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The detection of gravitational waves

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Abstract. On February 11 2016 the scientific community was surprised by the announcement of the first observation of gravitational waves made by the LIGO and Virgo collaborations. The signal was named GW150914 indicating that the astrophysical event that produced it occurred the 14 of September of 2015. The detected waveform is consistent with the predictions of general relativity for the spiral and merger of a binary black hole system and subsequent relaxation period known as *ring down* of the resulting black hole. That detection was the first direct evidence of the existence of gravitational waves and the first observation of the merger of a binary black hole system ¹.

1. Introduction

The first direct detection of a gravitational wave signal made by LIGO (Laser Interferometer Gravitational-Wave Observatory) represent a successful confirmation of a prediction by Einstein's theory of general relativity made more than one hundred years ago. The signal was detected by the two LIGO detectors located in Hanford, Washington and Livingston, Louisiana. The gravitational waves were produced by the collision of two black holes of approximately 36 and 29 solar masses [1, 2]. They were orbiting each other approximately 350km apart and subsequently merged to form another black hole. In order to extract information about the source that produced those gravitational waves many analytical and computational methods were required. Numerical Relativity templates and advanced data analysis algorithms were developed to pursuit that end [3].

In this respect, GW150914 allows to test General Relativity in the strong field. The signal was clear enough to allow us to perform some consistency tests with the predictions of General Relativity. The main results came from comparing the observed signal's amplitude and phase with numerical relativity predictions. These comparisons allowed to estimate the parameters of the binary, such as the mass of the individual black holes, their distance from the Earth and the spin of the final black hole [4].

Here, we present some of the basis of gravitational wave theory that will help us to understand the basic principles of one of the most important discoveries in physics.

2. Gravitational Waves. Foundations

Gravitational waves are produced by accelerating masses and they propagate at the speed of light. Detecting them has been very hard because they have almost no interaction with matter.

¹ Since then, there have been two more detections named GW151226 and GW170104 which are also consistent with the merger of two black holes [5, 6].



Even the strongest gravitational waves from extreme astrophysical events such as the merger of two black holes or neutron stars are only expected to produce relative length variations of order of 10^{-21} . As they travel, gravitational waves squash and stretch space-time in the plane perpendicular to their direction of propagation, hence are transversal waves.

General Relativity is a nonlinear theory. Therefore in general there is no clear distinction between waves and the rest of the metric. One can, however, use the notion of a wave in certain limiting situations: in a linearized theory, as small perturbations of a smooth background metric or in post-Newtonian theory. Perhaps the more straightforward way to introduce gravitational waves is through linearized theory around the space-time of Minkowski.

If the gravitational field strength is weak, then it is possible to find a Cartesian-like coordinate system such that the components of the metric tensor $g_{\mu\nu}$, will be small perturbations $h_{\mu\nu}$, of the flat Minkowski metric components $\eta_{\mu\nu}$, thus:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} , \quad |h_{\mu\nu}| \ll 1. \quad (1)$$

We will assume that $h_{\mu\nu}$ is a second rank tensor in the background metric and we will raise and lower the indices with $\eta^{\mu\nu}$ and $\eta_{\mu\nu}$. With this, we can linearize Einstein's field equations with respect to $h_{\mu\nu}$.

First, the linearized the Ricci tensor is

$$R_{\mu\nu} = \frac{1}{2} (\partial_\alpha \partial_\mu h^\alpha{}_\nu + \partial_\alpha \partial_\nu h^\alpha{}_\mu - \partial_\mu \partial_\nu h^\alpha{}_\alpha - \square h_{\mu\nu}) + O(h^2) , \quad (2)$$

where the d'Alembertian operator in the flat Minkowski space-time is $\square := \eta^{\mu\nu} \partial_\mu \partial_\nu$, the linearized Ricci scalar becomes

$$R = \eta^{\mu\nu} R_{\mu\nu} + O(h^2) = \partial_\mu \partial_\nu h^{\mu\nu} - \square h_{\mu\nu} + O(h^2) . \quad (3)$$

