Two-loop Electroweak Corrections to ϵ_K

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Abstract. The parameter ϵ_K measures CP violation in the neutral kaon system. It is a sensitive probe of new physics and plays a prominent role in the global fit of the Cabibbo-Kobabyashi-Maskawa matrix. As one of the first discovered sources of CP violation, it has been measured in experiment to permil precision. A simple reparametrization of the effective Hamiltonian has been recently shown to drastically reduce the perturbative errors coming from the charm-quark corrections, making the electroweak corrections relevant. In this proceedings, I will present the two-loop electroweak corrections to ϵ_K.

1. Introduction

Indirect CP violation in the neutral kaon system, known as ϵ_K, is one of the key observables in flavor physics as it is used to constrain the CKM matrix and the unitary triangle. It is also an excellent probe of new physics as it is supressed by the GIM mechanism. ϵ_K can be defined in terms of mass difference and decay rate difference of the kaons ΔM_K, $\Delta\Gamma_K$ and the off-diagonal matrix elements of the kaon mixing matrix M_{12}, Γ_{12} as

$$\epsilon_K \equiv e^{i\phi_\epsilon} \sin\phi_\epsilon \frac{1}{2} \arg\left(\frac{-M_{12}}{\Gamma_{12}}\right), \tag{1}$$

where $\phi_\epsilon = \arctan(2\Delta M_K/\Delta\Gamma_K)$ and $M_{12} = -\langle K^0|\mathcal{L}^{\Delta S=2}_{f=3}|\bar{K}^0\rangle/(2\Delta M_K)$. Indirect CP violation arises via kaon mixing as the mass eigenstates of the kaons do not correspond to the CP eigenstates. Leading corrections to the mixing can be calculated in terms of the box diagrams, shown in figure 1. Experimentally,

$$|\epsilon_K|_{exp} = (2.228 \pm 0.011) \times 10^{-3}, \tag{2}$$

with an uncertainty at the permil level [1]. In this work we will focus on the short-distance contributions to ϵ_K contained in M_{12}.

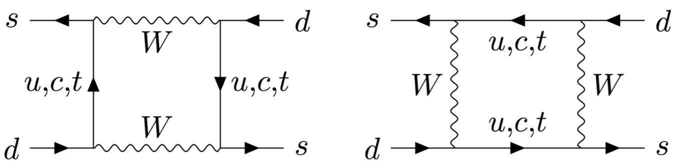

Figure 1. Kaon mixing diagrams.

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The effective $|\Delta S| = 2$ Lagrangian for ϵ_K at a hadronic scale $\mu = 2$ GeV and in three-flavor scheme is given by

$$\mathcal{L}_{f=3}^{|\Delta S|=2} = -\frac{G_F^2 M_W^2}{4\pi^2}\left[\lambda_u^2 C_{S2}^{uu}(\mu) + \lambda_t^2 C_{S2}^{tt}(\mu) + \lambda_u\lambda_t C_{S2}^{ut}(\mu)\right] Q_{S2} + \text{h.c.} + \dots \tag{3}$$

where $Q_{S2} = \left(\bar{s}_L^\alpha \gamma_\mu d_L^\alpha\right) \otimes \left(\bar{s}_L^\beta \gamma^\mu d_L^\beta\right)$ and $\lambda_i \equiv V_{is}^* V_{id}$. Here G_F is the Fermi constant, M_W is the W boson mass, V_{ij} are the CKM matrix elements, Q_{S2} is the operator corresponding to the box diagrams in figure 1 with W bosons integrated out and C_{S2}^{ij} are the Wilson coefficients.

It has recently been shown that considering the Lagrangian in (3) significantly reduces perturbative errors [2]. With reduced uncertainties we expect the electroweak corrections to become relevant. In particular, we focus on the second and the third terms in (3), i.e. the corrections coming from the top quark and the charm-top quark contributions to ϵ_K. As discussed in [2], the λ_u^2 term does not affect ϵ_K. Instead, it contributes to ΔM_K.

The rest of this publication is organised as follows. In section 2, a brief overview of the matching calculation is given. In section 3, the results are discussed, followed by conclusion. Further details on the work presented here can be found in [3, 4].

