)$$

[7] More precisely speaking, even if $T_{S\Phi\bar{\Phi}\bar{\Phi}}$ and its conjugate exist, they do not affect the inflationary trajectory we will discuss. However, $T_{S\Phi\bar{S}\bar{S}}$ and its conjugate do affect to the trajectory, and they may cause the instability of the trajectory. Therefore, we require that $T_{S\Phi\bar{S}\bar{S}}$ is sufficiently small as we will require that on $T_{SS\bar{S}\bar{S}}$.

This can be algebraically solved as

$$A = 3\sqrt{\frac{\beta}{\alpha}}\left(\sqrt{1+\frac{\alpha\gamma}{4\beta^3}}-\frac{\gamma}{2}\sqrt{\frac{\alpha}{\beta^3}}\right)^{\frac{1}{3}} - 3\sqrt{\frac{\beta}{\alpha}}\left(\sqrt{1+\frac{\alpha\gamma}{4\beta^3}}-\frac{\gamma}{2}\sqrt{\frac{\alpha}{\beta^3}}\right)^{-\frac{1}{3}}, \tag{4.34}$$

where

$$\alpha = \frac{1728 T_{SS\bar{S}\bar{S}}}{|D_S W|^2}, \tag{4.35}$$

$$\beta = K_{S\bar{S}} - 64 T_{\Phi S\bar{\Phi}\bar{S}}|\partial_\mu \Phi|^2, \tag{4.36}$$

$$\gamma = e^{\frac{K}{2}}|D_S W|^2. \tag{4.37}$$

On the other hand, the E.O.M of $F^{\bar{\Phi}}$ is given by

$$64\left(\frac{1}{64}K_{\Phi\bar{\Phi}} - T_{\Phi\Phi\bar{\Phi}\bar{\Phi}}|\partial_\mu\Phi|^2 + 2T_{\Phi S\bar{\Phi}\bar{S}}|F^S|^2 + T_{\Phi\Phi\bar{\Phi}\bar{\Phi}}|F^\Phi|^2\right)F^\Phi + e^{\frac{K}{2}}D_{\bar{\Phi}}\bar{W} = 0. \tag{4.38}$$

As in the case of the above discussion, the last term in the parentheses of the above equation is negligible. Then, this equation is approximated as a linear equation of F^Φ. Thus, we can conclude that $F^\Phi \propto |D_\Phi W| \sim 0$, and this ensures the approximation $|F^S| \gg |F^\Phi|$. Since the solution (4.34) is quite complicated, we expand it with respect to γ. Such an expansion is valid for $\gamma \ll 1$, which is reasonable during inflation. Then, we obtain

$$A \sim -\frac{\gamma}{\beta} = -\frac{e^{\frac{K}{2}}|D_S W|^2}{\beta} = -B^{S\bar{S}}e^{\frac{K}{2}}|D_S W|^2, \tag{4.39}$$

where $B^{S\bar{S}} = \beta^{-1}$. Thus, we find the final expression,

$$F^S = -e^{\frac{K}{2}}B^{S\bar{S}}D_{\bar{S}}\bar{W}. \tag{4.40}$$

Using these, we obtain the following on-shell expression of F-terms;

$$F^S \sim -B^{S\bar{S}}e^{\frac{K}{2}}D_{\bar{S}}\bar{W}, \tag{4.41}$$

$$F^\Phi \propto D_{\bar{\Phi}}\bar{W} = 0, \tag{4.42}$$

where the expression of F^S is given by the leading order of $|D_S W|$, $B^{S\bar{S}} = B^{-1}_{S\bar{S}}$, and $B_{S\bar{S}} = K_{S\bar{S}} - 64T_{\Phi S\bar{\Phi}\bar{S}}\partial_\mu\Phi\partial^\mu\bar{\Phi}$. These solutions are not singular in the limit of $T \to 0$.

By substituting them into the action (4.30), we obtain

$$S|_B^{\text{on-shell}} = \int d^4xe\Bigg[-(K_{\Phi\bar{\Phi}} + 64T_{\Phi S\bar{\Phi}\bar{S}}(B^{S\bar{S}})^2e^K|f(\Phi)|^2)\partial_\mu\Phi\partial^\mu\bar{\Phi}$$
$$+ 32T_{\Phi\Phi\bar{\Phi}\bar{\Phi}}|\partial_\mu\Phi\partial^\mu\Phi|^2 - (2B^{S\bar{S}} - K_{S\bar{S}}(B^{S\bar{S}})^2)e^K|f(\Phi)|^2$$
$$+ 32T_{SS\bar{S}\bar{S}}(B^{S\bar{S}})^4e^{2K}|f(\Phi)|^4\Bigg]. \tag{4.43}$$

In the following, we assume that $T_{IJ\bar{K}\bar{L}}$ is constant, and then, from the dimensional analysis, we can write $T_{IJ\bar{K}\bar{L}}$ as $M^{-4}C_{IJ\bar{K}\bar{L}}$ where M is a mass parameter and $C_{IJ\bar{K}\bar{L}}$ is a dimensionless parameter.

Let us focus on the "kinetic term" of Φ given by

$$-\left(K_{\Phi\bar{\Phi}} + 64C_{\Phi S\bar{\Phi}\bar{S}}(B^{S\bar{S}})^2\frac{e^K|f(\Phi)|^2}{M^4}\right)\partial_\mu\Phi\partial^\mu\bar{\Phi}$$
$$= -\left(K_{\Phi\bar{\Phi}} + 64C_{\Phi S\bar{\Phi}\bar{S}}(B^{S\bar{S}})^2\frac{V}{M^4}\right)\partial_\mu\Phi\partial^\mu\bar{\Phi}, \tag{4.44}$$

where $V \equiv e^K|f(\Phi)|^2$. In the case where $\frac{V}{M^4} \ll 1$ and $C_{\Phi S\bar{\Phi}\bar{S}} \sim \mathcal{O}(1)$, the kinetic coefficient is dominated by the first term in the parentheses, which is the standard kinetic term in SUGRA.

However, for $\frac{V}{M^4} \gg 1$, the situation becomes quite different. Before discussing it, we briefly summarize the scenario that will be considered below:

- We discuss the *slow-roll* inflation driven by Φ, and the energy is dominated by the "scalar potential" in the action (4.43) mixed with derivative terms through $B^{S\bar{S}}$.
- After the end of inflation, $V \to 0$, and then, the higher-order terms due to the SUSY higher-derivative terms decrease. Finally, the action effectively becomes the standard SUGRA one.

When $\frac{V}{M^4} \gg 1$, the kinetic term is effectively given by

$$-64C_{\Phi S\bar{\Phi}\bar{S}}(B^{S\bar{S}})^2\frac{V}{M^4}\partial_\mu\Phi\partial^\mu\bar{\Phi}. \tag{4.45}$$

In this case, the inflaton dynamics becomes far from the ordinary F-term inflation model owing to the nontrivial kinetic coefficient. The "canonically" normalized inflaton $\tilde{\Phi}$ is related to Φ as

$$d\tilde{\Phi} = 8\sqrt{\frac{C_{\Phi S\bar{\Phi}\bar{S}}(B^{S\bar{S}})^2V}{M^4}}d\Phi. \tag{4.46}$$

Therefore, we can rewrite $B_{S\bar{S}}$ as

$$\begin{aligned} B_{S\bar{S}} =& K_{S\bar{S}} - \frac{64C_{\Phi S\bar{\Phi}\bar{S}}}{M^4}\partial_\mu\Phi\partial^\mu\bar{\Phi} \\ =& K_{S\bar{S}} - \frac{\partial_\mu\tilde{\Phi}\partial^\mu\bar{\tilde{\Phi}}}{V(B^{S\bar{S}})^2}. \end{aligned} \tag{4.47}$$

On the other hand, the "scalar potential" in the action (4.43) is given by

$$V_{\rm inf} = (2B^{S\bar{S}} - K_{S\bar{S}}(B^{S\bar{S}})^2)V - 32C_{SS\bar{S}\bar{S}}(B^{S\bar{S}})^4\left(\frac{V}{M^4}\right)V. \tag{4.48}$$

We notice that the second term becomes negative for $C_{SS\bar{S}\bar{S}} > 0$, and its absolute value is larger than the first term for $C_{SS\bar{S}\bar{S}} \sim \mathcal{O}(1)$. Therefore, the potential becomes negative in such a case. One may think that this can be avoided for $C_{SS\bar{S}\bar{S}} < 0$, but that is not correct. The kinetic term of S omitted in the above is given by $\mathcal{L} \sim -(K_{S\bar{S}} + 64C_{SS\bar{S}\bar{S}}(B^{S\bar{S}})^2\frac{V}{M^4})\partial_\mu S\partial^\mu\bar{S}$. Therefore, for $C_{SS\bar{S}\bar{S}} < 0$, S obtains the ghost-like kinetic term, which is also problematic. Therefore, in order to retain a possibility to realize the scenario, we have to require

$$|C_{SS\bar{S}\bar{S}}| \ll \frac{M^4}{V}, \tag{4.49}$$

and then, the above problem can be avoided. We assume this condition in the following discussion, and neglect terms including $C_{SS\bar{S}\bar{S}}$. Under the assumption, the slow-role inflation is driven by $\tilde{\Phi}$ with the potential $\sim V$. In that case, the following relation holds; $\partial_\mu\tilde{\Phi}\partial^\mu\bar{\tilde{\Phi}} \sim \epsilon H^2 \sim \epsilon V$, where ϵ and H are the first slow-roll and the Hubble parameters respectively. Then, $\frac{\partial_\mu\tilde{\Phi}\partial^\mu\bar{\tilde{\Phi}}}{V(B^{S\bar{S}})^2} \sim \epsilon \ll 1$. We conclude that $B_{S\bar{S}} \sim K_{S\bar{S}}$ from Eq. (4.47), and obtain

$$\begin{aligned} \mathcal{L} \sim& -\partial_\mu\tilde{\Phi}\partial^\mu\bar{\tilde{\Phi}} + 32T_{\Phi\Phi\bar{\Phi}\bar{\Phi}}|\partial_\mu\Phi\partial^\mu\Phi|^2 - V \\ \sim& -\partial_\mu\tilde{\Phi}\partial^\mu\bar{\tilde{\Phi}} - V, \end{aligned} \tag{4.50}$$

where we have neglected the second term on the first line because of the same consideration as the above and assumed $K_{S\bar{S}} = 1$ for simplicity.

It is important to notice that, the functional form of $V(\Phi)$ looks different in terms of the canonically normalized inflaton $\tilde{\Phi}$. Because of the factor $\frac{\sqrt{V}}{M^2} \gg 1$, a large field variation of $\tilde{\Phi}$ becomes a small variation of Φ, which means that factor effectively enlarges the field space of Φ around the region $\frac{\sqrt{V}}{M^2} \gg 1$. Such an effect enables the large field inflation without a large field variation of Φ. From this observation, we find that it is useful to expand the potential around the point at which inflation ends. We denote the value of Φ at such a point as Φ_0. Then, the scalar potential can be expanded around Φ_0 as

$$V = V(\Phi_0) + (\partial_\Phi V(\Phi_0)\delta\Phi + \text{h.c.}) + \cdots , \tag{4.51}$$

where $\delta\Phi = \Phi - \Phi_0$ and ellipses denote the higher order terms. Due to the smallness of the variation $\delta\Phi$, we can discuss the inflaton dynamics with only the leading term of this expansion. This is an interesting feature of this model. The η-problem in SUGRA is avoided in this setup even if $\tilde{\Phi}$ realizes the large field inflation effectively.

Let us consider the case where the inflation is driven by $\phi = \text{Re}\delta\Phi$, and the orthogonal direction $\text{Im}\delta\Phi$ stays at its minimum.[8] In such a case, the effective Lagrangian becomes

$$\mathcal{L} \sim -64 C_{\Phi S\bar{\Phi}\bar{S}} \frac{V(\phi)}{M^4} \partial_\mu\phi\partial^\mu\phi - V(\phi), \tag{4.52}$$

where

$$V(\phi) = \sum_{n=1}^{\infty} V_n(0)\phi^n, \tag{4.53}$$

$V_n(0) = \frac{1}{n!}\partial_\phi^n V(0)$, and we have assumed that $V(0) \sim 0$. This condition is reasonble because $V(0)$ becomes the cosmological constant which should be negligibly smaller than the potential energy during inflation.

For the case in which the leading term of the potential is given by $V_m\phi^m$, the canonically normalized inflaton φ is related to ϕ as

$$\begin{aligned}\varphi =& \int \frac{8\sqrt{C_{\Phi S\bar{\Phi}\bar{S}} V_m(0)}}{M^2} \phi^{\frac{m}{2}} d\phi \\ =& \frac{16\sqrt{C_{\Phi S\bar{\Phi}\bar{S}} V_m(0)}}{(m+2)M^2} \phi^{\frac{m}{2}+1}.\end{aligned} \tag{4.54}$$

The potential $V \sim V_m\phi^m$ is represented as

$$V = \tilde{V}_m(0)\varphi^{\frac{2m}{m+2}}, \tag{4.55}$$

where

$$\tilde{V}_m(0) = V_m(0)\left(\frac{(m+2)M^2}{16\sqrt{C_{\Phi S\bar{\Phi}\bar{S}} V_m(0)}}\right)^{\frac{2m}{m+2}}. \tag{4.56}$$

[8] This situation is not trivial, and therefore, we will discuss the related topic on this assumption at the end of this section.

Surprisingly, the effective potential in this model is always represented by a monomial function with the fractional power $\frac{2m}{m+2}$.[9] As a consequence, this model predicts the cosmological parameters n_s and r as

$$n_s = 1 - \frac{2(m+1)}{(m+2)N_*}, \tag{4.57}$$

$$r = \frac{8m}{(m+2)N_*}, \tag{4.58}$$

where N_* is the number of e-foldings at the horizon exit.

In the above discussion, we have set $\mathrm{Im}\delta\Phi = 0$. However, the mass of $\mathrm{Im}\delta\Phi$ is almost the same as that of the inflaton. Therefore, as in the case of Ref. [80], a light scalar field may lead to the isocurvature perturbation which becomes an adiabatic mode after the field decays. If the field dominates the universe and decays after the time when it dominates the universe, the adiabatic mode leads to the non-Gaussianity of the scalar curvature perturbation, which is constrained by the latest Planck data [133]. However, in many cases, we can expect that such a situation does not occur due to the following reason: The light mode has the same mass with the inflaton, and no symmetry is imposed on the inflaton in our case. Then, we can naively expect that the inflaton and the light field decay into the other particles simultaneously. Therefore, the light field can never dominate the universe, and then, the non-Gaussianity predicted in this model may become small consistent with the Planck data. Such a discussion highly depends on more concrete setup, such as couplings between the inflaton and MSSM sector. And so, we do not continue to discuss it and will address this issue elsewhere.

4.4 Simplified model: F-term inflation without Kähler potential

We have discussed an F-term inflation model with SUSY higher-derivative terms (4.20) in the previous section. To clarify the consequence of the model, let us consider a specific limit of the model we have discussed, which corresponds to the one in Ref. [92].