The Einstein tensor, $G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is written as

$$G_{\mu\nu} = \frac{1}{2} \left(\partial_\alpha \partial_\mu h^\alpha{}_\nu + \partial_\alpha \partial_\nu h^\alpha{}_\mu - \partial_\mu \partial_\nu h^\alpha{}_\alpha - \square h_{\mu\nu} + \eta_{\mu\nu} (\square h - \partial_\alpha \partial_\beta h^{\alpha\beta}) \right) + O(h^2) . \quad (4)$$

It is possible to simplify the linearized Einstein tensor by introducing the trace reverse metric perturbation as

$$\bar{h}_{\mu\nu} := h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h , \quad (5)$$

and get

$$G_{\mu\nu} = \frac{1}{2} \left(\partial_\alpha \partial_\mu \bar{h}^\alpha{}_\nu + \partial_\alpha \partial_\nu \bar{h}^\alpha{}_\mu - \square \bar{h}_{\mu\nu} - \eta_{\mu\nu} \partial_\alpha \partial_\beta \bar{h}^{\alpha\beta} \right) + O(h^2) . \quad (6)$$

Using a infinitesimal coordinate transformation –known as gauge transformation– of the form

$$x'^\alpha = x^\alpha + \xi^\alpha(x^\beta) \quad (7)$$

one can choose coordinates for which the following additional (harmonic gauge) conditions are fulfilled:

$$\partial_\beta \bar{h}^{\beta\alpha} = 0 . \quad (8)$$

If these conditions are satisfied, the linearized Einstein tensor simplifies considerably:

$$G_{\mu\nu} = -\frac{1}{2} \square \bar{h}_{\mu\nu} + O(h^2) . \quad (9)$$

The linearized Einstein's field equations are:

$$\square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} . \quad (10)$$

As a result, time-dependent solutions of these equations can be interpreted as weak gravitational waves. The simplest solution of Eqs. (10) in vacuum ($T_{\mu\nu} = 0$) is a monochromatic plane wave of the form

$$\bar{h}_{\mu\nu} = A_{\mu\nu} \cos(k_\alpha x^\alpha) . \quad (11)$$

$A_{\mu\nu}$ is the constant amplitude and we have chosen the initial phase of the wave as zero for simplicity. It is straightforward to show that (11) are solutions of (10) only if

$$\eta^{\alpha\beta} k_\alpha k_\beta = 0 , \quad (12)$$

i.e. k^α are the components of a null four vector. Consequently gravitational waves travel at the speed of light.

Einstein's field equations take the simple form (10) only if the harmonic gauge conditions (8) are satisfied. This leads to the requirement that the plane wave solution $\bar{h}_{\mu\nu} k^\nu$ is orthogonal to k^ν :

$$\bar{h}_{\mu\nu} k^\nu = 0 . \quad (13)$$

Yielding to the conclusion that gravitational waves are transversal. If we additionally introduce the angular frequency as $\omega = ck^0$. The frequency f of the wave, measured in hertz, is related with ω by $\omega = 2\pi f$. The 3-spatial components of k^μ form the wave vector, which points to the direction in which the wave propagates. Its Euclidean length is related to the wavelength λ as $\lambda = 2\pi/|k^i|$. The actual degrees of freedom of the wave field $h_{\mu\nu}$, are best represented choosing a suitable coordinate system as follows.

