Fermilab Proposal

The CP/T Experiment

An Experimental Program to Study CP Violation and Search for CPT Violation in the $K_L - K_S$ System

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Abstract

This is a proposal for a Fermilab Main Injector experiment to carry out a program of measurements on the physics of K^0 mesons. The experiment is designed to maximize the interference between K_L and K_S mesons near their production target, and hence have excellent sensitivity to CP violation in many decay modes. The extremely accurate CP violation measurements we will be able to make will allow us to test CPT symmetry violation with sensitivity at the Planck scale.

We will measure four CP violation parameters that will test the Standard Model in new ways. Measurements of η_{+-0} and η_{000} will provide the first discovery of CP violation outside the K_L system, and a measurment of $\eta_{+-\gamma}$ will be a sensitive test of direct CP violation. We will measure the magnitude and phase of η_{+-} an order of magnitude better than it is presently known.

There are two ways that we will test CPT symmetry conservation which will be sensitive at the Planck scale: measuring the difference in phase between η_{+-} and ϵ, and evaluating the Bell-Steinberger relation. The measurements of η_{+-0} and η_{000} are an integral part of any program of CPT tests.

We will study rare K_S decays. $K_S \rightarrow \pi^0 e^+ e^-$ tests strangeness changing neutral currents and is necessary to disentangle direct CP violation in the K_L decay to the same final state. $K_{L,S} \rightarrow \pi^+\pi^- e^+ e^-$ is a new CP violating decay mode. The CP-conserving decay $K_S \rightarrow \pi^+\pi^-\pi^0$ is of interest in chiral perturbation theory.

The experiment will use an RF-separated K^+ beam striking a target at the entrance to a hyperon magnet to make the K^0 beam by charge exchange. The decay region, magnetic spectrometer, electromagnetic calorimeter, and muon detector follow immediately to observe interference between K_L and K_S near the target.

Contents

I. INTRODUCTION

This is a proposal for a program of experiments at the Fermilab Main Injector to study CP violation, test CPT symmetry conservation, and search for rare decays of the K_S meson.

One of the burning questions in high energy physics today is that of CP symmetry nonconservation. This is truly a dimly illuminated corner of the standard model which needs more experimental work. In addition, CP violation plays a crucial role in our understanding of the baryon asymmetry of the universe, and in the standard model the size of the effect seems to be too small to explain the observed asymmetry. This may indicate the presence of physics beyond the standard model. It is very important to perform more measurements of CP violating phenomena to search for possible deviations from the standard model.

All observations of CP violating effects have occurred in K_L decays. The $\pi^+\pi^-$ and $\pi^0\pi^0$ decays and semileptonic charge asymmetry were discovered in the 1960's [1]. CP violation in $K_L \to \pi^+\pi^-\gamma$ was discovered by Fermilab experiment E731 [2]. The angular asymmetry in $K_L \to \pi^+\pi^-e^+e^-$ has been found by the KTeV experiment. All are consistent with a single mechanism being at work: the K_L is a mixed state of the two CP eigenstates.

The standard model predicts additional CP violating effects should occur in K_S, D, and B meson decays [3]. The K_S effects test the basic ideas of mixing of CP eigenstates. The B meson decays are very useful for determining the angles of the unitarity triangle.

The standard model also predicts that direct CP violation should occur [4]. Experiments to measure ϵ'/ϵ search for this effect. Violation of the $\Delta I = 1/2$ rule reduces the size of ϵ'/ϵ. Cancellation between gluonic and electroweak penguin diagrams reduce it further, to the point where, within the theoretical uncertainties, it might be zero.

There are other manifestations of direct CP violation. It may contribute 10% to η_{+-0} and η_{000}, and 1% to $\eta_{+-\gamma}$. These effects would be seen as a difference between the above mentioned CP violation parameters and η_{+-}. The experiment proposed here will search for these effects. The easiest way to search for direct CP violation, next to ϵ'/ϵ, is to perform a simultaneous measurement of $\eta_{+-\gamma}$ and η_{+-}. Here there is no $\Delta I = 1/2$ rule suppression or destructive interference of penguin diagrams. The direct CP violation comes from the photon part of the decay amplitude (an electric dipole contribution to the direct emission term), and is fairly well understood. We should be able to measure this $\eta_{+-\gamma} - \eta_{+-}$ difference to 5σ accuracy.

CPT symmetry conservation is a subject under theoretical attack. Studies of Hawking radiation [5] and of string theory (the leading contender for a unified theory of all four forces of nature) [6] have shown the CPT theorem [7] to be invalid in real life (rather than in the three-force approximation we call the standard model). Many physicists are reluctant to accept the possiblity that CPT symmetry violation may occur at the Planck scale. One reason for this reluctance is that we have, so far, only theoretical hints that this is the case. Another reason is the great success of the standard model. It may be quite a few years before a theory that unifies all four of the forces of nature becomes mature enough so that convincing theoretical statements can be made about the CPT structure of the world.

Fortunately we don't have to wait. The $K_L - K_S$ system provides a way of testing the validity of CPT symmetry conservation where it is possible to perform extremely accurate experiments [8]. In this document we propose to do an experiment that will reach the Planck scale. Finding CPT symmetry nonconservation would be a major discovery that would change in a fundamental way how physicists view the world. If we don't find it we will strongly constrain several quantum theories of gravity [9] [11] and provide a powerful benchmark against which future theories must be measured.

Our goal is to perform the optimal experiment to study interference between K_L and K_S mesons near the production target. We will make an RF-separated K^+ beam, and produce a neutral kaon beam from it by charge exchange. In this process K^0's are produced copiously, but very few $\overline{K}^0$'s are made. This situation maximizes the size of the interference. We will use a hyperon magnet (a thinner version of the magnet in the Fermilab Proton Center beam line) to define the neutral beam and get as close to the target as possible. This "closed geometry" method of beam definition is excellent for collecting large amounts of data. The detector will consist of a standard Vee spectrometer, an electromagnetic calorimeter, and a muon detector. We own most of the apparatus we need, and hope to borrow a large fraction of the remainder to minimize the cost of the experiment.

The result of this design is an experimental arrangement that makes possible a program of measurements in this area. A rich harvest of Ph. D. theses would result from this program.

In this proposal we will describe the measurement of four CP violation parameters, two CPT symmetry conservation tests that are sensitive at the Planck scale, a test of the $\Delta S = \Delta Q$ rule, and studies of rare K_S decays. The four CP violation parameters are η_{+-0} and η_{000} which characterize CP violation in K_S decay and have never been measured, $\eta_{+-\gamma}$ which is important because its magnitude should show a large difference from that of η_{+-} which comes from direct CP violation, and η_{+-} itself. The measurements of η_{000} and η_{+-}, together with a measurement of $Im(x)$ (where x is the $\Delta S = \Delta Q$ rule violation parameter), will make possible a new evaluation of the Bell-Steinberger relation [12] with unprecedented accuracy. The two tests of CPT conservation come from a measurement of the phase difference between η_{+-} and ϵ, and from the evaluation of the Bell-Steinberger Relation.

The RF separated K^+ beam described below will be useful for other experiments as well. An experiment searching for the rare decay $K^+ \to \pi^+ \nu \overline{\nu}$ would find it ideal. The "CKM" experiment proposed by P. Cooper **et al.**, needs about 1/10 of the intensity that can be achieved with our design and requires the beam to have a very small divergence. These conditions can be achieved with our beam design. In addition, with a few modifications, the K^+ beam could be turned into a $\overline{p}$ beam, making other types of experiments possible.

II. CP THEORY AND PHENOMENOLOGY

In the 1970's the first electroweak theory was written down by S. Weinberg, A. Salam, and S. Glashow. It was a four quark theory, and correctly predicted all electromagnetic and weak interaction phenomena except for one: it was a CP conserving theory. Soon three generalizations of the theory were made to include CP violation. Weinberg's own idea [13] was to enlarge the Higgs sector and get CP violation through an interference between Higgs-exchange diagrams. R. Mohapatra and J. Pati [14] chose a larger gauge group: $SU(2)_L \times SU(2)_R \times U(1)$, called the "isosymmetric" model, where CP violation occurs through interference between left and right handed intermediate boson exchanges. M. Kobayashi and T. Maskawa [3] chose to enlarge the Weinberg, Salam, Glashow theory by adding a doublet of quarks resulting in CP violation through the familiar box diagram. The subsequent discovery at Fermilab of the b and t quarks promoted the Kobayashi - Maskawa theory to be part of the "standard model".

The focus of research in CP violation today is to perform tests in the dark corners, so to speak, of the standard model. One of the most important is the search for direct CP violation. All existing measurements of CP violating effects are decays of the K_L meson, and all measurements are consistent with the CP violation coming from the mixing between CP eigenstates in the K_L; i.e. from the K_1 term in the K_L part of Eq. 2.1.

$$\begin{cases} K_S = K_1 + (\epsilon + \Delta)K_2 \\ K_L = K_2 + (\epsilon - \Delta)K_1 \end{cases} \tag{2.1}$$

In Eq. 2.1 $K_1(K_2)$ is the CP-even (CP-odd) eigenstate, ϵ is the CP-violating mixing parameter, and Δ is the mixing parameter which is both CP and CPT violating. The standard model predicts that there should also be a contribution from the dominant K_2 term of the K_L, called direct CP violation. For K_L decays to two pions direct CP violation is parameterized by the quantity ϵ', and the ratio of the CP violation parameters describing the K_L decays to $\pi^0\pi^0$ and $\pi^+\pi^-$ is:

$$|\eta_{00}/\eta_{+-}| = 1 - 3Re(\epsilon'/\epsilon) \tag{2.2}$$

The effort to measure ϵ'/ϵ has been going on for almost 20 years, so far without any definitive results. The main cause of this difficulty is that ϵ'/ϵ is very small, and has been getting smaller as our knowledge of the top quark mass has improved. The difference between η_{00} and η_{+-} comes through a transition that violates the $\Delta I = 1/2$ rule and is thus suppressed. In addition in the standard model there is destructive interference between penguin diagrams that contribute to ϵ'/ϵ. The theoretical prediction is that $\epsilon'/\epsilon = (3 \pm 2) \times 10^{-4}$. Those of us in the KTeV experiment (T.A., D.R.B., A.E., S.R.S., S.V.S., R.S., G.B.T., and H.B.W.) hope to measure ϵ'/ϵ to an accuracy of $\pm 1 \times 10^{-4}$ (and have data in hand to reach a statistical accuracy of about $\pm 1.5 \times 10^{-4}$), and the aim of CERN experiment NA48 is to reach $\pm 2 \times 10^{-4}$. If the value of ϵ'/ϵ is near the lower edge of the theoretical uncertainty, the hope of both KTeV and NA48 to make a significant measurement may be disappointed.

If ϵ'/ϵ in the $\pi\pi$ decays is too small to measure we needn't despair, however. There are many other manifestations of direct CP violation that do not suffer from the supression factors that plague ϵ'/ϵ. There are several rare decays in which the contribution to the branching ratio of direct CP violation is large, $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \nu\overline{\nu}$ in particular, which are very interesting. Unfortunately these decays are VERY rare, with branching ratios expected to be of order 10^{-11}, and have significant background problems that, at this time, have not been solved.

We feel that a more promising approach to the study of direct CP violation is to look for differences between other CP-violation parameters and η_{+-}. In particular, measurements of $\eta_{+-\gamma}, \eta_{+-0}$, and η_{000}, if they yield values different from η_{+-}, would signify the existence of direct CP violation (or perhaps CPT violation). There are also predictions of large differences between these CP violation parameters and η_{+-} coming from physics beyond the standard model. Each of these CP violation parameters is written (taking $\eta_{+-\gamma}$ as an example) $\eta_{+-\gamma} = \epsilon + \epsilon'_{+-\gamma}$, with ϵ coming from indirect CP violation and $\epsilon'_{+-\gamma}$ being the contribution from direct CP violation.

There are two Feynman diagrams that contribute to the decays $K_{L,S} \to \pi^+\pi^-\gamma$, the inner bremsstrahlung (IB) diagram where the gamma ray comes from one of the charged pion legs, and the direct emission (DE) diagram where the gamma ray comes from the decay vertex. The K_S decay is dominated by the IB term, and the K_L has approximately equal contributions from IB and magnetic dipole (M1) DE terms. The former is CP violating (and should contribute ϵ to $\eta_{+-\gamma}$), and the latter is CP conserving. In $K_L - K_S$ interference the K_L M1 term does not interfere with the K_S IB term. On the other hand, if there were an electric dipole (E1) term in the K_L DE decay it would come from direct CP violation, and would show up in $K_L - K_S$ interference. This transition comes from the gluon penguin diagram, with the electromagnetic penguin diagram being one to two orders of magnitude smaller [16].

The prediction of the standard model is that the E1 term would make a contribution to $\epsilon'_{+-\gamma}$ which is of order $10^{-2} \times \eta_{+-}$. Several authors [15] estimate this difference to be between 1% and 2%. This difference varies as k^2, the square of the C.M. energy of the gamma ray, and the 10^{-2} estimates occur at the maximum energy. If one integrates over the Dalitz plot the interference term is smaller than the maximum value quoted above. Any observation of a variation of $\eta_{+-\gamma}$ with k would be a sign of direct CP violation.

In comparing different ways of searching for direct CP violation one might choose as a figure of merit the product of (relative difference from ϵ) $\times$ (square root of branching ratio). The method that uses $\eta_{+-\gamma}$ and η_{+-} has a figure of merit eight times better than the method that uses η_{00} and η_{+-}. In addition the acceptance of $\pi^+\pi^-\gamma$ events is higher than that of $\pi^0\pi^0$ decays, the $\pi^+\pi^-\gamma$ and $\pi^+\pi^-$ events are collected with the same trigger, and a simultaneous measurement of $\eta_{+-\gamma}$ and η_{+-} avoids the main difficulty of ϵ'/ϵ experiments, namely that the $\pi^0\pi^0$ decay is so difficult to understand.

