Primordial magnetic fields from second-order cosmological perturbations: Tight coupling approximation

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Abstract

We explore the possibility of generating large-scale magnetic fields from second-order cosmological perturbations during the pre-recombination era [1]. The key process for this is Thomson scattering between the photons and the charged particles within the cosmic plasma. To tame the multi-component interacting fluid system, we employ the tight coupling approximation. It is shown that the source term for the magnetic field is given by the vorticity, which signals the intrinsically second-order quantities, and the product of the first order perturbations. The vorticity itself is sourced by the product of the first-order quantities in the vorticity evolution equation. The magnetic fields generated by this process are estimated to be $\sim 10^{-29}$ Gauss on the horizon scale.

1 Introduction

Magnetic fields are known to be present on various scales in the universe [2, 3]. For example, magnetic fields are observed in galaxies and clusters, with intensity ~ 1 Gauss. Only an upper limit has been given for magnetic fields on cosmological scales, $< 10^{-9}$ Gauss. Primordial large-scale magnetic fields may be present and serve as seeds for the magnetic fields in galaxies and clusters, which are amplified through the dynamo mechanism after galaxy formation [4].

A number of models have been proposed for generating large-scale magnetic fields in the early universe. However, they rely more or less on some unknown physics. In the present paper, we discuss magnetogenesis in the pre-recombination era using only the conventional physics that has been established. The generation of magnetic fields in this era has been studied in Refs. [5, 6, 7, 8]. Now it is widely accepted that large-scale cosmological perturbations, generated from inflation in the early universe, evolve into a variety of structures such as the cosmic microwave background anisotropies and galaxies. However, inflation produces only density fluctuations (scalar perturbations) and gravitational waves (tensor perturbations); vector perturbations, and hence large-scale magnetic fields, are quite unlikely to be generated in the context of standard inflationary scenarios. Even if they were generated due to some mechanism, they only decays without any sources. This argument is based on *linear* perturbation theory, and so we will be studying second-order perturbations to overcome this difficulty. We consider a multi-fluid system composed of photons, electrons, and protons, which are tightly coupled via Thomson and Coulomb scattering but slightly deviate from each other [9]. Thomson scattering is important for the generation of large-scale magnetic fields in the pre-recombination era, because a rotational current will be produced by the momentum transfer due to the Thomson interaction. We shall see how this process occurs by doing the tight coupling expansion.

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2 Basic equations in second-order cosmological perturbation theory

The background spacetime is given by the spatially flat Friedmann-Lemaitre-Robertson-Walker metric. We write the perturbed metric in the Poisson gauge as

$$ds^2 = a^2(\eta)\left[-\left(1+2\phi^{(1)}+2\phi^{(2)}\right)d\eta^2 + 2\chi_i^{(2)}d\eta dx^i + \left(1-2\mathcal{R}^{(1)}-2\mathcal{R}^{(2)}\right)\delta_{ij}dx^i dx^j\right], \tag{1}$$

where a is the scale factor and η is the conformal time We have dropped the first order vector perturbations $\chi_i^{(1)}$ since they are not generated from inflation in the standard scenarios. We also neglect the tensor perturbations (gravitational waves) for simplicity.

We consider a multi-fluid system composed of photons (γ), electrons (e), and protons (p). We assume that the energy-momentum tensor for each fluid component is given by that of a perfect fluid (i.e., we neglect anisotropic stresses):

$$T_{(I)} \quad = (\rho_I + p_I)u_{(I)} \;\; u_{(I)} + p_I\delta \qquad (I = \gamma, p, e), \tag{2}$$

where $p_I = w_I\rho_I$ ($w_p = w_e = 0, w_\gamma = 1/3$) and $u_{(I)}$ is the 4-velocity of the fluid satisfying $g \quad u_{(I)} \;\; u_{(I)} \;\; = -1$. We define $\delta v_{(IJ)i} \equiv v_{(I)i} - v_{(J)i}$, $\beta \equiv m_e/m_p$ and $v_{(b)i}$ as the center of baryon's mass velocity.

The equations of motion governing the present 3-fluid system are given by

$$\begin{aligned}
\nabla \;\, T_{(\gamma)i} &= \kappa_i^{\gamma p} + \kappa_i^{\gamma e}, &(3)\\
\nabla \;\, T_{(p)i} &= e n_p E_i + \kappa_i^{pe} + \kappa_i^{p\gamma}, &(4)\\
\nabla \;\, T_{(e)i} &= -e n_e E_i + \kappa_i^{ep} + \kappa_i^{e\gamma}, &(5)
\end{aligned}$$

where we neglected the Lorentz force from the magnetic field because it will give rise to higher order contributions and scattering term is

$$\begin{aligned}
\kappa_i^{\gamma e} &= -\kappa_i^{e\gamma} = -\sigma_T n_e \rho_\gamma \left(u_{(\gamma)i} - u_{(e)i}\right), &(6)\\
\kappa_i^{\gamma p} &= -\kappa_i^{p\gamma} = -\tfrac{m_e^2}{m_p^2}\sigma_T n_p \rho_\gamma \left(u_{(\gamma)i} - u_{(p)i}\right), &(7)
\end{aligned}$$

where σ_T, ρ_γ, n_e, n_p and m_e, m_p are the Thomson cross section, the energy density of photons, the number density and mass of electrons and protons [6, 8]. The momentum transfer due to Coulomb scattering is written as

$$\kappa_i^{pe} = -\kappa_i^{ep} = -e^2 n_p n_e \eta_C \left(u_{(p)i} - u_{(e)i}\right), \tag{8}$$

where η_C is a electric resistivity which comes from Coulomb scattering.

