

# Charged polarized white dwarfs with finite temperature as a possible source of type Ia supernovae

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We investigate the structure of polarized charged white dwarfs with finite temperature as a possible type Ia supernovae source. The white dwarf is modeled considering an isothermal core with central temperature  $10^8$  [K] and an envelope where the temperature distribution depends on the mass density. Regarding the fluid, we assume that it is composed of nucleons and electrons. The structure of the polarized charged white dwarfs is obtained by solving the Einstein-Maxwell equations with charge densities represented by two Gaussians, forming an electric dipole layer at the stellar surface. We obtain larger and more massive white dwarfs when polarized charge and the Gaussians width are increased. We find that to appreciate effects in the white dwarf's structure, the electric polarized charge must be in the order of  $5.0 \times 10^{20}$ [C]. We obtain a maximum white dwarf mass of around  $2.4M_\odot$  for a polarized charge of  $1.5 \times 10^{21}$ [C]. This mass result can indicate that polarized charged white dwarfs are possible progenitors of superluminous type Ia supernovae. Furthermore, the mass-central density curves we obtain are very similar to the ones reached recently for ultra-magnetized white dwarfs.

*Keywords:* White dwarfs; charged white dwarfs; type Ia supernovae explosions

## Introduction

The mechanisms that increase the mass of a white dwarf (WD) have been extensively studied since supermassive WDs were associated with superluminous Type Ia supernovae (2003fg,<sup>1</sup> 2009dc,<sup>2</sup> 2006gz<sup>3</sup> and 2007if<sup>4</sup>). In what concerns the structure of WDs, in literature, there are works consider magnetic field, rotation, and electric field to explain the increase of such mass. To present considerable effects in the WD structure, recent works<sup>5,6</sup> present magnetic fields are the order of  $9 \times 10^{13}$ [G] and electric field of around  $10^{16}$  [V/cm].

We investigate charged white dwarfs made up by a non-degenerate fluid with isothermal core with central temperature  $T_c = 10^8$  [K] and envelope with temperature distribution.<sup>7</sup> Moreover, we assume a charge polarization<sup>8</sup> in the surface of WDs. We obtain very massive white dwarfs which can be the progenitors of Type Ia Supernovae. We note that the polarization generate notable effect in the structure of WDs and it decreases the electric field outside the star.

## 1. Structure equilibrium equations and boundary conditions

The line element considered to describe a charged WD, in Schwarzschild-like coordinates, is given by

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \quad (1)$$

where  $\nu$  and  $\lambda$  depend on the radial coordinate only. The inclusion of the electromagnetic tensor in the energy-momentum tensor leads to the nonzero Maxwell-Einstein equations:

$$\frac{dq}{dr} = 4\pi\rho_e r^2 e^{\lambda/2}, \quad (2)$$

$$\frac{dm}{dr} = 4\pi\varepsilon^2 + \frac{q}{r} \frac{dq}{dr}, \quad (3)$$

$$\frac{dP}{dr} = -(P + \varepsilon) \left[ 4\pi r p + \frac{m}{r^2} - \frac{q^2}{r^3} \right] e^\lambda + \frac{q}{4\pi r^4} \frac{dq}{dr}, \quad (4)$$

$$\frac{d\nu}{dr} = -\frac{2}{(p + \varepsilon)} \left[ \frac{dP}{dr} - \frac{q}{4\pi r^4} \frac{dq}{dr} \right], \quad (5)$$

with  $q$  and  $m$  representing respectively the electric charge and the mass within the radius  $r$ . The potential metric  $e^\lambda$  is described as

$$e^\lambda = \left[ 1 - \frac{2m}{r} + \frac{q^2}{r^2} \right]^{-1}. \quad (6)$$