The direct CP violation contributions to η_{+-0} and η_{000} are ϵ'_{+-0} and ϵ'_{000}, and share with $\epsilon'_{+-\gamma}$ the characteristics that they do not require violation of the $\Delta I = 1/2$ rule, and do not have cancellations between penguin diagrams. While in general ϵ'_{+-0} might be of order 10^{-3}, using PCAC and the soft pion theorem, one can relate the value of η_{+-0} at the center of the $\pi^+\pi^-\pi^0$ Dalitz plot to η_{+-}, with the result that ϵ'_{+-0} is much smaller. In ref. [18], the authors pointed out that the correction to this relation increases the size of ϵ'_{+-0} considerably. Various authors have estimated ϵ'_{+-0} and ϵ'_{000} to be between 2.5% and 10% of η_{+-} [17–19] in the standard model. In addition it is known that, in the standard model, ϵ'_{+-0} and ϵ'_{000} are purely imaginary.

The PCAC calculation is not valid for scalar exchange diagrams, so the multiple-Higgs boson model of CP violation could give much larger values of η_{+-0}, and this was the prediction from ref. [13]. But ref. [18] estimates ϵ'_{+-0} to be 5% of η_{+-} in this model. In the left-right gauge symmetric model there are also predictions of large values for ϵ'_{+-0} [14], but using a more modern limit on the mass of a right-handed W boson, ref. [18] calculates $|\epsilon'_{+-0}/\epsilon| = 0.10$. Two other interesting predictions of the left-right gauge symmetric model are that $\eta_{000} = \eta_{+-0}$ and that $\epsilon' = 0$.

In Eq. 2.1 the CP and CPT violating parameter Δ enters with opposite sign (by definition) in the K_S and K_L equations, so a measurement of η_{+-0} or η_{000} is sensitive to the existence of CPT violation. In particular, the current limit on the component of Δ which is parallel to ϵ is quite poor (about 25% of $|\epsilon|$) [20], so a comparison of $|\eta_{+-0}|$ and $|\eta_{+-}|$, for example, would be very interesting.

New tests of CP violation in the standard model can be performed by a compact experiment studying $K_L - K_S$ interference close to the kaon production target. Measuring $\eta_{+-\gamma}$ to high accuracy will provide a new testing ground for direct CP violation. Measuring η_{+-0} and η_{000} will prove for the first time that the mixing equation (Eqn. 2.1) is valid, will provide further new tests of direct CP violation, and will make possible a search for CPT violation and for effects beyond the standard model.

III. MEASUREMENT OF CP VIOLATION PARAMETERS

Our experiment is optimized to study $K_L - K_S$ interference near the kaon production target. This will allow us to measure several CP violation parameters not previously measured, and perform tests of the standard model not previously achievable.

In what follows we will quote sensitivities calculated by making the following assumptions:

- A beam of 6×10^{12} protons per spill at 120 GeV/c (about 1/5 of the expected initial Main Injector intensity), a 1 year-long run with the accelerator operating at 70% efficiency and the experiment being 75% efficient. This proton flux would produce a secondary K^+ beam of $2.4 \times 10^8 K^+$ per spill. The K^+ beam would strike a 10 cm tungsten target at 9 mrad.

- The measured charge exchange cross sections.
- A solid angle of 43.6 μster for the K^0 beam, similar to beams described in the KAMI Design Report [21].
- A hyperon magnet 1.5 meters thick.
- A decay region 9.5 meters long, followed by a vee spectrometer, lead glass electromagnetic calorimeter, and muon detector. Fig. 1 shows the hyperon magnet, decay region, spectrometer, and muon detector. The spectrum of neutral kaons is shown in Fig. 2

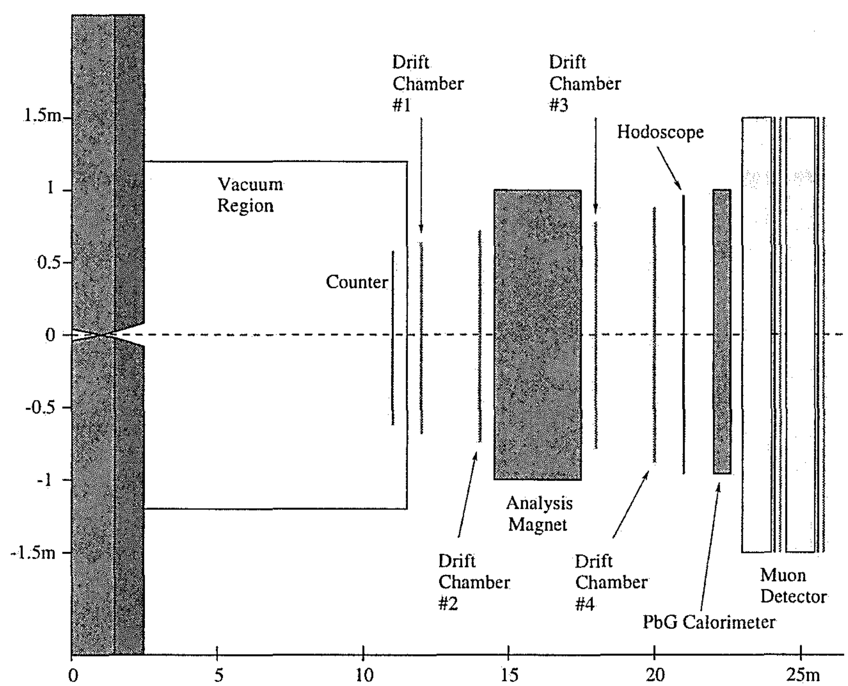

FIG. 1. Detector Layout

A. $\eta_{+-\gamma}$ and η_{+-}

The CP violation parameter for the decay $K_L \to \pi^+\pi^-\gamma$ is called $\eta_{+-\gamma}$. The contribution of direct CP violation in this decay is expected to be about 1%. The world's best measurement [22] of $\eta_{+-\gamma}$ was made in Fermilab experiment E773 of which three of us (S.R.S., S.V.S., and G.B.T.) took part. This result was $|\eta_{+-\gamma}| = [2.359 \pm 0.062(stat) \pm 0.040(syst)] \times 10^{-3}$, and it stands within about a factor of 3 of the expected difference from η_{+-}.

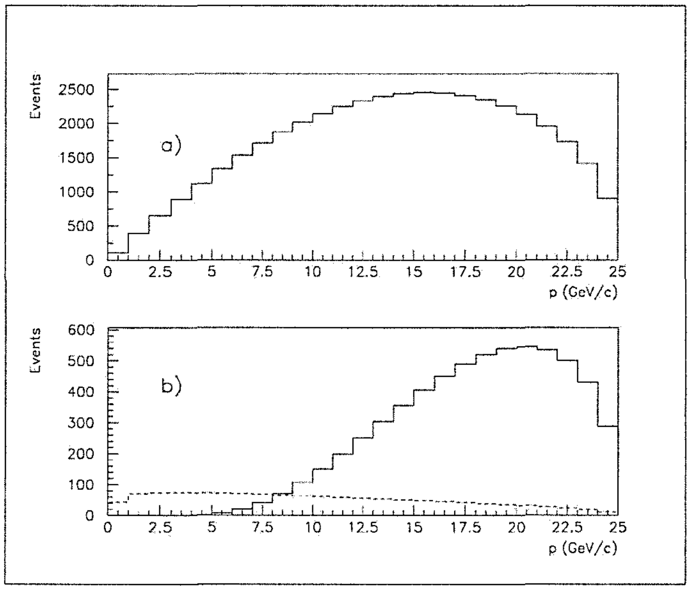

FIG. 2. Kaon Momentum Distributions. a) K^0's at the target. b) $K_S(K_L)$ solid (dashed) decays in the decay region. The flux from one spill of data is shown in this figure.

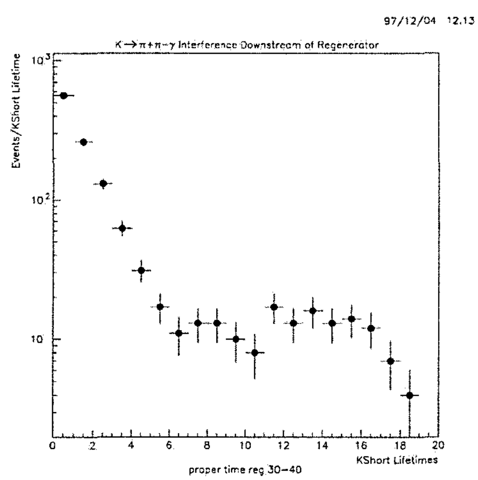

FIG. 3. Proper Time Distributions for $\pi^+\pi^-\gamma$ Decays from Fermilab Experiment E832. These events were in the "regenerator beam" and come from the decays of a mixture of K_S and K_L mesons. The oscillations visible come from $K_L - K_S$ interference.

Figure 3 shows the proper time dependence of $K_{L,S} \to \pi^+\pi^-\gamma$ decays downstream of the regenerator in the KTeV experiment. This is the complete data from the E832 1996 run, and comprises about 1/5 of the whole KTeV sample. In this figure the interference between K_L and K_S is seen. In KTeV we expect to measure the CP violation parameter $\eta_{+-\gamma}$ to better than 1% accuracy.

In the proposed experiment we will collect about 460M $\pi^+\pi^-\gamma$ events and 20 billion $\pi^+\pi^-$ events and perform a simultaneous fit for $\eta_{+-\gamma}$ and η_{+-} to measure their difference. For the purpose of estimation we have performed a simpler fit, to only the $\pi^+\pi^-\gamma$ events, using as fitting parameters, $|\eta_{+-\gamma}|, \phi_{+-\gamma}, D$, and three parameters describing the normalization and shape of the kaon momentum spectrum. The result is a measurement of $|\eta_{+-\gamma}|$ to $\pm 0.19\%$ accuracy, which will be a 5σ measurement of $\epsilon'_{+-\gamma}$ if it is of the expected size. Fig. 4 shows the proper time distribution of accepted $\pi^+\pi^-\gamma$ events.

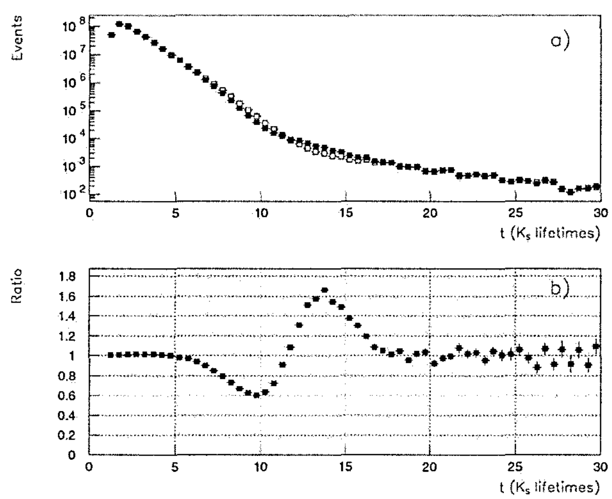

FIG. 4. Simulated Proper Time Distributions for $\pi^+\pi^-\gamma$ Decays. a) Distributions are shown both with interference (dark circles) and without (light circles). b) The ratio of the two distributions in part a).

B. η_{+-0} and η_{000}

The CP violation parameters η_{+-0} and η_{000} characterize CP violation in K_S decays to three pions. Every measured CP-violating decay has been of the K_L. Observation of

CP-violation in these decays would be the first discovery of CP violation outside the K_L system. These decays are expected to have large contributions from direct CP violation (up to about 10%), and are the places where CP violating effects beyond-the-standard-model may be the largest. Since in the standard model ϵ'_{+-0} and ϵ'_{000} are both purely imaginary, fitting the proper time distribution of $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$ events to extract these parameters from $K_L - K_S$ interference should be relatively simple. We expect to collect about 230 M $\pi^+\pi^-\pi^0$ decays and measure $Im(\eta_{+-0})$ to $\pm 0.35 \times 10^{-3}$ accuracy. We will simultaneously collect about 70 M $\pi^0\pi^0\pi^0$ decays and measure $Im(\eta_{000})$ to $\pm 0.64 \times 10^{-3}$ accuracy (note: $\eta_{+-} = 2.27 \times 10^{-3}$).

The most recent experiment at Fermilab to search for CP violation in K_S decay was E621, of which two of us (TJD and GBT) were members. The proper time distribution of $\pi^+\pi^-\pi^0$ decays observed in E621 is shown in Figure 5. The result of that experiment [23] was $Im(\eta_{+-0}) = -0.015 \pm 0.017(\text{stat}) \pm 0.025(\text{syst})$. E621 was done with an 800 GeV/c proton beam striking a target at the entrance to the hyperon magnet in the Fermilab P-Center beam line. The apparatus consisted of a magnetic spectrometer and lead glass electromagnetic calorimeter.

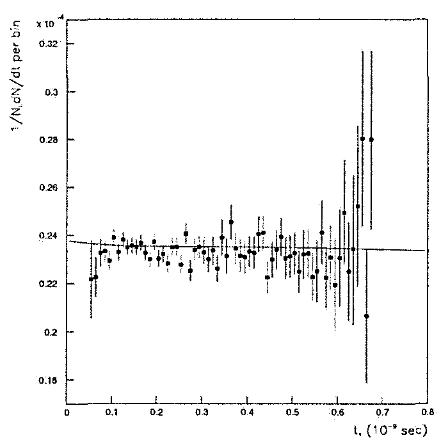

FIG. 5. Proper Time Distributions for $\pi^+\pi^-\pi^0$ Decays from Fermilab Experiment E621. The vertical axis is the absolutely normalized probability for the decay $K_L \to \pi^+\pi^-\pi^0$, and the horizontal axis is the proper lifetime of the kaons.

The design of the present experiment is vastly superior to that of E621: the K^+ beam yields a higher K^0 flux per incident beam particle, the showers from the charged beam particles are more easily contained and the rates in the detector are much lower, the K^0's are produced at much higher Feynman x, the size of the interference is much larger, the statistical accuracy will be much better, the acceptance will be larger, and the systematic uncertainties will be better.

The existing limit on η_{000} is $Im(\eta_{000})^2 < 0.1$, based on 632 events seen in a bubble chamber experiment [24]. The analysis of this experiment used the fact that ϵ'_{000} is imaginary, so that $Re(\eta_{000}) = Re(\epsilon)$, which equals $Re(\eta_{+-})$ to good accuracy.

The measurements of η_{+-0} and η_{000} are essential for a meaningful interpretation of tests of CPT symmetry conservation, including some tests already carried out. More details are given below in the context of CPT tests.

