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PROTON POLARIZATION IN RHIC WITH PARTIAL SIBERIAN SNAKES

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Abstract

In December 2021, damage to a couple of RHIC power supplies forced one of two Siberian Snakes in the Blue ring to operate as a partial Siberian Snake and with a different snake axis of rotation. The time-averaged polarization for that run actually ended up higher than in the Yellow ring, after casting the undamaged snake as a partial snake as well. In this work, we simulate polarization transmission through a series of increasingly realistic models of the Blue ring in the "dangerous region" of polarization loss. At first the bare lattice has a perfect closed-orbit and ideal magnet strengths. Then the measured magnet-to-magnet field strength variations were added to the lattice. Finally, the six Interaction Region 5mm closed orbit bumps were implemented. Each of these model lattices compared the use of a pair of partial snakes against a pair of a full snakes, and in simulations with realistic emittances, realistic polarization losses were not reproducible without inclusion of nonzero RMS lattice misalignments.

POLARIZED PROTON ACCELERATION

Isolated Resonance Model

In the limit of widely-separated spin-orbit resonances, the usual approach of calculating dynamical spin polarization is to treat the resonances as independent. Considering each resonance independently amounts to applying the Froissart-Stora equation sequentially to find the final polarization after crossing N resonances:

$$P_f = \prod_{i=1}^{N} \left(2\,\exp\left(-\frac{\pi}{2} \frac{\epsilon_i^2}{2\alpha} \right) - 1 \right) P_0$$

Each factor in the above equation gives the asymptotic polarization ratio after crossing a single resonance that satisfies the condition $\nu_0 \equiv Q$ mod1. In a flat ring without solenoids or Siberian snakes, we know $\nu_0 = G\gamma$, where $g \equiv 2 + 2G$ is the anomalous magnetic moment of the particle. This condition is satisfied twice for each integer of $G\gamma$.

Siberian Snakes

Using an even number of Siberian Snakes with appropriately chosen rotation axes fixes the closed-orbit spin tune to $1/2$. This opens the possibility of avoiding all first-order intrinsic resonances for appropriately chosen (sufficiently irrational) orbital tunes. This is indeed the present situation in RHIC, with the working point chosen to be $(28.695, 29.685)$.

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SPIN MOTION IN A MODEL OF RHIC

Closed Orbit Spin Dynamics

In the absence of Siberian Snakes, the closed-orbit in an ideal RHIC lattice shows no imperfection resonances (those resonances found at $\nu_0 = G\gamma \in \mathbb{Z}$). In a numerical tracking calculation, the only remaining source of errors is machine precision which is order 10^{-16} for a double.

Similarly, intrinsic spin-orbit resonance contributions arising from coherence of spin precession with betatron-frequency field oscillations should be zero to the order of machine precision for a particle on the closed-orbit and polarization would be strongly conserved [1].

Linear Orbital Motion

Near the closed orbit, linear terms of the orbital Hamiltonian dictate the motion of the particles. This motion through quadrupole magnets gives rise to strong first-order intrinsic resonances which are proportional to the amplitude of the betatron oscillations at coherent betatron and spin precession frequencies. In the absence of Siberian Snakes, up to 40% polarization can be lost in one turn at the strongest of these resonances for a 1σ particle.

Higher-Order Spin Resonances

When using ideal Siberian Snakes, the closed-orbit spin tune ν_0 is fixed to $\frac{1}{2}$ independently of magnetic rigidity, due to coherent cancellation of spin perturbations in the arcs between each snake [1]. This immediately sidesteps all integer "imperfection" resonances, and simultaneously avoids intrinsic resonances for appropriately chosen betatron tunes.

Nevertheless, in the vicinity of strong intrinsic resonances, the time-averaged equilibrium polarization of a particle or beam can still be very small for reasonable amplitudes. In RHIC, this is particularly evident near the 2 strongest intrinsic resonances found at $G\gamma = 393 + Q_y$ and $G\gamma = 411 - Q_y$. This effect is due to the coherence of spin perturbations from each focusing cell in all arcs of the lattice, and can overwhelm the coherent cancellations from the snakes for large enough amplitude. As such, the spin tune begins to deviate from 1/2 for particles away from the closed orbit, and for large enough amplitudes the amplitude-dependent spin tune (ADST) ν may cross a higher-order spin-orbit resonance condition at $\nu = j_0 + \vec{j} \cdot \vec{Q}$ where $|j_1| + |j_2| + |j_3| > 1$. Crossing higher-order resonances with snakes often causes depolarization due to how slow the ADST varies, since $\frac{d\nu}{dt}$ takes the place of $\frac{dG\gamma}{dt}$ as the effective depolarizing ramp rate [2].

