

Mass-Radius Relation for White Dwarfs Models at Zero Temperature

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2016 J. Phys.: Conf. Ser. 706 052016

(<http://iopscience.iop.org/1742-6596/706/5/052016>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 131.169.4.70

This content was downloaded on 16/06/2016 at 22:54

Please note that [terms and conditions apply](#).

Mass-Radius Relation for White Dwarfs Models at Zero Temperature

G. A. Carvalho^{1†}; R. M. Marinho Jr¹ and M. Malheiro¹

¹Departamento de Física, Instituto Tecnológico de Aeronáutica, Vila das Acácias, São José dos Campos, SP, 12228-900, Brazil

E-mail: [†]geanderson.araujo.carvalho@gmail.com

Abstract. In this work we investigate the structure of WD stars using the Tolman-Oppenheimer-Volkoff equations and compare with the Newtonian equations of gravitation in order to put in evidence the importance of the General Relativity in the study of these stars. We solved the equations using the exact relativistic energy equation for the model of completely degenerate electron gas and we also use the polytropic EoS for ultra and non-relativistic limit. We find a good fit of the TOV solution with the general EoS for the WD mass-radius diagram. We propose that our fit has to be used as relation between mass and radius for general relativistic WD instead of that Newtonian $M \sim 1/R^3$, this fit is given by $M = R/(a+bR+cR^2+dR^3+kR^4)$, where a, b, c and d are parameters and $1/k$ is the constant of the Newtonian mass-radius relation and it can be used in simulation study of binary systems that occurs accretion.

1. Introduction

In this work we did a continuation of the previous work[3] then a lot of this paper can be found in the old one.

Since their discovery WD constitute a laboratory of tests in general relativity[15][17][10]. We decide to study the WD mass-radius relation taking into account general relativity[1]. The WD are formed of matter in high densities, on average 10^6 g/cm³. In case of the white dwarfs they are constituted essentially of degenerate electron gas[15]. The typical mass is $0.5 - 1 M_{\odot}$ (M_{\odot} represents the solar mass) and they have radii of the order of 10000 km.

2. Differential Equations

Basically we analyze three cases. In general in our accounts the configuration of the mass of the star is static and spherically symmetric, in this case the pressure and the density of the star are functions of only the radial coordinate r [1].

In the first case we use the Newtonian equations, then we have[15]

$$\frac{dp}{dr} = -\frac{Gm(r)\rho}{r^2} \quad (1a)$$

$$\frac{dm}{dr} = 4\pi r^2 \rho, \quad (1b)$$



where p denotes pressure, G is the gravitational constant, ρ is the mass density, and m represents the mass inside of the radius r . We can improve the Newtonian case if we replace mass density by energy density and add the contribution of the electron energy density to the energy density of the star and write

$$\frac{dp}{dr} = -\frac{Gm(\rho c^2 + e_e)}{r^2 c^2} \quad (2a)$$

$$\frac{dm}{dr} = \frac{4\pi r^2(\rho c^2 + e_e)}{c^2}, \quad (2b)$$

where e_e is electron energy density and c is the speed of light. We call this as the improved Newtonian case. If taking into account the general relativistic effects, the structure of the star in hydrostatic equilibrium for a static and isotropic ideal fluid sphere is described by the TOV[18][13]

$$\frac{dp}{dr} = -\frac{Gm\epsilon}{c^2 r^2} \left[1 + \frac{p}{\epsilon}\right] \left[1 + \frac{4\pi r^3 p}{m c^2}\right] \left[1 - \frac{2Gm}{c^2 r}\right]^{-1} \quad (3a)$$

$$\frac{dm}{dr} = \frac{4\pi r^2 \epsilon}{c^2}, \quad (3b)$$

where ϵ represents the energy density. The new three terms in the equilibrium Eq.(3a) are corrections terms.

3. Equation of State EoS

To solve numerically the coupled differential equations in both cases, general relativistic and Newtonian, we must have an EoS. We use the well known politropic EoS and we use the EoS for completely degenerate electron gas given by[4]

$$p = \frac{m_e c^2}{\lambda_e^3} \frac{1}{24\pi^2} \left[(2x^3 - 3x)(1 + x^2)^{1/2} + 3 \sinh^{-1}(x) \right], \quad (4)$$

where x is a dimensionless variable $x = P_f/m_e c$ (P_f is the Fermi moment and m_e is the electron mass) and λ_e is the Compton wavelength of the electron. In terms of the parameter x the density is given by

$$\rho = 9.738 \times 10^5 \mu_e x^3 \text{ g/cm}^3, \quad (5)$$

μ_e is the ratio between the mass number and atomic number. If we assume that the white dwarf is made of ${}^4\text{He}$, ${}^{12}\text{C}$ or ${}^{16}\text{O}$, hence $\mu_e = 2$.