IV. CPT THEORY AND PHENOMENOLOGY

The CPT theorem [7] is based on the assumptions of locality, Lorentz invariance, the spin-statistics theorem, and asymptotically free wave functions. All quantum field theories (including the standard model of the elementary particles) assume CPT symmetry invariance.

There is a theoretical hint of the level at which CPT symmetry might be violated. This comes from the fact that gravity can't be consistently included in a quantum field theory, and the proof of the CPT theorem assumes Minkowski space [25,7]. To include gravity in a unified theory of all four forces of nature, many physicists think that a more general theory is needed, which would have quantum field theory embedded in it. In this more general theory the CPT theorem will be invalid.

One expects to see quantum effects of gravity at what is called the Planck scale: at energies of $M_{Planck}c^2 = \sqrt{\hbar c^5/G} = 1.2 \times 10^{19}$ GeV, or at distances of the order of 10^{-33} cm. The quantum effects of gravity are expected to be very small in ordinary processes.

However, in a place where the standard model predicts a null result, like CPT violation, quantum effects of gravity would stand out. Therefore, it would be very interesting to test CPT symmetry conservation at the Planck scale.

One might think that string theory, as a candidate for the more general theory that has quantum field theory embedded in it, would give us guidance. CPT conservation is artificially built into string theories, first by G. Veneziano [26].

Kostelecky and Potting [11] suggested that spontaneous CPT violation might occur in string theory; i.e., they put the CPT violation in the solutions rather than in the equations of motion. One of the problems with string theory in general is that it's not known how to relate string effects at the Planck scale to effects seen at current accelerator energies, and Kostelecky and Potting have the same difficulty. They have tried to remedy this by writing the most general additions to the Standard Model Lagrangian that maintain the SU(3) x SU(2) x U(1) effective structure of the theory but violate CPT symmetry. This allows them to classify the various types of CPT violation that might be seen (in the lepton sector, quark sector, etc.) and have a parameterization that includes all these effects. They find that the largest CPT violating effect is a change in quark propagators that has the opposite sign for antiquarks. This leads to a nonzero value of $|M_{K^0} - M_{\overline{K^0}}|$ coming from indirect CPT violation. This is much larger than any direct CPT violation effect. This is precisely the signature that this experiment would search for.

There have also been mechanisms proposed for the violation of quantum mechanics leading to CPT violating effects that would be seen in the $K^0 - \overline{K^0}$ system. J. Ellis **et al.** [9] introduced three new CPT violating parameters, α, β, and γ, describing this effect. In a recent analysis [27], Huet and Peskin pointed out that measurements of the magnitude and phase of η_{+-} and of the semileptonic charge asymmetry would be able to separately measure the three parameters. The experiment we will describe in this report would be the world's most sensitive [10] in this area.

The $K^0 - \overline{K}^0$ system provides us with an incredibly finely balanced interferometer that magnifies small perturbations such as CPT violating effects. It is a natural place to search for CPT symmetry violation since it exhibits C, P, and CP symmetry violation (and is the only place to date where CP violation has been seen). In the final analysis, the conservation or violation of CPT symmetry is an experimental question, and the search for this effect is of the utmost interest.

In K^0 physics, one can observe CPT violating effects through mixing or decays (called indirect or direct CPT violation respectively). In mixing, one introduces a parameter Δ which is both CP and CPT violating: All CP violation seen to date has been through the $(\epsilon - \Delta)$ term of the K_L. One can also have direct CPT violation, for example in semileptonic decays, where an amplitude y_l is introduced that is CPT violating [28]:

$$\begin{cases} \langle \pi^- l^+ \nu | T | K^0 \rangle = F_l(1 - y_l) \\ \langle \pi^+ l^- \overline{\nu} | T | \overline{K^0} \rangle = F_l^*(1 + y_l^*) \end{cases} \tag{4.1}$$

There are several measurements that would signify CPT violation: a difference between the phase of ϵ and the phase of η_{+-}, evidence for a non-zero Δ in the Bell-Steinberger relation, a difference between the phases of η_{+-} and η_{00}, or certain interference terms between K_L and K_S in semileptonic decays. In this report we will concentrate on the first two methods, measuring the phase of η_{+-} and comparing it to the calculated value of the phase of ϵ, and evaluating the Bell-Steinberger relation, since from them we can make the most accurate measurements.

We now consider the CPT test based on measuring the phase of η_{+-} and calculating the phase of ϵ. For what follows we adopt the Wu-Yang phase convention. Figure 6 shows the relationships between $\epsilon, \epsilon', \Delta$, and η_{+-}. ϵ' and Δ are shown greatly enlarged for clarity.

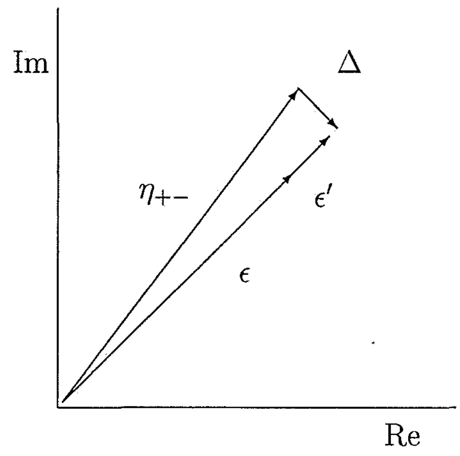

FIG. 6. The Wu-Yang Diagram

The size of $|\epsilon'/\epsilon|$ is of order 10^{-4}, and the phase of ϵ' is very close to that of ϵ, so the phase of the vector $\epsilon + \epsilon'$ is the same, to good accuracy, to the phase of ϵ (we show below that ϵ' is too small to have an affect on the calculation of the phase of ϵ at the level in which we are interested). We can see from the figure that the component of Δ perpendicular to ϵ, $\Delta_\perp$, is

$$\Delta_\perp = |\eta_{+-}|(\phi_{+-} - \phi_\epsilon) \tag{4.2}$$

where ϕ_{+-} (ϕ_ϵ) is the phase of $\eta_{+-}(\epsilon)$. In general, in terms of the elements of the kaon decay matrix Γ and mass matrix M, Δ is given by [29]:

$$\Delta = \frac{(\Gamma_{11} - \Gamma_{22}) + i(M_{11} - M_{22})}{(\Gamma_S - \Gamma_L) - 2i(M_L - M_S)} \tag{4.3}$$

The mass term has a phase perpendicular to ϕ_{SW}, the superweak phase, which is defined as $\tan\phi_{SW} = 2(M_L - M_S)/(\Gamma_S - \Gamma_L)$. ϕ_{SW} is approximately equal to ϕ_ϵ. The decay term is parallel to ϕ_{SW}. We can solve Eqns. 4.2 and 4.3 for $M_{11} - M_{22}$, which is the mass difference between the K^0 and $\overline{K^0}$ mesons, and get an equation which we can use to search for indirect CPT violation:

$$\frac{|M_{K^0} - M_{\overline{K^0}}|}{M_{K^0}} = \frac{2(M_L - M_S)}{M_{K^0}} \frac{|\eta_{+-}|}{\sin\phi_{SW}} |\phi_{+-} - \phi_\epsilon| \tag{4.4}$$

In Eqn. 4.4, Nature has been kind: the constant factors multiplying $|\phi_{+-} - \phi_\epsilon|$ are exceedingly small. $(M_L - M_S)$ is 10^{-6} eV, and when one divides by M_{K^0} the ratio is of order 10^{-15}. $|\eta_{+-}|$ is of order 10^{-3}. The product of all the factors multiplying $|\phi_{+-} - \phi_\epsilon|$ is 4×10^{-17}. By the Planck scale we mean

$$\frac{|M_{K^0} - M_{\overline{K^0}}|}{M_{K^0}} = \frac{M_{K^0}}{M_{Planck}} = 4.1 \times 10^{-20} \tag{4.5}$$

so a measurement of $|\phi_{+-} - \phi_\epsilon|$ accurate to 1 milliradian would test a CPT violating effect at the accuracy of the Planck scale.

Some of the present authors (SRS, SVS, and GBT) were part of Fermilab experiment E773. In this experiment we placed the limit (at 90% confidence level) [8],

$$\frac{|M_{K^0} - M_{\overline{K^0}}|}{M_{K^0}} < 1.3 \times 10^{-18} \tag{4.6}$$

so the result of Ref. [8] stands at 31 times the Planck scale.

That publication actually compared the phase of η_{+-} to the superweak phase (and stated clearly that in doing so the assumption was being made that CP violation would not be unexpectedly large in modes other than $\pi\pi$). In the calculation of the phase of ϵ, there are three corrections that should be made to the superweak phase: from $Im(x)$, the $\Delta S = \Delta Q$ rule violation parameter, from $Im(\eta_{+-0})$, and from $Im(\eta_{000})$. Together they have an uncertainty of 2.7 degrees which should be added in quadrature with the approximately 1 degree accuracy of Ref. [8].

The same three of us (plus D.B., H.W., T.A., and A.E.) are part of the KTeV experiment as well. There we expect to make an improvement of a factor of 3 to 5. Fig. 7 shows the z-dependence of $\pi^+\pi^-$ decays downstream of the regenerator in the KTeV experiment. The interference is seen very clearly. But the interference term from which ϕ_{+-} is measured, $2|\eta_{+-}||\rho|\cos(\Delta mt + \phi_\rho - \phi_{+-})\exp(-t/2\tau_s)$, is reduced by the regeneration amplitude $|\rho| \simeq 0.03$, and ϕ_{+-} and ϕ_ρ are hard to disentangle. Using the regeneration method will be difficult beyond the KTeV level [30].

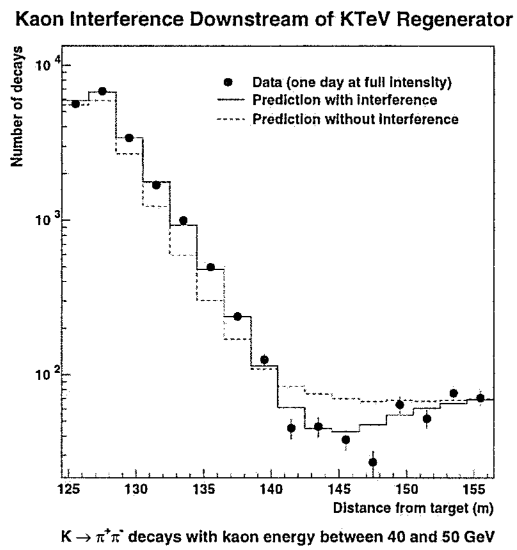

FIG. 7. $K_{L,S} \to \pi^+\pi^-$ Decays downstream from the regenerator in the KTeV Experiment. The longitudinal position of the kaon vertex is shown.

It should be understood clearly that measuring the phase of η_{+-} and comparing it to the superweak phase does not constitute a complete test of CPT symmetry conservation: the corrections to the superweak phase have larger uncertainties than existing experimental measurements of ϕ_{+-}. For example, if a significant difference between ϕ_{+-} and ϕ_{SW} were found in an experiment it would NOT prove that CPT symmetry was violated. More accurate measurements of $Im(x), Im(\eta_{+-0})$, and $Im(\eta_{000})$ must be made before this could be proved. An interference experiment located just downstream of the production target is needed for these measurements. In a regeneration experiment the interference in 3π decays is reduced in size by a factor of ρ, the regeneration amplitude, which is about 0.1 at most (at Main Injector energies), compared to an experiment near the production target, and it's extremely difficult for a regeneration experiment to measure $Im(x), Im(\eta_{+-0})$, and $Im(\eta_{000})$ to the required accuracy.

V. TWO TESTS OF CPT SYMMETRY CONSERVATION

A. The Phase Difference between η_{+-} and ϵ

After the KTeV experiment we expect to stand an order of magnitude above the Planck scale. To close that gap we will want to do an interference experiment near the kaon production target. The interference term is then $2D|\eta_{+-}|\cos(\Delta mt - \phi_{+-})\exp(-t/2\tau_s)$. Here ϕ_{+-} appears alone, and $|\rho|$ is replaced with the dilution factor, $D = (K^0 - \overline{K}^0)/(K^0 + \overline{K}^0)$ at the target. To maximize D and hence the interference, we choose to make our K^0 beam from a K^+ beam by charge exchange. Then at medium to high Feynman x, $D \simeq 1$. The charge exchange cross section is large, about 20% of the total cross section. To maximize the flux of K^+ made from the 120 GeV/c protons from the Fermilab Main Injector we choose a K^+ momentum of 25 GeV/c. We would use a hyperon magnet to define the K^0 beam, similar to the one in the Proton Center beam line. As in previous sections, in the calculations described below we assume the use of a vee spectrometer, a lead glass electromagnetic calorimeter, and a muon detector.

In Ref. [8] ϕ_{+-} was measured to 1^o accuracy. A CPT-violating mass difference exactly at the Planck scale would result in $|\phi_{+-} - \phi_\epsilon| = 0.06^o$. We set ourselves the goal of measuring ϕ_{+-} and ϕ_ϵ to sufficient accuracy to see such a CPT-violating effect.

We have calculated the statistical sensitivity of the CPT measurements using the same assumptions and programs used above in describing the sensitivity of this experiment for CP violation measurements.

