A parton model for hA interactions at high energies

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Abstract

A parton model for hA interactions at high energies is developed based on the assumption that parton amplitudes do not depend on virtualities provided only nonplanar diagrams are retained in the elastic amplitude. It is shown that although the AGK rules are exactly fulfilled in the model the interference between the direct and spectator mechanisms of particle production restores the conservation of energy. Conditions are studied under which Glauber-like formulas result for the amplitude and crosss-sections. The difference is analyzed between the predictions of the proposed model and current models of hA interactions based on the probabilistic interpretation of the Glauber theory.

1.Introduction There exist two versions of the parton model for hA interactions in the current literature. The original idea of [1] developed in [2] starts from a parton wave function of the incoming hadron, which gets slightly deformed in the process of interaction in its low-energy component part. The spectrum of produced states then essentially coincides with the rest part of the wave function, that is, the spectator part. Less ambitious constituent parton models (e.g. [3-5]) assume that partons which form the incoming hadron independently interact with nucleons of the target by interchanging some extended objects (e.g. coloured strings) with the spectrum of produced particles determined by their decay. However both type of parton models leave open some crucial questions. First, they do not produce in a convincing manner a Glauber-like form of the elastic amplitude (and the total cross-section), which agrees well with experimental facts. Rather the Glauber probabilities are postulated for n-fold hN interactions inside nuclei [5]. Second, they have problems with energy-momentum conservation.

It is the aim of the present paper to propose a slightly more general parton model for hA interactions, which clarifies upon these points [6,7]. The model is quite in the spirit of the original version of [2] with the only difference that elementary amplitudes are separated from the wave function not by their short range in rapidity but rather by the topology of the corresponding Feynman diagrams. This enables us to establish exat energy momentum conservation in the model. On the other hand, as a price, elementary amplitudes now become extended in rapidity and their spectrum overlaps with that of spectators. The model is thus an interpolation between the two extreme cases: that of the wave-function model of [2], where all the spectrum comes from spectators, and of constituent parton models with the spectrum due to interactions. Under some rather general assumptions the model leads to the Glauber formula for the hA elastic amplitude and the total cross-section. The inclusive cross- sections consist of two parts in our model. One, coming from spectators, behaves like $A^{2/3}$ for heavy nuclei. The direct part coming from interactions exactly satifies the AGK rules [8] and is proportional to A. A simple case with only one type of partons is discussed here. Generalizations to more realistic situations as well as corresponding numerical results are postponed for future publications.

2.The hA amplitude. As in [1,2] the starting point is that in the hA scattering long before the interaction the fast incoming hadron decays into weakly virtual partons each of which subsequently once interacts with one of the nucleons from the target nucleus and finally joins with the others to form the outgoing hadron (Fig. 1). By definition B_n is two particle irreducible with respect to each pair of lines p_i, p'_i. Separating the n-fold nuclear form factor from the hA amplitude in the standard manner we get for the hA elastic amplitude at fixed impact parameter

$$\mathcal{A} = \sum_1^A C_A^n T^n(b) W_n \qquad (1)$$

where T is the usual nuclear profile function and W_n the high-energy part, which is an integral of a product of B_n and n forward parton-nucleon amplitudes a over parton momenta p_i and transferred longitudinal momenta q_{iz}.

The amplitudes a depend on their energies and in general on parton virtualities. Our basic asumption is that the dependence of a's on parton virtualities may be neglected. It is implied that first one has to drop all planar diagrams taking the asymptotical value for the amplitude when $p_h \to \infty$ and only aftewards in the nonplanar contribution, which survives in that asymptotical limit, one considers partonic amplitudes as not depending upon virtualities.

We compare W_n with the n-fold form-factor F_n of the incoming hadron (Fig. 2) integrated over all q_{i-} and with $q_{i+} = q_{i\perp} = 0$. Instead of parton amplitudes a it contains parton form-factors f. We assume that the latter do not depend on virtualities either. Then F_n goes over to W_n if the form-factor f as a function of the light-cone variable p_+ coincides with $a(p_+)$. In a quantum field theory such a form-factor can be generated by a composite operator $j(x) = \phi^{(+)}(x)a_0(-i\partial_-)\phi^{(-)}(x)$ where $\pm$ components of the unrenormalized field ϕ are defined according to the frequency sign with respect to x_+ . The "bare" amplitude a_0 has to be 2-particle irreducible in the t-channel. In terms of j we find

$$\delta^3(p_h - p'_h)iW_n = m\sqrt{2} < p_h|(i\hat{a}(x_+))^n|p'_h > \tag{2}$$

where up to a normalization factor $\hat{a}(x_+)$ is a "charge" associated with the "current" j:

$$\hat{a}(x_+) = (m\sqrt{2})^{-1}\int dx_- d^2x_\perp j(x) \tag{3}$$

and $< p_h|$ ($|p'_h >$) is the final (inicial) hadron state.

