OPEN ACCESS
PAPER
Phys. Educ. **60** (2025) 055011 (7pp)
iopscience.org/ped

Time resolved investigations of momentum, energy and work during an elastic collision

Saskia Riedel, Jan Bauer* and Frank Stallmach

Institute for Physics Education, Leipzig University, Vor dem Hospitaltore 1, Leipzig 04103, Germany

E-mail: jan.bauer@uni-leipzig.de, saskia.riedel@uni-leipzig.de and stallmac@physik.uni-leipzig.de

Abstract
The conservation of momentum and energy are fundamental principles in physics, frequently introduced using illustrative examples of collisions. However, conventional methods often neglect the dynamic processes occurring during the collision itself. This paper presents a comprehensive experimental investigation of a central elastic collision between two carts. By utilizing digital sensors, the positions, interaction forces, and time throughout the collision process were measured with high resolution, enabling a precise examination of the conservation of momentum and mechanical energy, as well as the work-energy theorem. The experiment provides detailed insights into the dynamics of collision processes, revealing the transformation between kinetic and potential energy, highlighting the advantages of digital data acquisition for modern physics education.

Keywords: conservation of momentum, conservation of energy, elastic collision, time resolved, digital data acquisition, energy transfer, work-energy theorem

* Author to whom any correspondence should be addressed.

1. Introduction

The conservation of momentum and energy are fundamental concepts in physics. They regularly apply in current research and in everyday life, especially in relation to collision processes. Therefore, these concepts are important parts of physics education at school, where they are introduced and experimentally illustrated with common examples such as the collision of billiard balls or carts or glider on tracks. At college and university level science education, these concepts are generally part of the introductory physics courses and later extended to various subfields of physics [1–6]

In traditional physics education, experimental investigations of collision processes often focused solely on the states before and after the collision, leaving what happens during the collision largely inaccessible. For instance, Sawadthaisong *et al* [7] used photogates to measure the collision of gliders with rubber bumpers and picket fences on an air track, concluding due to the low temporal resolution of their measurement method that momentum and kinetic energy remained constant. Other implementations of such experiments investigate the conservation of momentum and energy, and the energy transfer during collisions of bodies. E.g. Bryan [8] employed video analysis to investigate the conservation of mechanical energy in typical motion experiments. Brunt and Brunt [9] verified momentum conservation only using motion sensors to examine elastic collisions of carts on a track with magnetic bumpers. Cross [10] estimated the impact forces during a billiard ball collision with a spring model and compared it to measurement with video analysis of a billiard ball dropping on a piezoelectric disk. Mora *et al* [11] explored multi-body collisions using gliders on an air track, showing momentum conservation throughout the collision while kinetic energy converted into 'elastic energy'. Pili and Violanda [12] validated the conservation of energy and the work-energy theorem through video analysis of a free-falling object. Using dynamic carts Larson [13] shows that momentum is always conserved, even when kinetic energy changes during collisions.

However, none of these experiments with higher temporal resolution thoroughly discussed both the conservation of momentum and energy throughout the collision process. They also did not consider energy transfer via interaction forces during the collision. As a result, it becomes challenging for learners to grasp the relationship between energy transfer, interaction forces, and conservation laws.

Also distinguishing between kinetic energy and momentum often represents a challenge. As the magnitude of both quantities are calculated via the mass and the velocity of the moving object. Thus, the fundamental difference between kinetic energy and momentum becomes not obvious for learners. In school-level education, one dimensional movements are often considered, so the vector nature of momentum, which is a key difference to kinetic energy, is prevalently apparent from the sign of the values.

The slightly misleading conflation of these two physical quantities is further reinforced by generalized mnemonics in schoolbooks, such as *'It is always the case that the sum of the momenta after the collision equals the sum of the momenta before the collision: […]. Furthermore, it is found that the sum of kinetic energies always remains constant'*, translation from [5]. Even in physics textbooks for science education, one can find phrasings that cause misunderstandings about the concepts of momentum and energy during elastic collisions. For example, *'[..] everyday collisions are inelastic but we can approximate some of them as elastic; [..] we can approximate that the total kinetic energy of the colliding bodies is conserved and is not transferred to other forms of energy,'* [6]

Therefore, it is important to analyse collisions processes in more detail and summarize the findings on the conservation of momentum and energy in precise scientific language. Additionally, modern, computer-based measurement techniques offer synchronized and digital acquisition of multiple data with a high temporal resolution. Such systems facilitate demonstration experiments or physics lab courses which enable learners to have a deeper understanding and more complex engagement with the subject matter.