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Tracking a particle with realistic amplitudes and ramping rates across the full RHIC energy range in the model lattice, however, shows no signs of any depolarization, with 100% polarization transmission [3].

CLOSED ORBIT DISTORTIONS

Magnetic Field Errors

Another factor of direct relevance to tracking particles are the differences between each magnet, which are well-documented due to cold-testing of many individual magnets in RHIC prior to installation. Incorporating these variations shows us the degradation of spin tune away from $\frac{1}{2}$, as well as a tilt of the closed-orbit invariant spin field away from vertical. Both of these effects would be absent on the closed-orbit in the absence of magnet-to-magnet variation. The ν_0 deviation alone increases the chances of crossing higher-order resonances for any particle. To measure this, we quantify polarization using $P_{\rm lim}$, which is defined as the maximum time-averaged polarization of a spin field [4]. An equivalent definition of $P_{\rm lim}$ is the phase-space average over the invariant spin field. A tune scan of $P_{\rm lim}$ is done at top energy, using an idealized lattice without errors in Fig. 1 and then including the measured magnetic field errors in Fig. 2. We see the introduction of all sorts of resonances, including imperfection, intrinsic, and snake resonances. Finally, in Fig. 3 we see that the snake resonances are mildly exacerbated, but mostly unaffected (including misalignments drastically worsens the snake resonances). This hints that operating with partial snakes is acceptable to zeroth-order.

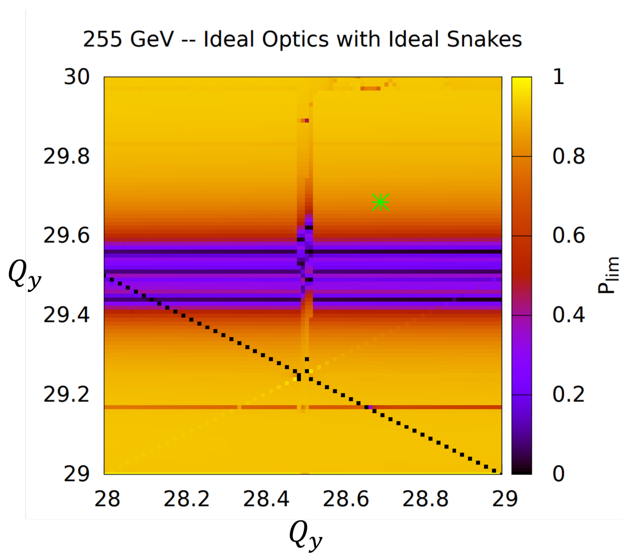

Figure 1: Tune scan of $P_{\rm lim}$ at storage energy using no errors and ideal snakes. Normalized action is $J_y = 5\pi$ mm mrad.

Snake Orbital Effects

Although in an ideal case snakes are optically transparent magnets for the purposes of orbital trajectories and beam focusing, in reality the extended helical dipole trajectories

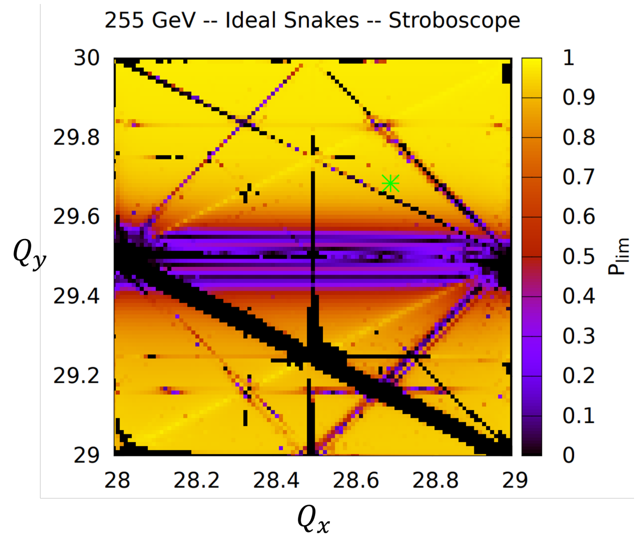

Figure 2: Tune scan of P_{lim} at storage energy using measured magnetic field errors and ideal snakes. Normalized action is $J_y = 5\pi$ mm mrad.