At each event in the space-time region covered by some almost Lorentzian and harmonic coordinates x^α let us choose time like unit vector u^μ and consider a gauge transformation generated by the functions ξ^α of the form:

$$\xi^\alpha = B^\alpha \cos(k_\beta x^\beta) . \quad (14)$$

It is possible to choose the quantities B^α in such a way that in the new harmonic coordinates $x'^\alpha = x^\alpha + \xi^\alpha$ the following conditions are fulfilled [8]:

$$\bar{h}'_{\mu\nu} u'^\nu , \quad \bar{h}'_{\mu}{}^\mu = 0 . \quad (15)$$

The transformations (14) preserve the conditions

$$\bar{h}'_{\mu\nu} k'^\nu = 0 , \quad (16)$$

and

$$\bar{h}'_{\mu\nu} = \bar{h}_{\mu\nu} . \quad (17)$$

Equations (15) define the *transverse and traceless* (TT) coordinate system related to the 4-vector u^μ . These equations comprise eight independent constraints on the components of the plane-wave solution $\bar{h}'_{\mu\nu}$: any plane monochromatic gravitational wave possesses two independent degrees of freedom, often also called waves polarizations. The simplest way to describe these two polarizations is by choosing u^μ of the form $u^\mu = (1, 0, 0, 0)$. Then the wave field satisfies

$$\bar{h}_{\mu 0} = 0 . \quad (18)$$

From now on, we drop the tilde in $h_{\mu\nu}$. If we assume that the gravitational wave propagates in the $+z$ direction. Then $k^\mu = (\omega/c, 0, 0, \omega/c)$ and

$$\bar{h}_{\mu 3} = 0 . \quad (19)$$

The traceless condition reduces to

$$\bar{h}_{11} + \bar{h}_{22} = 0 . \quad (20)$$

It is common to denote the independent components of the wave field as

$$h_+ = \bar{h}_{11} = -\bar{h}_{22} , \quad h_\times = \bar{h}_{12} = \bar{h}_{21} . \quad (21)$$

The functions h_+ and h_\times are called *plus* and *cross* polarizations of the wave. With this, the waveform in the TT gauge has the following matrix form:

$$h_{\mu\nu}^{TT}(t, \vec{x}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+(t - z/c) & h_\times(t - z/c) & 0 \\ 0 & h_\times(t - z/c) & -h_+(t - z/c) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} ,$$

where, for plane waves propagating in the $+z$ direction,

$$h_+ = A_+ \cos\left(\omega\left(t - \frac{z}{c}\right)\right) , \quad h_\times = A_\times \cos\left(\omega\left(t - \frac{z}{c}\right)\right) . \quad (22)$$

where A_+ and A_\times are constant amplitudes.

3. Response of test particles to gravitational waves

Gravitational waves cannot be detected locally due to their asymptotic character. According to the principle of equivalence the effects of gravity may be transformed away and the effect of a gravitational wave on a single point test particle has no measurable consequences. The coordinates of the particle remain unchanged when a gravitational wave passes [9]. Gravitational waves can be detected only by their influence on two or more test particle at different locations. Interferometer gravitational wave detectors are devices built to measure induced length changes [10]. The properties of the waves such as frequency and the amplitude are related to the motion of the test masses involved.

Gravitational waves represent propagating curvature perturbation and their effect on test particles is of tidal nature. In order to see the previous statement in detail, let us consider two nearby geodesics both parametrized by the same parameter λ , let ξ^μ be a vector connecting points on the two curves with the same value of λ . The connecting vector is said to measure the geodesic deviation of the curves. The equation that tell us how the geodesic deviation varies along the curves is given by

$$\frac{d^2\xi^\mu}{d\lambda^2} + R^\mu_{\alpha\nu\beta} u^\alpha \xi^\nu u^\beta = 0 , \quad (23)$$

where u^μ is the vector tangent to the geodesics [11]. The physical effect of gravitational waves is best explain with this equation, that, in the linearized limit for the comoving geodesic normal coordinates becomes:

$$\frac{d^2\xi^k}{dt^2} = -R^k_{0j0}\xi^j . \quad (24)$$

Substituting the components of the Riemann tensor in favor of the wave field in the TT gauge one gets:

$$\xi^k{}_{,00} = \frac{1}{2}\xi^k h_{ij,00}^{TT} . \quad (25)$$

Inserting the components of h_{ij} and considering a wave propagating in the $+z$ direction one gets a set of equations of the form:

$$\xi_{,tt}^x = \frac{1}{2}\xi^x h_{xx,tt} + \frac{1}{2}\xi^y h_{xy,tt} , \quad (26)$$

$$\xi_{,tt}^y = \frac{1}{2}\xi^x h_{xy,tt} - \frac{1}{2}\xi^y h_{xy,tt} , \quad (27)$$

$$\xi_{,tt}^z = 0 . \quad (28)$$

These equations show that only the x and y components of ξ^i of the nearby geodesics will be disturbed by the gravitational wave traveling in the $+z$ direction. Hence test particles are influenced in the directions perpendicular to the wave propagation as previously stated. We can use the above equations to describe the behavior of a ring of free stationary free particles in the $x - y$ plane when a gravitational wave passes in the $+z$ direction. Let us consider two particles separated in the x direction. At lowest order, we can neglect the terms with ξ^y to get:

$$\xi_{,tt}^x = \frac{1}{2}\xi^x h_{xx,tt} , \quad \xi_{,tt}^y = \frac{1}{2}\xi^x h_{xy,tt} . \quad (29)$$

Similarly, for two nearby particles separated in the y direction

$$\xi_{,tt}^y = \frac{1}{2}\xi^x h_{xy,tt} , \quad \xi_{,tt}^x = -\frac{1}{2}\xi^x h_{xx,tt} . \quad (30)$$

Let us assume that a wave with $h_{xy} = 0$, $h_{xx} \neq 0$ hits the particles. First the particles along the x direction will come towards each other and they move away as h_{xx} changes sign. This is the h_+ polarization. If on the other hand, $h_{xy} \neq 0$, $h_{xx} = 0$ the movement of the ring will be the same but rotated 45 degrees. This is the h_\times polarization.

Gravitational waves carry energy. This energy is released by accelerating masses and is taken away by gravitational waves. In the following we describe how to estimate the amount of energy radiated.

4. Energy and angular momentum carried by gravitational waves

The stress-energy carried by GWs cannot be localized within a wavelength [12]. Instead, one can say that a certain amount of stress energy is contained in a region of the space which extends over several wavelengths.

The energy-momentum tensor for gravitational waves $t_{\mu\nu}$, is obtained by averaging the squared gradient of the wave field over several wavelengths. In the TT gauge it has components:

$$t_{\mu\nu} = \frac{c^4}{32\pi G} \langle \partial_\mu h_{ij}^{\text{TT}} \partial_\nu h_{ij}^{\text{TT}} \rangle . \quad (31)$$

If one assumes that $h_{0\mu}^{\text{TT}} = 0$ then

$$t_{\mu\nu} = \frac{c^4}{32\pi G} \sum_{i,j=1}^3 \langle \partial_\mu h_{ij}^{\text{TT}} \partial_\nu h_{ij}^{\text{TT}} \rangle . \quad (32)$$

For a plane wave propagating in the z direction, the nonzero components of the tensor $t_{\mu\nu}$ are simply

$$t_{00} = -t_{0z} = -t_{z0} = t_{zz} = \frac{c^2}{16\pi G} \langle (\partial_t h_+)^2 + (\partial_t h_\times)^2 \rangle . \quad (33)$$

A very instructive result arises if we consider the monochromatic wave (11) and substitute in the previous equation

$$t_{00} = \frac{c^2 \omega^2}{16\pi G} (A_+^2 \langle (\sin^2(\omega(t - z/c))) \rangle + A_\times^2 \langle (\sin^2(\omega(t - z/c))) \rangle) . \quad (34)$$

The average of the sine squared over one period gives 1/2 and simplifying one obtains, in terms of the frequency measured in hertz:

$$t_{00} = \frac{\pi c^2 f^2}{8G} (A_+^2 + A_\times^2) . \quad (35)$$

This expression is very useful to give some orders of magnitude. For example, the energy flux in gravitational waves from the collapse of a supernova core to form a 10 solar masses black hole at a distance of 15 Mpc, will have frequencies about 10^3 Hz and amplitudes of the order of $A_+ \sim A_\times \sim 10^{-22}$. The computed flux is about 3 ergs/cm² sec. This is about ten orders of magnitude larger than the observed electromagnetic energy flux [8].