We get "Ohm's law" from Eq.(4) and Eq.(5):

$$E_i = \frac{1-\beta^3}{1+\beta}\frac{\sigma_T}{e}a\rho_\gamma(1-2\mathcal{R})\delta v_{(\gamma b)i}. \tag{9}$$

where we neglect $\delta v_{(pe)i}$ because $\delta v_{(pe)i} \ll \delta v_{(\gamma b)i}$ in our case [8]. This equation may be regarded as "Ohm's law" in some sense. In the standard Ohm's law, the electric field is proportional to the electric current density $\sim e\delta v_{(pe)i}$. It is reminded that the contribution from the electric current gives us the diffusion term in the evolution equation for the magnetic field, and then the source for the magnetic field cannot be induced. However, the electric field is proportional to $\delta v_{(\gamma b)i}$ in the above formula. The current in our "Ohm's law"is originated from the velocity difference between protons and electrons through the interaction with photons. Indeed, if one takes the same mass limit of $\beta \to 1$ (though it is not realized in the nature), the electric field cannot be generated.

Maxwell equation, $\nabla_{[\lambda}F_{\quad]} = 0$, is

$$\left(a^3 B^i\right)' = -\epsilon^{ijk}\partial_j\left[a\left(1+\phi\right)E_k\right] - \epsilon^{ijk}\left(a v_j E_k\right)'. \tag{10}$$

Thus we substitute Eq.(9) into Eq.(10) and get

$$\left(a^3B^i\right)' = -\frac{1-\beta^3}{1+\beta}\frac{\sigma_T}{e}\epsilon^{ijk}a^2\left[\partial_j\left(\rho_\gamma\delta v_{(\gamma b)k}\right)+\rho_\gamma\partial_j(\phi-2\mathcal{R})\delta v_{(\gamma b)k}+\frac{1}{a^2}\left(\rho_\gamma v_j a^2\delta v_{(\gamma b)k}\right)'\right]. \qquad (11)$$

Our remaining task is to evaluate $\delta v_{(\gamma b)i}$. The evolution equation of $\delta v_{(\gamma b)i}$ is $(4\rho_\gamma/3)^{-1}\times(3)-[m_p\,(1+\beta)\,n]^{-1}\times[(4)+(5)]$:

$$\frac{\rho'_\gamma}{\rho_\gamma}\left(v_{(\gamma)i}+\chi_i\right)-\frac{n'}{n}\left(v_{(b)i}+\chi_i\right)+\left(\delta v_{(\gamma b)i}\right)'+4\mathcal{H}\delta v_{(\gamma b)i}-(\phi+2\mathcal{R})\left[\frac{\rho'_\gamma}{\rho_\gamma}v_{(\gamma)i}-\frac{n'}{n}v_{(b)i}+\left(\delta v_{(\gamma b)i}\right)'+4\mathcal{H}\delta v_{(\gamma b)i}\right]$$
$$-5\mathcal{R}'\delta v_{(\gamma b)i}+\frac{1}{4}\frac{\partial_i\rho_\gamma}{\rho_\gamma}+\partial_j\left(v_{(\gamma)i}v^j_{(\gamma)}\right)-\frac{1}{1+\beta}\partial_j\left(v_{(p)i}v^j_{(p)}+\beta v_{(e)i}v^j_{(e)}\right)=-\alpha(1-2\mathcal{R})\delta v_{(\gamma b)i}, \qquad (12)$$

where α is defined as

$$\alpha := \frac{1+\beta^2}{1+\beta}(1+R)\frac{a\sigma_T\rho_\gamma}{m_p}\left(=\frac{\beta(1+\beta^2)}{1+\beta}(1+R)\frac{1}{\tau_T}\right) \qquad (13)$$

with $R := 3m_p\,(1+\beta)\,n/4\rho_\gamma$ and τ_T is the timescale of Thomson scattering for electrons.

3 Tight coupling approximation

We are to solve Eq. (12) using the tight coupling approximation (TCA) [9]. In this approximation the time scale of Thomson scattering, τ_T, is assumed to be much smaller than the wavelengths of the perturbations (k^{-1}). Thus, the small expansion parameter of the TCA is $k\tau_T$, which is dimensionless. During the pre-recombination era, photons, protons, and electrons are strongly coupled via Thomson scattering, and hence the TCA will be a good approximation.

