Calculation of Nuclear Reaction Matrix Elements

In our last article we reviewed recent progress in the problem of calculating the properties of finite nuclei using interactions consistent with information derived from the two-nucleon problem. The methods developed by Brueckner,[1] Bethe,[2] and Goldstone[3] provide a way of solving this problem with singular short-range nucleon-nucleon potentials, and the two-body "reaction matrix" introduced by these authors describes the interaction of two nucleons in the nuclear environment. One of the major practical difficulties in applying these methods is the solving of the equations for the reaction matrix. In this report we discuss new methods for solving the reaction-matrix equation which have been used recently. Many authors have contributed to these developments, but we will refer especially to recent work of C. W. Wong.[4] Köhler and McCarthy[5] have used a similar approach.

The two-body reaction matrix G_{12} is related to the two-body nucleon-nucleon potential v_{12} by the equation

$$G_{12} = v_{12} - v_{12}(Q/e_{12})G_{12}. \tag{1}$$

Here the quantity Q/e_{12} describes the propagation of two nucleons in the nuclear medium. The projection operator Q onto single-particle states which are unoccupied in the unperturbed ground state forbids scattering of the interacting nucleons into intermediate states which are already occupied. It represents the effect of the Pauli exclusion principle on the interaction of two nucleons in the nucleus. The energy denominator e_{12} gives the unperturbed energy of intermediate states relative to the initial state of the interacting nucleus. In many-body perturbation theory the two-body reaction matrix G_{12} plays the part of an effective nucleon-nucleon interaction, which is state-dependent because the unperturbed energies of the interacting nucleons occur in the energy denominator e_{12}. The perturbation series for various nuclear properties can be expressed in terms of the reaction matrix by summing classes of perturbation diagrams and, as the reaction matrix does not have excessive strength, the resulting series converges reasonably rapidly.

Until recently the main obstacle to applying perturbation methods in

calculating properties of finite nuclei has been the practical difficulty of solving the reaction-matrix equation. The usual methods for solving a two-body problem cannot be applied because the presence of the Pauli operator Q prevents the separation of relative and center-of-mass coordinates. Introducing the reference spectrum matrix of Bethe, Brandow, and Petschek[6] as an intermediate step overcomes this difficulty to a certain extent. The "reference matrix" is defined by the equation

$$G_{12}{}^R = v_{12} - v_{12}(1/e_{12}{}^R)G_{12}{}^R, \tag{2}$$

where $e_{12}{}^R = t_{\rm rel} + \gamma^2/m$, $t_{\rm rel}$ is the relative kinetic energy of the interacting nucleons (possibly with an effective mass), and γ^2 is a constant. The equation for G^R has a similar form to the reaction-matrix equation, but it can be solved more simply because there is no Pauli operator and because the energy denominator depends only on $t_{\rm rel}$. Equation (2) can be separated in relative and center-of-mass coordinates, and can be solved by reducing it to a differential equation in the relative coordinate which has the same form as the Schrödinger equation for nucleon-nucleon scattering (at negative energies) with an added inhomogeneous term.

The reference matrix is related to the reaction matrix by the equation

$$G = G^R + G^R(1/e^R - Q/e)G. \tag{3}$$

[This result may be obtained from Eqs. (1) and (2) by eliminating the potential v.] If the reference spectrum parameter γ is chosen carefully, then G^R is already a good approximation to G. This and related approximations have been used extensively for nuclear structure calculations (cf. Kuo and Brown[7]). When G^R is nearly equal to G, then Eq. (3) can be approximated by

$$G = G^R + \Delta G \approx G^R + G^R(1/e^R - Q/e)G^R. \tag{4}$$

If G is calculated from G^R using Eq. (3), then different choices of γ^2 should lead to different reference matrices, but to the same reaction matrix. Wong[4] has recently calculated the correction ΔG to the reference matrix using the approximate equation (4). He finds that the reference matrix G^R is quite sensitive to the choice of γ^2, but that the reaction matrix calculated from Eq. (4) changes by only a few per cent if γ^2 is varied over a wide range of values. This indicates that Eq. (4) is a very good approximation to the exact equation (3).

Wong gives two methods for calculating the correction term ΔG in Eq. (4), which differ in their treatment of the Pauli operator Q. The "global method" is essentially exact and is practicable for light nuclei. It involves dividing the correction into a part which does not contain the Pauli operator Q and a part which depends on $1 - Q$. The operator $(1 - Q)$ projects onto states in which at least one nucleon is in a single-particle

state which is occupied in the unperturbed ground state. The number of occupied single-particle states is small in a light nucleus, and the operator $1 - Q$ has a relatively small number of important matrix elements which may be enumerated and calculated explicitly. The second method makes a local density approximation by assuming that the effect of Q at a point in the nucleus is the same as in nuclear matter at the corresponding density. The local approximation can be used in both light and heavy nuclei. Comparison of result of the global and local approximations in light nuclei indicate that the local approximation is accurate to within about 5% for these nuclei. In heavy nuclei it should be better.

Recent calculations of the binding energies of light nuclei using the methods discussed above[4,5] give values which are somewhat too small. This is not surprising since similar calculations on the easier problem of nuclear matter also give low binding energies.[8] Meanwhile, further progress has been made in the nuclear-matter problem by (a) re-analyzing the two-body scattering data in terms of potentials which are assumed static, but dependent on angular momentum and spin,[9] (b) replacing the hard core by a "soft" core, i.e. a repulsive potential of Yukawa shape,[9] and (c) using a more accurate treatment of the corrections due to three-body correlations.[10] As a result the binding energy, equilibrium density, and compressibility of nuclear matter now seem in very satisfactory agreement with values derived from the empirical mass formula.[11] It is to be expected that the same improvements, when incorporated in the calculations for finite nuclei, will yield similarly good agreement.

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References

1. K. A. Brueckner, in *The Many-Body Problem,* edited by B. S. de Witt (Dunod, Paris, 1960) pp. 47–166.
2. H. A. Bethe, Phys. Rev. **103,** 1353 (1956).
3. J. Goldstone, Proc. Roy. Soc. (London) A**239,** 267 (1957).
4. C. W. Wong, preprint, Princeton Pac-937-215.
5. H. S. Köhler and R. J. McCarthy, Nucl. Phys. **86,** 611 (1966); H. S. Köhler, *ibid.* **88,** 529 (1966).
6. H. A. Bethe, B. H. Brandow, and A. G. Petschek, Phys. Rev. **129,** 225 (1963).
7. T. T. S. Kuo and G. E. Brown, Nucl. Phys. **85,** 40 (1966).
8. H. A. Bethe, Phys. Rev. **138,** B804 (1965).
9. R. V. Reid, Cornell thesis and other unpublished work, quoted in Ref. 11.
10. D. W. L. Sprung, P. C. Bhargava, and K. T. Dahlblom, Phys. Letters **21,** 538; **23,** 491 (1966).
11. H. A. Bethe, Lecture at American Physical Society meeting, New York, 1967 (unpublished).