5. The quadrupole formalism

Perhaps the simplest technique for computing the gravitational wave field in the presence of matter is the quadrupole formalism. Einstein himself derived the quadrupole formula for gravitational radiation by solving the linearized equations (10). See for instance [13].

This formalism is especially important because it is very accurate for many astrophysical sources of gravitational waves and is easy to calculate. It does not require for high accuracy in the computation on the strength of the source's internal gravity, but requires that internal motions inside the source are slow compared to the speed of light. This requirement implies that the wavelength of the gravitational waves emitted is much larger than the source's characteristic size, $\lambda \gg L_s$.

The lowest allowed multipole for gravitational radiation is the quadrupole. The monopole is forbidden as a result of mass conservation. In a similar way, dipole radiation is absent as a result of momentum conservation.

In order to gain some insight about the quadrupole formalism, let us introduce some coordinate system (t, \vec{x}) centered on the gravitational wave source. If we assume that the source has weak internal gravity and small internal stresses, Newtonian gravity is a good approximation to general relativity inside and near the source. Let us assume that an observer at rest (with respect to the source) measures the gravitational wave field $h_{\mu\nu}^{\text{TT}}$.

The quadrupole formalism allow us to get an expression for the gravitational wave field:

$$h_{ij}(t, x^i) = \frac{2}{R} \frac{G}{c^4} \ddot{Q}_{ij}^{\text{TT}} \left(t - \frac{R}{c} \right) , \quad (36)$$

where R is the euclidean distance from the point x^i to the center of the source and t is the proper time measured by the observer. The quantity Q_{ij} is the symmetric part of the second moment of the sources mass density ρ computed in a Cartesian coordinate system centered on the source evaluated at the retarded time $t_{ret} = t - \frac{R}{c}$

$$Q_{ij}^{\text{TT}} = \int d^3x \rho \left(x_i x_j - \frac{1}{3} \delta_{ij} r^2 \right) , \quad (37)$$

the superscript TT means that one needs to keep only the part that is transverse to the direction of propagation of the wave and is traceless. Furthermore Q_{ij} , is the coefficient of the $1/r^5$ term in the multipolar expansion of the source's Newtonian gravitational potential Φ [14].

$$\Phi = -\frac{GM}{r} - \frac{3G}{2} \frac{Q_{ij} x^i x^j}{r^5} - \frac{5G}{2} \frac{Q_{ijk} x^i x^j x^k}{r^7} + \dots . \quad (38)$$

From the quadrupole formula (37) and Eq. (32) for the energy-momentum tensor, one can compute the fluxes of energy and angular momentum carried by the wave.

The rates of emission of energy \mathcal{F}_E and angular momentum \mathcal{F}_{J_i} in the quadrupole approximation are [8].

$$\mathcal{F}_E = \frac{G}{5c^5} \sum_{i,j=1}^3 \langle (\ddot{Q}_{ij})^2 \rangle , \quad (39)$$

and

$$\mathcal{F}_{J_i} = \frac{2G}{5c^5} \sum_{j,k,l=1}^3 \epsilon_{ijk} \langle \ddot{Q}_{jl} \ddot{Q}_{kl} \rangle . \quad (40)$$

Because of the conservation of energy and angular momentum the source's energy and angular momentum should decrease at rates equal to minus rates given by Eqs. (39) and (40), thus one arrives to the energy and angular momentum loss equations:

$$\frac{dE^{\text{source}}}{dt} = -\frac{G}{5c^5} \sum_{i,j=1}^3 \langle (\ddot{Q}_{ij})^2 \rangle , \quad (41)$$

and

$$\frac{dJ_i^{\text{source}}}{dt} = -\frac{2G}{5c^5} \sum_{j,k,l=1}^3 \epsilon_{ijk} \langle \ddot{Q}_{jl} \ddot{Q}_{kl} \rangle . \quad (42)$$

The estimation of the energy and angular momentum radiated using (41) and (42) is very accurate for all sources as long as the wavelength is much longer than the source size L_s . In the following section we give an example of how to compute the energy radiated by a slow moving system of compact objects.