As we discussed, the Kähler potential of the inflaton multiplet is not important for the inflation. The important term is $\frac{1}{M^4}[C_{\Phi S\bar{\Phi}\bar{S}}|\mathcal{D}\Phi\mathcal{D}S|^2]_D$, which gives a nontrivial kinetic term of Φ. It is also important that S is strongly stabilized at the origin, which is achieved by the quartic term in Kähler potential $K \sim -\zeta|S|^4$. As we mentioned in Sec. 3.2, the Goldstino multiplet can be described by the nilpotent multiplet in the decoupling limit

[9] This result is valid as long as $|V_m| \sim |V_n|$ for all $n > m$ because $|\phi| < 1$.

of the sGoldstino. In our case, SUSY is broken by S during inflation, and the sGoldstino S becomes sufficiently heavy. Then S can be well described as the nilpotent multiplet. Taking these facts into account, we can simplify the action (4.26) as

$$S = -\frac{3}{2}[S_0\bar{S}_0 e^{-\frac{1}{3}|S|^2}]_D + [S_0^3 f(\Phi)S]_F + \frac{1}{M^4}[C|\mathcal{D}\Phi\mathcal{D}S|^2]_D, \tag{4.59}$$

where C is a real constant, and we have assumed that S satisfies the nilpotent condition $S^2 = 0$. The first term corresponds to the Kähler potential $K = |S|^2$, and there is no Kähler potential term of Φ. After superconformal gauge fixings, the bosonic part of the action (4.59) becomes

$$\begin{aligned} S|_B = \int d^4xe\Bigg[&\frac{1}{2}R + |F^S|^2 - 3|F^{S_0}|^2 + (f(\Phi)F^S + \text{h.c.}) \\ &+ \frac{32C}{M^4}|F^\Phi|^2|F^S|^2 - \frac{16C}{M^4}|F^S|^2\partial_\mu\Phi\partial^\mu\Phi\Bigg], \end{aligned} \tag{4.60}$$

where we have set $S = 0$ because it is a fermion bilinear due to the nilpotency condition. We can easily solve the E.O.M of F-terms, and obtain

$$F^{S_0} = F^\Phi = 0, \tag{4.61}$$

$$F^S = -\frac{\bar{f}(\bar{\Phi})}{1 - \frac{16C}{M^4}\partial_\mu\Phi\partial^\mu\bar{\Phi}}. \tag{4.62}$$

Substituting them into the action (4.60), we obtain

$$S|_B = \int d^4xe\left[\frac{1}{2}R - \frac{16B^2CV}{M^4}\partial_\mu\Phi\partial^\mu\bar{\Phi} - (2B - B^2)V\right], \tag{4.63}$$

where $B \equiv \left(1 - \frac{16C}{M^4}\partial_\mu\Phi\partial^\mu\bar{\Phi}\right)^{-1}$, and $V \equiv |f(\Phi)|^2$. This action precisely corresponds to the effective action discussed in the previous section.

From this discussion, we find some features of the model in Sec. 4.3. First, as we found, the η-problem is absent in the model. We can understand the reason as follows: In the limit $\frac{V}{M^4} \gg 1$, the Kähler potential can be effectively negligible with respect to the kinetic term. It is also important that in such a limit, the dominant part of the inflaton dynamics is the SUSY higher-derivative part $|\mathcal{D}\Phi\mathcal{D}S|^2$. The term has shift symmetry of Φ. That is the reason why the η-problem is absent even without the shift symmetry of Φ in the Kähler potential.

Secondly, the shift symmetry which the SUSY higher-derivative term has is under a complex shift $\Phi \to \Phi + Z$ where Z is a complex constant, because of the derivative operator $\mathcal{D}$. Such a symmetry makes both $\text{Re}\delta\Phi$ and $\text{Im}\delta\Phi$ light. That is the reason for the light mass of $\text{Im}\delta\Phi$ discussed in the previous section.

Chapter 5

Matter coupled DBI action in $\mathcal{N} = 1$ 4D conformal SUGRA

This chapter is based on Refs. [134, 135].

5.1 DBI action in $\mathcal{N} = 1$ 4D global SUSY

DBI action [7, 8] is a possible extension of the Maxwell action including higher order terms of a U(1) gauge field, which is given by

$$S_{\rm DBI} = \int d^4x \left(1 - \sqrt{-\det\left(\eta_{ab} + \frac{1}{M^2}F_{ab}\right)}\right), \tag{5.1}$$

where η_{ab} is the Minkowski metric, $F_{ab} = 2\partial_{[a}A_{b]}$, A_a is a gauge field under the U(1) symmetry, and M is a real parameter of mass dimension 1. This action is manifestly gauge invariant because it is given by the function of the invariant quantity F_{ab}. It is also important that the action does not produce the third order time derivative to the E.O.M of A_a, which means the absence of ghosts in spite of the higher order derivative couplings.

A natural question is how the SUSY DBI action can be written. The answer was first derived in Ref. [136], where the authors constructed a SUSY Lagrangian having the term (5.1) in its bosonic part. The authors of Ref. [136] also showed the superconformal realization of the DBI action, which we will derive in a different way. After that, it was shown that the Lagrangian in Ref. [136] is related to the partial breaking of 4D $\mathcal{N} = 2$ SUSY in Refs. [26, 27]. The partial breaking of $\mathcal{N} = 2$ SUSY is done by requiring a constraint on an $\mathcal{N} = 2$ superfield. As shown in Ref. [27], the SUSY DBI action of an $\mathcal{N} = 1$ vector multiplet V can be obtained as follows: We impose a constraint on

$\mathbf{W}$, which is an $\mathcal{N}=2$ superfield, given by $\mathbf{W}^2=0$. $\mathbf{W}$ consists of two $\mathcal{N}=1$ chiral superfields $W_\alpha=\bar{D}^2D_\alpha V$ and X, and in terms of these $\mathcal{N}=1$ superfields, the constraint $\mathbf{W}^2=0$ can be rewritten as [1]

$$X = X\bar{D}^2\bar{X} + \frac{1}{4}W^\alpha W_\alpha. \tag{5.2}$$

Then the DBI Lagrangian is given by

$$\mathcal{L} = \int d^2\theta \left(X\bar{D}^2\bar{X} + \frac{1}{4}W^\alpha W_\alpha \right) + \int d^2\theta \Lambda \left(\frac{1}{4}W^\alpha W_\alpha + X\bar{D}^2\bar{X} - X \right), \tag{5.3}$$

where Λ is a Lagrange multiplier chiral multiplet, whose equation of motion provides the constraint (5.2) on X and $W^\alpha W_\alpha$. The first two terms correspond to the kinetic terms of a vector and a chiral multiplet, V and X, of which an $\mathcal{N}=2$ vector multiplet consists. By the field redefinition $\Lambda \to \Lambda - 1$, the action can be rewritten as

$$\mathcal{L} = \left[\int d^2\theta X + \Lambda \left(\frac{1}{4}W^\alpha W_\alpha + X\bar{D}^2\bar{X} - X \right) + \text{h.c.} \right]. \tag{5.4}$$

We can solve the constraint (5.2) with respect to X algebraically and obtain[2]

$$X = W^2 \left[1 + \frac{1}{2}\bar{D}^2 \left(\frac{\bar{W}^2}{1 - \frac{1}{2}A + \sqrt{1 - A + \frac{1}{4}B^2}} \right) \right], \tag{5.5}$$

where $A \equiv \frac{1}{2}(D^2W^2 + \bar{D}^2\bar{W}^2)$ and $B = \frac{1}{2}(D^2W^2 - \bar{D}^2\bar{W}^2)$. Then, one can compute the Lagrangian and find its bosonic part

$$\mathcal{L}|_B = 1 - \sqrt{-\det(\eta_{ab} + F_{ab})}. \tag{5.6}$$

This gives exactly the action in Eq. (5.1) and one confirms that the Lagrangian (5.4) is the SUSY extension of the DBI action.

It is important to notice that there is an underlying condition on X given by $X^2=0$ because of the Grassmann nature of $W^\alpha W_\alpha$. As we can find from the solution (5.5), $X \propto W^2$, and therefore $X^2 \propto W^4 = 0$. Therefore, we find that X is a nilpotent superfield discussed in Sec. 3.2. This nilpotency is quite important by the following reason: The Lagrangian (5.4) can be rewritten as

$$\mathcal{L} = \int d^2\theta X + \int d^2\theta \Lambda \left(\frac{1}{4}W^\alpha W_\alpha + X\bar{D}^2\bar{X} - X \right) + \int d^2\theta \tilde{\Lambda} X^2 + \text{h.c.}, \tag{5.7}$$

[1] We use the convention in Ref. [26].

[2] When we solve the constraint, we obtain two branches of solutions and the solution (5.5) corresponds to one of them, with which the action (5.4) vanishes for $F_{\mu\nu} \to 0$.

where $\tilde{\Lambda}$ is the second Lagrange multiplier chiral multiplet, whose E.O.M is $X^2 = 0$. The equation $X^2 = 0$ is automatically satisfied after solving the first constraint (5.2), as discussed above. Therefore, the second term does not change the system, but it is important to show the nilpotentcy of X manifestly. By using the superspace identity $\int d^4\theta(\cdots) = -\frac{1}{4}\int d^2\theta\bar{D}^2(\cdots) + \text{tot. div}$, we can rewrite the action (5.7) as

$$\mathcal{L} = -\frac{1}{4}\int d^4\theta(\Lambda+\bar{\Lambda})|X|^2 + \left\{\int d^2\theta\left(X + \Lambda\left(\frac{1}{4}W^\alpha W_\alpha - X\right) + \tilde{\Lambda}X^2\right) + \text{h.c.}\right\}. \quad (5.8)$$

The first term can be regarded as the Kähler potential of Λ and X, and therefore, X and Λ seem to obtain their kinetic term given by $-K_{I\bar{J}}\partial_\mu\Phi^I\partial^\mu\bar{\Phi}^{\bar{J}}$. However, one notices that the kinetic mixing matrix $K_{I\bar{J}}$ has a negative eigenvalue because the determinant of it is given by $-\frac{1}{16}|X|^2$. Therefore, it seems that there is a ghost in the bosonic sector of the system, but this is not the case because of the nilpotency of X. As discussed in Sec. 3.2, X is described by a fermion bilinear, and then, $\det K_{I\bar{J}}|_B = 0$, which shows the absence of bosonic ghosts. Indeed, one can confirm that the purely bosonic part of the kinetic terms disappear by taking into account that X is a fermionic bilinear. Thus, one can show the absence of ghosts in the bosonic sector, which we can also check from the resultant Lagrangian (5.6). In this respect, the nilpotency condition on X is quite important, and we will extend the SUSY DBI action in such a way that X satisfies the condition.

We will discuss the DBI action of a single U(1) vector multiplet in the following although there are other types of DBI extensions in global SUSY. Let us briefly comment on such extensions. In Refs. [27, 122, 123], the SUSY DBI action of a real linear (tensor) multiplet is constructed. The strategy is almost the same as that discussed above, and the authors pointed out that the action is dual to the DBI action of a chiral multiplet, which would be the effective action of the position moduli of D3-brane in six dimension. In Refs. [137, 138], the DBI type action with multiple U(1) gauge superfields are constructed by extending the case with a single U(1) vector multiplet. The massive SUSY DBI action of a gauge and a real linear multiplets is discussed in Ref. [139].

There are also the DBI actions in extended ($\mathcal{N} \geq 2$) global SUSY with the superspace or the component formulations. In Refs. [140, 141, 142, 143, 144, 145], the DBI action in $\mathcal{N}=2$ superspace is discussed although the complete form of it is unknown.

On the other hand, the SUSY Dp-brane action, which is a kind of the DBI type actions, also has been constructed and developed within the component formalism in Refs. [146, 147, 148, 149, 150, 151]. The relation between D-branes and the DBI action is reviewed e.g. in Ref. [152].

5.2 superconformal extension of DBI action

In this section, we discuss the superconformal realization of the DBI action as in the previous chapter. As we have found, the DBI action can be expressed in terms of a set of chiral multiplets, which are related with each other by a nontrivial constraint. From this viewpoint, to construct the DBI action in SUGRA, what we have to do is the superconformal extension of the constraint (5.2). Then, we need to realize the superconformal version of terms in Eq. (5.2). The second term on the right-hand side of Eq. (5.2) is trivial, and we know the corresponding superconformal chiral multiplet $\mathcal{W}^2$ with $(w,n)=(3,3)$. In the first term on the right-hand side of Eq. (5.2), we notice that $\bar{D}^2$ makes $\bar{X}$ a chiral multiplet. We know such an operator, that is, the chiral projection Σ defined in Eq. (2.51). To use the operator on a multiplet, it should satisfy the constraint $w-n=2$, and therefore, the Weyl and the chiral weights of an antichiral multiplet $\bar{X}$ are uniquely determined as $(w,n)=(1,-1)$. The chiral multiplet $\Sigma(\bar{X})$ has the weights $(w,n)=(2,2)$, and $X\Sigma(\bar{X})$ has the weights $(w,n)=(3,3)$. Thus, we find that the superconformal extension of the right-hand side of Eq. (5.2) consists of $\mathcal{W}^2$ and $X\Sigma(\bar{X})$ with the weights $(w,n)=(3,3)$. However, the left-hand side of Eq. (5.2) is given by X, and a naive extension is not applicable because of the difference of the weights between both sides. This means that the superconformal extension of the constraint (5.2) is impossible without introducing other multiplets.

A possible candidate for the multiplet introduced in the constraint is a compensator multiplet. Here, let us consider the old minimal formulation, that is, the case with a chiral compensator S_0 with $(w,n)=(1,1)$. Then, we find the following superconformal constraint, which is expected to be an extension of Eq. (5.2),

$$S_0^2X=\mathcal{W}^2-aX\Sigma(\bar{X}), \tag{5.9}$$

where a is a real constant parameter. For simplicity of the later discussion, we redefine X as S_0X, and then X has the weights $(w,n)=(0,0)$ as the usual matter chiral multiplets, and then the constraint becomes

$$S_0^3X=\mathcal{W}^2-aS_0X\Sigma(\bar{S}_0\bar{X}). \tag{5.10}$$

It is important to note that the solution of this constraint is also a nilpotent multiplet, since we can formally express the constraint as

$$X=\frac{\mathcal{W}^2}{S_0^3+aS_0\Sigma(\bar{S}_0\bar{X})}, \tag{5.11}$$

which is proportional to $\mathcal{W}^2$ as in the global SUSY case. Although we can solve this equation in almost the same way as the global SUSY case, the calculation is rather complicated.

Instead, we use the superconformal version of the action (5.7). Such an action can be written as

$$S = [bS_0^3 X]_F + [\Lambda(\mathcal{W}^2 - aS_0 X\Sigma(\bar{S}_0\bar{X}) - S_0^3 X)]_F + [S_0^3\tilde{\Lambda}X^2]_F + [-cS_0\bar{S}_0]_D, \tag{5.12}$$

where b and c are real parameters, and we have introduced Lagrange multiplier chiral multiplets Λ and $\tilde{\Lambda}$ with $(w,n) = (0,0)$. The last term gives the kinetic term of the gravity multiplet. In the following, we focus on the bosonic part of the action, and then, we need to recall that the third term gives the nilpotency condition of X, which yields $X = \frac{\bar{\psi}^X P_L \psi^X}{2F^X}$. Taking it into account, the bosonic part of the action (5.12) is given by

$$\begin{aligned} S|_B = \int d^4xe\bigg[& a|S_0|^2(\Lambda+\bar{\Lambda})|F^X|^2 + \{S_0^3(b-\Lambda)F^X + \text{h.c.}\} + \frac{1}{2}(\Lambda+\bar{\Lambda})F_{\mu\nu}F^{\mu\nu} \\ & - \frac{1}{2}(\Lambda-\bar{\Lambda})F_{\mu\nu}\tilde{F}^{\mu\nu} - (\Lambda+\bar{\Lambda})D^2 + \frac{c}{3}|S_0|^2 R(e,b) - c(|F^{S_0}|^2 - D_\mu S_0 D^\mu \bar{S}_0)\bigg], \end{aligned} \tag{5.13}$$

where $F_{\mu\nu} = 2\partial_{[\mu}B_{\nu]}$, B_μ is a gauge field, $\tilde{F}^{\mu\nu} = -\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$, and $D_\mu S_0 = \partial_\mu S_0 - b_\mu S_0 - iA_\mu S_0$. We choose $c = \frac{3}{2}$, and then, we can obtain the Einstein frame action by choosing the superconformal gauge fixing conditions as $S_0 = \bar{S}_0 = 1$ and $b_\mu = 0$. Thus, the action (5.13) becomes

$$\begin{aligned} S|_B^E = \int d^4xe\bigg[& 2a(\text{Re}\Lambda)|F^X|^2 + \{(b-\Lambda)F^X + \text{h.c.}\} + (\text{Re}\Lambda)F_{\mu\nu}F^{\mu\nu} \\ & - i(\text{Im}\Lambda)F_{\mu\nu}\tilde{F}^{\mu\nu} - 2(\text{Re}\Lambda)D^2 + \frac{1}{2}R - \frac{3}{2}(|F^{S_0}|^2 - A_\mu A^\mu)\bigg]. \end{aligned} \tag{5.14}$$