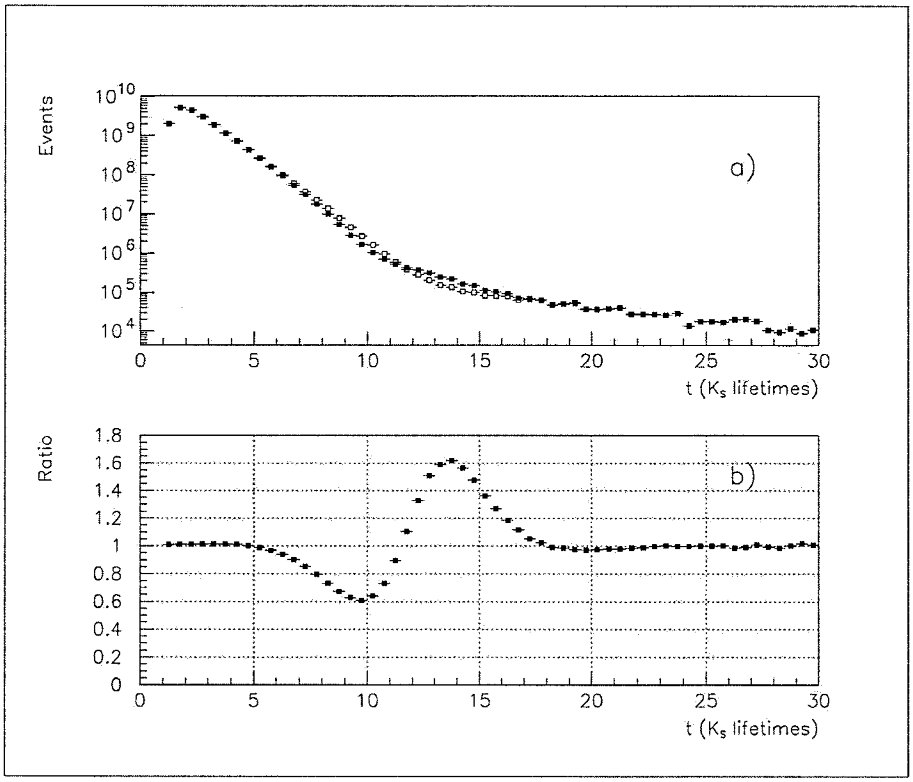

FIG. 8. Proper Time Distributions for $\pi^+\pi^-$ Decays a) Distributions are shown both with interference (dark squares) and without (light squares). b) The ratio of the two distributions in part a).

Fig. 2 shows the momentum spectrum of kaons exiting from the target, and of K_S and K_L decaying in the decay region. Fig. 8 shows the proper time distribution of accepted events. The figure shows the actual proper time distribution and also what the distribution would look like if there were no interference. The second part of the figure shows the ratio of those two curves. Between 5 and 20 K_S lifetimes the interference is first a 40% destructive effect then is a 65% constructive effect.

We calculated the distribution of events in momentum and proper time for the resulting 20 billion events and fit this distribution using MINUIT, with fitting parameters $|\eta_{+-}|$, ϕ_{+-}, D, and three parameters describing the normalization and shape of the kaon momentum spectrum. The interference term from which ϕ_{+-} is measured also depends on τ_S and Δm, the K_S lifetime and the $K_L - K_S$ mass difference. Although the latter parameters are not relevant for the purposes of the fit, they are strongly correlated with ϕ_{+-}. We can minimize the effects of these correlations by making a better measurement of Δm using our K_{e3} and $K_{\mu 3}$ data (this measurement of Δm will be about 20 times more accurate than the current PDG value), then use this result as a constraint in our ϕ_{+-} fit. The uncertainty that results from this fit is 0.040 degrees. This will meet our goal of testing CPT symmetry conservation at the Planck scale. This number ± 0.040 degrees has another meaning: it is the statistical (including fitting) uncertainty of this measurement, and sets the scale against which all other aspects of the $|\phi_{+-} - \phi_\epsilon|$ measurement should be compared.

In this experiment we measure ϕ_{+-}, but we must also determine ϕ_ϵ. The leading contribution to ϕ_ϵ is the superweak phase, ϕ_{SW}, given by $\tan(\phi_{SW}) = 2\Delta m/\Delta\Gamma$. The superweak phase will be measured by KTeV to accuracy sufficient for our purposes here. We next describe some corrections to this contribution.

For this experiment, ϵ' will have no meaningful effect. Assuming CPT invariance, the phase of ϵ' is known to be (48 ± 4) degrees [31]. Its magnitude is unknown, but if we assume it to be the central value from E731 we find that the maximum possible difference it can provide between ϕ_{+-} and ϕ_ϵ is 0.003 degrees, a factor of 20 smaller than the contribution of CPT violation at the Planck scale.

The full formula for ϕ_ϵ is [32]

$$\tan\phi_\epsilon = \frac{2\Delta m}{\Gamma_S - \Gamma_L}\cos\xi + \frac{\sin\xi}{\delta} \tag{5.1}$$

where $\xi = \arg(\Gamma_{12} A_0 \overline{A}_0{}^*)$ and $\delta = 2Re(\epsilon)$. Here A_0 is the isospin 0 part of the $\pi^+\pi^-$ decay amplitude. In the Wu-Yang phase convention, A_0 is real, and Γ_{12} gives contributions from two sources: semileptonic decays through $Im(x)$, the $\Delta S = \Delta Q$ violation parameter, and 3π decays through $Im(\eta_{+-0})$ and $Im(\eta_{000})$.

In the standard model we expect $x \simeq 10^{-7}$, which is too small to affect this experiment, but $Im(x)$ is known experimentally only to an accuracy of ± 0.026. This results in an uncertainty in ϕ_ϵ of 1.7 degrees. To prove that an observed difference between ϕ_{+-} and ϕ_ϵ were due to CPT violation one would have to measure $Im(x)$ about 40 times more accurately than today's level. The way we will do this is described below.

The contribution to ϕ_ϵ from the 3π modes in the standard model is 0.017 degrees, which is smaller than the accuracy we are trying to obtain. But if one takes into account the current world's knowledge, the uncertainty these decay modes contribute is 2.2 degrees. So they also have to be measured better.

The best experimental approach to measuring these three quantities, x, η_{+-0}, and η_{000}, is the same: choose an experiment with high dilution factor and observe interference between K_L and K_S close to the target; i.e. the experiment described here. These measurements should be thought of as being an integral part of this experiment. We have performed a calculation of the sensitivity of this experiment for these quantities, and we estimate that we can reach at least the required sensitivity. We conclude that we can determine ϕ_ϵ to the required accuracy. More details are given below.

We used the same Monte Carlo and fitting programs to estimate the sensitivity of our experiment to the measurements necessary for the calculation of ϕ_ϵ, $Im(x), Im(\eta_{+-0})$, and $Im(\eta_{000})$, and conclude that we will have the required sensitivity. We find that the uncertainty in $Im(x)$ contributes much more than $Im(\eta_{+-0})$ and $Im(\eta_{000})$ to the uncertainty in ϕ_ϵ. The detailed results are as follows.

- In the same time we are collecting $K_{\pi 2}$ decays for the measurement of the phase of η_{+-}, we will also collect 980 M K_{e3} decays and 990 M $K_{\mu 3}$ decays with which to measure $Im(x)$. Because there is a missing neutrino in each of these decays, there is a twofold ambiguity in the reconstruction of the the parent kaon's momentum. Using the spectrum of accepted K_{e3} events, we chose the more probable solution. This method gives the correct solution 83% of the time. We fit the resulting momentum vs. proper time distribution of the two semileptonic decays for $Re(x)$ and $Im(x)$. The resulting uncertainty in $Im(x)$ from the fit is $\pm 0.5 \times 10^{-3}$. This should be compared with $\pm 0.6 \times 10^{-3}$ which is needed to determine ϕ_ϵ to .04 degrees. We are thus within the accuracy regime we require.

- We will collect about 230 M $\pi^+\pi^-\pi^0$ decays. These would allow us to measure

$Im(\eta_{+-0})$ to $\pm 0.35 \times 10^{-3}$, which is smaller than the $\pm 4 \times 10^{-3}$ needed to determine ϕ_ϵ to ± 0.04 degrees. This will result in a 6.5σ measurement of η_{+-0}.

- We will collect about 70 M $\pi^0\pi^0\pi^0$ decays and measure $Im(\eta_{000})$ to $\pm 0.64 \times 10^{-3}$ accuracy. This is also better than the $\pm 4 \times 10^{-3}$ we need for the ϕ_ϵ determination, and will result in a 3.5σ measurement of η_{000}. This decay is technically the most difficult one to measure. It drives the design of our detector in several ways. Because the decay γ-rays spread out so much we need to have an electromagnetic calorimeter covering an area of about 4 square meters. A hole must be left in the center of the calorimeter to let the neutral beam pass through, but the fact that ours is not a high-rate experiment will make it possible to place a small supplementary calorimeter downstream, covering the hole region, to catch the gamma rays in that region.

We have estimated the systematic uncertainties expected in this experiment.

To deal with the background under our $\pi^+\pi^-$ events due to semileptonic decays, particularly K_{e3}'s, we have calculated that we must correct for this effect to the 0.1% level. This is five times less stringent than the 0.02% level that is sought for in the KTeV experiment.

Another background comes from $\pi^+\pi^-$ decays of kaons that are produced either in the hyperon magnet collimator or regenerated from K_L's that pass through the edges of the collimator (jointly called collimator production). We calculated this rate using GEANT and the geometry of the collimator (to be described below), and find it to be about 1.5% of the flux of kaons that pass through the collimator. This is an acceptable rate, as we can measure this background to 5% of itself and correct for it.

The accuracy with which one must know the acceptance is an important parameter of the experiment. We have estimated that accuracy by generating the distribution of $\pi^+\pi^-$ events in momentum and proper time, then introduced a z-dependent bias into the acceptance before fitting. The size of the bias that results in a change in the central value of ϕ_{+-} of 0.04 degrees is then a measure of the accuracy needed in the experiment. That acceptance bias was .07%. This is also less stringent than what we expect to achieve in KTeV, and we expect to do better than $\pm 0.07\%$ in our experiment.

If the phase of η_{+-} were to change by 0.04 degrees, the interference term would shift in space, for the average momentum kaon, by a distance of 1.3 mm. We must know the decay point and t=0 point (on the average) to better than this distance. For the decay point, we estimate that we can determine drift chamber wire positions to 0.2 mm, as well as measure the location of the center of the target to that accuracy. In the latter determination we are helped by the fact that the incoming K^+ and outgoing K^0 are being absorbed in the target with the same cross section, making the center of the target the average production point. To test whether scattering of the K^0's in the target could contribute to a change of the average production point we performed a Monte Carlo simulation that follows individual K^+ mesons through the target, produces K^0's through charge exchange, and allows the K^0's to scatter elastically. The outgoing K^0 then has a different energy than when it was produced, so when we observe it and calculate the production point as the center of the target we are making a small error. In performing this Monte Carlo calculation we learned that there is a solid angle effect which moves the average production point downstream by 0.9 mm, but that the scattering effect we were concerned with is only an 0.3 mm effect. It is clear that we must understand the size and divergence of the beam to good accuracy to be able to perform this experiment. Measurements of these quantities must be part of our experiment. Running with two different target thicknesses would provide a good test of our target-length correction calculation.

B. CPT Test via the Bell-Steinberger Relation

The next test of CPT symmetry conservation comes through an evaluation of the Bell-Steinberger relation. Our ability to measure CP violation parameters (and also $Im(x)$) very accurately will make it possible to reduce the uncertainties in the Bell-Steinberger relation by two orders of magnitude, which will make this CPT test be sensitive at the Planck scale also.

The Bell-Steinberger relation [12] is a statement of the conservation of probability in $K^0 - \overline{K}^0$ decays, in which, through Eq. 2.1, Δ appears. It is usually written as:

$$(1 + i\tan\phi_{SW})[Re(\epsilon) - iIm(\Delta)] = \sum_f \alpha_f \tag{5.2}$$

where the sum runs over all decay channels f, and $\alpha_f = \frac{1}{\Gamma_S} A^*(\mathrm{K_S} \to f) A(\mathrm{K_L} \to f)$. Table I shows all of the decays that contribute to the states, f, and the formulas for each α_f. In addition the table lists $\delta Re(\alpha_f)$ and $\delta Im(\alpha_f)$, the uncertainties in the real and imaginary parts of each contribution. These uncertainties are taken from the 1996 Particle Data Group compilation [31]. The most recent published evaluation of the Bell-Steinberger relation is ref. [20].

Decay Mode	α_f	$10^6 \times \delta Re(\alpha_f)$	$10^6 \times \delta Im(\alpha_f)$
$\mathrm{K}_L \to \pi^+\pi^-$	$\alpha_{+-} = B^{(S)}_{+-}\eta_{+-}$	15.5	15.5
$\mathrm{K}_L \to \pi^0\pi^0$	$\alpha_{00} = B^{(S)}_{00}\eta_{00}$	10.7	10.8
$\mathrm{K}_L \to \pi^+\pi^-\gamma$	$\alpha_{+-\gamma} = B^{(S)}_{+-\gamma}\eta_{+-\gamma}$	0.64	0.66
$\mathrm{K}_L \to \pi e\nu$ and $\pi\mu\nu$	$\alpha_{l3} = \frac{\tau_S}{\tau_L}[B^{(L)}_{\pi e\nu} + B^{(L)}_{\pi\mu\nu}]$		
	$\times[\delta_l(1 + 2Re(x)) - 2iIm(x)]$	0.20	59.2
$\mathrm{K}_S \to \pi^+\pi^-\pi^0$	$\alpha_{+-0} = \frac{\tau_S}{\tau_L} B^{(L)}_{+-0}\eta^*_{+-0}$	0.01	6.51
$\mathrm{K}_S \to \pi^0\pi^0\pi^0$	$\alpha_{000} = \frac{\tau_S}{\tau_L} B^{(L)}_{000}\eta^*_{000}$	65.6	98.5

TABLE I. Contributions of kaon decay modes to the Bell-Steinberger relation

The biggest uncertainties in the Bell-Steinberger relation at this time come from η_{000}, $Im(x)$, and δ_l (the charge asymmetry in K_L semileptonic decays). Although δ_l doesn't explicitly appear in Table I, it is the best way of evaluating $Re(\epsilon)$. The proposed experiment will be able to make excellent measurements of the first two of these quantities, and KTeV will measure δ_l quite accurately. For the next level of accuracy in the Bell-Steinberger relation the uncertainties of the α_{+-} and α_{00} terms must be reduced. These uncertainties depend on those of $|\eta_{+-}|, Re(\epsilon'/\epsilon)$, and $\Delta\phi = \phi_{00} - \phi_{+-}$. The latter two quantities will be measured by the KTeV experiment to sufficient accuracy for our purposes here.

We will have good sensitivity for the $|\eta_{+-}|$ measurement. In our fits to the proper time dependence of $\pi^+\pi^-$ events we have excellent statistical sensitivity for measuring $|\eta_{+-}|$. In the interference term, however, it is highly correlated with D, the dilution factor. We will measure D using semileptonic decays. The semileptonic charge asymmetry at zero proper time equals D. We calculate that we will be able to measure D to better than 0.1% for momenta above 13 GeV/c. We should be able to measure $|\eta_{+-}|$ to 0.1% accuracy, about 10 times better than it is currently known.