In this article, we propose an experiment and its data analysis, which allows us to study the conservations and transfers of momentum and energy during a central elastic collision between two bodies with high temporal resolution. In our experiment two carts of equal mass collide on a track. Positions and interaction force are measured simultaneously throughout the entire collision process. Based on these experimental data we investigate the time dependencies of mechanical energies and momenta as well as the relationship between mechanical work and energy change.

2. Theory

For a central elastic collision of two bodies in a closed system, the total momentum $\vec{p}$ and the total energy E are conserved. The following equations apply where the potential energy depends on the position and the interaction of the bodies with each other,

$$\vec{p} = \vec{p}_1 + \vec{p}_2 \tag{1}$$

$$E = E_{\text{kin},1} + E_{\text{kin},2} + E_{\text{pot}} \,. \tag{2}$$

The momentum and kinetic energy of a body are calculated via its mass m and velocity $\vec{v}$ by the equations

$$\vec{p}_i = m_i \cdot \vec{v}_i \quad i = 1,\, 2 \tag{3}$$

and

$$E_{\text{kin},i} = \frac{m_i}{2} v_i^2 \quad i = 1,2 \tag{4}$$

respectively. The bodies involved in the collision exert forces on each other, which change the momenta and their kinetic energies. During the collision process, work is done, which can be calculated using the work integral,

$$W_i = \int \vec{F}_i\,(\vec{s}_i)\ \mathrm{d}\vec{s}. \tag{5}$$

In accordance with the conservation of energy, the total work done must satisfy the following condition:

$$W_1 + W_2 = 0. \tag{6}$$

Figure 1. Experimental setup for the central elastic collision with carts on a track.

3. Experimental setup

On a horizontally levelled aluminium track, two smart carts (PASCO Scientific Inc., USA) with force sensors are positioned facing each other (see figure 1).

Both carts are equipped with permanent magnet bumpers connected to their force sensors. After initiating time-synchronized data recording with 250 Hz acquisition rate for both the force and the position sensors, the left cart (cart 1, blue) is set in motion by being pushed manually towards the right cart (cart 2, red). All experimental sensor data were digitally recorded and analysed using the software capstone (PASCO Scientific Inc., USA). This includes the numerical differentiation of the position data to obtain the velocities and the numerical integration of the force data to calculate the work.

4. Results and data analysis

The momenta of both carts and their sum are calculated using equations (1) and (3). The results are plotted as functions of time (see figure 2). During a central elastic collision between two carts of equal mass, the momentum of cart 1 (blue dots in figure 2), which is initially in motion, is fully transferred to cart 2 (red dots in figure 2). Before the collision starts at t_s cart 2 is at rest and the total momentum (black dots in figure 2) is solely determined by the momentum of cart 1; similarly, when the collision ended at t_e, the total momentum corresponds to the momentum of cart 2. t_s and t_e were determined by the synchronized force sensors. During interval $[t_s,\ t_e]$ both force sensors reach values deviating from zero Newton, indicating the mutual interaction of both carts. At time t_m cart 1 transferred half of its momentum to cart 2 (see figure 2). Also, friction influences the momenta

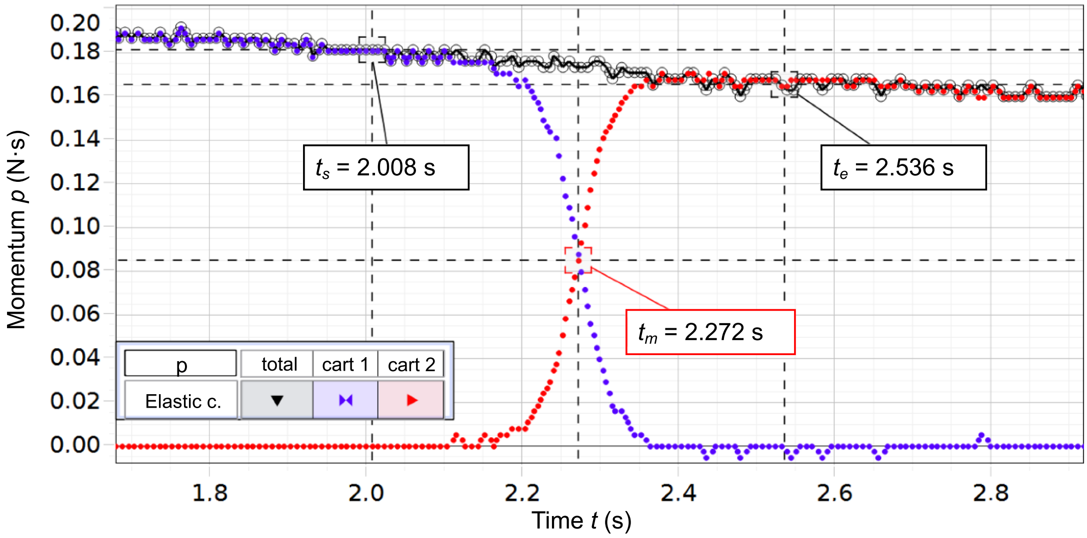

Figure 2. Time dependence of the momenta during a central elastic collision between two carts of equal mass. The colours of the dots represent the values for the carts 1 (blue) and 2 (red) and their sum (black).