2. Calculation of electroweak corrections

The Lagrangian (3) is valid below the charm-quark scale. Its Wilson coefficients are obtained by matching from the effective four- and five-flavor Lagrangians, after the appropriate renormalization group evolution. For the top quark contribution, the Lagrangian is given by

$$\mathcal{L}_{f=5}^{|\Delta S|=2} = -\frac{G_F^2 M_W^2}{4\pi^2}\lambda_t^2 C_{S2}^{\prime\prime tt}(\mu) Q_{S2}^{\prime\prime} + \text{h.c.} + \dots , \tag{4}$$

where $Q_{S2}^{\prime\prime} = \left(\bar{s}_L^\alpha \gamma_\mu d_L^\alpha\right) \otimes \left(\bar{s}_L^\beta \gamma^\mu d_L^\beta\right)$. For the charm-top quark contribution, the relevant Lagrangians can be written as

$$\begin{aligned}\mathcal{L}_{f=4,5}^{\text{eff}} = &\left[\sum_{q,q'=u,c} V_{qs}^* V_{q'd}(C_+ Q_+^{qq'} + C_- Q_-^{qq'}) - \lambda_t \sum_{i=3,6} C_i Q_i\right] - \frac{G_F^2 M_W^2}{4\pi^2}\lambda_t^2 C_{S2}^{\prime\prime tt} Q_{S2}^{\prime\prime} \\ &- 8G_F^2\left(\lambda_u\lambda_t + \lambda_t^2\right)\tilde{C}_7\tilde{Q}_7 + \text{h.c.},\end{aligned} \tag{5}$$

where $Q_\pm^{qq'} = \frac{1}{2}\left((\bar{s}_L^\alpha \gamma_\mu q_L^\alpha)(\bar{q}_L^{\prime\beta}\gamma^\mu d_L^\beta) \pm (\bar{s}_L^\alpha \gamma_\mu q_L^\beta)(\bar{q}_L^{\prime\beta}\gamma^\mu d_L^\alpha)\right)$, $\tilde{Q}_7 = \frac{m_c^2}{g_s^2\mu^{2\epsilon}}(\bar{s}_L\gamma_\mu d_L)(\bar{s}_L\gamma_\mu d_L)$ and Q_i are the QCD penguin operators. In both cases the matching procedure is similar. We first perform the matching at the weak scale μ_t, where the top quark, W and Z bosons are integrated out and then using renormalization group equations evolve the result down to 2 GeV, at each quark mass scale decoupling the quarks and taking into account threshold corrections.

As we employ dimensional regularization and work in $d = 4 - 2\epsilon$ dimensions certain dimension 4 relations (such as Dirac algebra and Fierz transformations) become inconsistent. This results in the evanescent operators arising in intermediate stages of the calculation. These operators are defined such that they vanish at $d \to 4$ and for $|\Delta S| = 2$ operators are given by

$$E_{S2}^{(1)} = \left(\bar{s}_L^\alpha \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} d_L^\alpha\right) \otimes \left(\bar{s}_L^\beta \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} d_L^\beta\right) - (16 - a_{11}\epsilon - 4\epsilon^2) Q_{S2}\,, \tag{6}$$

$$E_{S2}^{(2)} = \left(\bar{s}_L^\alpha \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} d_L^\alpha\right) \otimes \left(\bar{s}_L^\beta \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} d_L^\beta\right) - \left(256 - a_{21}\epsilon - \tfrac{108816}{325}\epsilon^2\right) Q_{S2}\,. \tag{7}$$

where the coefficients in front of the ϵ constitute a particular choice of scheme. As a result, the Wilson coefficients depend on the scheme of the evanescent operators. This then has to cancel against the corresponding scheme dependence of the hadronic matrix element, $\langle Q_{S2}\rangle$.

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In obtaining our results we performed several checks. For the top quark contribution we computed the $\mathcal{O}(30,000)$ two-loop diagrams independently. We checked that both IR and UV divergences cancel in the matching, yielding a finite result. We verified that our results agree when using different normalizations for the effective Lagrangian and several renormalization schemes for the electroweak input parameters. For the charm-top contribution we performed the calculation in generalized R_ξ gauge for gluons and photon and verified the gauge-parameter drops out of our results. We analytically checked that our results are independent of all matching scales, and that the dependence on the renormalization scheme is canceling by the corresponding scheme dependence of the hadronic matrix element.

3. Results

To obtain a numerical estimate of the size of the electroweak corrections, as well as an estimate of the remaining perturbative uncertainties, the Wilson coefficients are evaluated at electroweak (top and charm-top quark contribution) and charm-threshold (charm-top quark contribution) matching scales μ_t and μ_c respectively.

The corresponding plot for the top quark contribution is presented in figure 2. Further discussion on the EW renormalization 'hybrid' scheme used can be found in [3]. The plots for the charm-top contribution are shown in figure 3. As we can see, in both cases the EW corrections do not introduce additional dependence on the matching scales and simply shift the Wilson coefficients. In both cases a very small dependence on the scheme of the evanescent operators is found. For the top quark contribution we have

$$C_{S2}^{\prime\prime tt}(2\,\mathrm{GeV}) = (3.90 - 0.0003\,a_{11}) \times 10^{-8}\,, \tag{8}$$

and for the charm-top quark contribution we get

$$\tilde{C}_{S2}^{ut}(2\,\mathrm{GeV}) = -14.075 + 0.001\,a_{11}. \tag{9}$$

This dependence will exactly cancel once the QED matrix element is known. As the QED corrections to the matrix element are not logarithmically enhanced ($\tilde{C}_{S2}^{ut}(\mu)$) or come with smaller couplings, e.g. α vs. top-Yukawa ($C_{S2}^{\prime\prime tt}(\mu)$), we neglect the dependence on a_{11} in the final result.