To calculate (2) we introduce a complete set of bare N-parton states and the corresponding wave-function $\Psi(k_i)$. Then we find

$$iW_n = \int \prod_1^n (d\alpha_i i a_0(\alpha_i))\rho_n(\alpha_i) \tag{4}$$

Here $\alpha_i = k_{i+}/p_{h+}$. The quantity

$$\rho_n(\alpha_i) = \sum_{N\geq n} N!/(N-n)! \int \prod_1^n \frac{d^2k_{i\perp}}{2\alpha_i} \prod_{n+1}^N \frac{d^3k_i}{2k_{i+}} 2p_{h+}\delta^3(p_h - \sum k_i)|\Psi_N(k_i)|^2 \tag{5}$$

is the probability for n bare partons to have their "+" components of momenta $k_{i+} = \alpha_i p_{h+}$. From (5) it follows that ρ_n obey the energy conservation sum rule:

$$\int \alpha d\alpha \rho_{n+1}(\alpha, \alpha_i) = (1 - \sum_1^n \alpha_i)\,\rho_n(\alpha_i) \tag{6}$$

Another sum rule can be deduced for a separate term ρ_{nN} in the sum over N in (5):

$$\int d\alpha \rho_{n+1,N}(\alpha, \alpha_i) = (N-n)\,\rho_{nN}(\alpha_i) \tag{7}$$

The probabilities ρ are evidently real. Therefore in our model the AGK cutting rules will be exactly fulfilled in spite of the dependence of the amplitudes $a_0(s)$ on energy (see [9]).

3.Inclusive cross-sections. Various contributions to the inclusive cross-sections can be obtained by cutting diagrams for the forward elastic scattering amplitude and fixing the registered particle in one of these cuts. It divides into a direct part coming from observing the particle in the cut parton-nucleon amplitude a_0 and a spectator part with the particle in the cut ρ or lines of active partons. According to the AGK cutting rules in the direct part all contributions with additional parton interactions cancel and we are left with the impulse approximation proportional to A:

$$J_A^{(1)}(\alpha) = A\int d\beta \rho_1(\beta) j_0(\beta, \alpha) \tag{8}$$

Here $j_0(\alpha_1, \alpha)$ is a bare inclusive cross-section to observe a particle with the longitudinal momentum αp_{h+} produced by a hadron with the longitudinal momentum $\alpha_1 p_{h+}$ incident on a nucleon at rest.

For the spectator contribution we have to consider the same structure as (2) with an additional particle in the initial and final hadronic states $|p_h, k>$ where k refers to the observed particle. As one can see from Fig. 3 only the disconnected part of the corresponding partonic wave function $\Phi_N(k_i)$ contributes to the absorptive part of interest. So we represent Φ_N as a sum over $i = 1, ...N$ of disconnected parts with $k_i = k$. We then split Φ_N in two terms containig summation over i in the limits from 1 to n and from n+1 to N respectively. Then the product of terms with $i \geq n+1$ gives rise to the absorptive part corresponding to the observation of spectators:

$$D_n^{(1)}(\alpha) = -2\,\mathrm{Re} \int \prod_1^n (d\alpha_i i a_0(\alpha_i))\rho_{n+1}(\alpha, \alpha_i) \tag{9}$$

The mixed product of terms with $i \leq n$ and $i \geq n+1$ describes the observation of active partons:

$$D_n^{(2)}(\alpha) = -2\,\mathrm{Re} n i a_0(\alpha) \int \prod_1^{n-1} (d\alpha_i i a_0(\alpha_i))\rho_n(\alpha, \alpha_i) \tag{10}$$

The last contribution with $i \leq n$ in both inicial and final states corresponds to a planar diagram contribution and we neglect it. As a consequence of (6) the sum D_n of (9) and (10) satisfies the sum rule

$$\int \alpha d\alpha D_n(\alpha) = 2\mathrm{Im} W_n \tag{11}$$

For the cross-sections it implies that particles described by the spectator contribution carry all the energy available.

The contribution $D_n^{(2)}$ does not exist for $n = 1$ as in that case there is no interaction in one of the parts of the cut amplitude. This leads to a correction term which results in an additional contribution to the cross-section proportional to A. In (8) $J_1^{(1)}$ has to be replaced by ,

$$J_1(\alpha) = \int d\alpha_1 \rho_1(\alpha_1) j_0(\alpha_1, \alpha) - \rho_1(\alpha)\sigma_0(\alpha) \tag{12}$$

where $\sigma_0 = 2\,\mathrm{Im} a_0$ is a cross-section for parton- nucleon interaction. The elementary inclusive cross-section j_0 has to satisfy the energy conservation sum rule: $\int \alpha d\alpha j_0(\alpha_1, \alpha) = \alpha_1 \sigma_0(\alpha_1)$ With this sum rule we find from (12)

$$\int \alpha d\alpha J_1(\alpha) = 0 \tag{13}$$

Together with (11) this demonstrates the conservation of energy in our inclusive cross-sections.