of the rolling carts. Thus, the total momentum is observed to decrease continuously by about 20% in the total time interval of the displayed data of 1.1 s.

During the collision, the carts interact via their magnets, continuously altering their velocities and momenta. However, the experimental data clearly show that the total momentum remains conserved throughout the experiment. It is only reduced due to friction. Especially, it does not indicate any major change in its time dependence during the collision process where the individual carts completely exchange their momenta.

The kinetic energy of both individual carts and their sum are calculated using equations (2) and (4). The results are plotted in dependence on time in figure 3. The three inserts in figure 3 report the total kinetic energies for the selected three instants of time. The total kinetic energy before the collision corresponds to the kinetic energy of cart 1, as cart 2 is still at rest. Similarly, the total kinetic energy after the collision equals the kinetic energy of cart 2. As the total momentum, also the total kinetic energy is observed to decrease due to friction. However, during the collision, the total kinetic energy initially decreases down to about 50% of the value just before the start of the collision until it increases back to the value expected due to dissipation by friction. This observation clearly indicates that kinetic energy is not conserved during the collision process. Obviously, the conservation of kinetic energy only holds if the momentum transfer from cart 1 to cart 2 is completed.

The decrease of the total kinetic energy at time t_m down to $1/2$ of its value just before the collision is explained in the following way: At time t_m both carts of equal masses have the same velocity and momenta (compare figure 2). Thus,

$$v_1(t_m) = v_2(t_m) = \frac{1}{2} v_1(t_s).$$

Calculating the sum of the kinetic energies of both carts for this instant of time leads to

$$E_{\mathrm{kin},1}(t_m) + E_{\mathrm{kin},2}(t_m) = \frac{1}{2} \cdot E_{\mathrm{kin}}(t_s).$$

During the collision the mutual interaction forces between the two carts continuously accelerate cart 2 and reduce the speed of cart 1. Thus, the accelerating force of cart 1 acting on cart 2 performs work, which increases the kinetic energy of

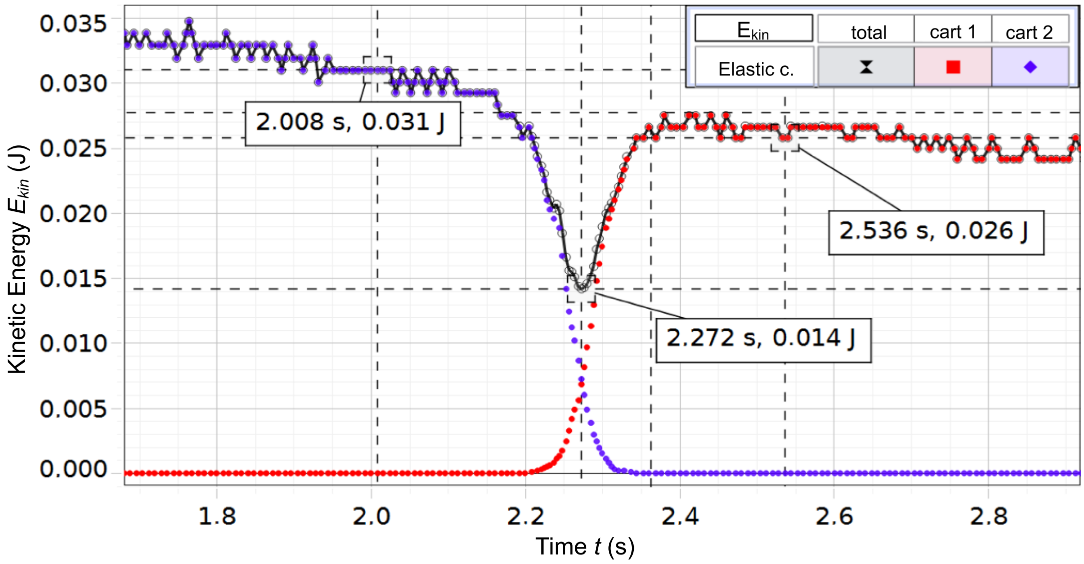

Figure 3. Time dependence of the kinetic energy during a central elastic collision between two carts of equal mass. The colours of the dots represent the values for the carts 1 (blue) and 2 (red) and their sum (black).

cart 2. Simultaneously, the force of cart 2 acting on cart 1 decelerates cart 1 and reduces its kinetic energy.