The most accurate way to determine $|\eta_{00}|$ will be by using the KTeV value of ϵ'/ϵ and our measurement of $|\eta_{+-}|$. The most accurate way of determining ϕ_{00} will use the KTeV value of $\Delta\phi$ and our measurement of ϕ_{+-}.

We should be able to reduce the uncertainties in the Bell-Steinberger relation by about two orders of magnitude from their present values. The limit on $Re(\Delta)$ will be about 5×10^{-6}, about twice the contribution of CPT violation at the Planck scale, and will be set by the uncertainty in δ_l. For $Im(\Delta)$ the limit will be about 1×10^{-6}, dominated by the uncertainty in $Im(x)$, which would allow us to place a 2σ limit at the Planck scale. Since the Bell-Steinberger measurement is sensitive to $Re(\Delta)$ and $Im(\Delta)$ independently, these limits would be valid even if Δ is parallel to ϵ, in contrast to the CPT violation limits from $|\phi_{+-} - \phi_\epsilon|$, which are sensitive only to the component of Δ perpendicular to ϵ.

Item	Existing Uncertainty	Anticipated Measurements
ϕ_{+-}	$\pm 1^o$	$\pm 0.040^o$
$Im(x)$	± 0.026	± 0.0005
$Im(\eta_{+-0})$	± 0.017	± 0.0003
$Im(\eta_{000})$	± 0.3	± 0.0006
$\lvert\eta_{+-}\rvert$	$\pm 1\%$	$\pm 0.1\%$
ϕ_{SW}	$\pm 0.12^o$	$\pm 0.024^o$ (KTeV)
δ_l	$\pm 2.7\%$	$\pm 0.3\%$ (KTeV)
$Re(\epsilon'/\epsilon)$	$\pm 6 \times 10^{-4}$	$\pm 1 \times 10^{-4}$ (KTeV)
$\Delta\phi$	$\pm 1^o$	$\pm 0.25^o$ (KTeV)

TABLE II. Measurements for CPT Conservation Tests

Table II summarizes all of the quantities we would measure to make the two tests of CPT symmetry conservation. We also list the important contributions to be made to this experiment by measurements the KTeV experiment will make.

VI. STUDY OF RARE K_S DECAYS

A great deal of attention has been paid to searches for rare decays of the K_L meson, but little work has been done on the K_S. Some of the interesting K_S searches are for decays that test strangeness-changing neutral currents. One such decay is $K_S \to \pi^0 e^+ e^-$. This is a rare but CP conserving decay. Its branching ratio must be measured in order to unambiguously disentangle the indirect CP violating contribution to $K_L \to \pi^0 e^+ e^-$ decay. The latter decay is one of the main candidates for finding direct CP violation in a rare K^0 decay. It has contributions from direct and indirect CP violating processes, and from a CP conserving two-photon exchange diagram as well. An accurate theoretical calculation of the CP conserving rate can be made, but the uncertainties of the theoretical calculation of the indirect CP violating contribution are quite large. The branching ratio for the equivalent K_S decay can be used to calculate this indirect CP violating contribution. Once these two rates are known, the direct CP violating part can be extracted once the branching ratio of the K_L decay is measured.

The $K_S \to \pi^0 e^+ e^-$ branching ratio is expected to be between 5×10^{-10} and 5×10^{-9} [33]. Since our single event sensitivity for this decay will be 1.0×10^{-10}, we expect to collect between 5 and 50 of these events.

As an example of how this measurement could be used to contribute to the determination of the direct CP violating part of the $K_L \to \pi^0 e^+ e^-$ decay, let's assume that we have 16 K_S events and have thus measured the indirect CP violating part of the K_L to 25% of itself. Let's further assume that each of the three contributions to the K_L decay are equal, so the calculation of the CP conserving contribution and the measurement of the indirect CP violation part add up to 67% of the total, with uncertainty of $\pm 8\%$ for the indirect CP violating part (and much smaller uncertainty for the CP conserving part). Then this sum would be $(67 \pm 8)\%$, and a measurement of the K_L decay accurate to $\pm 5\%$ would determine the direct CP violating part to 3.5σ significance.

In this simple model the uncertainty in the K_S decay has reduced the significance of the measurement of the direct CP violating part of the K_L by a factor of 1.9. Since the measurement of the K_L branching ratio is so difficult one would not want to reduce the significance of a measurement in this way, hence a more accurate K_S measurement would be desirable. This could be made by using the pions in the secondary beam: by turning off the RF cavities and removing the stopper from the beam, an increase in beam intensity of a factor of 10 could be achieved. The dilution factor of these kaons would be decreased by a large factor, but that is not important for this measurement.

It is worth mentioning that an extension of this experiment with higher flux (but with high dilution factor) would be a good vehicle for finding direct CP violation in the $K_L \to \pi^0 e^+ e^-$ decay through $K_L - K_S$ interference. In this method the "Greenlee background" (the K_L radiative Dalitz decay) is not a problem as it is in a conventional K_L experiment.

One way to get this additional intensity would be to significantly increase the solid angle of the collimator in the hyperon magnet. We have examined this scenario only in a preliminary way, but it might be a possibility.

Another rare decay of interest is $K_S \to \gamma\gamma$. This decay exhibits an interesting set of interference terms, due to indirect and direct CP violation, that have never been seen. The K_S (K_L) branching ratio is 2×10^{-6} (6×10^{-4}). Our experimental design does not have the set of veto counters for photons that would be needed to make a clean identification of the $\gamma\gamma$ decay (due to the large background from $\pi^0\pi^0$ decays where two photons are lost), so we would concentrate on the Dalitz decay, $K_S \to e^+e^-\gamma$, which has not been observed previously. We would lose rate, but would still see several hundred K_S events, all at small proper times. We would see ten thousand K_L events, spread approximately evenly over the 30 τ_S proper time range of the experiment. The interference should stand out clearly.

Another Dalitz decay with interesting CP properties is $K_S \to \pi^+\pi^-e^+e^-$. In addition to time-dependent interference between K_S and K_L, there is a CP-violating asymmetry in the angle between the decay planes of the two pions and two electrons. This decay-angle asymmetry has been discovered by the KTeV experiment. Our experiment would be the first observation of the K_S decay.

The decay $K_S \to \pi^+\pi^-\pi^0$ has a CP conserving part as well as a CP violating part. The final state isospin of the three pions must be 0, 1, 2, or 3, and the $I = 0$ and 3 states are strongly suppressed. The CP of the final state is $(-1)^I$ so the $I = 1$ state is CP violating for K_S decays and the $I = 2$ state is CP conserving. These two final states are easily separated since the Dalitz plot distribution is symmetric (antisymmetric) under the exchange of the π^+ and π^- for the CP violating (conserving) decay. For a detector which is charge symmetric, simply integrating over the Dalitz plot picks out the CP violating decay very precisely. Forming the asymmetry between the decays where the π^+ has more energy in the center of mass system, and where the π^- is more energetic, picks out the CP conserving decay. Then one can observe the $K_L - K_S$ interference as a function of proper time, fit for the K_S decay amplitude, and compute the branching ratio. This technique was used in

Fermilab experiment E621, which discovered [35] this decay. They measured the branching ratio to be $(4.8^{+2.2}_{-1.6}(\text{stat})\pm 1.1(\text{syst})) \times 10^{-7}$. Figure 9 shows the proper time graph from which this branching ratio was computed. The decay amplitude can be predicted by chiral perturbation theory, and with better experimental precision an interesting test can be made of that theory.

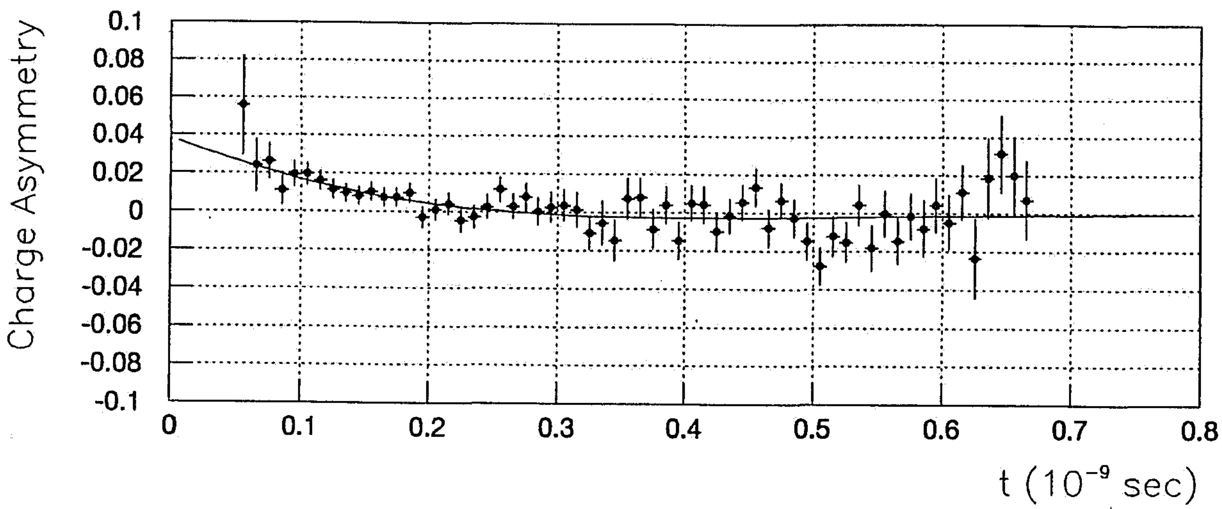

FIG. 9. Proper Time Distributions for $K_{L,S} \to \pi^+\pi^-\pi^0$ CP-conserving Decays from Fermilab Experiment E621. The asymmetry between events where the π^+ is more energetic in the center of mass, and events where the π^- is more energetic, is shown, as a function of the proper time of the decay.

VII. K^+ BEAM DESIGN

Our experiment requires an RF-separated K^+ beam. There are two technical issues to be addressed before such a beam can be made: how to design the optics of the beam, and how to build the necessary RF cavities. Our progress in tackling these issues is outlined below.

One of us (J. D.) designed the optics of an RF separated K^+ beam for the TRIUMF KAON accelerator [34], and has modified his design for the present experiment's needs. Our design has superconducting C-band RF cavities, operating at 5.79710 GHz to perform the separation. In the past a separated K^+ beam was built at CERN using superconducting S-band (2.79 GHz) cavities, and normal S-band cavities have been used to make K^+ beams at the Brookhaven AGS and at the Argonne ZGS. The superconducting RF cavities used for the CERN beam have been loaned to the Serpukhov Accelerator Laboratory, where they will be used in the near future to make an RF separated K^+ beam.

In general, the way the RF separation works is that the first cavity imposes a transverse momentum kick on the beam of 10 MeV/c per meter of cavity length, creating a waving-fan of particles exiting from it. The second RF cavity is located 75 m downstream, and in between is a system of quadrupoles with a transformation matrix of (-1). The phase of the second cavity is tuned so π^+ mesons arrive with the same phase that they had in the first cavity. Since the quadrupoles have reversed the π^+ momenta they get the same kick from the second cavity and the π^+ end up with no net kick. With this design choice of RF frequency and distance between the cavities, the π^+ and protons are 360 degrees out of phase at the second cavity, so protons also receive no net kick. K^+ mesons are 90 degrees out of phase from the π^+ so they get a net bend of 1.7 mrad in our design. A beam plug downstream of the second cavity destroys the π^+ and protons, and passes most of the K^+. So our design separates K^+ from both π^+ and protons. We choose the direction of the RF cavities' kick to be vertical to decouple from the momentum selection, which is in the horizontal direction.

Fig. 10 shows a plan view of the beam line, and indicates how the CPT and CKM experiments could share the same K^+ beam. Fig. 11 shows the beam line optical elements and beam size in x and y, as a function of z. The first quadrupoles collect the K^+'s and focus them in x at a focus between bending magnets B2 and B3, where a momentum slit selects the central momentum and momentum width of the beam.