As the scheme dependence of our result cannot be cancelled against the QED ADM (as it is scheme independent), we cannot define the η factors as done in QCD. Instead, we suggest a temporary prescription. For the top quark contribution one should multiply η_{tt} by the electroweak correction factor $(1 - \Delta_{tt})$, with $\Delta_{tt} = 0.01 \pm 0.004$. For the charm-top contribution

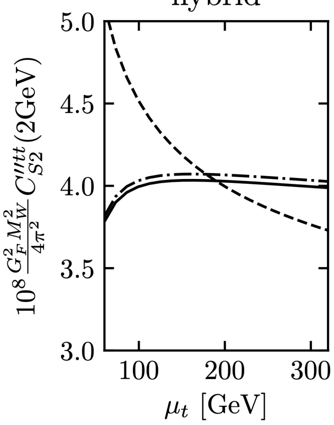

Figure 2. Residual matching scale dependence of the Wilson coefficient for the top quark contribution. The dashed line shows the LO result, the dash-dotted line shows the result including NLO QCD corrections, while the solid line show the full (QCD and QED) NLO result.

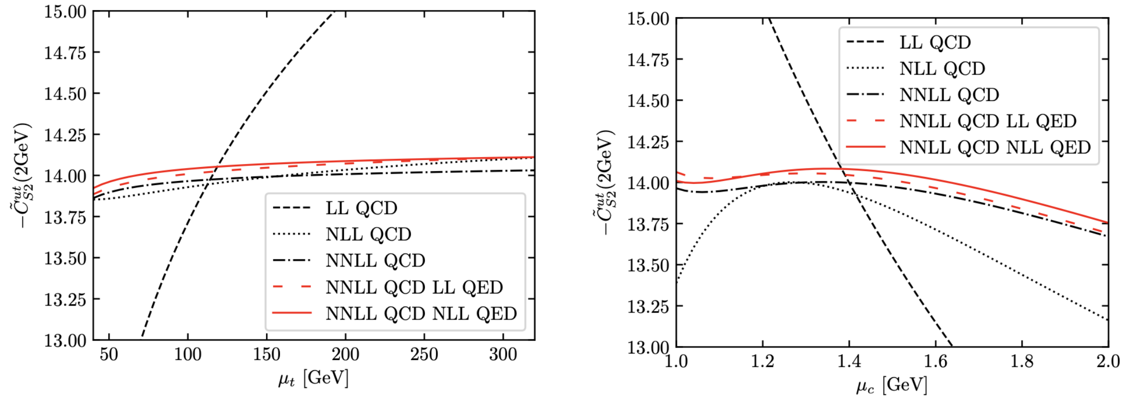

Figure 3. Residual dependence of the Wilson coefficient for the charm-top contribution on the electroweak (left panel) and charm-threshold (right panel) matching scales. The short-dashed, dotted, and dash-dotted lines show the LL, NLL, and NNLL QCD results, respectively. The long-dashed and solid lines show the results including also the LL and NLL electroweak corrections.

one should rescale the NNLL QCD value of η_{ut} by a factor of 1.005. Collecting all contributions, including the EW corrections yields

$$|\epsilon_K| = 2.170(65)_{\text{pert.}}(76)_{\text{non-pert.}}(153)_{\text{param.}} \times 10^{-3}, \tag{10}$$

with the errors corresponding to short-distance, long-distance, and parametric uncertainties.

4. Conclusion

In summary, we have presented the two-loop electroweak corrections to the top-quark contribution and the leading and next-to-leading electroweak corrections to the charm-top contribution to ϵ_K. We discussed the corresponding dependences on the matching scales and the evanescent operator scheme. We found -1.0% shift in central value of Wilson coefficient for the top quark contribution and a -0.5% shift for the charm-top quark contribution. A systematic estimate of the QED corrections to the hadronic matrix element would complete our analysis.

The aim of this work is to provide a further step in the prediction of ϵ_K with residual theoretical uncertainty at the percent level. Further important directions of improvement are the calculation of the three-loop QCD corrections in the top-quark sector of the effective Lagrangian, and the NLO scheme conversion from RI/SMOM to $\overline{\text{MS}}$ for the hadronic matrix element of the local $|\Delta S| = 2$ operator.

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