Figures 4(a) and (b) plot the forces acting on each cart depending on their distance travelled during the collision. After cart 1 has travelled 0.36 m, the interaction begins (cf figure 2: t_s) with the still stationary cart 2 via the magnets. The magnitudes of the interaction forces recorded for both carts increase. While the travelled distance of cart 1 increases, the systems of the two magnets gain their potential energy up to a maximum value at the minimum distance between the two carts. At this position the repulsion forces reach their maximum extreme values. Their values of approximately ± 1.90 N agree within the measurement

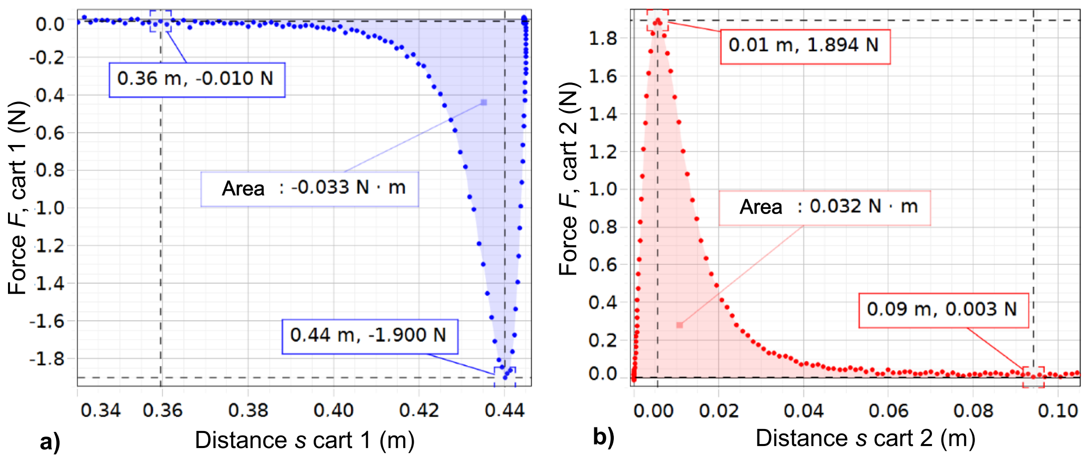

Figure 4. Force distance plots for cart 1 (left, blue) and cart 2 (right, red) during a central elastic collision. The highlighted areas represent the numerically calculated works performed on the respective carts during the collision.

uncertainties of the force sensors. While the collision process continues the distance between the carts increases, causing the interaction force to decrease back to zero (see figure 4).

Thus, during the collision, a part of the kinetic energy is transformed into potential energy of the two opposing magnets on the carts. This potential energy reaches its maximum value at the closest approximation of both carts where the repulsing forces also reach their maximum amplitudes, and the kinetic energy of the systems shows its minimum. The potential energy is then reconverted into kinetic energy as the magnetic bumpers move away from each other until the collision process is completed.

The numerical integrations of the curves in figures 4(a) and (b) yield the work performed on each of the two carts due to the collision. The resulting values are given in the figures. It is noticeable that the sum of both values is found to be -0.01 Nm and, thus very close to zero as expected from (6). Since cart 1 performs work on cart 2, its work carries a negative sign. Conversely, the work, which is done on cart 2, receives a positive sign.

When comparing the magnitudes of the works to the total kinetic energy at time t_s, it becomes evident that they are also almost of equal magnitude. This corresponds to the expectation of kinetic energy conservation before and after the collision, as cart 1 transfers its entire kinetic energy to cart 2 via the work performing on cart 2 during the collision. We conclude that energy conservation is confirmed in our experiment by numerical integration of two experimentally observed force-distance curves (5).