Due to temperature, the white dwarfs are considered to have a partial degeneracy, thus the contribution of electrons in the EoS is related to Fermi-Dirac integrals. Once the temperature distribution is defined,<sup>7</sup> these integrals are solved by using an adaptive quadrature method.<sup>9</sup> Next, we proceed to integrate Eqs. (2)-(5) by using the fourth-order Runge-Kutta method for the uncharged star, i.e.,  $Q = 0$ . For this aim, in the center  $r = 0$ , it is considered the boundary conditions:

$$m(0) = 0, \quad \varepsilon(0) = \varepsilon_0, \quad \text{and} \quad \nu(0) = \nu_0, \quad (7)$$

and the star's surface is determined at  $P(r = R^+) = 0$ . This first integration enables us to find the values of the total star mass  $M$  and total radius  $R^+$ . The location of the second and negative Gaussian at  $R^-$  is determined through the relation  $R^- = R^+ + \Delta R$ , where  $\Delta R = 4b$  and  $b = 5$  [km]. The value of  $b$  employed is the

smallest necessary to obtain considerable effects on the structure of the white dwarf. Once found the polarized charge, by using the charge density, we integrate again Eqs. (2)-(5) to obtain the charged white dwarf structure. The boundary conditions in Eq. (7) are used together with  $q(0) = 0$  and  $\rho_e(0) = \rho_{e0}$ .

## 2. Equation of state and charge density profile

Regarding the stellar fluid, we consider that the equation of state is made of by nucleons, electrons, and photons.<sup>10,11</sup> Due to our consideration of very high central temperatures ( $T_c = 10^8$  [K]), we neglect the lattice interactions. Thus, the pressure  $P$  and energy density  $\varepsilon$  is depicted by

$$P = P_e + P_N + P_\gamma, \quad (8)$$

$$\varepsilon = \varepsilon_e + \varepsilon_N + \varepsilon_\gamma, \quad (9)$$

where the subindexes  $e$ ,  $N$  and  $\gamma$  represent the electrons, nucleons, and photons contributions, respectively.

We assume that electrons and protons are susceptible to a centrifugal force responsible for accelerating heavy particles away from light ones. Consequently, polarization forms in the stellar envelope. Inspired in Negreiros work,<sup>8</sup> we consider the polarization to be defined with the inclusion of two Gaussians in the charge density:

$$\rho_e = e^{-\lambda/2} \left[ k^+ \exp \left( -\frac{(r - R^+)^2}{b^2} \right) + k^- \exp \left( -\frac{(r - R^-)^2}{b^2} \right) \right], \quad (10)$$

where  $r$  represents the radial coordinate,  $b = 5$  [km] is the Gaussian width, and with  $k^+$  and  $k^-$  being constants that depend on the global charge neutrality. The  $R^+$  is the radius for the uncharged configuration. Moreover,  $R^- = R^+ + \Delta R$ , being  $\Delta R = 4b$  a fixed value for all the central energy densities.

To obtain  $k$ , we solve the integration by considering a total zero charge

$$Q_t = \int_0^\infty e^{\lambda/2} 4\pi r^2 \rho_e dr = 0. \quad (11)$$

By using Eq. (10) in this last relation, we find

$$k^+ = \frac{Q}{8\pi} \left( \frac{\sqrt{\pi}b(R^+)^2}{2} + \frac{\sqrt{\pi}b^3}{4} \right)^{-1}, \quad (12)$$

$$k^- = \frac{-Q}{8\pi} \left( \frac{\sqrt{\pi}b(R^-)^2}{2} + \frac{\sqrt{\pi}b^3}{4} \right)^{-1}. \quad (13)$$

With the intent to better illustrate the stellar charge profile, we present the charge density as a function of radial coordinate in Fig. 1.