The E.O.M of F^X, F^{S_0}, D and A_μ can be algebraically solved as

$$F^X = -\frac{b-\lambda+i\phi}{2a\lambda}, \tag{5.15}$$

$$F^{S_0} = D = 0, \tag{5.16}$$

$$A_\mu = 0, \tag{5.17}$$

where $\lambda \equiv \text{Re}\Lambda$ and $\phi \equiv \text{Im}\Lambda$. Then, we obtain the on-shell action

$$S|_B^E = \int d^4xe\bigg[\frac{1}{2}R + \lambda F_{\mu\nu}F^{\mu\nu} - i\phi F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{(b-\lambda)^2+\phi^2}{2a\lambda}\bigg]. \tag{5.18}$$

The E.O.M of the Lagrange multipliers are

$$\frac{\phi}{\lambda} = ai F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{5.19}$$

$$\frac{b^2}{\lambda^2} = 1 + 2aF_{\mu\nu}F^{\mu\nu} + a^2(F_{\mu\nu}\tilde{F}^{\mu\nu})^2. \tag{5.20}$$

We can solve these equations with respect to ϕ and λ, and then, by substituting the solutions, we obtain

$$\begin{aligned} S|_B^E =& \int d^4x e\left[\frac{1}{2}R - \frac{b}{a} + \frac{b}{a}\sqrt{1 + aF_{\mu\nu}F^{\mu\nu} + a^2(F_{\mu\nu}\tilde{F}^{\mu\nu})^2}\right] \\ =& \int d^4x\left[\frac{1}{2}\sqrt{-g}R + \frac{b}{a}\sqrt{-\det(g_{\mu\nu} + 2\sqrt{a}F_{\mu\nu})} - \frac{b}{a}\sqrt{-g}\right], \end{aligned} \tag{5.21}$$

where we have substituted one of the solutions for Eq. (5.20) so that the second and the third terms cancel with each other for $F_{\mu\nu} = 0$. This is the DBI action (5.6) coupled to supergravity, for $a = -b = \frac{1}{4}$. Thus, we confirm that the following action is the superconformal extension of the DBI action,

$$S = \left[-\frac{1}{4}S_0^3X\right]_F + \left[\Lambda\left(\mathcal{W}^2 - \frac{1}{4}S_0X\Sigma(\bar{S}_0\bar{X}) - S_0^3X\right)\right]_F + [S_0^3\tilde{\Lambda}X^2]_F - \left[\frac{3}{2}S_0\bar{S}_0\right]_D. \tag{5.22}$$

Before discussing the matter couplings, let us consider the DBI action in the new and the non-minimal SUGRA. We know the old minimal version (5.22) and it can be rewritten as

$$S = \left[-\frac{1}{4}\tilde{X}\right]_F + \left[\Lambda\left(\mathcal{W}^2 - \frac{1}{4}\Sigma\left(\frac{\tilde{X}\bar{\tilde{X}}}{(S_0\bar{S}_0)^2}\right) - \tilde{X}\right)\right]_F + [\hat{\Lambda}\tilde{X}^2]_F - \left[\frac{3}{2}S_0\bar{S}_0\right]_D, \tag{5.23}$$

where $\tilde{X} = S_0^3X$, $\hat{\Lambda} = S_0^{-3}\tilde{\Lambda}$, and we have used the identity $S_0X\Sigma(\bar{S}_0\bar{X}) = \Sigma(S_0\bar{S}_0X\bar{X})$. We notice that the action contains S_0 and $\bar{S}_0$ in the form of a real combination $S_0\bar{S}_0$. Therefore, we can replace the combination with a real linear multiplet L_0 or a combination of complex linear compensators $\mathbf{L}_0\bar{\mathbf{L}}_0$ with an appropriate power of them. More concretely, the DBI action with a real linear multiplet and with a complex linear multiplet are given by

$$S^{\rm new}=\left[-\frac{1}{4}\tilde{X}\right]_F+\left[\Lambda\left(\mathcal{W}^2-\frac{1}{4}\Sigma\left(\frac{\tilde{X}\bar{\tilde{X}}}{L_0^2}\right)-\tilde{X}\right)\right]_F+[\hat{\Lambda}\tilde{X}^2]_F-\left[\frac{3}{2}L_0\ln\left(\frac{L_0}{\mathcal{S}\bar{\mathcal{S}}}\right)\right]_D, \tag{5.24}$$

$$S^{\rm NM}=\left[-\frac{1}{4}\tilde{X}\right]_F+\left[\Lambda\left(\mathcal{W}^2-\frac{1}{4}\Sigma\left(\frac{\tilde{X}\bar{\tilde{X}}}{(\mathbf{L}_0\bar{\mathbf{L}}_0)^{\frac{2}{w}}}\right)-\tilde{X}\right)\right]_F+[\hat{\Lambda}\tilde{X}^2]_F-\left[\frac{3}{2}(\mathbf{L}_0\bar{\mathbf{L}}_0)^{\frac{1}{w}}\right]_D, \tag{5.25}$$

where $\mathcal{S}$ is a chiral multiplet with $(w,n)=(1,1)$, the last term in each action denotes the pure SUGRA action, and we have assumed that the weights of $\mathbf{L}_0$ are $(w,n)=(w,w-2)$. In the above expressions, one should consider $\tilde{X}$ as a chiral multiplet with $(w,n)=(3,3)$. Note that, the construction of the DBI action coupled to a chiral compensator or a real linear compensator is discussed also in Refs. [153, 154].

5.3 DBI action coupled with chiral matter

We can also extend the DBI action (5.22) to that coupled with matter chiral multiplets. Such a situation is more realistic than the pure DBI action discussed so far. As shown in the previous section, the nilpotency condition on X is important for avoiding ghosts. A possible generalization of the constraint (5.10) is

$$S_0^3X=\mathcal{W}^2-S_0X\Sigma(\omega(\Phi^I,\bar{\Phi}^{\bar{J}})\bar{S}_0\bar{X}) \tag{5.26}$$

where $\omega(\Phi^I,\bar{\Phi}^{\bar{J}})$ is a real function of matter chiral multiplets Φ^I and its conjugates $\bar{\Phi}^{\bar{J}}$, whose weights are $(w,n)=(0,0)$. The matter couplings can be added also in the action and the general matter coupled extension is given by

$$\begin{aligned}S=&-\frac{1}{4}[S_0^3f(\Phi^I)X]_F+[\Lambda(\mathcal{W}^2-S_0X\Sigma(\omega(\Phi^I,\bar{\Phi}^{\bar{J}})\bar{S}_0\bar{X})-S_0^3X)]_F\\&+[S_0^3\tilde{\Lambda}X^2]-\frac{3}{2}\left[S_0\bar{S}_0\exp\left(-\frac{1}{3}K(\Phi^I,\bar{\Phi}^{\bar{J}})\right)\right]_D+[S_0^3W(\Phi^I)]_F,\end{aligned} \tag{5.27}$$

where $K(\Phi^I,\bar{\Phi}^{\bar{J}})$ and $W(\Phi^I)$ are Kähler and super-potentials of chiral multiplets respectively, and $f(\Phi^I)$ is a holomorphic function of chiral multiplets. The meaning of $f(\Phi^I)$ can be understood by considering the limit $\omega\to 0$. In the limit, the constraint (5.26) becomes $S_0^3X=\mathcal{W}^2$, and then, the first term in Eq. (5.27) is $-\frac{1}{4}[f(\Phi^I)\mathcal{W}^2]_F$, which is the standard kinetic term of a gauge multiplet. Therefore, we find that $f(\Phi^I)$ corresponds to the gauge kinetic function.

Let us consider the component expression of the bosonic part of this action. That is given by

$$S|_B = \int d^4xe\Bigg[\frac{1}{2}|S_0|^2 e^{-\frac{K}{3}} R(e,b) + \omega|S_0|^2(\Lambda+\bar{\Lambda})|F^X|^2 - \left\{S_0^3\left(\frac{f}{4}+\Lambda\right)F^X + \text{h.c.}\right\} \\ + (\text{Re}\Lambda)F_{\mu\nu}F^{\mu\nu} - i(\text{Im}\Lambda)F_{\mu\nu}\tilde{F}^{\mu\nu} - 2(\text{Re}\Lambda)D^2 \\ + ie^{-\frac{K}{3}}|S_0|^2 K_I k^I D + \mathcal{L}_{\text{ordinary}}\Bigg], \tag{5.28}$$

where k^I is the Killing vector for U(1) isometry on the manifold spanned by Φ^I. $\mathcal{L}_{\text{ordinary}}$ is given by

$$\mathcal{L}_{\text{ordinary}} = \mathcal{N}_{\hat{I}\bar{\hat{J}}}(F^{\hat{I}}\bar{F}^{\bar{\hat{J}}} - D^a\Phi^{\hat{I}}D^a\bar{\Phi}^{\bar{\hat{J}}}) + (F^{\hat{I}}\hat{W}_{\hat{I}} + \text{h.c.}), \tag{5.29}$$

where the indices $\hat{I},\bar{\hat{J}}$ run over Φ^I and S_0, I is an index of matter chiral multiplets , $\mathcal{N} = -3|S_0|^2 e^{-\frac{K}{3}}$, $\hat{W} = S_0^3 W$, and D_a denotes the covariant derivative. Here we take the standard superconformal gauge fixing conditions to obtain the Einstein frame action,

$$S_0 = \bar{S}_0 = e^{\frac{K}{6}}, \quad b_\mu = 0. \tag{5.30}$$

Then, we obtain the following action,

$$S|_B^E = \int d^4xe\Bigg[\frac{1}{2}R + 2\omega(\text{Re}\Lambda)e^{\frac{K}{3}}|F^X|^2 - \left\{e^{\frac{K}{2}}\left(\frac{f}{4}+\Lambda\right)F^X + \text{h.c.}\right\} + \mathcal{L}^E_{\text{ordinary}} \\ + (\text{Re}\Lambda)F_{\mu\nu}F^{\mu\nu} - i(\text{Im}\Lambda)F_{\mu\nu}\tilde{F}^{\mu\nu} - 2(\text{Re}\Lambda)D^2 + iK_Ik^ID\Bigg], \tag{5.31}$$

where $\mathcal{L}^E_{\text{ordinary}}$ is $\mathcal{L}_{\text{ordinary}}$ under the superconformal gauge conditions (5.30). It is straightforward to solve the E.O.Ms of F^X, $F^{\hat{I}}$, D and A_μ. After solving them, the action becomes

$$S|_B^{\text{on-shell}} = \int d^4xe\Bigg[\frac{1}{2}R + \lambda F_{\mu\nu}F^{\mu\nu} - i\phi F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{(K_Ik^I)^2}{2\lambda} + \mathcal{L}^{\text{on-shell}}_{\text{ordinary}} \\ - \frac{1}{2\omega\lambda}e^{\frac{2K}{3}}\left\{\left(\frac{1}{4}p+\lambda\right)^2 + \left(\frac{1}{4}q+\phi\right)^2\right\}\Bigg], \tag{5.32}$$

where $\lambda = \mathrm{Re}\Lambda$, $\phi = \mathrm{Im}\Lambda$, $p = \mathrm{Re}f(\Phi^I)$, $q = \mathrm{Im}f(\Phi^I)$, and

$$\mathcal{L}^{\mathrm{on-shell}}_{\mathrm{ordinary}} = -K_{I\bar{J}} D_\mu \Phi^I D^\mu \bar{\Phi}^{\bar{J}} - V_F. \tag{5.33}$$

In the above equation, $D_\mu \Phi^I = \partial_\mu \Phi^I - B_\mu k^I$ is a covariant derivative with respect to the internal U(1) symmetry.

In the same way as in the case without matters, we can integrate λ and ϕ out and obtain the following solutions with respect to them,

$$\lambda = \pm \frac{p e^{\frac{K}{3}}}{4\sqrt{\omega}} \left(1 - \frac{4\omega (K_I k^I)^2}{p^2 e^{\frac{2K}{3}}}\right)^{\frac{1}{2}} \left(\frac{e^{\frac{2K}{3}}}{\omega} + 2F_{\mu\nu}F^{\mu\nu} + \frac{\omega}{e^{\frac{2K}{3}}}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right)^{-\frac{1}{2}}, \tag{5.34}$$

$$\chi = -\frac{q}{8} + i\frac{\lambda\omega}{e^{\frac{2K}{3}}} F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{5.35}$$

We find that there are two branches in the solution of λ. We choose the one with negative sign so that the DBI action should vanish when $k^I = F_{\mu\nu} = 0$. Substituting the solutions into Eq. (5.32), we obtain the final form of the action as

$$\begin{aligned} S|_B^{\mathrm{final}} =& \int d^4x e \left[\frac{1}{2}R + \mathcal{L}^{\mathrm{on-shell}}_{\mathrm{ordinary}} - \frac{i}{4}(\mathrm{Im}f)F_{\mu\nu}\tilde{F}^{\mu\nu}\right] \\ &+ \int d^4x \frac{e^{\frac{2K}{3}}(\mathrm{Re}f)}{4\omega}\left\{\sqrt{-g} - \sqrt{P}\sqrt{-\det(g_{\mu\nu} + 2e^{-\frac{K}{3}}\sqrt{\omega}F_{\mu\nu})}\right\}, \end{aligned} \tag{5.36}$$

where

$$P \equiv 1 - \frac{4\omega(K_I k^I)^2}{e^{\frac{2K}{3}}(\mathrm{Re}f)^2} = 1 + \frac{8\omega}{e^{\frac{2K}{3}}(\mathrm{Re}f)} V_D. \tag{5.37}$$

By expanding the second line of the action with respect to ω, we find

$$\int d^4x e \left[-\frac{1}{4}(\mathrm{Re}f)F_{\mu\nu}F^{\mu\nu} + \frac{1}{2\mathrm{Re}f}(K_I k^I)^2 + \mathcal{O}(\omega^2)\right]. \tag{5.38}$$

The first and the second terms are the kinetic term of the gauge field and the D-term potential in usual cases, respectively. This means that the couplings between the compensator and the gauge multiplet only appear as the higher order terms, which can be confirmed also by the discussion below Eq. (5.27). It is possible to remove the matter coupling in the square root term by choosing $\omega = \alpha e^{\frac{2K}{3}}$, where α is a real constant. This choice is called Kähler covariant form [136] because in this case the DBI action is invariant under the Kähler transformation.

The DBI action in Eq. (5.36) has an interesting feature. The scalar potential in the action is obtained, by setting $F_{\mu\nu}=0$, as

$$\begin{aligned} V =& \frac{e^{\frac{2K}{3}}(\mathrm{Re}f)}{4\omega}(\sqrt{P}-1) \\ =& \frac{e^{\frac{2K}{3}}(\mathrm{Re}f)}{4\omega}\left(\sqrt{1+\frac{8\omega}{e^{\frac{2K}{3}}(\mathrm{Re}f)}V_D}-1\right). \end{aligned} \tag{5.39}$$

This deformation of the scalar potential is due to the higher order corrections associated with the DBI extension. We will discuss the effects of this deformation on an inflation model in the next section.

Let us comment on the relation between our DBI type extension and superstring. In the type II superstring theory, the D3-brane action is given by

$$S=\int d^4\sigma\sqrt{-\det(\hat{g}_{\mu\nu}+F_{\mu\nu}+B_{\mu\nu})}, \tag{5.40}$$

σ^μ ($\mu=0\cdots 3$) is the world volume coordinate, $\hat{g}_{\mu\nu}$ is the induced metric on the world volume, $F_{\mu\nu}$ and $B_{\mu\nu}$ are the field strength of a U(1) gauge field and an antisymmetric tensor field called B-field, respectively. The induced metric is related to the 10 dimensional metric G_{MN} as $\hat{g}_{\mu\nu}=\partial_\mu X^M(\sigma)G_{MN}\partial_\nu X^N(\sigma)$, where $X^M(\sigma)$ is the world volume scalar field describing the position of the D3-brane in 10 dimensional spacetime. Note that this action has the general coordinate transformation invariance on the world volume. Comparing this action and that in Eq. (5.21), we find that the latter corresponds to the former under the condition: $X^\mu(\sigma)=\sigma^\mu$ for $\mu=0,\cdots,3$, $X^M=0$ for $M=4,\cdots,9$, $G_{\mu\nu}=g_{\mu\nu}$, and $B_{\mu\nu}=0$.