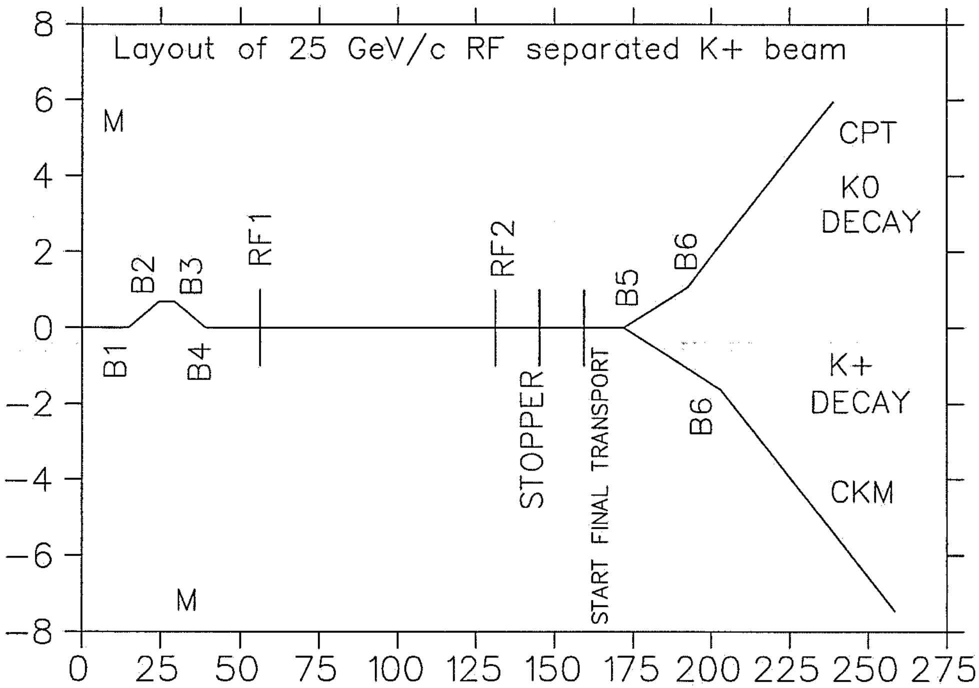

FIG. 10. Plan View of the K^+ Beam

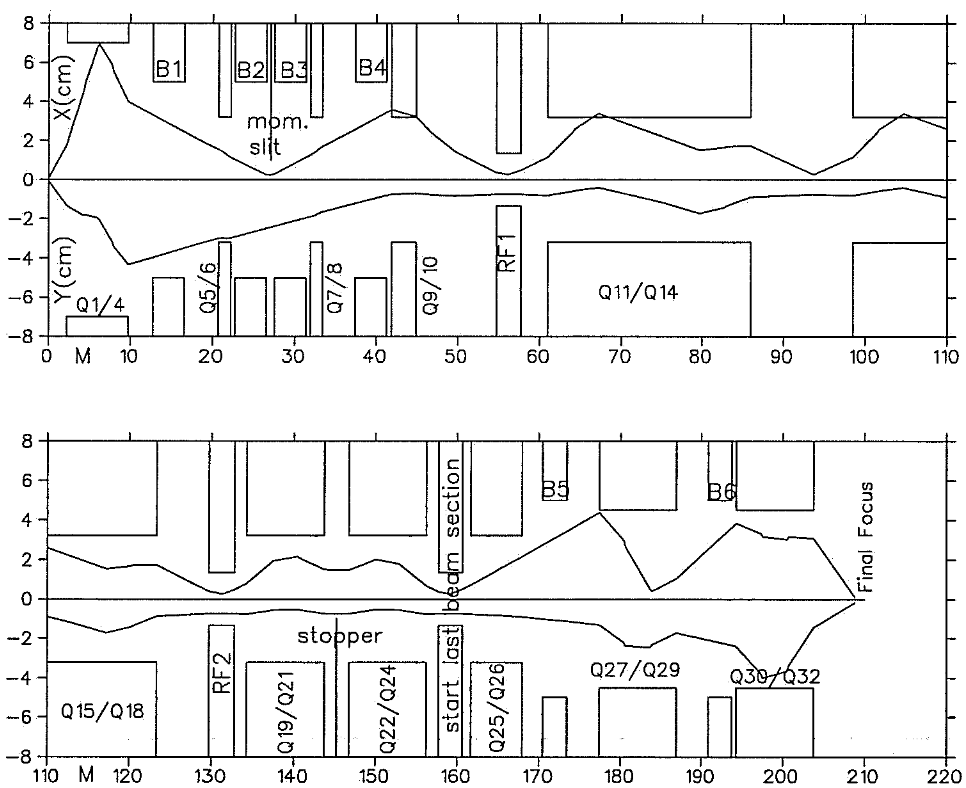

FIG. 11. K^+ Beam Line Elements and Beam Sizes

The proton dump is located after the first bending magnet. The quadrupoles before the first RF cavity transform the large y' acceptance into a small divergence (0.6 mrad) so the 1.2 mrad kick of the first RF cavity dominates the particles' angles. Two (-1) matrix quadrupole strings follow which give a net (+1) transformation, and the second RF cavity (RF2) is run at opposite phase (for pions) from the first. The next quadrupoles transform y' into y at the stopper. After that the beam is cleaned up with bending magnets and focused on the K^0 production target. The spot size at the target is about 5 mm in diameter.

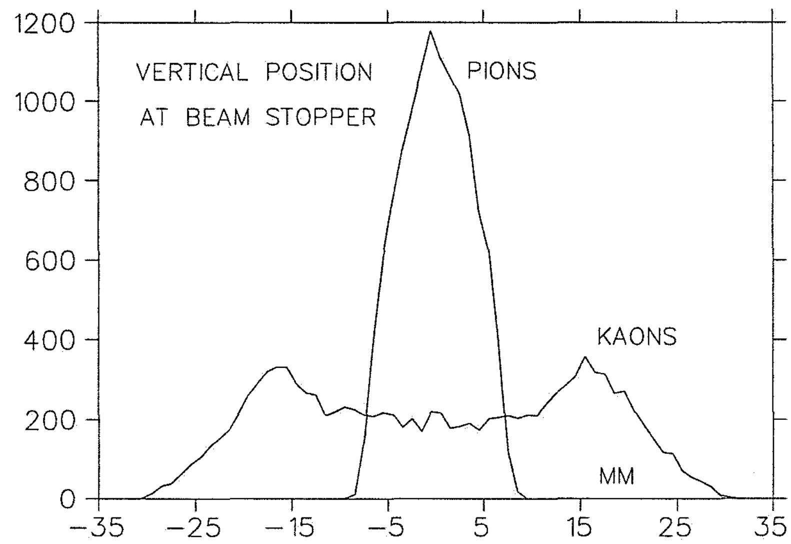

FIG. 12. y Coordinate of Pions and Kaons at the Stopper

This is a highly symmetric design which minimizes higher order aberrations. Using the TRIUMF Monte Carlo program REVMOC, which has been tested to give identical results to DECAY TURTLE, we calculated the acceptance of the beam, taking into account kaon decays. Fig. 12 shows the y coordinate of pions and kaons at the location of the stopper. It can be seen from the figure that the physical size of the stopper should be about 16 mm (± 8 mm), and that about half of the kaons are transported past it. With 6×10^{12} protons per pulse at 120 GeV/c striking the target at 0 mrad, we got a flux of 2.4×10^{8} kaons per pulse, which is what we used in our statistical significance calculations.

The prediction of TRANSPORT and REVMOC is that the beam will be very clean. One might expect that the main source of backgrounds would be interactions in the stopper. To test this we put the stopper and the magnets following it in GEANT, and used the phase space of pions, protons, and kaons determined from REVMOC as the input parameters of beam particles. We found that there was very little background from pions hitting the stopper, probably because they are heading into the material, but kaons that just clip the edge did produce pions of only slightly lower energy that were closer to the beam phase space. They can be removed by collimators just before the B5 bending magnet, however. The largest source of background that we have found so far comes from scattering in the vicinity of the first RF cavity. This produces about 2% background in the beam, which is well within the cleanliness we need. These GEANT calculations are not yet complete, but the results so far are quite promising.

The second technical question is how to build the RF cavities. We need two cavities each 3 m long, which produce a deflecting field of about 10 MV/m. They must be superconducting, since the Main Injector spill length would require enormous RF power otherwise. The superconducting cavities built at CERN in 1977 had a field strength of 1.3 MV/m, so are too weak for our application. The state of the art in achieving field strength has advanced considerably since then, and cavities reaching 25 MV/m are regularly produced at DESY for the TESLA project.

A group in the Fermilab Beams Division, headed by Helen Edwards, is working on developing the lab's capability for building superconducting RF cavities. They are interested in carrying out the R&D for the cavities that we need, and three of us (T.K. full time, plus G.B.T. and H.B.W.) have joined Dr. Edwards' group to work on this project. Al Moretti of Fermilab has estimated that an initial period of R&D would be required that would last a year, and would cost about $750k. Building the actual cavities would take about 1 1/2 years and would cost about $1800k. The time scale of this work is reasonably matched to the Main Injector schedule, and the costs are reasonable for a facility that would initially serve two experiments and perhaps others in the future.

The first cavity that we are building was designed by Mark Champion and Mike McAshan of Fermilab, based on the design that has been used for the TESLA prototype cavities. It has a 3 cm iris, and is designed to operate in the $TM_{110} - \pi$ mode at 3.9 GHz (exactly three times higher frequency than TESLA cavities). The first cavity, of two cells with cylindrical end pieces, will be made by the Rutgers Physics Department Machine Shop, and will be copper. This will be tested at Dr. Edwards' lab at A0. Upon building a successful copper prototype, we will move to niobium, and test it in its superconducting state. For this prototype we will make use of the infrastructure at other labs (such as Cornell, Argonne, Jefferson Lab, or DESY) for electron beam welding, and the acid etching and high power/high purity water washing to produce the extremely clean niobium surface necessary to achieve high fields.

Although it is not a big project to construct superconducting RF cavities that deflect rather than accelerate a beam, and achieve the modest fields that we require at a somewhat higher frequency than people have used previously, it is the largest R&D project necessary for this experiment. The CP/T collaboration recognizes the importance of this project, and is investing appropriate person power in this project.

VIII. HYPERON MAGNET DESIGN

For the hyperon magnet, which cleans up the region where the K^+ beam strikes the K^0 production target, we have chosen a design similar to that of the working hyperon magnet now in the Fermilab Proton Center beam. This conventional magnet uses tapered pole pieces to make a 35 kGauss field in its central region. A plan view of the collimator in this magnet is shown in Fig. 13. The K^+ beam strikes a 10 cm tungsten target at 9 mrad, and is bent to the side to be destroyed in the collimator. Neutral particles follow a straight path through the waist of the collimator to form the beam. The magnetic field region is 1.5 m long, and the collimator shown is made of tungsten. After the magnet is a further meter of tungsten absorber which absorbs particles which punch through the collimator in the magnet. The jaws of this absorber point toward the edges of the first drift chamber, so they provide the maximum of absorption power without reducing the acceptance of the detector.

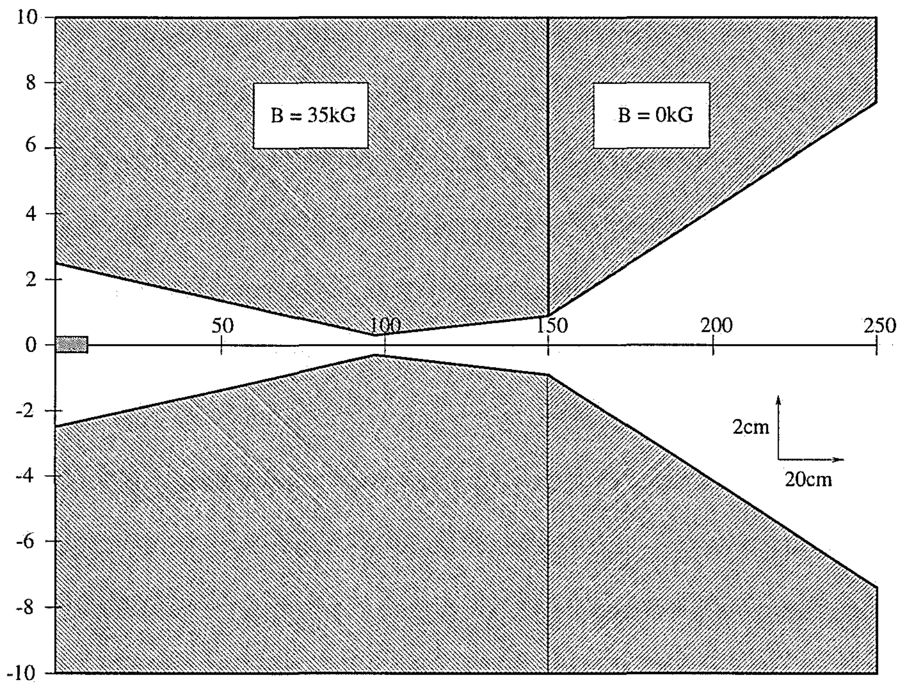

FIG. 13. Plan View of the Collimator in the Hyperon Magnet

IX. RATES AND TRIGGERING

Using the hyperon magnet design shown above we have performed a GEANT calculation of the resulting beam and particle intensities. GEANT reproduces the kaon flux described above in our subsection on the sensitivity of the experiment. We calculated the rates in the first drift chamber, which is the detector that will be exposed to the largest particle flux. With 2.4×10^8 K^+/sec the first drift chamber will see 120 kHz of charged particles, approximately evenly distributed over its face (which adds up to about 8 Hz per centimeter of sense wire). This rate is considerably lower than that seen in KTeV. In addition there is 1.4 MHz of γ-rays, most less than 10 MeV, striking it in the beam region. These photons are the remnants of electromagnetic showers in the high-Z target. Taking account of the material in the front end of the spectrometer, this leads to 4 kHz of conversions, or 50 Hz per centimeter of sense wire. This is much less than the space charge limit of 20 kHz per centimeter of sense wire.

The neutron/K_L ratio is 2.0 (a little better than KTeV), with average neutron momentum about 1 GeV/c. The Λ/K_S ratio is 0.1 (with average Λ momentum of 3 GeV/c), which is much lower than previous K_S experiments at Fermilab.

At these low rates the correct strategy would be to trigger on all 2-track events. This would result in 4 kHz of $K_{\pi 2}$ events per spill, and a data volume of about 2 MBytes/second. Again using KTeV for comparison, this is 1/10 of its capabilities. To estimate the total trigger rate we must add about 10% for Λ's, 20% for K_L decays, and perhaps 30% more for triggers that do not reconstruct into good Vees. We will also need a trigger for decays to $\pi^0\pi^0\pi^0$ final states. Here we will use similar strategies to our previous experiments and have a hardware trigger processor that counts clusters of hit blocks in the lead glass array, and another one that measures the total energy in the array. The Rutgers Physics Department Electronics Shop has made preliminary designs for both of these trigger processors.

These are quite low rates, easily handled by detectors and a data acquisition system. This comes from the fact that we are working in a tertiary K^0 beam. If we were to increase the proton beam intensity to 2×10^{13} protons per pulse (the Main Injector is expected to produce 3×10^{13}), all rates would still be less than those expected for KTeV, and the 12 months of data collection time would become only 3 months. This would be an important running consideration.

X. SITING THE EXPERIMENT IN THE MESON LAB

Tom Kobilarcik of the Fermilab Beams Division has looked into how to fit our beam and experiment into the existing beam lines of the Meson Lab. He has found that our very straight K^+ beam must be modified to fit the gently curving tunnels. This can be done without compromising the beam's quality by placing bending magnets at the locations of the RF cavities. There are two beam lines that seem appropriate for our K^+ beam, M-East and M-Test. There is an existing target pile in M-Test, and the beam is almost the perfect length if we use C-band RF cavities. For the M-East beam line, there is a target hall, currently empty, called the pion target hall, about 850 feet from the entrance to the Meson

Detector Building. The currently existing Meson East target pile could be mined for most of the elements we would need. The experiment would go into the east end of the Meson Detector Building. There is room in this beam line and experimental hall for the beam, the experiment, plus the CKM experiment, with whom we would share the beam.

The M-East beam line is about 200 feet longer than the optimal length if we were to use C-band RF cavities at 5.79 GHz. However, it may turn out that the cheapest cavities to build would use a somewhat lower RF frequency. If this is the case we may need this extra 200 feet.