4. Mass-radius relation

By solving the Eq.(1) and Eq.(3) with the ultra-relativistic we see that in both solutions the stars are unstable due to secular instability $\partial M/\partial \rho_c < 0$ and the non-relativistic limit for EoS is good only below $0.25M_\odot$ (fig.(1)). We also solved the set of Eq.(1), (2) and (3) with the EoS (4).

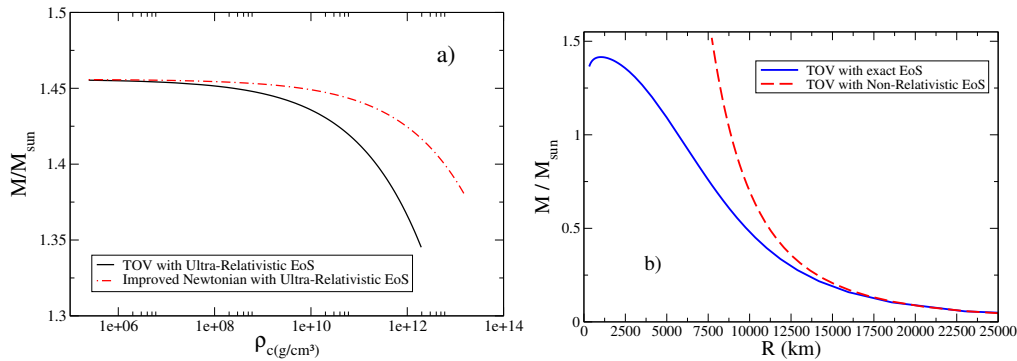


Figure 1. a) TOV and Improved Newtonian case using politropic equation with index $n = 3$
 b) TOV with EoS (4) and politropic with index $n = 1.5$.

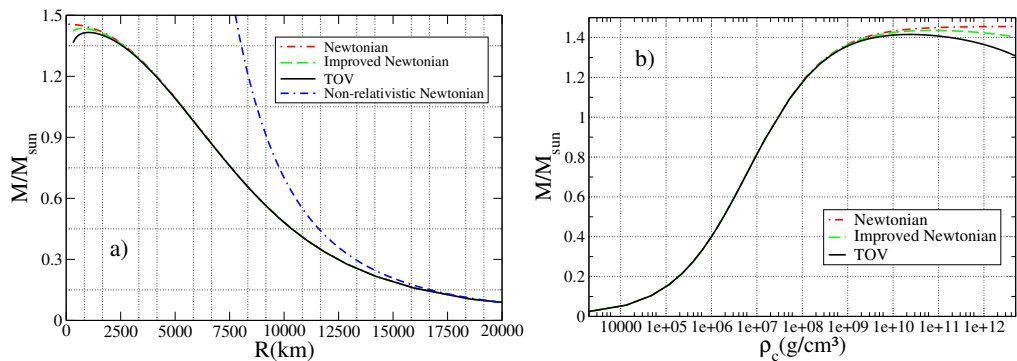


Figure 2. a) Mass-radius relation for general relativistic and Newtonian ODE and b) Mass-Central density relation for the same models.

In Fig.(2) we show the numerical results for mass-radius relation and mass-density relation. The purely Newtonian Eq.(1), i.e., without special relativity corrections don't have the secular instability $\partial M/\partial R > 0$. While TOV and even "Improved Newtonian" case presents instability, when pass from stability to instability we compute the mass and the correspondent radius (tab.(1)).

The differences between the maximum masses are small, however the minimum radii are different.

Table 1. Maximum mass for the models of white dwarfs stars.

Model	Mass	Radius [km]
Newtonian $P = P(\rho)$	$1.4546 M_{\odot}$	329
Improved Newtonian $P = P(\epsilon)$	$1.4358 M_{\odot}$	739
TOV $P = P(\epsilon)$	$1.4154 M_{\odot}$	1021
Non-relativistic Newtonian $P = K\rho^{5/3}$	$1.4564 M_{\odot}$	7833

As we can see in tab.(1) the "Newtonian" minimum radius is about three times larger than "TOV" one.

We use the expression below to fit the TOV curve in fig.(2).

$$\frac{M}{M_{\odot}} = \frac{R}{a + bR + cR^2 + dR^3 + kR^4}. \quad (6)$$

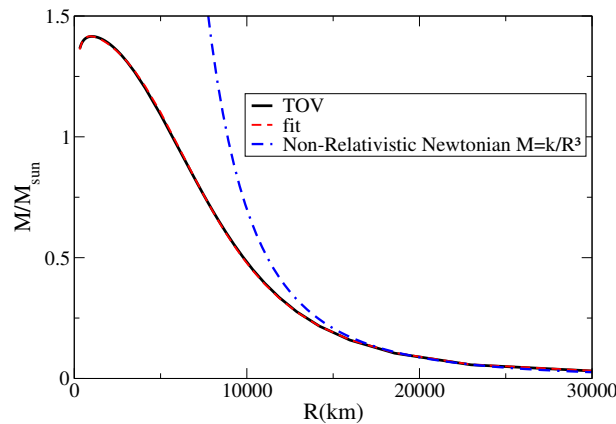


Figure 3. General relativistic mass-radius diagram fit.