From this correspondence, we can expect that the D3-brane action takes the form (5.36) if matter multiplet exists. Although such an action has never been known in the superstring theory, it is rather natural to consider the case with matter couplings. We need to investigate more about the D-brane action in the presence of matter multiplets, from string theoretical side.

Let us close this section with some additional comments on $\mathcal{N}=2$ SUSY breaking. As we reviewed at the first of this chapter, the SUSY DBI action is related to the partial breaking of $\mathcal{N}=2$ SUSY, and then, the vector multiplet (or more precisely $\mathcal{W}_\alpha$) corresponds to the Goldstino multiplet for broken $\mathcal{N}=2$ SUSY. Therefore, in the SUGRA-extended case discussed above, $\mathcal{W}$ also seems to be a Goldstino multiplet, which should be eaten by the super-Higgs mechanism. If it is the case, the action we have constructed should vanish in the unitary gauge of $\mathcal{N}=2$ SUSY. Although such a possibility exists, there is also another possibility that the "Goldstino" multiplet becomes physical.

Such a situation realizes if there are other breaking sectors of $\mathcal{N}=2$ SUSY. Since only a linear combination of the $\mathcal{N}=2$ SUSY breaking multiplets can be eaten by the super-Higgs mechanism, in the presence of multiple SUSY breaking $\mathcal{N}=1$ superfields, the other modes become physical massless $\mathcal{N}=1$ superfields. To prove this statement, we have to realize our models from $\mathcal{N}=2$ SUGRA. However, this issue is beyond the scope of this thesis.

5.4 Massive vector multiplet inflation with DBI extension

In this section, we discuss the DBI extension of the inflation model with a massive vector multiplet. As we reviewed in Sec. 3.4, the massive vector multiplet is a reducible multiplet, which can be decomposed into a chiral multiplet and a gauge multiplet. From this perspective, the DBI extension of a massive vector multiplet action can be realized with the matter coupled DBI action (5.27). Let us recall the massive vector multiplet action

$$S=\left[-\frac{3}{2}S_0\bar{S}_0\exp\left(-\frac{2}{3}J\right)\right]_D+\left[-\frac{1}{4}\mathcal{W}^2(V)\right]_F, \tag{5.41}$$

where $J=J(\frac{1}{2}(\Phi+\bar{\Phi}+gV))$. We find that $2J(\frac{1}{2}(\Phi+\bar{\Phi}+gV))$ corresponds to the Kähler potential. Therefore, what we need for the extension is just extending the second term in this action to the DBI type one. In the way shown in the previous section, we obtain the DBI extension as

$$\begin{aligned}S=&\left[-\frac{3}{2}S_0\bar{S}_0\exp\left(-\frac{2}{3}J\right)\right]_D+\left[-\frac{1}{4}S_0^3X\right]_F\\&+[\Lambda(\mathcal{W}^2-S_0X\Sigma(\omega(\Phi+\bar{\Phi})\bar{S}_0\bar{X})-S_0^3X)]_F,\end{aligned} \tag{5.42}$$

where we have chosen the function ω in a U(1) gauge invariant way. This is the DBI action of the massive vector multiplet since it reproduces the action (5.41) in the limit $\omega\to 0$.

This action corresponds to that in Eq. (5.27) with

$$f(\Phi^I)=1,\quad K=2J\left(\frac{1}{2}(\Phi+\bar{\Phi})\right),\quad W(\Phi^I)=0,$$

and $k^\Phi=ig$, which is the Killing vector of Φ under the U(1) symmetry. Then, the bosonic part of the action (5.42) is easily derived as

$$\begin{aligned} S|_B =& \int d^4xe\left[\frac{1}{2}R - \frac{1}{2}J''(C)\partial_\mu C\partial^\mu C - \frac{1}{2}J''(C)(\partial_\mu\theta - gA_\mu)^2\right] \\ &+ \int d^4x\frac{e^{\frac{4J}{3}}}{4\omega}\left\{\sqrt{-g} - \sqrt{P}\sqrt{-\det(g_{\mu\nu} + 2e^{-\frac{2J}{3}}\sqrt{\omega}F_{\mu\nu})}\right\}, \end{aligned} \tag{5.43}$$

where $C = \mathrm{Re}\Phi$, $\theta = \mathrm{Im}\Phi$. The function P in this case is given by

$$P = 1 + \frac{4\omega g^2(J'(C))^2}{e^{\frac{4J}{3}}}. \tag{5.44}$$

For simplicity, we choose

$$\omega = \frac{1}{4M^4}e^{\frac{4J}{3}}, \tag{5.45}$$

where M is a real parameter. Then, the action is simply given by

$$\begin{aligned} S|_B =& \int d^4xe\left[\frac{1}{2}R - \frac{1}{2}J''(C)\partial_\mu C\partial^\mu C - \frac{g^2}{2}J''(C)(A_\mu - \frac{1}{g}\partial_\mu\theta)^2\right] \\ &+ \int d^4xM^4\left\{\sqrt{-g} - \sqrt{P}\sqrt{-\det\left(g_{\mu\nu} + \frac{1}{M^2}F_{\mu\nu}\right)}\right\}, \end{aligned} \tag{5.46}$$

with $P = 1 + \frac{g^2(J'(C))^2}{M^4}$. Note that the mass term of A_μ, which is the third term on the first line of Eq. (5.46), is not affected by the DBI extension. This is consistent with the global SUSY result in Ref. [139].

Let us discuss the inflationary trajectory of the inflaton C. The scalar potential is given by

$$\begin{aligned} V =& M^4(\sqrt{P} - 1) \\ =& M^4\left(\sqrt{1 + \frac{g^2(J'(C))^2}{M^4}} - 1\right). \end{aligned} \tag{5.47}$$

Of course, this potential reproduces the original potential $V = \frac{g^2}{2}(J'(C))^2$ in the limit $M \to \infty$. In the case with $M \sim 1$, the higher order corrections propotional to g^{2n} ($n \geq 2$) are negligible because g characterizes the scale of the inflation and should be much smaller than 1 to explain the amplitude of the scalar curvature perturbation. In such a case, the higher order corrections which originate from the DBI extension do not change the prediction of the model with $V = \frac{g^2}{2}(J'(C))^2$, as discussed in Ref. [155].

From the consideration above, the higher order correction seems negligible in general. However, it is not the case if $M \ll 1$. Let us consider such a case in the following. The important quantity characterizing the deformation is $\beta \equiv \frac{g^2}{M^4}$. For $\beta \gg 1$, which corresponds to the case with a sufficiently small M, the potential behaves as

$$\tilde{V} = \frac{g^2}{\beta}(\sqrt{1+\beta(J'(C))^2}-1) \sim \frac{g^2}{\sqrt{\beta}}|J'(C)|. \tag{5.48}$$

This effect obviously makes the potential flatter than the original one. To show this analytically, let us compare the slow-roll parameters in the original and this cases. In the original case where the potential is $V = \frac{g^2}{2}(J')^2$, the slow-roll parameters ϵ and η are given by

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \frac{2(J'')^2}{(J')^2}, \tag{5.49}$$

$$\eta = \frac{V''}{V} = \frac{2\{(J'')^2 + J'J'''\}}{(J')^2}. \tag{5.50}$$

On the other hand, in the case with the potential (5.48), the slow-roll parameters, denoted by $\tilde{\epsilon}$ and $\tilde{\eta}$, are

$$\tilde{\epsilon} = \frac{1}{2}\left(\frac{\tilde{V}''}{\tilde{V}}\right)^2 = \frac{1}{2}\frac{(J'')^2}{(J')^2} = \frac{1}{4}\epsilon, \tag{5.51}$$

$$\tilde{\eta} = \frac{\tilde{V}''}{\tilde{V}} = \frac{\{(J'')^2 + J'J'''\}}{(J')^2} - \frac{(J'')^2}{2(J')^2} = \frac{1}{2}\eta - \frac{1}{4}\epsilon, \tag{5.52}$$

where ϵ and η are the ones in the original case. Therefore, we find that the higher order corrections in this specific case make potential flat rather than steep, whereas one naively expects that they make it steeper. Note that it is not correct to think that the values of cosmological parameters obey the above relation between those in the original and the deformed cases. That is because the moment of which inflation ends is also affected by the flatness of the potential.

The minimum of the potential is given by the point at $J' = 0$ as in the original case. Therefore, even with the DBI extension, this model does not break SUSY at the minimum.

Here, let us discuss some concrete examples with specific choices of $J(C)$. First, we consider a model with $J(C) = \frac{1}{2}C^2$, and then, the scalar potential becomes

$$V = \frac{g^2}{\beta}(\sqrt{1+\beta C^2}-1). \tag{5.53}$$

The kinetic term of C is originally canonical in this case. The shapes of the potential with different values of β are shown in Fig. 5.1. As expected, the original form $V \propto C^2$ becomes like $V \sim |C|$ when $\beta = 10$ (see Fig. 5.1). Owing to the deformation, the cosmological parameters n_s and r predicted in this model are also changed, and for comparison, we show those values in the cases with $\beta = 10^{-5}$ and $\beta = 10$:

$$(n_s, r) = \begin{cases} (0.967, 0.132) & (\beta = 10^{-5}), \\ (0.975, 0.0666) & (\beta = 10), \end{cases} \tag{5.54}$$

where we have shown the values at $N = 60$, N being the number of e-foldings at the horizon exit.

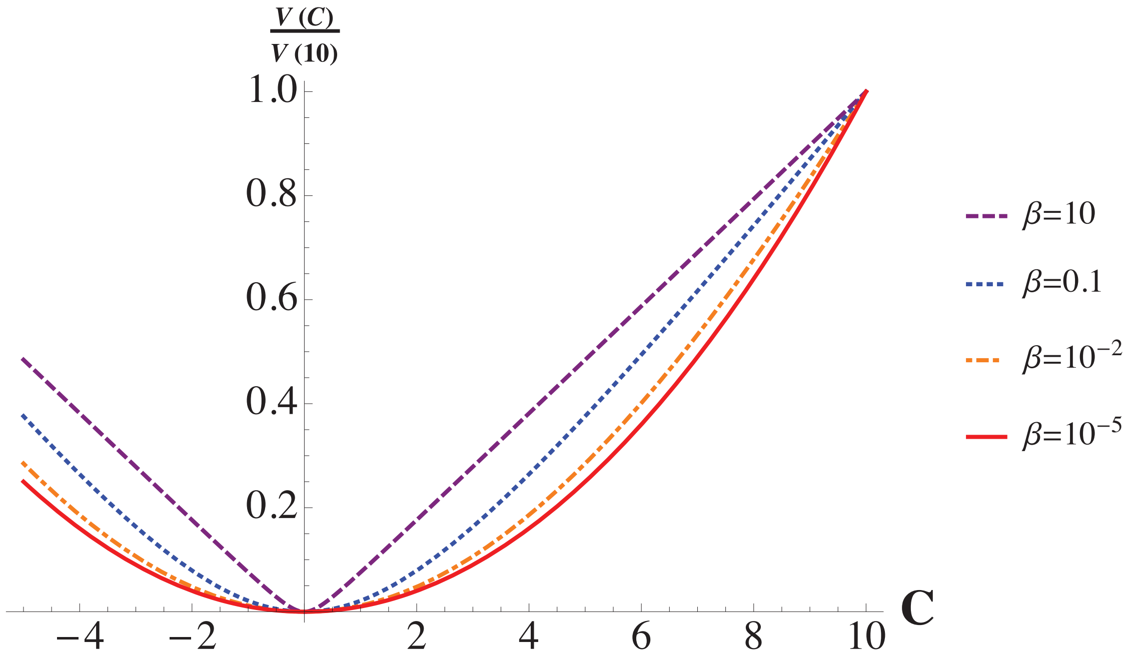

Figure 5.1: The forms of the scalar potential (5.53) with different values of β are shown. The potential is normalized at $C = 10$.

The second example is the Starobinsky type model discussed in Sec. 3.3.2, where $J = -\frac{3}{2}[\ln(-\frac{1}{3}Ce^C)]$. In this case, the kinetic coefficient is $J'' = -\frac{3}{2C^2}$, and the canonically normalized inflaton is $\phi = \sqrt{\frac{3}{2}}\ln(-C)$. In terms of ϕ, the scalar potential can be written as

$$\tilde{V}=\frac{g^2}{\beta}\left[\sqrt{1+\frac{9\beta}{4}(1-e^{-\sqrt{\frac{2}{3}}\phi})^2}-1\right]. \tag{5.55}$$

We have to be aware that this is *not* the DBI extension of the Starobinsky model in the new minimal SUGRA. We will discuss it in the next section. As in the model with $J=\frac{1}{2}C^2$, we show the form of the potential (5.55) in Fig. 5.2. The cosmological parameters in this

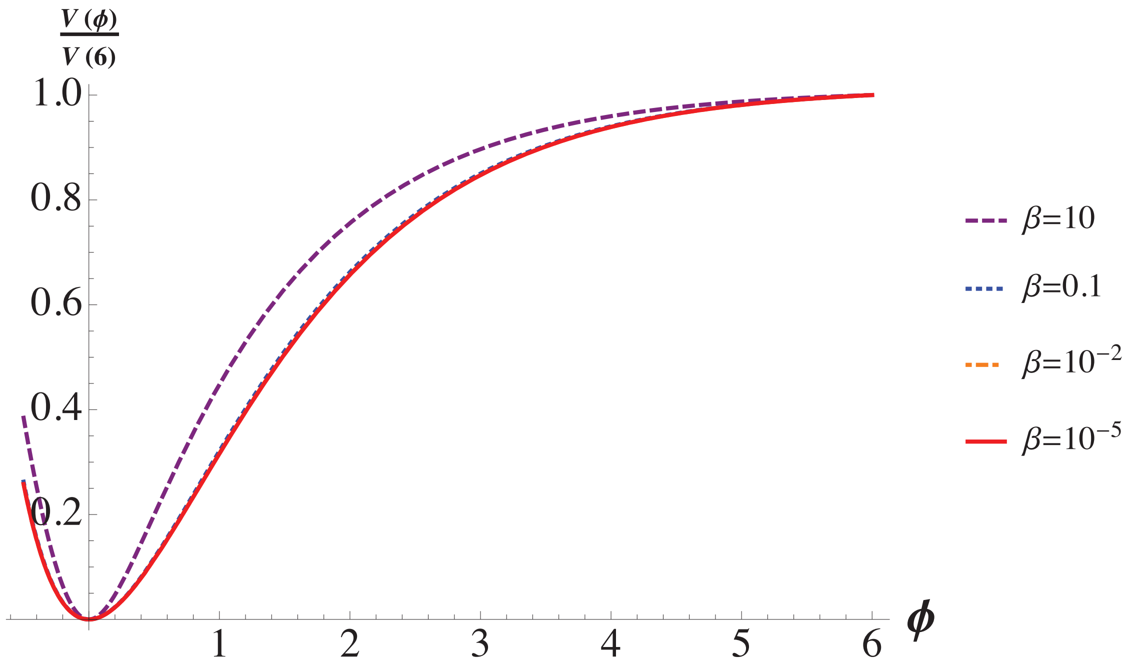

Figure 5.2: The forms of the scalar potential (5.55) with different values of β are shown. The potential is normalized at $\phi=6$.

model are also affected by the deformation. However, as read from the Fig. 5.2, the effect is less significant than one in the previous case:

$$(n_s,r)=\begin{cases}(0.968,0.00296) & (\beta=10^{-5}),\\ (0.968,0.00280) & (\beta=10),\end{cases} \tag{5.56}$$

where these are the values at $N=60$. This is because the leading terms of the potential are almost the same in both cases, which take the form $1-\alpha e^{-\sqrt{\frac{2}{3}}\phi}$ with different values of α.

In this section, we have discussed the DBI extension of the massive vector superfield inflation. With our specific choice of ω (5.45), the action takes a relatively simple form (5.46). Interestingly, the resultant potential of the inflaton always becomes flatter than the original one, and then, it makes the predicted tensor to scalar ratio smaller. This effect seems to be favored by the recent results of the CMB observation, such as Planck 2015 [133], since the smaller value of the tensor to scalar ratio looks better to fit into the data.

