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USING OCTUPOLES TO CREATE UNIFORM ELECTRON BEAM PRODUCED BY IRRADIATION ACCELERATORS

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Abstract

Uniform beams are widely used and essential in linear accelerator (linac) applications. In this paper, we proposed a method utilizing permanent quadrupole magnets and octupole magnets to uniform a Gaussian beam generated by linac. The principle of uniformization using octupole fields has been discussed in first section, the octupole strength and the beam distribution required for octupole field to uniformization are expressed using transport matrix which also shows the extent of the resultant uniform region. For an initial Gaussian beam distribution, we demonstrate the optical design that combines quadrupole and octupole fields to achieve one-dimensional beam uniformity. The particle-tracking simulations were also conducted to confirm the validity of the formulas, and we got a uniform region with 20 cm side length. A physical prototype is currently being fabricated, experimental results will be detailed in future work.

INTRODUCTION

In charged-particle beam applications, especially in radiotherapy and scientific research, a uniform radiation field is always required, which means the number of particles per unit area in the yield is consistent.To the best of our knowledge, two types of methods are widely employed to generate a uniform-irradiation field at present.

The most commonly used method is utilizing scatterers of specific geometric design and specific materials.Beams transport through different attenuation lengths of material, and its intensity could be weaken at various levels.As a matter of course, some particles will loss and adjustment of the field size is difficult. Another is *beam scanning technology* [1], which use a deflecting magnetic or electric field to rapidly scan a Gaussian beam over the sample. The uniformity of total dose depends on scanning frequency and possibly still does not meet requirement within tiny area. This approach increases the complexity and cost of system, and requires high synchronization of the electrical system.

The *nonlinear-focusing method* using multipole magnet to perform high-uniformity irradiation without lossing beam [2], base on this point, we propose a compact system that using permanent octupole magnets to uniformize the beam generated by linac. For beams with a Gaussian distribution, particles at different radial positions affected by varying focusing strength in the octupole field, result in beam uniformization. If the transverse size of the beam is smaller than the inner diameter of the octupole magnet, this method does not cause any beam loss, and the target size can be conveniently adjusted by repositioning the octupole magnet without altering the strength. Yosuke et al. used the beam's Twiss parameters to express the odd-order field strength and the size of the resulting uniform area, employing several quadrupoles to ensure the final uniform effect [3]. This work can be applied to the transport system of large accelerators, however, it is difficult to know the Twiss parameters of the beam in compact linac system, making it important to establish a simpler theoretical analysis system.

In this paper, we use the transfer matrix to theoretically analyze the uniformization effect of the octupole field, and the octupole field strength and uniform area size can be determined from the initial beam bunch phase space. We also conducted particle dynamics simulations, demonstrating that uniformization can be simply achieved using only two quadrupoles and two octupoles, validating the analytical formulas presented in the second section.

OPTICAL ANALYSIS AND DESIGN

The theory of particle dynamics in multipole fields is well-developed. Theoretically, all odd-order fields can be employed to uniform beams with Gaussian transverse profile. Here, we focus on the effects of octupole fields on beam distributions and we also discussed how to use octupole fields to achieve beam uniformity within a compact distance.

Beam Transformation in Octupole Field

First, taking the motion of a single particle as an example, suppose we use a nonlinear magnet that generate an ideal field. In the horizontal and vertical coordinates, x and y, the field can be expanded into a power series. From this, we can derive the particle's transverse coupled nonlinear equations of motion:

$$\begin{cases} x'' + K_4(s)x + \sum_{n=3}^{\infty} \frac{K_{2n}}{(n-1)!} \operatorname{Re}\left[(x+iy)^{n-1}\right] = 0, \\ y'' - K_4(s)y + \sum_{n=3}^{\infty} \frac{K_{2n}}{(n-1)!} \operatorname{Re}\left[i(x+iy)^{n-1}\right] = 0, \end{cases} \tag{1}$$

where K_4 is the strength of quadrupole and K_{2n} is the strength of 2n-pole nonlinear component of the multipole field. It is hard to solve Eq. (1), thus we expand the higher-order and only consider the octupole component:

$$\begin{cases} x'' + \frac{K_8}{3!}x^3\left[1 - 3\left(\frac{y}{x}\right)^2\right] = 0, \\ y'' + \frac{K_8}{3!}y^3\left[1 - 3\left(\frac{x}{y}\right)^2\right] = 0. \end{cases} \tag{2}$$

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as we can see, the Eq. (1) has been transformed to decouple the vertical motion from the horizontal one. It is not difficult to find out that we can care about only one direction separately without considering the other one when $|y/x| \ll 1$ or $|x/y| \ll 1$. In other words, the octupole magnet needs to be set at the position where the beam size in one direction is much larger than that in the other, and the chosen direction of beam can be uniformed.