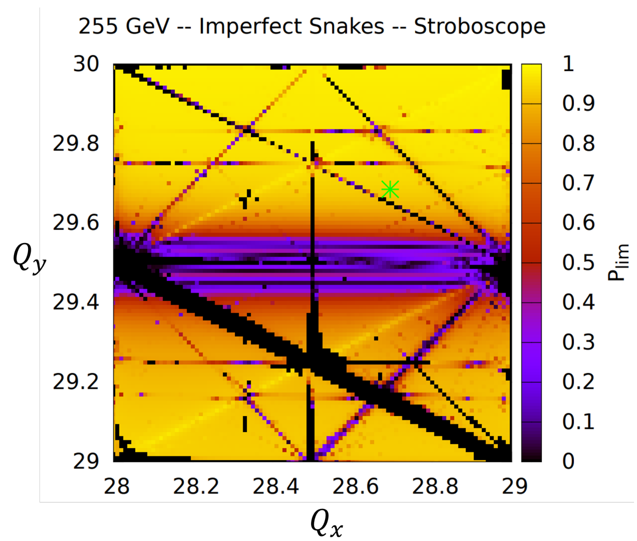

Figure 3: Tune scan of P_{lim} at storage energy with field errors and snake errors. Normalized action is $J_y = 5\pi$ mm mrad.

that a beam follows through will contribute significant coupling [5]. Such coupling allows for the apparent exchange of the projected horizontal and vertical emittances, which should be safe for a proton beam if it were not also associated with tune shift. Tune shift in particular is very dangerous for snakes because of strong snake resonances in the vicinity. Furthermore, the dipole field in the snakes is inherently weak-focusing, but there are even more amplitude dependent effects than in an ordinary dipole due to the slight non-uniformity of the snake fields along the axis. Thus effects like focusing and chromaticity come into play [3]. Luckily, these effects are adiabatically damped along with the emittance of the beam to a significant extent, such that the snake compensation settings are off at storage energy.

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IR Bumps

In the RHIC accelerator complex, the blue and yellow rings have 6 interaction points, labeled IR6, IR8, IR10, IR12, IR2, IR4, referencing a clock configuration. To ramp beams in both rings simultaneously, it is necessary to apply a closed-orbit bump surrounding the IRs. During energy ramping, the IR bumps are set so the closed orbit of each beam is vertically displaced by 5mm in opposite directions [6], which is then decreased to 3mm at flat-top before colliding mode is engaged. The longitudinal extent of these bumps is around 150-200m for each bump, which can lead to a buildup of quadrupole kicks to the spin, as shown in Fig. 4. Thus it is important to include these bumps in tracking simulations of a real ramp.

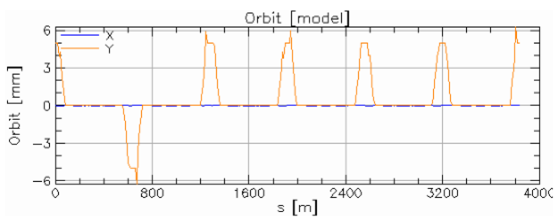

Figure 4: IR bumps during critical region of ramp, contributing imperfection resonance terms

RAMPING SIMULATIONS

Using the measured magnetic field errors as well as IR bumps, we track a $1\sigma = 5\pi$ mm mrad particle through the most dangerous depolarizing region $G\gamma \in [370, 432]$ using ideal snakes. We then repeat this but replace the ideal snakes with their optimized, but broken, counterparts and compare results. Since we are only tracking one particle, statistics is not in our favor, so we apply a moving average over 500 turns instead. The results are seen in Figs. 5 and 6, where at the 1σ level there is no observable difference in polarization between the two cases at many energies, in the absence of misalignments. However, it is clear that at the 4σ level, the broken snake underperforms in comparison by completely flipping the 4σ spin by the end of the ramp.