5.5 DBI-Starobinsky model in new minimal SUGRA

In this section, we consider another application of the DBI extension developed so far, that is, the DBI extension of the Starobinsky model in the new minimal SUGRA. We call it the DBI-Starobinsky model here. As we discussed in Sec. 3.3.2, the Starobinsky model in the new minimal SUGRA has the same structure as the massive vector inflation model. Then, it is natural to extend the model as a possible generalization of the higher order gravity action. It is worth recalling that any higher curvature extension R^n ($n \geq 3$) in old minimal SUGRA always has at least one ghost mode. However, as we discussed and will revalidate, the DBI type extension does not lead to such a ghost instability in spite of the presence of higher curvature terms.

Let us recall the R^2 action in the new minimal SUGRA

$$S = \left[\frac{3}{2}L_0 V_R\right]_D + [-h\mathcal{W}^2(V_R)]_F, \tag{5.57}$$

where h is a real constant. Since the second term looks like the field strength term of a "gauge" multiplet V_R, we can find the following extension,

$$S = \left[\frac{3}{2}L_0 V_R\right]_D + [-hX]_F + \left[\mathcal{W}^2(V_R) - \kappa\Sigma\left(\frac{X\bar{X}}{L_0^2}\right) - X\right]_F, \tag{5.58}$$

where X is a chiral multiplet with $(w, n) = (3, 3)$, and κ is a real constant. We need to remember that, with a real linear compensator, it is impossible to compensate the weights of chiral multiplets in the F-term formula. Therefore, the action above is the general form of the DBI-Starobinsky model without matter multiplets.

Under the superconformal gauge fixing conditions $L_0 = 1$, $b_\mu = 0$, the bosonic part of the action (5.58) becomes

$$S|_B = \int d^4xe\bigg[\frac{1}{2}R + \frac{3}{4}B_\mu B^\mu - \frac{3}{2}A_\mu^{(R)}B^\mu + 2\kappa\lambda|F^X|^2 - 2\lambda D_{(R)}^2 + \lambda F_{\mu\nu}^{(R)}F^{(R)\mu\nu} - i\chi F_{\mu\nu}^{(R)}\tilde{F}^{(R)\mu\nu} - \{(h+\Lambda)F^X + \text{h.c.}\}\bigg], \tag{5.59}$$

where $\lambda = \text{Re}\Lambda$, $\chi = \text{Im}\Lambda$, $A_\mu^{(R)}$ is a vector component of V_R, $D_{(R)} = \frac{1}{3}(R + \frac{3}{2}B_\mu B^\mu)$ and $F_{\mu\nu}^{(R)} = 2\partial_{[\mu}A_{\nu]}^{(R)}$. We can eliminate auxiliary fields other than B_μ by solving their E.O.M and obtain

$$S = \int d^4xe\left[\frac{1}{2}R + \frac{3}{4}B_\mu B^\mu - \frac{3}{2}A_\mu^{(R)}B^\mu - \frac{h}{\kappa}\{1 - \sqrt{4\kappa D_{(R)}^2 - \det(\eta_{ab} + \sqrt{\kappa}F_{ab}^{(R)})}\}\right]. \tag{5.60}$$

Since $D_{(R)}$ contains $B_\mu B^\mu$, it is difficult to solve the E.O.M of B_μ, which is a complicated equation with respect to B_μ. However, on the hypersurface $A_\mu^{(R)} = 0$,[3] we can obtain the simple solution $B_\mu = 0$ for the E.O.M of B_μ, since all the terms of B_μ become the function of $B_\mu B^\mu$ and the E.O.M of B_μ proportional to B_μ. Then, the action becomes a purely gravitational action given by

$$S = \int d^4xe\left[\frac{1}{2}R - \frac{h}{\kappa}\left\{1 - \sqrt{1 + \frac{4\kappa}{9}R^2}\right\}\right]. \tag{5.61}$$

This is a higher curvature action and it is known that this kind of action does not produce any ghost modes (see e.g. Ref. [106]).

To discuss the inflationary trajectory in this model, we consider the duality transformation of the action (5.58). Using a real linear Lagrange multiplier multiplet U, we rewrite the action (5.58) as

$$S = \left[\frac{3}{2}L_0V_R\right]_D + [-hX]_F + \left[\Lambda\left(\mathcal{W}^2(V) - \kappa\Sigma\left(\frac{X\bar{X}}{L_0^2}\right) - X\right)\right]_F + [U(V - V_R)]_D, \tag{5.62}$$

[3] We consider the hypersurface to integrate B_μ out otherwise it is difficult to do the integration. On the other hand, in the dual Lagrangian discussed below, such a difficulty does not exist and it is possible to integrate out all auxiliary fields in the dual one.

where V is a gauge multiplet. The second and the third terms describe the DBI action of V. The variation of U gives $V = V_R$ up to chiral multiplets, and it reproduces the original action (5.58). Here, we follow the procedure performed from Eq. (3.43) to Eq. (3.45). We can rewrite the action (5.62) as

$$S = \left[\frac{1}{4}S_0\bar{S}_0(\Phi+\bar{\Phi}+gV)e^{\frac{1}{2}(\Phi+\bar{\Phi}+gV)}\right]_D + \left[-\frac{g^2h}{4}X\right]_F + \left[\Lambda\left(\mathcal{W}^2(V) - \frac{9\kappa g^2}{4}\Sigma\left(\frac{X\bar{X}}{(S_0\bar{S}_0)^2 e^{(\Phi+\bar{\Phi}+gV)}}\right) - X\right)\right]_F, \tag{5.63}$$

where, in addition to the procedure, we have made the following redefinitions: $\Lambda \to \frac{4}{g^2}\Lambda$ and $X \to \frac{g^2}{4}X$. More familiar expression is derived by a further redefinition $X \to S_0^3 X$, and then, the action becomes

$$S = \left[\frac{1}{4}S_0\bar{S}_0(\Phi+\bar{\Phi}+gV)e^{\frac{1}{2}(\Phi+\bar{\Phi}+gV)}\right]_D + \left[-\frac{g^2h}{4}S_0^3X\right]_F + \left[\Lambda\left(\mathcal{W}^2(V) - S_0X\Sigma\left(\frac{9\kappa g^2}{4e^{(\Phi+\bar{\Phi}+gV)}}\bar{S}_0\bar{X}\right) - S_0^3X\right)\right]_F. \tag{5.64}$$

We can regard this action as a special case of that in Eq. (5.27) with the following choices of functions,

$$K(\Phi^I,\bar{\Phi}^{\bar{J}}) = -3\ln\left[-\frac{1}{6}(\Phi+\bar{\Phi}+gV)e^{\frac{1}{2}(\Phi+\bar{\Phi}+gV)}\right], \quad f(\Phi^I) = g^2h,$$
$$\omega(\Phi^I,\bar{\Phi}^{\bar{J}}) = \frac{9\kappa g^2}{4e^{(\Phi+\bar{\Phi}+gV)}}, \quad W(\Phi^I) = 0. \tag{5.65}$$

Taking them into account, we obtain the bosonic part of the action (5.64) in the unitary gauge $\mathrm{Im}\Lambda = 0$ as

$$S = \int d^4xe\left[\frac{1}{2}R - \frac{3}{4C^2}\partial_\mu C\partial^\mu C - \frac{3g^2}{4C^2}A_\mu A^\mu\right] + \int d^4x\frac{M^4}{C^2}\left(\sqrt{-g} - \sqrt{P}\sqrt{-\det\left(g_{\mu\nu} - \frac{C}{M^2}F_{\mu\nu}\right)}\right), \tag{5.66}$$

where $C = \mathrm{Re}\Phi$, we have chosen parameters h and κ as $h = \frac{1}{g^2}$, $\kappa = \frac{1}{g^2M^4}$, and

$$P = 1 + \frac{9g^2C^2}{M^4}\left(1+\frac{1}{C}\right)^2. \tag{5.67}$$

The canonically normalized inflaton ϕ is related to C as $C=-e^{\sqrt{\frac{2}{3}}\phi}$, and the potential term is

$$\begin{aligned} V =& M^4 e^{-2\sqrt{\frac{2}{3}}\phi}\left(\sqrt{1+\frac{9g^2e^{2\sqrt{\frac{2}{3}}\phi}}{M^4}(1-e^{-\sqrt{\frac{2}{3}}\phi})^2}-1\right) \\ =& \frac{g^2}{\beta}e^{-2\sqrt{\frac{2}{3}}\phi}\left(\sqrt{1+9\beta e^{2\sqrt{\frac{2}{3}}\phi}(1-e^{-\sqrt{\frac{2}{3}}\phi})^2}-1\right), \end{aligned} \tag{5.68}$$

where $\beta=\frac{g^2}{M^4}$. The forms of the potential are shown in Fig. 5.3.

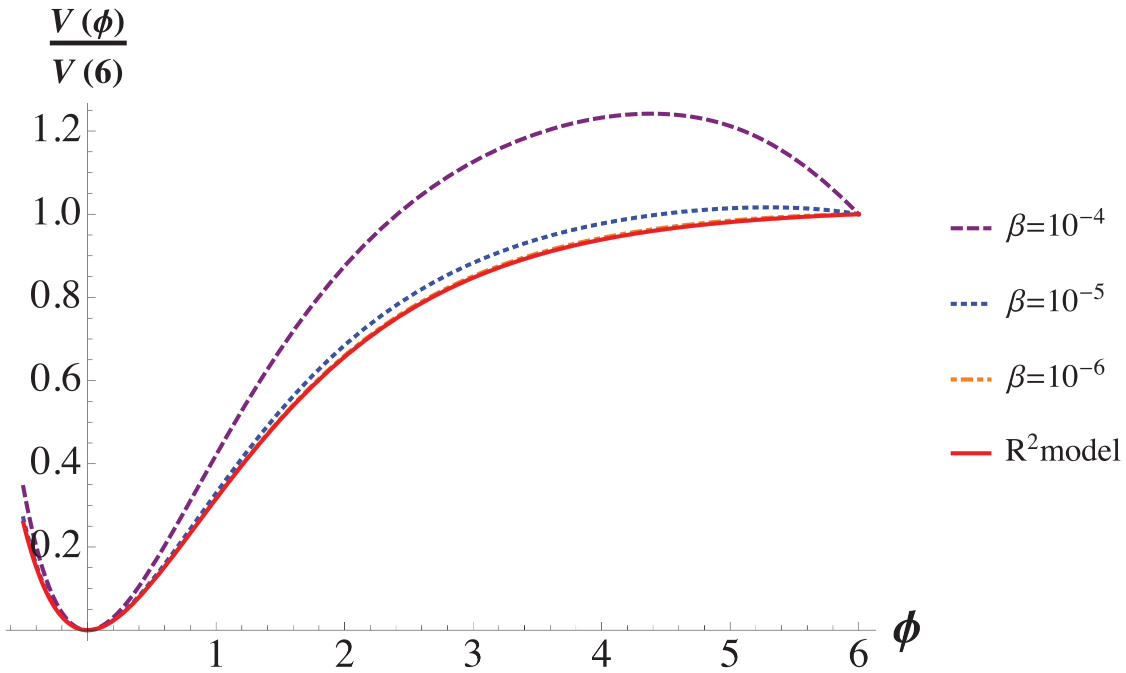

Figure 5.3: The forms of the scalar potential (5.68) with different values of β are shown. The potential is normalized at $\phi=6$. The red line shows the potential in the original Starobinsky model.

From Fig. 5.3, we find that the potential is highly sensitive to the higher order corrections. Although the DBI type correction makes the potential flatter in the previous models, the flatness is spoiled by the corrections in this model. The reason can be understood as follows: The plateau potential in the Starobinsky model is understood as the consequence of the conformal symmetry of R^2 term. However, when $\kappa R^2 \gg 1$, the second term in Eq. (5.61), behaves as $\frac{2f}{3\sqrt{\kappa}}R$, which does not have conformal symmetry. Therefore, when β becomes large, the potential with the DBI extension is not flat anymore.

Chapter 6

Summary and conclusions

We have discussed two classes of SUGRA models with SUSY higher-derivative terms. One is a model with chiral multiplets and the other with a massive vector multiplet. In both cases, the higher-derivative interactions can be related to D-branes in superstring.

In Chapter 4, we have discussed the ghost-free higher-derivative action of chiral multiplets. As shown in Sec. 4.2, we have embedded the terms, which are known in global SUSY and the old minimal SUGRA, into conformal SUGRA. We have found that if the **u**-associated derivatives are absent, the ghost-free term takes a universal form given in Eq. (4.20). In such a case, we can perform the duality transformation of compensators, by which we can convert the action in one SUGRA formulation to another. The SUSY higher-derivative term contains not only the higher-derivative interaction of the scalar but also some other contributions, the quartic term of the F-term, and the F-term dependent kinetic term of the scalar. In particular, we focus on the latter as a new contribution to the inflaton dynamics.

In Sec. 4.3, we have proposed an inflation model with such SUSY higher-derivative terms. The model is quite similar to the ordinary F-term chaotic inflation models. However, we have found that the shift symmetry of the inflaton superfield in Kähler potential is not required to realize inflation. Usually, such a situation leads to the η-problem as discussed in Sec. 3.1. However, the large field inflation occurs in our model. That is because the field variation of inflaton becomes small in view of the original field space, even if the canonically normalized inflaton takes a large field variation. This difference between the original and the normalized inflaton is caused by the F-term dependent kinetic term from the SUSY higher-derivative term. Such a nontrivial kinetic term has an interesting feature: The kinetic coefficient is proportional to the F-term potential and the effective potential of the inflaton φ takes a form $V \propto \varphi^{\frac{2n}{n+2}}$ where n is a positive integer. Therefore, our model predicts a very narrow region in the n_s-r plane read from CMB data, and it can be confirmed or excluded by the near future experiments.

We have also discussed the reason why the η-problem is absent by using the simplified action in Sec. 4.4. We have found that the action has a complex shift symmetry with respect to the inflaton superfield, in the case where the kinetic term is dominated by the SUSY higher-derivative term. In such a case, not only the inflaton but also its scalar superpartner become light during inflation, which may lead to the isocurvature perturbation or the non-Gaussianity of the primordial curvature perturbation. This is an interesting feature of our model, and we need to construct a more concrete model to discuss the thermal history of the universe after inflation.

We have developed the DBI action of gauge multiplets in Chapter 5. As we have reviewed in Sec. 5.1, the DBI action can be derived as a result of the partial breaking of $\mathcal{N} = 2$ SUSY. It can be done by imposing the nonlinear constraint on two $\mathcal{N} = 1$ superfields, of which an $\mathcal{N} = 2$ superfield consists. We have found that the underlying nilpotency of a superfield plays an important role to eliminate ghost modes. Then, the constraint in superspace seems to provide a good guiding principle for constructing the higher-derivative action. In Sec. 5.2, we have promoted the superspace constraint to that in conformal SUGRA and confirmed the correspondence between them. The superconformal version of the constraint should contain the compensator superfield, which implies the presence of the gravitational coupling between a gauge multiplet and the other multiplet in the Einstein-frame action. Such couplings prominently appear in the case with matter chiral multiplets. We have also discussed the other possible matter couplings by adding some functions to the DBI action, the gravity action, and the constraint. Our modifications preserve the underlying nilpotency, which ensures the absence of ghost modes. The resultant action takes nontrivial form and we have found that the D-term potential is also deformed by the DBI extension.

For an investigation of the effect of DBI extension, we have constructed the DBI extended massive vector multiplet action in Sec. 5.4. What we have found is that the scalar component of the multiplet is affected by the extension only in its scalar potential term. For a concrete discussion, we have chosen the simplest case where the DBI correction can be characterized by one parameter M. When the effect of corrections becomes relatively larger, the deformed scalar potential becomes flatter than the original one in general. It leads to the smaller tensor-to-scalar ratio r, which is favored by the latest CMB data since the upper bound of r is strongly constrained.