Uniformization using Quadrupole and Octupole Field

When beam transport through a ideal quadrupole field, particles are defocused in one direction and focused in the other direction, forming a linear distribution after drifting the focal length which is exactly required for octupole field to uniformize. So we consider the transport system as shown in Fig. 1.

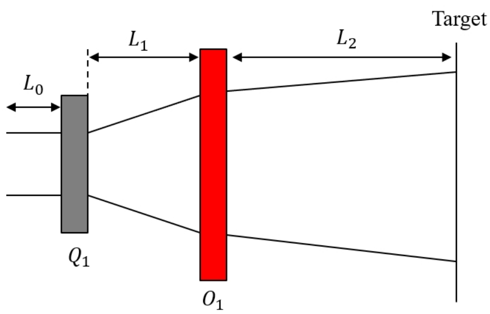

Figure 1: Q_1 and O_1 represent quadrupole and octupole magnet respectively. The focal length of quadrupole is f_1, and the strength of octupole field is K_8 where its gradient of field is r. The black line demonstrates the approximate envelope of beam.

At the entrance of the quadrupole magnet, the coordinate and momentum vectors (regardless of the z direction) of an arbitrary particle is written as $[x_0, p_{x0}, y_0, p_{y0}]$. The initial distribution is Gaussian,i.e., $\rho_0 = N/\left(\sqrt{2\pi}\sigma_0\right) \times \exp\left[-x_0^2/\left(2\sigma_0^2\right)\right]$, where ρ_0 is the initial particle density function and σ_0 is the root-mean-squared radius of the beam envelope. And at the target, the final distribution is uniform, i.e., $\rho_t = N/(r_t)$, where ρ_t is the particle density function at target and r_t is the length of uniform region. The beam loss in process assumed to be zero and the magnets are regarded as thin lens. At the entrance of octupole magnet, we have the new coordinate and momentum of this particle:

$$\begin{bmatrix} x_0 \\ p_{x0} - \frac{x_0}{f_1} \\ y_0 \\ p_{y0} + \frac{y_0}{f_1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{f_1} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f_1} & 1 \end{bmatrix} \times \begin{bmatrix} x_0 \\ p_{x0} \\ y_0 \\ p_{y0} \end{bmatrix} \quad (3)$$

As we proposed above, the position of octupole magnet L_1 need to satisfy the expression: $L_1 = x_0/\left(\frac{x_0}{f_1} - p_{x0}\right)$. At the entrance of octupole magnet, the x coordinate of all particles nearly equals to zero, and the y coordinate $y_1 = y_0 + L_1 \cdot \left(p_{y0} + \frac{y_0}{f_1}\right)$. For nonlinear field, matrix operation is not available, so we analyzed the momentum transformation by the method mentioned in Eq. 2. Note that, beams neglect the octupole field in the horizontal direction, after drifting length of L_2 we have:

$$\begin{bmatrix} x_2 \\ \frac{x_0}{f_1} - p_{x0} \\ y_2 \\ p_{y2} \end{bmatrix} = \begin{bmatrix} 1 & L_2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ \frac{x_0}{f_1} - p_{x0} \\ y_1 \\ p_{y2} \end{bmatrix} \quad (4)$$

where,

$$\begin{cases} x_2 = \left(\frac{x_0}{f_1} - p_{x0}\right) \cdot L_2, \\ p_{y2} = p_{y0} + \frac{y_0}{f1} - \frac{r}{6} \cdot y_1^3, \\ y_2 = y_1 + p_{y2} \cdot L_2. \end{cases} \quad (5)$$

Particles number N is constant in the process of transportation, on this premise, we have $\rho_0 dx_0 = \rho_t dx_t$. The last equation in Eq. 5 shows the relationship between y_2 and y_0, combined with the particle density functions equation, we have the solvable equations:

$$\begin{cases} \rho_2 = \rho_0 \cdot \left(\frac{dy_2}{dy_0}\right)^{-1} \\ y_2 = y_1 + \left(p_{y0} + \frac{y_0}{f_1} - \frac{r}{6} \cdot y_1^3\right) \cdot L_2 \\ y_1 = y_0 + L_1 \cdot \left(p_{y0} + \frac{y_0}{f_1}\right) \end{cases} \quad (6)$$

Substituting the initial distribution in y direction, i.e., Gaussian, and the size of irradiation region at the target into Eq. 6, we are definitely able to solve out the required strength of octupole and quadrupole magnet.