All of the magnets needed for this beam exist at Fermilab, and have been "tagged" for our experiment.

XI. DETECTOR APPARATUS

A. Detectors We Own

We will use the lead glass electromagnetic calorimeter owned by the IHEP, Protvino group. This consists of 2916 blocks, each 3.8×3.8 cm$^2 \times$ 18 radiation lengths in size. They would be stacked in a 54×54 array. Although the rate of hadrons in the beam region is only about 190 kHz, and the particles are spread over about 50 counters, the transverse spread of neutron and K_L showers would raise the rates in the central third of the calorimeter blocks. Therefore we plan to leave a hole in the center of the array to let the neutral beam pass through. To catch gamma rays in the beam region a secondary array of radiation-hard lead glass blocks (also owned by the IHEP group) will be stacked behind the main array.

We made a preliminary design of a muon detector using GEANT. It consists of two modules, each consisting of an iron filter 2 m vertically $\times$ 3 m horizontally $\times$ 1 m thick followed by a hodoscope of scintillation counters and a drift tube chamber. This array would identify 98% of muons above 1 GeV/c and reject 99.8% of pions. We will use the drift tube chambers built by the IHEP group as the D0 small angle muon detectors, which are being replaced in the ongoing D0 upgrade. We will build new muon detector hodoscopes.

The counter in the vacuum at the end of the decay region will be built new. In previous K_S and hyperon experiments we have used scintillation counters 1 mm thick in this type of situation.

The two hodoscopes of counters for triggering would be built new. They would be similar to the trigger counters of the KTeV experiment.

B. Detectors We Could Borrow

We are looking into the question of borrowing other detectors we need. A system of drift chambers for our charged particle spectrometer is the main item in question. There are several analysis magnets at Fermilab that would work for our experiment, for example the Jolly Green Giant.

There are several drift chamber systems that will be available at the time of the first 120 GeV/c Main Injector fixed target run. One that many of us in the CPT collaboration are familiar with is the KTeV system. KTeV will metamorphose into KAMI on this time scale, and build a new tracking system in the process. The KTeV chamber frames are owned by the University of Chicago, and the wires and electronics are owned by Fermilab. The electronics was built by the Rutgers group, and one of us (SVS) was responsible for the entire drift chamber system management in KTeV. As these are the chambers we know best, it is they who are in our Monte Carlo program, and their sizes and resolution have gone into all the calculations of acceptances and resolutions for the present proposal. We are negotiating for their use.

Table III lists all of the items needed for our beam line and detector, the lead time needed to produce them, and their costs. The net cost of new items is $3960k. The longest lead-time items are the RF cavities for the K^+ beam, which will take 2.5 years for R&D and construction.

Location	Item	Lead Time	New Costs
Beam Line	Target Pile	0.5 yr	$1000k
	Magnets for K^+ Beam	exist	0
	RF Cavities	2.5 yrs	$2550k
	Hyperon Magnet	1 yr	$250k
Detector	Counter in vacuum	0.5 yr	$10k
	Drift Chambers	exist	0
	Analysis Magnet	exist	0
	Trigger Hodoscope	0.5 yr	$50k
	Lead Glass Calorimeter	exist	$50k
	Muon Detector	0.5 yr	$50k

TABLE III. Costs and Leadtimes

XII. OTHER EXPERIMENTS AROUND THE WORLD

Another experiment that aims to search for CPT symmetry violation is the KLOE experiment at the Frascati DAΦNE Φ factory. This is an "open geometry" experiment limited by e^+e^- beam luminosity. They expect to achieve a sensitivity to the fractional $K^0 - \overline{K}^0$ mass difference of about 1×10^{-18} [36], by studying interference in K_{e3} decays. This is 25 times worse than what we propose here.

The KLOE experiment expects to produce $15 \times 10^9 K_L + K_S$ events per year if the DAΦNE luminosity were $10^{33} cm^{-2} sec^{-1}$. For the decay $K_S \to \pi^0\pi^0\pi^0$, with a branching ratio expected to be 2×10^{-9}, this would amount to 30 events produced per year. Their best way to measure $|\eta_{000}|$ is by tagging K_S decays by observing an unambiguous K_L decay, since their $K_L - K_S$ interference is very small in this channel. If they achieve that luminosity, their acceptance were 100%, there were no tagging inefficiencies or backgrounds, and their poor resolution allowed them to reconstruct the 6γ decays properly, this would result in a measurement of $|\eta_{000}|$ accurate to about 10%, which would be excellent. However, the maximum luminosity of the DAΦNE storage ring is expected to be a factor of 2 lower, about half of the K_L decays will occur outside their fiducial region, and inefficiencies of all kinds will result in their finding many fewer events. Using their method for η_{+-0} is harder because the branching ratio is half as big. So the KLOE experiment will not be competitive to the present proposal.

The CPLEAR experiment has concluded its data collection, and their results will not be competitive.

The KAMI experiment at Fermilab, as proposed, does not have a regenerator with which to measure ϕ_{+-}. However the KTeV collaboration does own a beautiful regenerator, which was built by a collaborator on the present proposal (SVS), and that could be put into service. The $K_L - K_S$ interference term for decays downstream of the regenerator is smaller than that in the present proposal. In that term there is a factor $\cos(\Delta mt + \phi_\rho - \phi_{+-})$, from which ϕ_{+-} can be measured. Unfortunately ϕ_{+-} is 100% correlated with ϕ_ρ, which has been measured previously to only 1 degree accuracy. In KTeV it is hoped to measure ϕ_ρ to about 1/4 degree accuracy using K_{e3} decays, but an additional factor of 17 is needed to make a competitive measurement. This will be extremely difficult.

The importance of measuring in the same experiment the phase of ϵ and the phase of η_{+-} cannot be stressed too heavily. In an experiment with a regenerator x, η_{+-0}, and η_{000} cannot be measured to the required accuracy, therefore neither can the phase of ϵ.

The tests of the standard model using η_{+-0}, and η_{000} cannot be performed in a regenerator experiment. Studies of rare K_S decays cannot be performed in a regenerator experiment.

Due to the excellent photon veto system that the KAMI collaboration plans to build, there is one CPT conservation test that can be performed better there than with our detector, namely the measurement of $\Delta\phi = \phi_{00} - \phi_{+-}$. In the standard model this quantity is zero to the precision of a few tenths of a degree, and might deviate from zero at this level due to uncertainties in final state interaction phase shifts. A nonzero value of $\Delta\phi$ which is larger than this would be a signal of direct CPT violation. In our chapter on CPT phenomenology we pointed out the way that Nature has been incredibly kind in the search for indirect CPT violation by providing a free 17 order of magnitude factor in Eqn. 4.4. However, no such enhancement factor exists for $\Delta\phi$ and direct CPT violation. For example, in the model of Ref. [11] direct CPT violation is much too small for the KAMI experiment to see.

Encouraged by the work of A. Kostelecky [11], the B-factory experiments, Belle and BaBar, intend to perform CPT symmetry conservation tests similar to the $|\phi_{+-} - \phi_\epsilon|$ one described here [37]. The $B^0/\overline{B}^0$ system does not provide as sensitive a test as the kaon system, however.

At JHF, the 50 GeV/c proton accelerator proposed to be built at KEK in Japan, there is a proposal in preparation for an experiment very similar to ours to perform CP and CPT symmetry tests and measurements [38]. They plan to use the higher intensity of their proton beam to make a more intense K^+ beam and collect higher statistics than can be amassed at Fermilab.

XIII. CONCLUSION

We have described an experiment to carry out a systematic program of measurements in $K_S - K_L$ interference physics. We will search for CPT symmetry violation in the decays of K^0 mesons with the sensitivity to reach the Planck scale, measure CP violation parameters to test the detailed predictions of the Standard Model, and study rare kaon decays.

Our design uses protons from the Fermilab Main Injector to make an RF separated K^+ beam. With this we make a tertiary neutral kaon beam created in just the way to maximize the interference between K_S and K_L while maintaining high flux. We use a "closed geometry" hyperon magnet for beam definition. A standard Vee spectrometer, with drift chambers, an electromagnetic calorimeter, and a muon detector, is used to make the measurement.

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this paper of why their data is wrong. We do not mean to take the limits they quote as a benchmark against which the CP/T experiment should be compared, but the CP/T experiment will be able to place limits two orders of magnitude better than those quoted by CPLEAR.

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Fermilab Proposal P894

The CP/T Experiment

An Experimental Program to Study CP Violation and Search for CPT Violation in the $K_L - K_S$ System

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Abstract

This is a proposal for a Fermilab Main Injector experiment to carry out a program of measurements on the physics of K^0 mesons. The experiment is designed to maximize the interference between K_L and K_S mesons near their production target, and hence have excellent sensitivity to CP violation in many decay modes. The extremely accurate CP violation measurements we will be able to make will allow us to test CPT symmetry violation with sensitivity at the Planck scale.

We will measure four CP violation parameters that will test the Standard Model in new ways. Measurements of η_{+-0} and η_{000} will provide the first discovery of CP violation outside the K_L system, and a measurment of $\eta_{+-\gamma}$ will be a sensitive test of direct CP violation. We will measure the magnitude and phase of η_{+-} an order of magnitude better than it is presently known.

There are two ways that we will test CPT symmetry conservation which will be sensitive at the Planck scale: measuring the difference in phase between η_{+-} and ϵ, and evaluating the Bell-Steinberger relation. The measurements of η_{+-0} and η_{000} are an integral part of any program of CPT tests.

We will study rare K_S decays. $K_S \to \pi^0 e^+ e^-$ tests strangeness changing neutral currents and is necessary to disentangle direct CP violation in the K_L decay to the same final state. $K_{L,S} \to \pi^+\pi^-e^+e^-$ is a new CP violating decay mode. The CP-conserving decay $K_S \to \pi^+\pi^-\pi^0$ is of interest in chiral perturbation theory.

The experiment will use an RF-separated K^+ beam striking a target at the entrance to a hyperon magnet to make the K^0 beam by charge exchange. The decay region, magnetic spectrometer, electromagnetic calorimeter, and muon detector follow immediately to observe interference between K_L and K_S near the target.

Contents

I. INTRODUCTION

A. The CP/T Experiment

This is a proposal for a program of experiments at the Fermilab Main Injector to study CP violation, test CPT symmetry conservation, and search for rare decays of the K_S meson.

One of the burning questions in high energy physics today is that of CP symmetry nonconservation. This is truly a dimly illuminated corner of the standard model which needs more experimental work. In addition, CP violation plays a crucial role in our understanding of the baryon asymmetry of the universe, and in the standard model the size of the effect seems to be too small to explain the observed asymmetry. This may indicate the presence of physics beyond the standard model. It is very important to perform more measurements of CP violating phenomena to search for possible deviations from the standard model.

All observations of CP violating effects have occurred in K_L decays. The $\pi^+\pi^-$ and $\pi^0\pi^0$ decays and semileptonic charge asymmetry were discovered in the 1960's [1]. CP violation in $K_L \to \pi^+\pi^-\gamma$ was discovered by Fermilab experiment E731 [2]. The angular asymmetry in $K_L \to \pi^+\pi^-e^+e^-$ has been found by the KTeV experiment. All of these measurements are consistent with the following mechanism being at work: the K_L is a mixed state of the two CP eigenstates. Recently (February, 1999) the KTeV collaboration at Fermilab has confirmed a previous CERN result that direct CP violation (again of the K_L meson) exists. This is a second mechanism for CP violation.

The standard model predicts additional CP violating effects should occur in K_S, D, and B meson decays [3]. The K_S effects test the basic ideas of mixing of CP eigenstates. The B meson decays are very useful for determining the angles of the unitarity triangle. The discovery of CP violation in B meson decays may be only a year or two off.

The standard model also predicts that direct CP violation should occur [4]. Experiments to measure ϵ'/ϵ search for this effect. Violation of the $\Delta I = 1/2$ rule reduces the size of ϵ'/ϵ. Cancellation between gluonic and electroweak penguin diagrams reduce it further, so it is a small effect and hard to measure.

There are other manifestations of direct CP violation. It may contribute 10% to η_{+-0} and η_{000}, and 1% to $\eta_{+-\gamma}$. These effects would be seen as a difference between the above mentioned CP violation parameters and η_{+-}. The experiment proposed here will search for these effects. The easiest way to search for direct CP violation, next to ϵ'/ϵ, is to perform a simultaneous measurement of $\eta_{+-\gamma}$ and η_{+-}. Here there is no $\Delta I = 1/2$ rule suppression or destructive interference of penguin diagrams. The direct CP violation comes from the photon part of the decay amplitude (an electric dipole contribution to the direct emission term), and is fairly well understood. We should be able to measure this $\eta_{+-\gamma} - \eta_{+-}$ difference to 12σ accuracy.

CPT symmetry conservation is a subject under theoretical attack. Studies of Hawking radiation [5] and of string theory (the leading contender for a unified theory of all four forces of nature) [6] have shown the CPT theorem [7] to be invalid in real life (rather than in the three-force approximation we call the standard model). Many physicists are reluctant to accept the possiblity that CPT symmetry violation may occur at the Planck scale. One reason for this reluctance is that we have, so far, only theoretical hints that this is the case. Another reason is the great success of the standard model. It may be quite a few years before a theory that unifies all four of the forces of nature becomes mature enough so that convincing theoretical statements can be made about the CPT structure of the world.

Fortunately we don't have to wait. The $K_L - K_S$ system provides a way of testing the validity of CPT symmetry conservation where it is possible to perform extremely accurate experiments [8]. In this document we propose to do an experiment that will reach the Planck scale. Finding CPT symmetry nonconservation would be a major discovery that would change in a fundamental way how physicists view the world. If we don't find it we will strongly constrain several quantum theories of gravity [9] [11] and provide a powerful benchmark against which future theories must be measured.

Our goal is to perform the optimal experiment to study interference between K_L and K_S mesons near the production target. We will make an RF-separated K^+ beam, and produce a neutral kaon beam from it by charge exchange. In this process K^0's are produced copiously, but very few $\overline{K}^0$'s are made. This situation maximizes the size of the interference. We will use a hyperon magnet (a thinner version of the magnet in the Fermilab Proton Center beam line) to define the neutral beam and get as close to the target as possible. This "closed geometry" method of beam definition is excellent for collecting large amounts of data. The detector will consist of a standard Vee spectrometer, an electromagnetic calorimeter, and a muon detector. We own most of the apparatus we need, and hope to borrow a large fraction of the remainder to minimize the cost of the experiment.