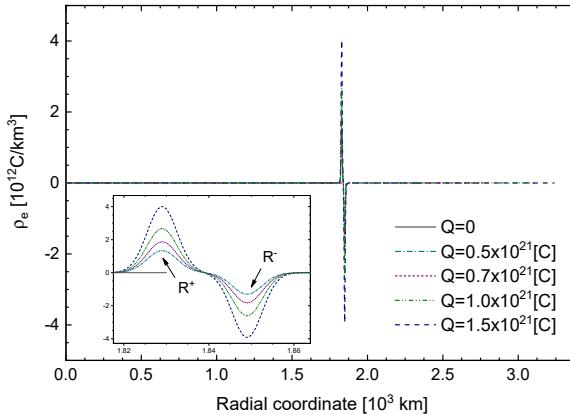


Fig. 1. The charge density as a function of the radial coordinate for a star with central energy density  $\varepsilon_c = 10^{10} [\text{g cm}^{-3}]$  and several polarized charge values.

### 3. White dwarf configurations

Consequently to the polarized charge, we find a very high electric field in the stellar envelope and no electric field outside the star (in contrast to previous charge white dwarfs studies<sup>6</sup>). In our calculations, considering a star with central density  $\varepsilon_c = 5 \times 10^{10} [\text{g/cm}^3]$ , we find maximum internal electric fields of around  $10^{16} [\text{V/cm}]$ . Under these conditions, the radius is increased in 77% and the mass 63% for  $Q = 1.5 \times 10^{21} [\text{C}]$ . This electric field induces an increase in the internal star pressure.

The pressure in the stellar interior enhancing due to the charge density was already noticed in previous charge white dwarfs studies.<sup>6</sup> The pressure profile found by Carvalho and collaborators increases with charge and decreases due to self-gravitation. In our work, the neutralization of charge is the main responsible to decrease the stellar charge pressure. Thus, compared to Carvalho's work,<sup>6</sup> the charge density we propose affects more the mass than the radius.

In Fig. 2 we show the mass as a function of central energy density. This figure is important to analyze the relation between polarized charge white dwarfs and Type Ia Supernovae. We can note that for the uncharged sequence, the mass increases with central energy density until it reaches a maximum mass (pink dot) and decreases. This behavior is different for sequences of charged stars. These curves increase mass with central energy density (the same pattern observed for ultra-magnetized white dwarfs<sup>5</sup>). Another important observation in this figure is related to the maximum mass obtained.

For the maximum polarized charge we find maximum masses around  $\approx 2.4 M_\odot$  for  $1.5 \times 10^{21} [\text{C}]$ . This increase in mass is obtained for the first time for charged white dwarfs with no electric field outside the star, which is the main novelty of this work.

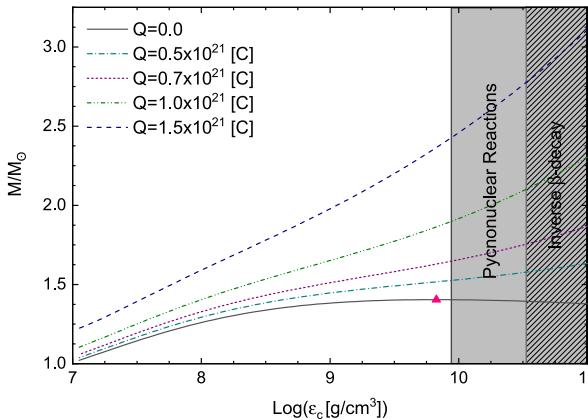


Fig. 2. The mass as a function of the energy density for several sequences of constant polarized charge and  $T_c = 10^8$ [K].

#### 4. Conclusion

In this work, we investigated the structure of charged hot white dwarfs to associate them with superluminous type Ia supernovae. In what concerns to the stellar fluid, we consider a central temperature of  $T_c = 10^8$ [K] and a temperature distribution in the envelope.<sup>11</sup> Besides, due to the high central temperature considered, we neglected the particles' interactions. We used Maxwell-Einstein equations to find the structure of these charged polarized white dwarfs. We found that more massive and bigger white dwarfs are found with our predictions.

Our calculations enable the finding of maximum masses around  $2.4M_\odot$  for stable charge white dwarfs configurations. Albeit we reach very high electric fields to obtain these results ( $\approx 10^{16}$  [V/cm]), the electric field outside the star is zero. For the first time, isolated charged white dwarfs are obtained with a zero electric field outside.

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