CONCLUSIONS

These results, together with previous results on the tilt of the invariant spin field due to the broken snake, are part of a growing pool of evidence that slightly asymmetric snakes snakes may not hurt polarization as much as previously thought [7, 8]. It's noted that the asymmetry in the snakes contributes a small resonance driving term whose phase could happen to cancel existing resonance driving terms. Furthermore, it is clear that the simulation of snake axis errors should be done in conjunction with misalignment error simulations, since misalignments are the only remaining avenue to reproduce the missing polarization.

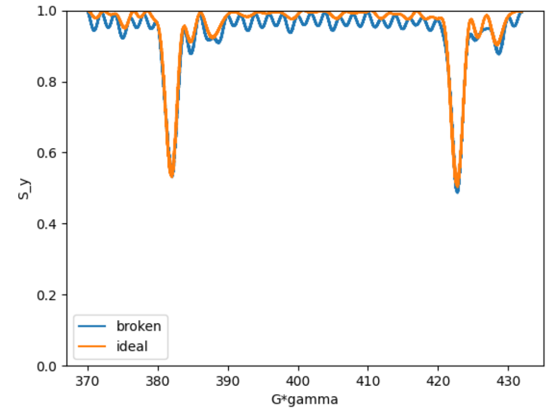

Figure 5: Tracking a 1σ particle with $J_y = 5\pi$ mm mrad

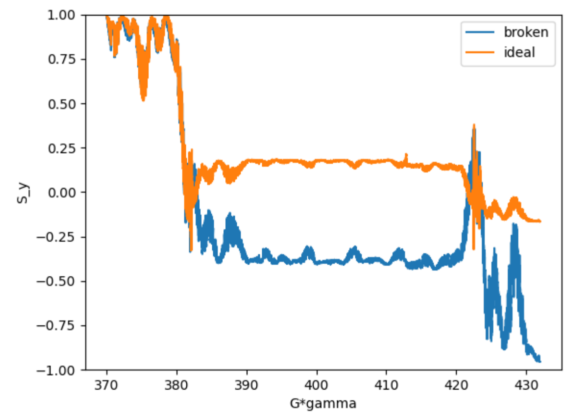

Figure 6: Tracking a 4σ particle with $J_y = 20\pi$ mm mrad

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REFERENCES

[1] G. H. Hoffstaetter, *High-Energy Polarized Proton Beams, A Modern View*. Springer New York, 2006.

[2] E. Hamwi and G. H. Hoffstaetter, "Higher-order spin depolarization analysis," in *Proc. IPAC'23*, Venice, Italy, 2023, pp. 3133–3135. doi:10.18429/JACoW-IPAC2023-WEPL014

[3] M. Xiao and T. Katayama, "Spin tracking with real siberian snakes in rhic," Brookhaven National Lab., Upton, NY, USA, Tech. Rep., 2000.

[4] G. H. Hoffstaetter and M. Vogt, "Strength of higher-order spin-orbit resonances," *Phys. Rev. E*, vol. 70, p. 056 501, 5 2004. doi:10.1103/PhysRevE.70.056501

ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-MOPS04

[5] N. Z. Khalil and V. Ptitsyn, “Coupling Spin Resonances With Siberian Snakes,” in *Proc. NAPAC’13*, Pasadena, CA, USA, Sep.-Oct. 2013, pp. 577–579. https://jacow.org/PAC2013/papers/TUPBA26.pdf

[6] V. Ptitsyn and T. Satogata, “The RHIC Orbit Control,” in *Proc. EPAC’02*, Paris, France, 2007, pp. 353–355. https://jacow.org/e02/papers/MOPLE073.pdf

[7] F. Méot *et al.*, “RHIC Run 22, 9 o’clock, a Snake in the Blue,” Brookhaven National Lab., Upton, NY, USA, Tech. Rep. BNL-223081-2022-TECH, 2022. doi:10.2172/1873174

[8] V. Schoefer, E. Aschenauer, H. Huang, V. Ptitsyn, F. Meot, and V. Ranjbar, “Stable spin direction measurements at RHIC with polarized proton beams,” in *Proc. IPAC’23*, Venice, Italy, 2023, pp. 515–518. doi:10.18429/JACoW-IPAC2023-MOPL004