5. Conclusion

In our experiment, we demonstrated that high temporal resolution of digital data acquisition allows for a detailed, step-by-step examination of physical phenomena, during a central elastic collision. The detailed analysis of the experimental data enables the precise calculation of momenta and energies. The conservation of momentum can be verified at any instant of time during the interaction of the two bodies. It also allows us to demonstrate that the conservation of kinetic energy only holds if the momentum transfer is completed, meaning no potential energy remains stored in the system. In agreement to what has been already clearly stated by *Feynman* [3] and to avoid misconceptions or misleading generalized mnemonics regarding the conservation of momentum and kinetic energy, we recommend to use the following phrasing for to characterize elastic collisions: *Momentum and mechanical energy are always conserved in an elastic collision. Kinetic energy is only conserved if the collision process is fully completed.*

There are now a number of ideas [9–13] for demonstration and physics lab experiments available that allow teachers to empower their learners to study the momentum and energy transfer and the development of potential energy during the collision. Our idea to simultaneously measure the position and the interaction forces may easily be extended to other mass ratios and other interaction forces using, i.e. different bumpers, if they allow time resolved data acquisition during the interaction process.

These types of experiments open modern didactic approaches making fundamental physical laws tangible for students through experimentation and analysis. Moreover, such digitally based experiments align with global demands for the digitalization of education to develop competencies for life in the digital world [14].

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Acknowledgment

We would like to thank the State Ministry of Education and Cultural Affairs of Saxony (Germany) for deputing Saskia Riedel to the University of Leipzig and for the support of the research project on the digitalization of physics teaching.

Received 26 May 2025, in final form 20 June 2025
Accepted for publication 1 July 2025
https://doi.org/10.1088/1361-6552/adeb2b

References

[1] Coletta V 2010 *Physics Fundamentals* 2nd edn (Physics Curriculum & Instruction)
[2] Demtröder W 2021 *Experimentalphysik 1* 9th edn (Springer) pp 116f
[3] Feynman R, Leighton R B and Sands M 2013 (California Institute of Technology) (available at: www.feynmanlectures.caltech.edu/) (Accessed 26 May 2025)
[4] Hewitt P G 2015 *Conceptual Physics* 12th edn (Pearson)
[5] Schön L-H, Winter R, Lindner K, Diehl B, Erb R, Schmalhofer C and Tillmanns P 2009 *Physik Oberstufe* (Cornelson Verlag) p 51
[6] Walker J, Halliday D and Resnick R 2014 *Principles of Physics* 10th edn (Wiley) p 205
[7] Sawadthaisong N, Chitaree R and Barufaldi J P 2011 Demonstration of momentum and kinetic energy during an entire collision *Phys. Teach.* **49** 56–57
[8] Bryan J A 2010 Investigating the conservation of mechanical energy using video analysis: four cases *Phys. Educ.* **45** 50
[9] Brunt M and Brunt G 2013 Introducing conservation of momentum *Phys. Educ.* **46** 664
[10] Cross R 2020 Impact force between two colliding billiard balls *Phys. Educ.* **55** 065002
[11] Mora D Y R, Idárraga S D and Montoya P C C 2020 Video analysis of one-dimensional multi-body collisions: linear momentum, impulse and energy *Phys. Educ.* **55** 065023
[12] Pili U B and Violanda R R 2022 Tracker-assisted modelling: simultaneous validation of the conservation law of energy and the work-energy theorem *Phys. Educ.* **57** 015012
[13] Larson R F 2024 Sensor cart collisions and the conservation laws *Phys. Teach.* **62** 599–603
[14] International Society for Technology in Education n.d. ISTE Standards: For Students (available at https://iste.org/standards/students) (Accessed 14 October 2024)

Saskia Riedel holds a Bachelor's degree in laboratory and Process Engineering with a focus on Biotechnology and completed her teacher training in Physics and Chemistry at the University of Leipzig with the State Examination (2021). After working as a secondary school teacher, she has been deputed by the Saxon State Ministry of Education to serve as a lecturer at the Institute for Physics Education at the University of Leipzig. Since January 2024, she has been pursuing a PhD at the Faculty of Physics and Earth System Sciences. Her research focuses on the development and evaluation of Development of activating teaching and learning sequences for a multifaceted physics education.

Jan Bauer received his PhD for solar cell characterization in 2009 from University of Halle-Wittenberg and Max Planck Institute of Microstructure Physics (MPIMSP), Halle, Germany. After being RnD researcher with CaliSolar GmbH, Berlin, Germany, he was scientist at MPIMPS and later team leader at Fraunhofer Center for Silicon Photovoltaics, Halle, Germany. Starting in 2019 he does technical education of future physics teachers at the Institute for Physics Education at Leipzig University until now.

Frank Stallmach has been an adjunct professor at the Institute for Physics Education at the University of Leipzig since 2022, where he heads the Department of university physics education. He regularly gives lectures in mechanics, quantum mechanics and astronomy for physics teacher training students. His current research focus is on the knowledge-based improvement of physics education at universities through the use of digital and multimedia tools.