4. Glauber cross-sections. Two conditions are needed to secure that the hA amplitude (1) coincide with the Glauber one. First one has to assume that the elementary amplitude a_0 does not depend on energy, that is on α. As we shall see this does not imply that the resulting hadron-nucleon cross-section is independent of energy. With constant a_0 using (7) we obtain

$$iW_n = (ia_0)^n \rho_n^{(0)} \tag{14}$$

where numbers $\rho_n^{(0)}$ are defined according to $\rho_n^{(0)} = \sum_{N \geq n} N!/(N-n)! \rho_{0N}$ The probabilities ρ_{0N} give the distribution in the number of partons in the projectile. If we impose a second condition, namely, that ρ_{0N} are Poissonian, then the sum over N gives $\rho_n^{(0)} = \lambda^n$ where λ is the mean number of partons. Together with (14) it leads to the Glauber formula for the hA amplitude with the hN amplitude given by λa_0. The hN cross-section will depend on energy if λ depends on it.

The spectator part of the inclusive cross-section also simplifies for constant a_0. For the corresponding absorptive part we get

$$D_n(\alpha) = -\text{Re}(ia_0)^n \rho_n^{(1)}(\alpha) \tag{15}$$

where the distribution $\rho_n^{(1)}$ is defined as $\rho_n^{(0)}$ with $\rho_{1N}(\alpha)$ instead of ρ_{0N}. If we assume the Poissonian form for the latter then we get sum rules

$$\int d\alpha \rho_n^{(1)}(\alpha) = \lambda^n(\lambda + n); \quad \int \alpha d\alpha\, \rho_n^{(1)}(\alpha) = \lambda^n \tag{16}$$

With a parametrization

$$\rho_n^{(1)}(\alpha) = c_n \lambda^n \alpha^\beta (1-\alpha)^{\gamma_n} \tag{17}$$

the conditions (16) require: $\gamma_n = (n+\lambda-1)\beta - 1, c_n^{-1} = \text{B}(\beta+1, \gamma_n+1)$. Assuming (17) one can perform the summation over all n for the spectator part of the inclusive cross section. For heavy nucleus $A >> 1$ one obtains the cross-section

$$I_A(\alpha) = \alpha^\beta (1-\alpha)^{\gamma_0} \sigma_A^{tot} / \text{B}(\beta+1, \gamma_0+1) \tag{18}$$

which means that the spectator part is proportional to $A^{2/3}$.

Thus we come to the conclusion that at least in this simplified case and choice of $\rho_n^{(1)}$ the total inclusive cross-section contains two contributions: the direct one, proportional to A, and the spectator one, proportional to $A^{2/3}$.

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Figures

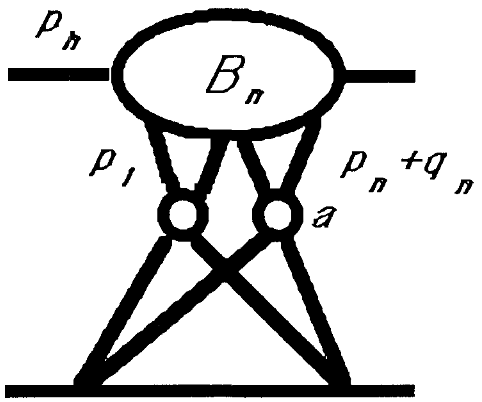

Fig. 1
The hA elastic scattering amplitude

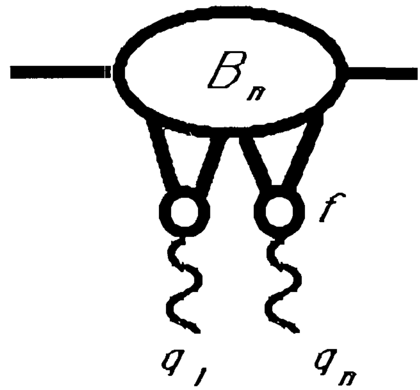

Fig. 2
The n-fold form-factor of the projectile

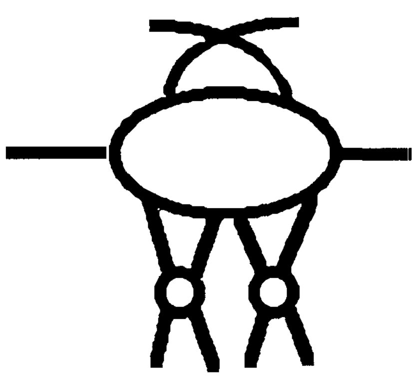

Fig. 3

The diagram for the spectator mechanism of particle production. Only the disconnected part of the wave function contributes