The constants a , b , c and d in TOV case using $\mu_e = 2$ (Fig. (3)) are

$$\begin{aligned} a &= 20.86 \text{ km} \\ b &= 0.66 \\ c &= 2.48 \times 10^{-5} \text{ km}^{-1} \\ d &= -2.43 \times 10^{-5} \text{ km}^{-2}. \end{aligned} \quad (7)$$

the Eq.(6) with above constants is a better expression that we found in[3] to calculate the mass in function of the radius considering GR effects in static case.

5. Concluding Remarks

We show that the general relativity can be relevant to mass-radius relation of WD. Eq.(6) can be applied in another formulas like magnetic dipole field, gravitational potential[4][9] and surface gravity. We shown also that is better to use the exact relativistic EoS.

6. Acknowledgments

I would like to thank CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior) for the financial support.

References

- [1] ADLER, R., BAZIN, M., AND SCHIFFER, M. *Introduction to general relativity*. McGraw-Hill, 1965.
- [2] ALTHAUS, L., CRSICO, A., ISERN, J., AND GARCA-BERRO, E. Evolutionary and pulsational properties of white dwarf stars. *The Astronomy and Astrophysics Review* 18 (2010), 471–566.
- [3] CARVALHO, G. A., MARINHO JR, R. M., AND MALHEIRO, M. Mass-radius diagram for compact stars. *Journal of Physics: Conference Series* 630 (2015), 012058.
- [4] CHANDRASEKHAR, S. *An Introduction to the Study of Stellar Structure*. Dover Publications, 1957.

- [5] HAMADA, T., AND SALPETER, E. E. Models for zero-temperature stars. *The Astrophysical Journal* *134* (1961), 683.
- [6] HOWELL, D. A., SULLIVAN, M., NUGENT, P. E., ELLIS, R. S., CONLEY, A. J., LE BORGNE, D., CARLBERG, R. G., GUY, J., BALAM, D., BASA, S., FOUCHEZ, D., HOOK, I. M., HSIAO, E. Y., NEILL, J. D., PAIN, R., PERRETT, K. M., AND PRITCHET, C. J. The type ia supernova snls-03d3bb from a super-chandrasekhar-mass white dwarf star. *Nature* *443* (2006), 308–311.
- [7] JACKSON, C. B., TARUNA, J., POULIOT, S. L., ELLISON, B. W., LEE, D. D., AND PIEKAREWICZ, J. Compact objects for everyone: I. white dwarf stars. *European Journal of Physics* *26*, 5 (2005), 695.
- [8] KEPLER, S. O., KLEINMAN, S. J., NITTA, A., KOESTER, D., CASTANHEIRA, B. G., GIOVANNINI, O., COSTA, A. F. M., AND ALTHAUS, L. White dwarf mass distribution in the sdss. *Monthly Notices of the Royal Astronomical Society* *375* (2007).
- [9] KEPLER, S. O., AND SARAIVA, M. F. O. *Astronomia e Astrofísica*. Departamento de Astronomia, Instituto de Física, Universidade Federal do Rio Grande do Sul, 2012.
- [10] LOBATO, R. V., COELHO, J., AND MALHEIRO, M. Particle acceleration and radio emission for sgrs/axps as white dwarf pulsars. *Journal of Physics: Conference Series* *630* (2015), 012015.
- [11] MARSH, T. R., NELEMANS, G., AND STEEGHS, D. Mass transfer between double white dwarfs. *Monthly Notices of the Royal Astronomical Society* *350* (2004), 113–128.
- [12] MATHEW, A., AND NANDY, M. K. General relativistic calculations for white dwarf stars. *ArXiv e-prints* (2014).
- [13] OPPENHEIMER, J. R., AND VOLKOFF, G. M. On massive neutron cores. *Physical Review* *55* (1939), 374–381.
- [14] POSTNOV, K. A., AND YUNGELSON, L. R. The evolution of compact binary star systems. *Living Reviews in Relativity* *17* (2014), 1–166.
- [15] SAGERT, I., HEMPEL, M., GREINER, C., AND SCHAFFNER-BIELICH, J. Compact stars for undergraduates. *European Journal of Physics* *27* (2006), 577.
- [16] SHAPIRO, S., AND TEUKOLSKY, S. *Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects*. 2008.
- [17] SILBAR, R. R., AND REDDY, S. Neutron stars for undergraduates. *American Journal of Physics* *72*, 7 (2004), 892–905.
- [18] TOLMAN, R. C. Static solutions of einstein’s field equations for spheres of fluid. *Physical Review* *55* (1939), 364–373.