We have also constructed the DBI-Starobinsky model in the new minimal SUGRA in Sec. 5.5. That is a possible higher-curvature action in SUGRA without ghost modes. Such an extension has been done in the framework of the DBI extension since the Starobinsky model in the new minimal SUGRA is dual to a model with a massive vector multiplet. We have seen that the DBI-Starobinsky model action is uniquely determined and it is dual to the DBI extended action of a massive vector multiplet with a special choice of functions ω and K in Eq. (5.36). In the dual model, we have found that the inflaton potential loses its plateau due to the DBI-corrections, and then, successful inflation in the original case does not occur. The reason for it is related to the conformal symmetry of the term R^2. In the DBI-extended case, the higher curvature term appears in the form $\sqrt{1+\beta R^2}$, which does not have the conformal symmetry. The symmetry can be effectively restored if $\beta \ll 1$ and such a limit corresponds to the original Strarobinsky model.

From the observations in this thesis, we have found that SUSY higher-derivative terms can play important roles in the inflationary universe. Interestingly enough, the terms discussed in this thesis have a possibility to appear in the low energy effective theory of superstring. To make sure the possibility, we need to clarify the relation between our models and their UV completion in superstring. In particular, the effective action of D-branes seems to be the most important ingredient for it. Although a part of the action has been understood, more realistic situation, where the system would be more complicated, should be considered to construct models describing our universe. This requires further investigations from both the SUGRA and the string theoretical sides.

Acknowledgements

First of all, I would like to express my sincerest gratitude to my supervisor Hiroyuki Abe for educating me during my master and doctor courses, collaborations on various works, hearty encouragements, and invaluable discussions.

I am also very grateful to Yutaka Sakamura and Shuntaro Aoki for collaborations on the main parts of this thesis and related topics. Without their collaborations, this thesis would have never been completed.

I would like thank also for other collaborators, Hajime Otsuka, Fuminori Hasegawa, Takahiro Terada, Yuki Watanabe, and Jun'ichi Yokoyama for collaborations on related works, which are stimulating and helpful.

I am grateful to members of the my doctoral thesis committee, Taichiro Kugo, Hiromichi Nakazato, and Kei-ichi Maeda for careful reading of the manuscript and various comments, which are useful for improving this thesis.

I would like to thank all the members of Nakazato-Abe lab. at Waseda university for useful discussions. I express my sincere gratitude to Shuntaro Aoki and Fuminori Hasegawa for stimulating and interesting discussions on various topics. I have learned much from discussions with them.

I acknowledge the Japan Society for the Promotion of Science (JSPS) for fellowships. I have been supported by Research Fellowships for Young Scientists (No. 26-4236), which is from JSPS, since Apr. 2014.

Finally, I would like to thank my family for their support and hospitality.

Appendix A

Notation and some formulae

We use the notation and the convention in Ref. [38], which are briefly summarized below.

We take the natural unit convention $\hbar = c = 1$ where $\hbar$ is the Dirac constant and c is the speed of light. The Minkowski metric is $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$.

The γ-matrices satisfy the Clifford algebra $\{\gamma_a, \gamma_b\} = 2\eta_{ab}$ where $\{\cdot, \cdot\}$ denotes the anti-commutator. We sometimes use the higher-rank γ-matrices defined by

$$\gamma^{\mu_1 \cdots \mu_r} \equiv \gamma^{[\mu_1} \cdots \gamma^{\mu_r]} \tag{A.1}$$

where $[\cdots]$ denote the antisymmetrization of the indices with total weight 1. For example, $\gamma^{\mu\nu} = \frac{1}{2!}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$. The highest γ-matrix in 4D is $\gamma_{\mu_1 \cdots \mu_4}$, with which we can define γ_* as

$$\gamma_{\mu_1 \cdots \mu_4} = -i\epsilon_{\mu_1 \cdots \mu_4}\gamma_*, \tag{A.2}$$

where $\epsilon_{\mu_1 \cdots \mu_4}$ is the Levi-Civita antisymmetric tensor satisfying $\epsilon_{0123} = 1$. Since γ_* satisfies $\gamma_*^2 = 1$, we can define the chirality projection operator P_L and P_R as

$$P_L = \frac{1}{2}(1 + \gamma_*), \tag{A.3}$$

$$P_R = \frac{1}{2}(1 - \gamma_*). \tag{A.4}$$

The charge conjugation matrix C is defined so that $C^T = -C$, $(\gamma^\mu)^T = -C\gamma^\mu C^{-1}$ and C is a unitary matrix, where T denotes the transpose operation.

With C, we define the "Majorana" conjugation of a spinor ψ as

$$\bar{\psi} \equiv \psi^T C. \tag{A.5}$$

Then, one can find the following property called "*Majorana flip*" for any two spinors ψ_1 and ψ_2,

$$\bar{\psi}_1 \gamma_{\mu_1 \cdots \mu_r} \psi_2 = t_r \bar{\psi}_2 \gamma_{\mu_1 \cdots \mu_r} \psi_1, \tag{A.6}$$

where t_r takes values ± 1 depending on r, $t_r = 1$ for $r = 0, 3$ (mod4) and $t_r = -1$ for $r = 1, 2$ (mod4).

By using the set of γ-matrices $\{\Gamma^A\} = \{1, \gamma^\mu, \gamma^{\mu_1 \mu_2}, \gamma^{\mu_1 \mu_2 \mu_3}, \gamma^{\mu_1 \mu_2 \mu_3 \mu_4}\}$, we can derive the Fierz identity given by

$$(\bar{\psi}_1 \psi_2)(\bar{\psi}_3 \psi_4) = -\frac{1}{4} \sum_A (\bar{\psi}_1 \Gamma^A \psi_4)(\bar{\psi}_3 \Gamma_A \psi_2), \tag{A.7}$$

where the set Γ_A should be understood as $\{\Gamma_A = 1, \gamma_\mu, \gamma_{\mu_2 \mu_1}, \gamma_{\mu_3 \mu_2 \mu_1}, \gamma_{\mu_4 \mu_3 \mu_2 \mu_1}\}$ in which each component has indices with the reverse ordered compared to that in $\{\Gamma^A\}$.

Although we do not use the spinor index practically, we can formally introduce it as follows. For an index of a spinor ψ, we use a subscript α and denote it as ψ_α. The index can be raised by a charge conjugation $(C^T)_{\alpha\beta}$ as $\psi^\alpha = (C^T)^{\alpha\beta} \psi_\beta$. Then the Lorentz invariant combination $\bar{\psi}\psi$ can be expressed as $\bar{\psi}\psi = \psi^\alpha \psi_\alpha$. And also the γ-matrix γ_μ is expressed as $(\gamma_\mu)_\alpha^\beta$. The index of γ-matrices can also be lowered as $(\gamma_\mu)_{\alpha\beta} = (\gamma_\mu)_\alpha^\gamma (C^{-1})_{\gamma\beta}$.

Appendix B

Transformation law of a general multiplet

In this appendix, we show the transformation law of a general multiplet $\mathcal{C} = [C, \zeta, H, K, B_a, \lambda, D]$.

$$\begin{aligned} C : \delta_Q C &= \frac{i}{2}\bar{\epsilon}\gamma_*\zeta, \quad \delta_M C = 0, \quad \delta_D C = w\lambda_D C, \quad \delta_A C = in\theta C \\ &\delta_S C = 0, \quad \delta_K C = 0, \end{aligned} \tag{B.1}$$

$$\begin{aligned} \zeta : \delta_Q \zeta &= \frac{1}{2}(iH\gamma_* - K - \not{B} - i\gamma_*\not{D}C)\epsilon, \quad \delta_M\zeta = -\frac{1}{4}\lambda^{ab}\gamma_{ab}\zeta, \\ &\delta_D\zeta = \left(w + \frac{1}{2}\right)\lambda_D\zeta, \quad \delta_A\zeta = i\left(n - \frac{3}{2}\gamma_*\right)\theta\zeta, \\ &\delta_S\zeta = -i(w\gamma_* + n)\eta C, \quad \delta_K\zeta = 0, \end{aligned} \tag{B.2}$$

$$\begin{aligned} H : \delta_Q H &= \frac{i}{2}\bar{\epsilon}\gamma_*(\lambda + \not{D}\zeta), \quad \delta_M H = 0, \\ &\delta_D H = (w+1)\lambda_D H, \quad \delta_A H = i\theta(nH - 3iK), \\ &\delta_S H = \frac{i}{2}\bar{\eta}\{(w-2)\gamma_* + n\}\zeta, \quad \delta_K H = 0, \end{aligned} \tag{B.3}$$

$$\begin{aligned} K : \delta_Q K &= -\frac{1}{2}\bar{\epsilon}(\lambda + \not{D}\zeta), \quad \delta_M K = 0, \\ &\delta_D K = (w+1)\lambda_D K, \quad \delta_A K = i\theta(-nK + 3iH), \\ &\delta_S K = \frac{1}{2}\bar{\eta}\{(w-2) + n\gamma_*\}\zeta, \quad \delta_K K = 0, \end{aligned} \tag{B.4}$$

$$\begin{aligned} B_a : \delta_Q B_a &= -\frac{1}{2}\bar{\epsilon}(\gamma_a\lambda + D_a\zeta), \quad \delta_M B_a = -\lambda_a^b B_b, \\ &\delta_D B_a = (w+1)\lambda_D B_a, \quad \delta_A B_a = in\theta B_a, \\ &\delta_S B_a = \frac{1}{2}\bar{\eta}\{(w+1) + n\gamma_*\}\gamma_a\zeta, \quad \delta_K B_a = -2i\lambda_{Ka} nC, \end{aligned} \tag{B.5}$$

$$\begin{aligned}
\lambda : \delta_Q\lambda &= \left(\frac{i}{2}\gamma_* D + \frac{1}{4}\gamma^{ab}\hat{F}_{ab}\right)\epsilon, \quad \delta_M\lambda = -\frac{1}{4}\lambda^{ab}\gamma_{ab}\lambda, \\
\delta_D\lambda &= \left(w + \frac{3}{2}\right)\lambda_D\lambda, \quad \delta_A\lambda = i\theta\left(n + \frac{3}{2}\gamma_*\right)\lambda, \\
\delta_S\lambda &= \frac{1}{2}\gamma_*(-iH\gamma_* - K + \not{B} + i\gamma_*\not{D}C)(w\gamma_* + n)\eta, \\
\delta_K\lambda &= \lambda_K^a(w + n\gamma_*)\gamma_a\zeta, \qquad \text{(B.6)} \\
D : \delta_Q D &= \frac{i}{2}\bar{\epsilon}\gamma_*\not{D}\lambda, \quad \delta_M D = 0, \\
\delta_D D &= (w+2)\lambda_D D, \quad \delta_A D = in\theta D, \\
\delta_S D &= i\bar{\eta}(w\gamma_* + n)\left(\lambda + \frac{1}{2}\not{D}\zeta\right), \quad \delta_K D = 2\lambda_K^a(wD_aC + inB_a), \qquad \text{(B.7)}
\end{aligned}$$

where

$$\hat{F}_{ab} \equiv 2D_{[a}B_{b]} + \epsilon_{abcd}D^cD^dC. \qquad \text{(B.8)}$$

In a more general case, a lowest component C^I transforms under an internal symmetry as $C^I \to \theta^A k_A^I(C)$, where θ^A is a gauge transformation parameter. Then, we have to add the following terms to the above expressions:

$$\delta_Q B_a^I = \frac{i}{2}\bar{\epsilon}\gamma_a\gamma_*(\lambda^G)^A k_A^I(C), \qquad \text{(B.9)}$$

$$\delta_Q\lambda^I = \left[-\frac{1}{2}D^A k_A^I(C) + \frac{1}{4}((\bar{\lambda}^G)^A\gamma^a\zeta^J)\partial_J k_A^I\gamma_a + \frac{1}{4}((\bar{\lambda}^G)^A\gamma_*\gamma_a\zeta^J)\partial_J k_A^I\gamma_*\gamma_a\right]\epsilon, \qquad \text{(B.10)}$$

$$\delta_Q D^I = \frac{1}{2}\bar{\epsilon}\zeta^J\partial_J k_A^I D^A + \frac{i}{2}\bar{\epsilon}\gamma_*\not{B}^I\lambda^A\partial_J k_A^I - \frac{1}{2}\bar{\epsilon}\not{D}(k_A^I\lambda^A). \qquad \text{(B.11)}$$

Appendix C

Brief review of inflationary cosmology

In this appendix, we briefly review inflation in the early universe. The inflationary universe was proposed as a possible solution to the horizon, the flatness, and the monopole problems [13, 14, 15, 16, 17].[1] Inflation is the accelerated expansion of the universe. In particular, the slow-roll inflation models reviewed in the following predicts the scalar curvature perturbation, whose spectrum is almost scale independent. Such a prediction is nicely consistent with the CMB observation results today.

The slow-roll inflation models are realized with at least one bosonic field. In general, multiple scalar and vector fields can contribute to the dynamics, however, we focus on the model with a single scalar field. We consider the following system:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu - V(\phi)\right], \tag{C.1}$$

where ϕ is a real scalar field, and $V(\phi)$ is a real function of ϕ. We assume that the background metric is the flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric given by

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \tag{C.2}$$

where $a(t)$ is the so-called scale factor. At the leading order, we assume all the quantities are homogeneous with respect to space, and then, $\phi = \phi(t)$. The Einstein equation in

[1] We do not address how such problems can be solved in the inflationary models. For reviews, see e.g. [156].

this system can be rewritten as

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \tag{C.3}$$

$$\frac{\ddot{a}}{a} = \frac{1}{3}\left(V(\phi) - \dot{\phi}^2\right), \tag{C.4}$$

where $H \equiv \frac{\dot{a}}{a}$ and the dot denotes the time derivative. We notice that if $\dot{\phi}^2$ is negligible compared to $V(\phi)$, the acceleration rate of the scale factor $\frac{\ddot{a}}{a}$ can be positive. Therefore, with slowly varying ϕ, we can realize the accelerated expansion of the universe.

It is also known that, to solve the problems in the Big-Bang model, the inflationary era should be sufficiently long. Let us discuss the condition to achieve such a requirement. The E.O.M of the inflaton is given by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{C.5}$$

where prime denotes the derivative with respect to ϕ. $\dot{\phi}$ should be small during the time scale of the inflation, which is given by H^{-1}. Equivalently, $|\ddot{\phi} \times H^{-1}| \ll |\dot{\phi}|$, and under the condition, Eq. (C.5) is approximately rewritten as

$$\dot{\phi} \sim -\frac{V'}{3H}. \tag{C.6}$$

Then, the assumption $\dot{\phi}^2 \ll V$ is equivalent to

$$\epsilon \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1, \tag{C.7}$$

where ϵ is called the first slow-roll parameter. This is because $\frac{\dot{\phi}^2}{V} \sim \frac{2}{3}\epsilon$ where we have used the reduced E.O.M (C.6). We also find the following relation,

$$\ddot{\phi} \sim -\frac{V''}{3H}\dot{\phi} + \frac{V'}{3H^2}\dot{H}, \tag{C.8}$$

where we have differentiated both sides of Eq. (C.6). $\dot{H}$ is approximately given by $\frac{V'\dot{\phi}}{6H}$ from Eq. (C.3), and then, we find

$$\frac{\ddot{\phi}}{H\dot{\phi}} \sim -\frac{V''}{V} + \epsilon. \tag{C.9}$$

Therefore, the requirement $|\ddot{\phi} \times H^{-1}| \ll |\dot{\phi}|$ is equivalent to $|\eta| \ll 1$, where

$$\eta \equiv \frac{V''}{V}. \tag{C.10}$$

η is called the second slow-roll parameter. We define the end of inflation by the conditions $\epsilon \sim 1$ or $\eta \sim 1$. If one of the conditions is satisfied, we can not use the approximated dynamics, and the accelerated expansion ends soon after that.