At zeroth order in the TCA, all fluid components have the same velocity v_i and the density fluctuations are adiabatic. Following Ref. [8], we define the deviation from the adiabatic distribution for baryons by $n_b = \bar{n}_b\,(1+\Delta_b)$. Then, we expand various quantities such as Δ_b and $v_{(I)i}$ in terms of the tight coupling parameter $k\tau_T$:

$$\Delta_b = \Delta_b^{(I)}+\Delta_b^{(II)}+\cdots, \qquad (14)$$
$$v_{(\gamma)i}=v_i, \qquad v_{(b)i}=v_i+v^{(I)}_{(b)i}+v^{(II)}_{(b)i}+\cdots, \qquad (15)$$
$$\delta v_{(\gamma b)i}=\delta v^{(I)}_{(\gamma b)i}+\delta v^{(II)}_{(\gamma b)i}+\cdots, \qquad (16)$$

where v_i is the common velocity of photons and baryons in the tight coupling limit. Our notation is that Roman and Arabic numerals stand for the order of TCA and that in cosmological perturbation theory, respectively. Here, we adopt the photon frame, so that $\Delta_\gamma^{(I)}=\Delta_\gamma^{(II)}=0$. In other words, the quantities associated with photons give the "background" in the TCA. Note that we consider cosmological perturbation theory and the TCA simultaneously. The following analysis includes cosmological perturbations up to second-order and the tight coupling expansion up to TCA(II), where TCA(n) denotes the tight coupling approximation at n-th order.

4 Generation of magnetic fields

We solve Eq.(12) for $\delta v_{(\gamma b)i}$ using TCA and substitute the results into Eq.(11). When we consider up to TCA(I), the equation of the magnetic field is

$$(a^3B^i)' = 2\frac{1-\beta^3}{1+\beta}\frac{\sigma_T}{e}a^4\bar{\rho}^{(0)}_\gamma\frac{\mathcal{H}}{\bar{\alpha}^{(0)}}\omega^{(2)i}. \qquad (17)$$

where $\omega^{(2)i}$ is photon's vorticity, $\omega^i \equiv -\frac{1}{2}\varepsilon^{i\ \ \rho\lambda}u_{(\gamma)\lambda}\nabla\ \ u_{(\gamma)\rho}$. The evolution equation of the vorticity is obtained by taking the curl of the total momentum conservation:

$$\left(2a^2\bar{\rho}^{(0)}_T\omega^{i(2)}\right)'+8a^2\mathcal{H}\bar{\rho}^{(0)}_T\omega^{i(2)}=0. \qquad (18)$$

It is important to note here that there is no source for the vorticity at TCA(I). Therefore, the vorticity decays and the magnetic field is not generated at TCA(I).

Next we consider up to TCA(II). Then the equation of the magnetic field is

$$(a^3 B^i)' = \frac{1-\beta^3}{1+\beta}\frac{\sigma_T}{e}a^2\bar{\rho}^{(0)}_\gamma\left[\frac{2a^2\mathcal{H}}{\bar{\alpha}^{(0)}}\omega^{(2)i} + \epsilon^{ijk}\frac{\bar{R}^{(0)}}{1+\bar{R}^{(0)}}\partial_j\Delta_b^{(I,1)}\delta v_{(\gamma b)k}^{(I,1)}\right]. \tag{19}$$

In the same way the evolution equation of the vorticity is

$$(a^2\omega^{(2)i})' + \frac{\mathcal{H}\bar{R}^{(0)}}{1+\bar{R}^{(0)}}a^2\omega^{(2)i} = \frac{\bar{R}^{(0)}}{2(1+\bar{R}^{(0)})^2}\epsilon^{ijk}\partial_j\Delta_b^{(I,1)}\delta v_{(\gamma b)k}^{(I,1)}. \tag{20}$$

Since the right hand side in Eq. (20) contain the source term for the vorticity at TCA(II), the vorticity can be generated at this order. Thus the magnetic field is generated. We estimate roughly the value of the generated magnetic field at the recombination epoch. When we consider the horizon scale, $B \sim 10^{-29}$ Gauss. According to [4], this will be amplified enough to explain the present observed magnetic fields.

5 Summary and future works

We have derived an analytic formula for the magnetic fields generated from second-order cosmological perturbations in the pre-recombination era. Photons and charged particles are strongly coupled via Thomson scattering within the cosmic plasma, and hence the system behaves almost as a single fluid. In this single-fluid description magnetic fields are never generated, and therefore the tiny deviation from the single-fluid description is crucial here. Using the tight coupling approximation (TCA) to treat the small difference between photons and charged particles, we have seen how magnetic fields are generated from cosmic inhomogeneities. It was found that magnetic fields are not generated at first order in the TCA. Therefore, we conclude that magnetogenesis requires both the second-order cosmological perturbations and the second-order TCA. The resultant magnetic fields are expressed in terms of the vorticity and the product of the first-order perturbations. The latter can be computed by solving the linear Einstein equations, while the former is governed by the vorticity evolution equation, with the source term given by the product of the first order terms.

We have not included the effect of anisotropic stresses of photon fluids and the recombination process, though they will be equally important for magnetogenesis on small scales [7]. Taking into account these effects and computing the power spectrum of the generated magnetic fields require detailed numerical calculations, which are left for future works.

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