6. The two-body problem

One of the most important applications of the quadrupole formalism as an approximation to general relativity is the two-body problem, i.e. the problem of finding the motion and gravitational radiation of self gravitating relativistic systems of two extended bodies. This calculation is based on a Newtonian description of the dynamics of the two bodies. The Newtonian dynamics will depart from the relativistic description when the relative velocity of the binary is comparable with the speed of light or when the gravitational energy becomes large compared to the rest mass energy of the system.

Relativistic binary systems exist in the universe, examples of this are neutron stars or black holes binaries. These systems emit gravitational waves and are the most prominent sources of gravitational radiation that can be detected by LIGO. Surprisingly one can learn a lot from the collision of two bodies using the quadrupole formalism presented above.

A Newtonian binary system is illustrated in Fig. 1. For simplicity let us assume that the system follow circular orbits and the masses are equal. Thus

$$M_1 = M_2 = M , \quad \text{and} \quad R_1 = R_2 = R . \quad (43)$$

Introducing cylindrical coordinates

$$x(t) = R \cos \Omega t , \quad y(t) = R \sin \Omega t , \quad z(t) = 0 . \quad (44)$$

The nonzero components of the quadrupole moment (37) are

$$Q_{xx} = MR^2 \left(\frac{1}{3} + \cos 2\Omega t \right) , \quad Q_{yy} = MR^2 \left(\frac{1}{3} - \cos 2\Omega t \right) , \quad Q_{zz} = -\frac{2}{3} MR^2 . \quad (45)$$

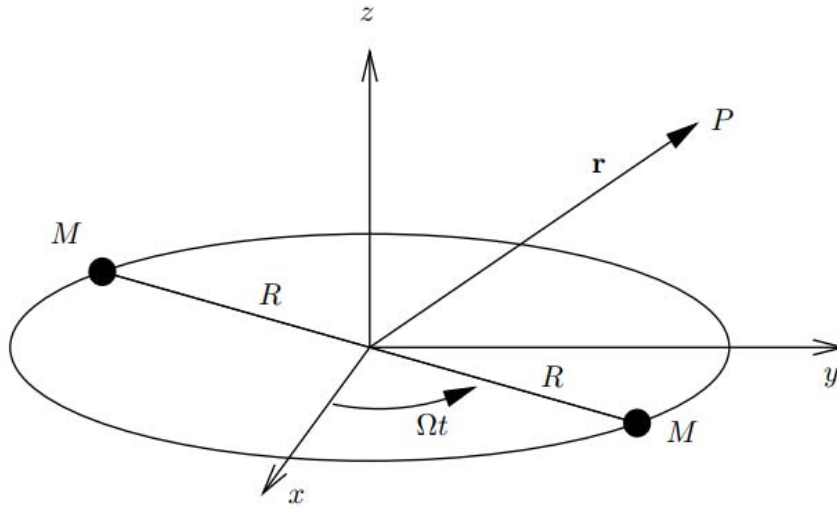


Figure 1. Two bodies with equal masses orbiting around their center of mass in circular orbits.

Taking the third time derivative and replacing into the expression to compute the gravitational wave luminosity as the energy lost by the source $L_{gw} = -\frac{dE^{\text{source}}}{dt}$

$$L_{gw} = \frac{G}{5c^5} \langle \ddot{Q}^{ij} \ddot{Q}_{ij} \rangle \quad (46)$$

$$= \frac{G}{5c^5} \langle (\ddot{Q}_{xx})^2 + 2(\ddot{Q}_{xy})^2 + (\ddot{Q}_{yy})^2 + (\ddot{Q}_{zz})^2 \rangle \quad (47)$$