The cosmological parameters, which we can observe by the cosmological observation experiments, are expressed by the slow-roll parameters. We do not review the detailed derivation of them, but the reason why the perturbation appears can be understood intuitively as follows: The field value of the inflaton quantum-mechanically fluctuates during inflation. Then, for each point in space, the value of the potential is also different due to the fluctuation. Such a difference of the potential causes the perturbation of spacetime metric. Detailed review of the cosmological perturbation can be found in Ref. [157]. We just show the cosmological observables here. The power spectrum of the scalar curvature perturbation $\mathcal{P}_\zeta(k)$ is given by

$$\mathcal{P}_\zeta(k) = \frac{1}{24\pi^2}\frac{V}{\epsilon}\bigg|_{k=aH}, \tag{C.11}$$

where $|_{k=aH}$ means that the value is evaluated when the corresponding mode k exits the horizon. The scale dependence of the power spectrum n_s is also important, which is given by

$$n_s - 1 = \frac{d\ln\mathcal{P}_\zeta}{d\ln k}\bigg|_{k=aH} = (2\eta - 6\epsilon)|_{k=aH}. \tag{C.12}$$

The other observable is the ratio between the power spectrum of the tensor and the scalar modes, which is called the tensor-to-scalar ratio r given by

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} = 16\epsilon|_{k=aH}, \tag{C.13}$$

where $\mathcal{P}_T$ is the power spectrum of the tensor perturbation. There are other cosmological observables, such as the running spectral index $\alpha = \frac{dn_s}{d\ln k}$, its running $\beta = \frac{d\alpha}{d\ln k}$, and the tensor spectral index $n_T = \frac{d\ln\mathcal{P}_T}{d\ln k}$, etc. We can also calculate them with the information of the scalar action at the horizon exit scale, but we do not consider them in this thesis. Such observables are expected to be important in future observations.

From the observables given above, we can read off the information of inflation at a specific "time" during inflation. The "time" depends on the detail of the cosmological history after the end of inflation. However, under reasonable assumptions, we can estimate the "time" as $N_* = 40$ to $N_* = 60$, where N_* is the so called *e-folding number* defined by $N = \int_{t_*}^{t_e} H dt$. t_e denotes the time when inflation ends and t_* denotes the time when the observed perturbation mode exited the horizon. It is practically useful to express N by the field value of ϕ at each time. That can be done by the following transformation,

$$N = \int_{t_*}^{t_e} H dt = \int_{\phi_*}^{\phi_e} d\phi \frac{dt}{d\phi} H = \int_{\phi_e}^{\phi_*} d\phi \frac{V}{V'}, \tag{C.14}$$

where we have used Eq. (C.6) in the third equality.

Appendix D

Components of $\mathcal{D}_\alpha \Phi^I$

D.1 Component expression of $\mathcal{D}_\alpha \Phi$

We give the components of a superconformal multiplet $\mathcal{D}_\alpha \Phi$. With a constant spinor ξ^α, the multiplet $\xi^\alpha \mathcal{D}_\alpha \Phi^I$ can be treated as a Lorentz scalar. Although, in Chapter 3, we consider the case where a chiral multiplet Φ^I is a singlet under all the internal gauge symmetries, we give here an expression in the case where Φ^I has gauge charges, for generality. We assume Φ^I transforms as $\Phi^I \to \theta^A k_A^I$ under internal symmetries. Then, the components of $\xi^\alpha \mathcal{D}_\alpha \Phi^I$ are summarized as follows:

$$\xi^\alpha \mathcal{D}_\alpha \Phi^I|_C = \bar{\xi} P_L \chi^I, \tag{D.1}$$

$$\xi^\alpha \mathcal{D}_\alpha \Phi^I|_\zeta = -\sqrt{2}(F^I + \not{D}\Phi^I) P_L \xi, \tag{D.2}$$

$$\xi^\alpha \mathcal{D}_\alpha \Phi^I|_H = \sqrt{2}\bar{\xi} P_L (\lambda^G)^A k_A^I, \tag{D.3}$$

$$\xi^\alpha \mathcal{D}_\alpha \Phi^I|_K = -\sqrt{2} i \bar{\xi} P_L (\lambda^G)^A k_A^I, \tag{D.4}$$

$$\xi^\alpha \mathcal{D}_\alpha \Phi^I|_{B_a} = i\bar{\xi}\gamma_{ab} D^b (P_L \chi^I) + \sqrt{2} i \bar{\xi} \gamma_a P_R (\lambda^G)^A k_A^I, \tag{D.5}$$

$$\begin{aligned} \xi^\alpha \mathcal{D}_\alpha \Phi^I|_\lambda =& \sqrt{2} i (\not{D} F^I + \Box \Phi + \gamma^{ab} D_a D_b \Phi^I) P_L \xi + \frac{i}{\sqrt{2}} P_L \gamma^{ab} \xi \hat{F}_{ab}^A k_A^I, \\ & + \sqrt{2} P_L \xi D^A k_A^I - 2i P_L \chi^J (\bar{\xi} P_L (\lambda^G)^A) \partial_J k_A^I, \end{aligned} \tag{D.6}$$

$$\begin{aligned} \xi^\alpha \mathcal{D}_\alpha \Phi^I|_D =& -\bar{\xi} \Box P_L \chi^I - i \bar{\xi} \gamma^{ab} P_L \chi^I \tilde{R}_{ab}(A) - \sqrt{2} \bar{\xi} P_L \not{D} \lambda^A k_A^I, \\ & + i \bar{\xi} P_L \chi^J D^A \partial_J k_A^I - \sqrt{2} F^J \bar{\xi} P_L (\lambda^G)^A \partial_J k_A^I, \end{aligned} \tag{D.7}$$

where $|_C$ stands for the C component in terms of a general multiplet (2.44) and so on.

Bibliography

[1] M. Ostrogradski: Mem. Ac. St. Petersbourg VI 4, 385 (1850).

[2] R. P. Woodard, Lect. Notes Phys. **720**, 403 (2007) [astro-ph/0601672].

[3] G. W. Horndeski, Int. J. Theor. Phys. **10**, 363 (1974).

[4] T. Kobayashi, M. Yamaguchi and J. Yokoyama, Prog. Theor. Phys. **126**, 511 (2011) doi:10.1143/PTP.126.511 [arXiv:1105.5723 [hep-th]].

[5] M. Koehn, J. L. Lehners and B. A. Ovrut, Phys. Rev. D **86**, 085019 (2012) doi:10.1103/PhysRevD.86.085019 [arXiv:1207.3798 [hep-th]].

[6] F. Farakos, C. Germani, A. Kehagias and E. N. Saridakis, JHEP **1205**, 050 (2012) doi:10.1007/JHEP05(2012)050 [arXiv:1202.3780 [hep-th]].

[7] M. Born and L. Infeld, Proc. Roy. Soc. Lond. A **144**, 425 (1934).

[8] P. A. M. Dirac, Proc. Roy. Soc. Lond. A **268**, 57 (1962).

[9] M. Kaku, P. K. Townsend and P. van Nieuwenhuizen, Phys. Rev. D **17**, 3179 (1978).

[10] M. Kaku and P. K. Townsend, Phys. Lett. B **76**, 54 (1978).

[11] P. K. Townsend and P. van Nieuwenhuizen, Phys. Rev. D **19**, 3166 (1979).

[12] T. Kugo and S. Uehara, Nucl. Phys. B **226**, 49 (1983).

[13] A. H. Guth, Phys. Rev. D **23**, 347 (1981).

[14] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980)

[15] K. Sato, Mon. Not. Roy. Astron. Soc. **195**, 467 (1981).

[16] A. D. Linde, Phys. Lett. B **108**, 389 (1982).

[17] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).

[18] A. D. Linde, Phys. Lett. B **129**, 177 (1983).

[19] M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. Lett. **85**, 3572 (2000) [hep-ph/0004243].

[20] R. Kallosh, A. Linde and T. Rube, Phys. Rev. D **83**, 043507 (2011) [arXiv:1011.5945 [hep-th]].

[21] S. Cecotti, Phys. Lett. B **190**, 86 (1987).

[22] R. Kallosh and A. Linde, JCAP **1306**, 028 (2013) [arXiv:1306.3214 [hep-th]].

[23] F. Farakos, A. Kehagias and A. Riotto, Nucl. Phys. B **876**, 187 (2013) [arXiv:1307.1137 [hep-th]].

[24] T. Kawano, Prog. Theor. Phys. **120**, 793 (2008) [arXiv:0712.2351 [hep-th]].

[25] S. Ferrara, R. Kallosh, A. Linde and M. Porrati, Phys. Rev. D **88**, no. 8, 085038 (2013) [arXiv:1307.7696 [hep-th]].

[26] J. Bagger and A. Galperin, Phys. Rev. D **55**, 1091 (1997) doi:10.1103/PhysRevD.55.1091 [hep-th/9608177].

[27] M. Rocek and A. A. Tseytlin, Phys. Rev. D **59**, 106001 (1999) doi:10.1103/PhysRevD.59.106001 [hep-th/9811232].

[28] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Nucl. Phys. B **212**, 413 (1983).

[29] S. Ferrara, L. Girardello, T. Kugo and A. Van Proeyen, Nucl. Phys. B **223**, 191 (1983).

[30] L. Randall and R. Sundrum, Nucl. Phys. B **557**, 79 (1999) [hep-th/9810155].

[31] G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP **9812**, 027 (1998) [hep-ph/9810442].

[32] J. A. Bagger, T. Moroi and E. Poppitz, JHEP **0004**, 009 (2000) [hep-th/9911029].

[33] C. Cheung, F. D'Eramo and J. Thaler, Phys. Rev. D **84**, 085012 (2011) [arXiv:1104.2598 [hep-ph]].

[34] T. Kugo and S. Uehara, Nucl. Phys. B **222**, 125 (1983).

[35] R. Kallosh, L. Kofman, A. D. Linde and A. Van Proeyen, Class. Quant. Grav. **17**, 4269 (2000) [Class. Quant. Grav. **21**, 5017 (2004)] [hep-th/0006179].

[36] S. Ferrara, R. Kallosh, A. Linde, A. Marrani and A. Van Proeyen, Phys. Rev. D **82**, 045003 (2010) [arXiv:1004.0712 [hep-th]].

[37] H. Abe, S. Aoki, F. Hasegawa and Y. Yamada, JHEP **1501**, 026 (2015) [arXiv:1408.4875 [hep-th]].

[38] D. Z. Freedman and A. Van Proeyen, 2012, "Supergravity", Cambridge University Press

[39] D. Butter, Annals Phys. **325**, 1026 (2010) [arXiv:0906.4399 [hep-th]].

[40] L. Castellani, P. Fre and P. van Nieuwenhuizen, Annals Phys. **136**, 398 (1981).

[41] T. Kugo and S. Uehara, Prog. Theor. Phys. **73**, 235 (1985).

[42] A. Van Proeyen, CERN-TH-3579, C83-02-14.

[43] R. Holman, P. Ramond and G. G. Ross, Phys. Lett. B **137**, 343 (1984).

[44] S. Kachru, R. Kallosh, A. D. Linde, J. M. Maldacena, L. P. McAllister and S. P. Trivedi, JCAP **0310**, 013 (2003) [hep-th/0308055].

[45] R. Allahverdi, K. Enqvist, J. Garcia-Bellido, A. Jokinen and A. Mazumdar, JCAP **0706**, 019 (2007) [hep-ph/0610134].

[46] A. D. Linde and A. Westphal, JCAP **0803**, 005 (2008) [arXiv:0712.1610 [hep-th]].

[47] M. Badziak and M. Olechowski, JCAP **0807**, 021 (2008) [arXiv:0802.1014 [hep-th]].

[48] J. P. Conlon, R. Kallosh, A. D. Linde and F. Quevedo, JCAP **0809**, 011 (2008) [arXiv:0806.0809 [hep-th]].

[49] M. Badziak and M. Olechowski, JCAP **0902**, 010 (2009) [arXiv:0810.4251 [hep-th]].

[50] Y. Yamada, JHEP **1307**, 039 (2013) [arXiv:1211.1455 [hep-ph]].

[51] M. Cicoli, F. Muia and F. G. Pedro, arXiv:1509.07748 [hep-th].

[52] M. Bolz, A. Brandenburg and W. Buchmuller, Nucl. Phys. B **606**, 518 (2001) [Nucl. Phys. B **790**, 336 (2008)] [hep-ph/0012052].

[53] J. Pradler and F. D. Steffen, Phys. Rev. D **75**, 023509 (2007) [hep-ph/0608344].

[54] J. Pradler and F. D. Steffen, Phys. Lett. B **648**, 224 (2007) [hep-ph/0612291].

[55] M. Kawasaki, F. Takahashi and T. T. Yanagida, Phys. Lett. B **638**, 8 (2006) [hep-ph/0603265].

[56] M. Kawasaki, F. Takahashi and T. T. Yanagida, Phys. Rev. D **74**, 043519 (2006) [hep-ph/0605297].

[57] M. Endo, M. Kawasaki, F. Takahashi and T. T. Yanagida, Phys. Lett. B **642**, 518 (2006) [hep-ph/0607170].

[58] K. Kohri, T. Moroi and A. Yotsuyanagi, Phys. Rev. D **73**, 123511 (2006) [hep-ph/0507245].

[59] M. Endo, K. Hamaguchi and F. Takahashi, Phys. Rev. D **74**, 023531 (2006) [hep-ph/0605091].

[60] M. Endo, F. Takahashi and T. T. Yanagida, Phys. Rev. D **76**, 083509 (2007) [arXiv:0706.0986 [hep-ph]].

[61] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross, Phys. Lett. B **131**, 59 (1983).

[62] B. de Carlos, J. A. Casas, F. Quevedo and E. Roulet, Phys. Lett. B **318**, 447 (1993) [hep-ph/9308325].

[63] T. Banks, D. B. Kaplan and A. E. Nelson, Phys. Rev. D **49**, 779 (1994) [hep-ph/9308292].

[64] M. Endo, K. Hamaguchi and F. Takahashi, Phys. Rev. Lett. **96**, 211301 (2006) [hep-ph/0602061].

[65] S. Nakamura and M. Yamaguchi, Phys. Lett. B **638**, 389 (2006) [hep-ph/0602081].

[66] T. Asaka, S. Nakamura and M. Yamaguchi, Phys. Rev. D **74**, 023520 (2006) [hep-ph/0604132].

[67] M. Dine, R. Kitano, A. Morisse and Y. Shirman, Phys. Rev. D **73**, 123518 (2006) [hep-ph/0604140].

[68] R. Kitano, Phys. Lett. B **641**, 203 (2006) [hep-ph/0607090].

[69] J. L. Evans, M. A. G. Garcia and K. A. Olive, JCAP **1403**, 022 (2014) [arXiv:1311.0052 [hep-ph]].

[70] R. Kallosh, A. Linde and A. Westphal, Phys. Rev. D **90**, no. 2, 023534 (2014) [arXiv:1405.0270 [hep-th]].

[71] A. B. Goncharov and A. D. Linde, Phys. Lett. B **139**, 27 (1984).

[72] R. Kallosh, A. Linde and D. Roest, JHEP **1311**, 198 (2013) [arXiv:1311.0472 [hep-th]].

[73] J. J. M. Carrasco, R. Kallosh, A. Linde and D. Roest, Phys. Rev. D **92**, no. 4, 041301 (2015) doi:10.1103/PhysRevD.92.041301 [arXiv:1504.05557 [hep-th]].

[74] D. Roest and M. Scalisi, Phys. Rev. D **92**, no. 4, 043525 (2015) [arXiv:1503.07909 [hep-th]].

[75] S. V. Ketov and T. Terada, Phys. Lett. B **736**, 272 (2014) [arXiv:1406.0252 [hep-th]].

[76] S. V. Ketov and T. Terada, JHEP **1412**, 062 (2014) [arXiv:1408.6524 [hep-th]].

[77] A. Linde, JCAP **1505**, no. 05, 003 (2015) [arXiv:1504.00663 [hep-th]].

[78] S. V. Ketov and T. Terada, arXiv:1509.00953 [hep-th].

[79] A. Hebecker, S. C. Kraus and L. T. Witkowski, Phys. Lett. B **737**, 16 (2014) [arXiv:1404.3711 [hep-th]].

[80] V. Demozzi, A. Linde and V. Mukhanov, JCAP **1104**, 013 (2011) [arXiv:1012.0549 [hep-th]].

[81] H. M. Lee, JCAP **1008**, 003 (2010) [arXiv:1005.2735 [hep-ph]].

[82] S. Ferrara, R. Kallosh, A. Linde, A. Marrani and A. Van Proeyen, Phys. Rev. D **83**, 025008 (2011) [arXiv:1008.2942 [hep-th]].

[83] D. V. Volkov and V. P. Akulov, Phys. Lett. B **46**, 109 (1973).