PARTICLE-TRACKING SIMULATION

The magnetic and charged-particle-tracking simulation were performed using multiphysics software COMSOL [4]. The whole transport system includes two Halbach quadrupole magnets and two Halbach octupole magnets to which using 16 pieces of permanent magnet in different remanence directions. And the tranverse distribution of initial beam is ideal Gaussian generating in the code COMSOL, meanwhile, we neglected the space charge force as it is far smaller compared with magnetic force. The beam source is set at position of 0mm in longitudinal direction, and the other parameters of magnets are shown in Table.1. The absolute value of remanence are all set to 1.41 T.

Table 1: Fundamental Parameters of Magnets

Magnets Number	Position [mm]	Inner Radius [mm]	Outer Radius [mm]	Length [mm]
Q1	13	7	10	10
O1	50	9	20	12
Q2	192	23	40	13
O2	260	58	90	25

We noticed that the inner diameter of the magnet gradually increases with longitudinal distance since we need to ensure

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as little particle loss as possible due to the beam defocusing of the quadrupole magnet. Figure 2 shows the shape and distribution of initial beam.

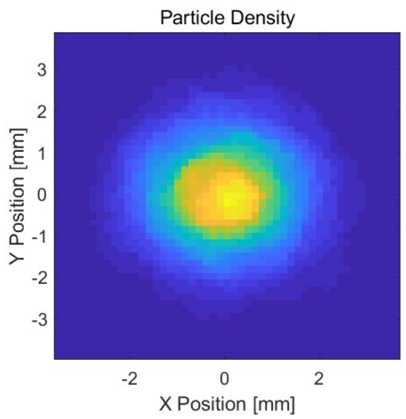
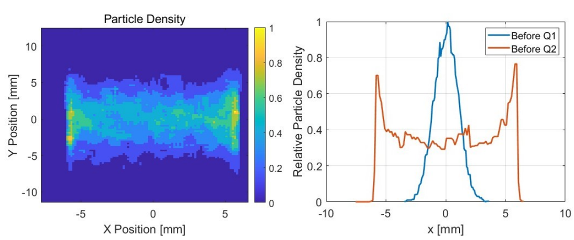


Figure 2: The pcolor image of initial profile of Gaussian beam.

The particle energy was set to 10 MeV, and the maximum transverse position is 2 mm. A screen was located at the entrance of the second quadrupole magnet to observe the real-time beam spot and evaluate the uniformization in one dimension as illustrated in Fig. 3. Due to a strong magnetic field near magnet, the edge part of the Gaussian beam is folded inside result in density peaks at edges of uniform region, shown in Fig. 3b.

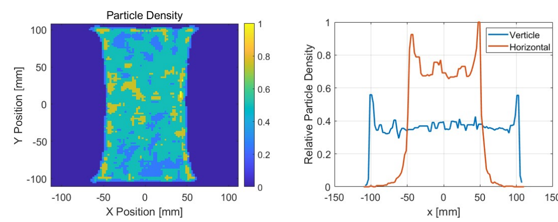


(a) (b)

Figure 3: (a) Beam profile at the entrance of Q2, (b) the density distribution in x direction before Q1 and before Q2 respectively

When the strength of octupole magnet increase, on the one hand, more particles are pushed in internal part and the edge peak rises wtihout affecting flat region.On the other hand, the width of uniform area reduces. These edge peaks can be eliminated using higher odd-order field, e.g. dodecapole [3].

The final distribution at the target is shown in Fig. 4. It seems that the irradiation is not so flat in uniform region and too much noise existing, since only 10,000 electrons were calculated. The width and length of final area are different as we using the parameters shown in Table 1. The uniform area can be optimized into a square by adjusting the strength and distance of the second set of magnets.



(a) (b)

Figure 4: (a) Beam profile at the target, (b) the density distribution in x direction and y direction respectively

CONCLUSION

In this paper, we proposed a method to homogenize beams within a compact distance by using two quadrupole magnets and two octupole magnets to transform the initial Gaussian distribution into a [10 cm × 20 cm] square region with two directions uniformed.This work could provide simpler solutions for the requirement of uniform beam area in future medical, industrial and security applications.

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