$$= \frac{128G}{5c^5} \Omega^6 M^2 R^4 . \quad (48)$$

The frequency Ω and the period T are related by $\Omega = 2\pi/T$. Furthermore, using Kepler's third law: $R^3 = \frac{GMT}{16\pi^2}$ the gravitational wave luminosity can be expressed in terms of T

$$L_{gw} = \frac{128}{5} 2^{2/3} \frac{c^5}{G} \left(\frac{\pi GM}{c^3 T} \right)^{10/3} . \quad (49)$$

$$(50)$$

This may be an enormous luminosity, For example, the estimated luminosity for the event GW150914 is $L \sim 0.2 \times 10^{-3} L_0$, where the Planck luminosity is $L_0 := c^5/G \sim 10^{59}$ erg/s. By comparison, the luminosity of the sun is $L_\odot = 3.839 \times 10^{33}$ erg/s.

The Newtonian binding energy of the binary is $E = -\frac{1}{2} \frac{GmM}{R}$, and taking the derivative

$$\frac{dE}{dt} = \frac{GmM}{2R^2} \frac{dR}{dt} , \quad (51)$$

thus as the gravitating system losses energy by emitting radiation, the distance between the two bodies decreases at a rate

$$\frac{dR}{dt} = -\frac{64}{5} \frac{G^3}{c^5} \frac{M^3}{R^3} . \quad (52)$$

The size of the binary orbit decreases and the components move faster leading to emission of gravitational waves with increasing amplitude and frequency. This last stage is known as the *chirp* signal.

The orbital frequency increases accordingly to .

$$\frac{1}{T} \frac{dT}{dt} = \frac{3}{2} \frac{1}{R} \frac{dR}{dt} , \quad (53)$$

and the system will merge after a time t_{merger}

$$t_{\text{merger}} = \frac{5}{256} \frac{c^5}{G^3} \frac{R_0^4}{M^5} , \quad (54)$$

where R_0 is the initial separation.

The previous analysis can be generalized to consider elliptic orbits, but it is possible to shown that gravitational wave emission circularize the orbits faster than the coalescence timescale, making the study of circular orbits quite useful [8]. For instance, applying this formalism to the binary Pulsar of Hulse and Taylor ²: with parameters $R_0 = 5\text{Kpc}$, $M_1 \sim M_2 \sim 1.4M_\odot$, $dT/dt = 7\text{h } 45\text{min}$. one gets:

$$T \sim 2.4 \times 10^{-12} \text{sec/sec}, \quad f_{\text{gw}} \sim 7 \times 10^{-5} \text{Hz}, \quad t_{\text{merger}} \sim 3.5 \times 10^8 \text{yr}. \quad (55)$$

which is in excellent agreement with the observed values for $dT/dt = (2.30 \pm 0.22) \times 10^{-12} \text{sec/sec}$ [16].

7. Outlook

Gravitational-wave detection will allow, in a near future, new and more precise measurements of astrophysical events. We may be able to determine the formation mechanism of two merging black holes by measuring accurately their spins. Although LIGO was not able to measure the magnitude of the spins of the colliding black holes very accurately, better measurements might be possible with more sensitive detectors and better data analysis techniques.

The implications of gravitational wave astronomy for astrophysics are impressive. Multiple detections may allow us to determine the rate at which black holes merge and to test the models used to describe the formation of binary systems. Furthermore, electromagnetic counterparts of gravitational waves may be used to have a better understanding of astrophysical phenomena [17].

With the detection of LIGO, we are entering in a new age of gravitational-wave astronomy. With this new window we will be able to detect phenomenon that was not possible to detect using the standard tools. Surprisingly the first detection of LIGO came from the merger of two black holes, the objects we cannot see with electromagnetic radiation and represent one of the most precious jewels of General Relativity. Certainly, we are in the verge of dazzling discoveries.

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² The Hulse and Taylor binary neutron star system provided the first indirect astrophysical evidence of the existence of gravitational waves [15].

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