[84] S. Deser and B. Zumino, Phys. Rev. Lett. **38**, 1433 (1977).

[85] E. A. Ivanov and A. A. Kapustnikov, J. Phys. A **11**, 2375 (1978).

[86] M. Rocek, Phys. Rev. Lett. **41**, 451 (1978).

[87] U. Lindstrom and M. Rocek, Phys. Rev. D **19**, 2300 (1979).

[88] Z. Komargodski and N. Seiberg, JHEP **0909**, 066 (2009) [arXiv:0907.2441 [hep-th]].

[89] I. Antoniadis, E. Dudas, S. Ferrara and A. Sagnotti, Phys. Lett. B **733**, 32 (2014) [arXiv:1403.3269 [hep-th]].

[90] S. Ferrara, R. Kallosh and A. Linde, JHEP **1410**, 143 (2014) [arXiv:1408.4096 [hep-th]].

[91] R. Kallosh and A. Linde, JCAP **1501**, no. 01, 025 (2015) [arXiv:1408.5950 [hep-th]].

[92] S. Aoki and Y. Yamada, Phys. Rev. D **90**, no. 12, 127701 (2014) [arXiv:1409.4183 [hep-th]].

[93] G. Dall'Agata and F. Zwirner, JHEP **1412**, 172 (2014) [arXiv:1411.2605 [hep-th]].

[94] R. Kallosh, A. Linde and M. Scalisi, JHEP **1503**, 111 (2015) [arXiv:1411.5671 [hep-th]].

[95] R. Kallosh and T. Wrase, JHEP **1412**, 117 (2014) [arXiv:1411.1121 [hep-th]].

[96] E. A. Bergshoeff, K. Dasgupta, R. Kallosh, A. Van Proeyen and T. Wrase, JHEP **1505**, 058 (2015) [arXiv:1502.07627 [hep-th]].

[97] R. Kallosh, F. Quevedo and A. M. Uranga, arXiv:1507.07556 [hep-th].

[98] F. Farakos and A. Kehagias, Phys. Lett. B **724**, 322 (2013) [arXiv:1302.0866 [hep-th]].

[99] E. A. Bergshoeff, D. Z. Freedman, R. Kallosh and A. Van Proeyen, arXiv:1507.08264 [hep-th].

[100] F. Hasegawa and Y. Yamada, arXiv:1507.08619 [hep-th].

[101] R. Kallosh, arXiv:1509.02136 [hep-th].

[102] R. Kallosh and T. Wrase, arXiv:1509.02137 [hep-th].

[103] F. Farakos, A. Kehagias and K. Koutrolikos, Nucl. Phys. B **894**, 569 (2015) [arXiv:1501.07562 [hep-th]].

[104] B. Whitt, Phys. Lett. B **145**, 176 (1984).

[105] K. i. Maeda, Phys. Rev. D **37**, 858 (1988).

[106] K. i. Maeda, Phys. Rev. D **39**, 3159 (1989).

[107] F. Hasegawa and Y. Yamada, arXiv:1509.04987 [hep-th].

[108] E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti, JHEP **1509**, 217 (2015) [arXiv:1507.07842 [hep-th]].

[109] I. Antoniadis and C. Markou, arXiv:1508.06767 [hep-th].

[110] G. D. Diamandis, A. B. Lahanas and K. Tamvakis, arXiv:1509.01065 [hep-th].

[111] S. Cecotti and R. Kallosh, JHEP **1405**, 114 (2014) [arXiv:1403.2932 [hep-th]].

[112] S. Cecotti, S. Ferrara, M. Porrati and S. Sabharwal, Nucl. Phys. B **306**, 160 (1988).

[113] S. Mukhi, Phys. Rev. D **20**, 1839 (1979).

[114] A. Van Proeyen, Nucl. Phys. B **162**, 376 (1980).

[115] F. Farakos and R. von Unge, JHEP **1408**, 168 (2014) [arXiv:1404.3739 [hep-th]].

[116] M. B. Green and J. H. Schwarz, Phys. Lett. B **149**, 117 (1984).

[117] S. Aoki and Y. Yamada, JCAP **1507**, no. 07, 020 (2015) [arXiv:1504.07023 [hep-th]].

[118] S. Ohashi, N. Tanahashi, T. Kobayashi and M. Yamaguchi, JHEP **1507**, 008 (2015) [arXiv:1505.06029 [gr-qc]].

[119] J. Gleyzes, D. Langlois, F. Piazza and F. Vernizzi, Phys. Rev. Lett. **114**, no. 21, 211101 (2015) [arXiv:1404.6495 [hep-th]].

[120] J. Khoury, J. L. Lehners and B. Ovrut, Phys. Rev. D **83**, 125031 (2011) doi:10.1103/PhysRevD.83.125031 [arXiv:1012.3748 [hep-th]].

[121] J. Wess and J. Bagger. Supersymmetry and supergravity - 1992. Univ. Pr.. Princeton, USA: r. 259 p.

[122] J. Bagger and A. Galperin, Phys. Lett. B **412**, 296 (1997) [hep-th/9707061].

[123] F. Gonzalez-Rey, I. Y. Park and M. Rocek, Nucl. Phys. B **544**, 243 (1999) [hep-th/9811130].

[124] F. Farakos, C. Germani and A. Kehagias, JHEP **1311**, 045 (2013) [arXiv:1306.2961 [hep-th]].

[125] A. Nicolis, R. Rattazzi and E. Trincherini, Phys. Rev. D **79**, 064036 (2009) [arXiv:0811.2197 [hep-th]].

[126] M. Nitta and S. Sasaki, Phys. Rev. D **90**, no. 10, 105001 (2014) [arXiv:1406.7647 [hep-th]].

[127] M. Nitta and S. Sasaki, Phys. Rev. D **91**, 125025 (2015) [arXiv:1504.08123 [hep-th]].

[128] D. Ciupke, J. Louis and A. Westphal, arXiv:1505.03092 [hep-th].

[129] A. Pasquinucci and S. Penati, Int. J. Mod. Phys. A **4**, 1913 (1989).

[130] C. Germani and A. Kehagias, Phys. Rev. Lett. **105**, 011302 (2010) [arXiv:1003.2635 [hep-ph]].

[131] C. Germani and A. Kehagias, Phys. Rev. Lett. **106**, 161302 (2011) [arXiv:1012.0853 [hep-ph]].

[132] C. Germani and Y. Watanabe, JCAP **1107**, 031 (2011) [JCAP **1107**, A01 (2011)] [arXiv:1106.0502 [astro-ph.CO]].

[133] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1502.02114 [astro-ph.CO].

[134] H. Abe, Y. Sakamura and Y. Yamada, Phys. Rev. D **92**, no. 2, 025017 (2015) doi:10.1103/PhysRevD.92.025017 [arXiv:1504.01221 [hep-th]].

[135] H. Abe, Y. Sakamura and Y. Yamada, Phys. Rev. D **91**, no. 12, 125042 (2015) doi:10.1103/PhysRevD.91.125042 [arXiv:1505.02235 [hep-th]].

[136] S. Cecotti and S. Ferrara, Phys. Lett. B **187**, 335 (1987).

[137] S. Ferrara, M. Porrati and A. Sagnotti, JHEP **1412**, 065 (2014) [arXiv:1411.4954 [hep-th]].

[138] S. Ferrara, M. Porrati, A. Sagnotti, R. Stora and A. Yeranyan, Fortsch. Phys. **63**, 189 (2015) [arXiv:1412.3337 [hep-th]].

[139] S. Ferrara and A. Sagnotti, JHEP **1504**, 032 (2015) [arXiv:1502.01650 [hep-th]].

[140] S. V. Ketov, Mod. Phys. Lett. A **14**, 501 (1999) [hep-th/9809121].

[141] S. V. Ketov, Nucl. Phys. B **553**, 250 (1999) [hep-th/9812051].

[142] S. V. Ketov, Class. Quant. Grav. **17**, L91 (2000) [hep-th/0005126].

[143] S. Bellucci, E. Ivanov and S. Krivonos, Phys. Lett. B **502**, 279 (2001) [hep-th/0012236].

[144] S. Bellucci, E. Ivanov and S. Krivonos, Phys. Rev. D **64**, 025014 (2001) [hep-th/0101195].

[145] S. Bellucci, S. Krivonos, A. Shcherbakov and A. Sutulin, Phys. Lett. B **721**, 353 (2013) [arXiv:1212.1902 [hep-th]].

[146] A. Achucarro, J. M. Evans, P. K. Townsend and D. L. Wiltshire, Phys. Lett. B **198**, 441 (1987).

[147] P. S. Howe and E. Sezgin, Phys. Lett. B **390**, 133 (1997) [hep-th/9607227].

[148] M. Aganagic, C. Popescu and J. H. Schwarz, Phys. Lett. B **393**, 311 (1997) [hep-th/9610249].

[149] E. Bergshoeff and P. K. Townsend, Nucl. Phys. B **490**, 145 (1997) [hep-th/9611173].

[150] M. Aganagic, C. Popescu and J. H. Schwarz, Nucl. Phys. B **495**, 99 (1997) [hep-th/9612080].

[151] E. Bergshoeff, F. Coomans, R. Kallosh, C. S. Shahbazi and A. Van Proeyen, JHEP **1308**, 100 (2013) [arXiv:1303.5662 [hep-th]].

[152] A. A. Tseytlin, In *Shifman, M.A. (ed.): The many faces of the superworld* 417-452 [hep-th/9908105].

[153] S. M. Kuzenko and S. Theisen, JHEP **0003**, 034 (2000) [hep-th/0001068].

[154] S. M. Kuzenko and S. A. McCarthy, JHEP **0505**, 012 (2005) [hep-th/0501172].

[155] S. Ferrara, R. Kallosh, A. Linde and M. Porrati, JCAP **1311**, 046 (2013) [arXiv:1309.1085 [hep-th]].

[156] D. Baumann, arXiv:0907.5424 [hep-th].

[157] Y. Wang, Commun. Theor. Phys. **62**, 109 (2014) doi:10.1088/0253-6102/62/1/19 [arXiv:1303.1523 [hep-th]].

早稲田大学　博士（理学）学位申請 研究業績書

氏 名　山田　悠介　　　　　　　印

（2016　年　　2月　現在）

種 類 別	題名、　発表・発行掲載誌名、　発表・発行年月、　連名者（申請者含む）
論文	“SUSY flavor structure of generic 5D supergravity models”, European Physical Journal C 72 (2012) 2018, 2012 年 5 月発行, Hiroyuki Abe, Hajime Otsuka, Yutaka Sakamura, Yusuke Yamada
論文	“Instant uplifted inflation: A solution for a tension between inflation and SUSY breaking scale”, Journal of High Energy Physics 1307(2013)039, 2013 年 7 月発行, Yusuke Yamada
論文	“Impacts of non-geometric moduli on effective theory of 5D supergravity”, Journal of High Energy Physics 1311(2013)090, Yutaka Sakamura, Yusuke Yamada
論文	“Natural realization of a large extra dimension in 5D supergravity”, Progress of Theoretical and Experimental Physics 2014(2014)9 039B02,2014 年 9 月,Yutaka Sakamura, Yamada Yusuke
論文	“Inflation in supergravity without Kahler potential”,Physics Review D 90 127701,2014 年 12 月, Shuntaro Aoki, Yusuke Yamada
論文	“Illustrating SUSY breaking effects on various inflation mechanisms”, Journal of High Energy Physics 1501(2015)026,2015 年 1 月発行, Hiroyuki Abe, Shuntaro Aoki, Fuminori Hasegawa, Yusuke Yamada
論文	“Reheating processes after Starobinsky inflation in old-minimal supergravity”, Journal of High Energy Physics 1502(2015)105,2015 年 2 月発行, Takahiro Terada, Yuki Watanabe, Yusuke Yamada, Jun’ichi Yokoyama
論文	“N=1 superfield description of vector-tensor couplings in six dimensions”, Journal of High Energy Physics 1504(2015)035,2015 年 4 月発行, Hiroyuki Abe, Yutaka Sakamura, Yusuke Yamada
○論文	“Massive vector multiplet inflation with Dirac-Born-Infeld type action”, Physical Review D 91 125042,2015 年 6 月発行, Hiroyuki Abe, Yutaka Sakamura, Yusuke Yamada
○論文	“Impacts of supersymmetric higher deriveative terms on inflation models in supergravity”, Journal of Cosmology and Astroparticle Physics 1507(2015)020,2015 年 7 月発行,Shuntaro Aoki, Yusuke Yamada
○論文	“Matter coupled Dirac-Born-Infeld action in four dimensional N=1 conformal supergravity”,Physical Review D 92 025017,2015 年 7 月発行, Hiroyuki Abe, Yutaka Sakamura, Yusuke Yamada

早稲田大学　博士（理学）学位申請 研究業績書

種 類 別	題名、　発表・発行掲載誌名、　発表・発行年月、　連名者（申請者含む）
論文	“N=1 superfield description of six-dimensional supergravity”, Journal of High Energy Physics 1510(2015)181, 2015 年 11 月発行, Hiroyuki Abe, Yutaka Sakamura, Yusuke Yamada
論文	“Component action of nilpotent multiplet coupled to matter in 4 dimensional N=1 supergravity”, Journal of High Energy Physics 1510(2015)106, 2015 年 10 月発行, Fuminori Hasegawa, Yusuke Yamada
論文	“de Sitter vacuum from R^2 supergravity”,Physical Review D 92 105027, 2015 年 11 月発行, Fuminori Hasegawa, Yusuke Yamada
講演	Yutaka Sakamura and Yusuke Yamada, “LARGE volume scenario in 5D SUGRA”,Summer Institute 2013, Jirisan National Park Hanhwa Resort, 08/2013
講演	Yutaka Sakamura and Yusuke Yamada, “LARGE volume scenario in 5D SUGRA models”, PASCOS 2013, GIS international Convention Center,11/2013
講演	Hiroyuki Abe, Shuntaro Aoki, Fuminori Hasegawa and Yusuke Yamada, “Illustrating SUSY breaking effects on various inflation models”, PASCOS 2014, Warsaw university, 06/2014
講演	Shuntaro Aoki and Yusuke Yamada, “Inflation in supergravity without Kaehler potential”, B-mode cosmology, KEK, Japan, 02/2015
講演	安倍博之, 大塚啓, 阪村豊,山田悠介, “Effective theory of 5D SUGRA models with multi moduli”,日本物理学会第 67 回年次大会, 関西学院大学, 03/2012
講演	安倍博之, 大塚啓, 阪村豊,山田悠介, “Inflation in 5D SUGRA models with multi-moduli”,日本物理学会第 67 回秋季大会,京都産業大学, 09/2012
講演	安倍博之, 青木俊太朗,山田悠介,“Phenomenology after the instant uplifted inflation in 5D SUGRA models”,日本物理学会第 68 回年次大会,広島大学, 03/2013
講演	安倍博之, 阪村豊,山田悠介, “LARGE volume scenario in 5D SUGRA”,日本物理学会第 68 回秋季大会, 高知大学, 09/2013
講演	安倍博之,山田悠介, “Matter coupled minimal supergravity inflation”,日本物理学会第 69 回年次大会, 東海大学, 03/2014
講演	安倍博之, 青木俊太朗, 長谷川史憲, 山田悠介, “Illustrating SUSY breaking effects on various inflation models”, 基研研究会 素粒子物理学の進展 2014, 京都大学, 07/2014

早稲田大学　博士（理学）学位申請 研究業績書

種 類 別	題名、　発表・発行掲載誌名、　発表・発行年月、　連名者（申請者含む）
講演	青木俊太朗,山田悠介,“Cosmological application of higher derivative conformal supergravity”,日本物理学会第69回秋季大会,佐賀大学, 09/2014
講演	青木俊太朗,山田悠介, “Inflation in supergravity without Kahler potential”, Workshop on geometry,extra dimensions and string phenomenology in Miyazaki, ホリデイ イン リゾート宮崎, 11/2014
講演	安倍博之, 阪村豊,山田悠介, “Matter coupled DBI action in 4 dimensional N=1 SUGRA and its application to inflation”,日本物理学会第 70 回秋季大会, 大阪市立大学, 09/2015