The result of this design is an experimental arrangement that makes possible a program of measurements in this area. A rich harvest of Ph. D. theses would result from this program.

In this proposal we will describe the measurement of four CP violation parameters, two CPT symmetry conservation tests that are sensitive at the Planck scale, a test of the $\Delta S = \Delta Q$ rule, and studies of rare K_S decays. The four CP violation parameters are η_{+-0} and η_{000} which characterize CP violation in K_S decay and have never been measured, $\eta_{+-\gamma}$ which is important because its magnitude should show a large difference from that of η_{+-} which comes from direct CP violation, and η_{+-} itself. The measurements of η_{000} and η_{+-}, together with a measurement of $Im(x)$ (where x is the $\Delta S = \Delta Q$ rule violation parameter), will make possible a new evaluation of the Bell-Steinberger relation [12] with unprecedented accuracy. The two tests of CPT conservation come from a measurement of the phase difference between η_{+-} and ϵ, and from the evaluation of the Bell-Steinberger Relation.

The RF separated K^+ beam described below will be useful for other experiments as well. An experiment searching for the rare decay $K^+ \to \pi^+\nu\overline{\nu}$ would find it ideal. The "CKM" experiment proposed by P. Cooper **et al.**, needs about 1/10 of the intensity that can be achieved with our design and requires the beam to have a very small divergence. These conditions can be achieved with our beam design. In addition, with a few modifications, the K^+ beam could be turned into a $\overline{p}$ beam, making other types of experiments possible.

B. Version 2 of the Proposal

This is the second version of the CP/T experiment proposal. The new items that have been added can be fit into the following categories.

- Improvements in the Monte Carlo program. We have made substantial improvements in our Monte Carlo program in many areas. These are described more fully in Appendix A.

- Calculations of systematic uncertainties. We have performed many calculations of systematic uncertainties due to backgrounds, determined the uncertainty in ϕ_{+-} due to the uncertainty in the acceptance, and also made improvements in the methods of analyzing the data. These are listed in the table of contents and described in the body of the text.

- Improvements in our understanding of the beam and detector. We have made much progress in our understanding of the beam and detector. There are two areas that stand out: one is the result of our work on R&D on superconducting RF cavities done in collaboration with the Fermilab "A0" group headed by Helen Edwards. Here we have performed calculations and constructed prototypes of RF cavities and have come to understand how they work much better. The second is the realization that our lead glass electromagnetic calorimeter does not need to have a hole in it for the beam to pass through. This has had major implications for the design of our beam and detector, which are described below.

II. CP THEORY AND PHENOMENOLOGY

In the 1970's the first electroweak theory was written down by S. Weinberg, A. Salam, and S. Glashow. It was a four quark theory, and correctly predicted all electromagnetic and weak interaction phenomena except for one: it was a CP conserving theory. Soon three generalizations of the theory were made to include CP violation. Weinberg's own idea [13] was to enlarge the Higgs sector and get CP violation through an interference between Higgs-exchange diagrams. R. Mohapatra and J. Pati [14] chose a larger gauge group: $SU(2)_L \times SU(2)_R \times U(1)$, called the "isosymmetric" model, where CP violation occurs through interference between left and right handed intermediate boson exchanges. M. Kobayashi and T. Maskawa [3] chose to enlarge the Weinberg, Salam, Glashow theory by adding a doublet of quarks resulting in CP violation through the familiar box diagram. The subsequent discovery at Fermilab of the b and t quarks promoted the Kobayashi - Maskawa theory to be part of the "standard model".

The focus of research in CP violation today is to perform tests in the dark corners, so to speak, of the standard model. One of the most important is the search for direct CP violation. All nonzero measurements of CP violating effects are decays of the K_L meson, and all measurements before 1999 were consistent with the CP violation coming from the mixing between CP eigenstates in the K_L; i.e. from the K_1 term in the K_L part of Eq. 2.1.

$$\begin{cases} K_S = K_1 + (\epsilon + \Delta)K_2 \\ K_L = K_2 + (\epsilon - \Delta)K_1 \end{cases} \tag{2.1}$$

In Eq. 2.1 $K_1(K_2)$ is the CP-even (CP-odd) eigenstate, ϵ is the CP-violating mixing parameter, and Δ is the mixing parameter which is both CP and CPT violating. The standard model predicts that there should also be a contribution from the dominant K_2 term of the K_L, called direct CP violation. For K_L decays to two pions direct CP violation is parameterized by the quantity ϵ', and the ratio of the CP violation parameters describing the K_L decays to $\pi^0\pi^0$ and $\pi^+\pi^-$ is:

$$|\eta_{00}/\eta_{+-}| = 1 - 3Re(\epsilon'/\epsilon) \tag{2.2}$$

The effort to measure ϵ'/ϵ has been going on for almost 20 years, perhaps only recently arriving at results consistent among the various experiments. The main cause of this difficulty is that ϵ'/ϵ is very small, and has been getting smaller as our knowledge of the top quark mass has improved. The difference between η_{00} and η_{+-} comes through a transition that violates the $\Delta I = 1/2$ rule and is thus suppressed. In addition in the standard model there is destructive interference between penguin diagrams that contribute to ϵ'/ϵ. The theoretical prediction is that $\epsilon'/\epsilon = (3 \pm 2) \times 10^{-4}$. Those of us in the KTeV experiment (T.A., D.R.B., A.E., S.R.S., S.V.S., R.S., G.B.T., and H.B.W.) hope to measure ϵ'/ϵ to an accuracy of $\pm 1 \times 10^{-4}$ (and have data in hand to reach a statistical accuracy of about $\pm 1.5 \times 10^{-4}$), and the aim of CERN experiment NA48 is to reach $\pm 2 \times 10^{-4}$.

The announcement of the first results from the KTeV collaboration, in February, 1999, that $Re(\epsilon'/\epsilon) = (28 \pm 4) \times 10^{-4}$, was based on 20% of their data. This result is in close agreement with the previous CERN result of the NA31 experiment but contradicts the previous Fermilab result of E731. It seems to contradict the previous theoretical estimates as well: some authors [16] have reverted to mechanisms beyond the standard model to get $Re(\epsilon'/\epsilon)$ to be in the 10^{-3} range. As more KTeV data is analyzed and as results from NA48 (NA31's successor experiment) are announced, and as more theoretical work is done, the situation hopefully will become clearer.

There are many other manifestations of direct CP violation that do not suffer from the suppression factors that plague ϵ'/ϵ. There are several rare decays in which the contribution to the branching ratio of direct CP violation is large, $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \nu \overline{\nu}$ in particular, which are very interesting. Unfortunately these decays are VERY rare, with branching ratios expected to be of order 10^{-11}, and have significant background problems that, at this time, have not been solved.

We feel that a more promising approach to the study of direct CP violation is to look for differences between other CP-violation parameters and η_{+-}. In particular, measurements of $\eta_{+-\gamma}, \eta_{+-0}$, and η_{000}, if they yield values different from η_{+-}, would signify the existence of direct CP violation (or perhaps CPT violation) in these decay modes. There are also predictions of large differences between these CP violation parameters and η_{+-} coming from physics beyond the standard model. Each of these CP violation parameters is written (taking $\eta_{+-\gamma}$ as an example) $\eta_{+-\gamma} = \epsilon + \epsilon'_{+-\gamma}$, with ϵ coming from indirect CP violation and $\epsilon'_{+-\gamma}$ being the contribution from direct CP violation.

Our approach to the study of CP violation is particularly interesting if the preliminary result of E832 on ϵ'/ϵ is correct and that quantity is much larger than most theoretical estimates. To quote Gino Isidori, "the measurement of $\eta_{+-\gamma}$ becomes more interesting after the KTeV result (it could help to understand the nature of CP violation in the K system, e.g. discriminating between SM and possible extensions)" [17]. John Donoghue commented that if the large value of ϵ'/ϵ comes from less complete cancellation between gluonic and electroweak penguin diagrams then $\epsilon'_{+-\gamma}$ "could change by a factor of two or so. Of greater interest is the possibility that it is not the Standard Model. In this case, the +−gamma result could change in unknown ways depending on the model" [18].

There are two Feynman diagrams that contribute to the decays $K_{L,S} \rightarrow \pi^+\pi^-\gamma$, the inner bremsstrahlung (IB) diagram where the gamma ray comes from one of the charged pion legs, and the direct emission (DE) diagram where the gamma ray comes from the decay vertex. The K_S decay is dominated by the IB term, and the K_L has approximately equal contributions from IB and magnetic dipole (M1) DE terms. The former is CP violating (and should contribute ϵ to $\eta_{+-\gamma}$), and the latter is CP conserving. In $K_L - K_S$ interference the K_L M1 term does not interfere with the K_S IB term. On the other hand, if there were an electric dipole (E1) term in the K_L DE decay it would come from direct CP violation, and would show up in $K_L - K_S$ interference. This transition comes from the gluon penguin diagram, with the electromagnetic penguin diagram being one to two orders of magnitude smaller [19].

The prediction of the standard model is that the E1 term would make a contribution to $\epsilon'_{+-\gamma}$ which is of order $10^{-2} \times \eta_{+-}$. Several authors [15] estimate this difference to be between 1% and 2%. This difference varies as k^2, the square of the C.M. energy of the gamma ray, and the 10^{-2} estimates occur at the maximum energy. If one integrates over the Dalitz plot the interference term is smaller than the maximum value quoted above. Any observation of a variation of $\eta_{+-\gamma}$ with k would be a sign of direct CP violation.

The phase of $\epsilon'_{+-\gamma}$ is predicted to be $45° \pm 5°$ [20], making the $\epsilon'_{+-\gamma}$ vector essentially parallel to that of ϵ. This means that direct CP violation will only change the magnitude of $\eta_{+-\gamma}$, and not its phase.

In comparing different ways of searching for direct CP violation one might choose as a figure of merit the product of (relative difference from ϵ) $\times$ (square root of branching ratio). The method that uses $\eta_{+-\gamma}$ and η_{+-} has a figure of merit eight times better than the method that uses η_{00} and η_{+-}. In addition the acceptance of $\pi^+\pi^-\gamma$ events is higher than that of $\pi^0\pi^0$ decays, the $\pi^+\pi^-\gamma$ and $\pi^+\pi^-$ events are collected with the same trigger, and a simultaneous measurement of $\eta_{+-\gamma}$ and η_{+-} avoids the main difficulty of ϵ'/ϵ experiments, namely that the $\pi^0\pi^0$ decay is so difficult to understand.

The direct CP violation contributions to η_{+-0} and η_{000} are ϵ'_{+-0} and ϵ'_{000}, and share with $\epsilon'_{+-\gamma}$ the characteristics that they do not require violation of the $\Delta I = 1/2$ rule, and do not have cancellations between penguin diagrams. While in general ϵ'_{+-0} might be of order 10^{-3}, using PCAC and the soft pion theorem, one can relate the value of η_{+-0} at the center of the $\pi^+\pi^-\pi^0$ Dalitz plot to η_{+-}, with the result that ϵ'_{+-0} is much smaller. In ref. [21], the authors pointed out that the correction to this relation increases the size of ϵ'_{+-0} considerably. Various authors have estimated ϵ'_{+-0} and ϵ'_{000} to be between 2.5% and 10% of η_{+-} [20–22] in the standard model. In addition it is known that, in the standard model, ϵ'_{+-0} and ϵ'_{000} are purely imaginary.

The PCAC calculation is not valid for scalar exchange diagrams, so the multiple-Higgs boson model of CP violation could give much larger values of η_{+-0}, and this was the prediction from ref. [13]. But ref. [21] estimates ϵ'_{+-0} to be 5% of η_{+-} in this model. In the left-right gauge symmetric model there are also predictions of large values for ϵ'_{+-0} [14], but using a more modern limit on the mass of a right-handed W boson, ref. [21] calculates $|\epsilon'_{+-0}/\epsilon| = 0.10$.

In Eq. 2.1 the CP and CPT violating parameter Δ enters with opposite sign (by definition) in the K_S and K_L equations, so a measurement of η_{+-0} or η_{000} is sensitive to the existence of CPT violation. In particular, the current limit on the component of Δ which is parallel to ϵ is quite poor (about 25% of $|\epsilon|$) [23], so a comparison of $|\eta_{+-0}|$ and $|\eta_{+-}|$, for example, would be very interesting.

New tests of CP violation in the standard model can be performed by a compact experiment studying $K_L - K_S$ interference close to the kaon production target. Measuring $\eta_{+-\gamma}$ to high accuracy will provide a new testing ground for direct CP violation. Measuring η_{+-0} and η_{000} will prove for the first time that the mixing equation (Eqn. 2.1) is valid, will provide further new tests of direct CP violation, and will make possible a search for CPT violation and for effects beyond the standard model.

III. MEASUREMENT OF CP VIOLATION PARAMETERS

Our experiment is optimized to study $K_L - K_S$ interference near the kaon production target. This will allow us to measure several CP violation parameters not previously measured, and perform tests of the standard model not previously achievable.

In what follows we will quote sensitivities calculated by making the following assumptions:

- A beam of 3×10^{12} protons per spill at 120 GeV/c (about 1/10 of the expected initial Main Injector intensity), a 1 year-long run with the accelerator operating at 70% efficiency and the experiment being 75% efficient. This proton flux would produce a secondary K^+ beam of $1.2 \times 10^8 K^+$ per spill. The K^+ beam would strike a 10 cm tungsten target at 9 mrad.
- The measured charge exchange cross sections.
- A solid angle of 172 μster for the K^0 beam, a bit larger than beams described in the KAMI Design Report [24].
- A hyperon magnet 1.5 meters thick followed by 1 meter of tungsten absorber.
- A decay region 9.5 meters long, followed by a Vee spectrometer, lead glass electromagnetic calorimeter, and muon detector. Fig. 1 shows the hyperon magnet, decay region, spectrometer, and muon detector. The spectrum of neutral kaons is shown in Fig. 2.

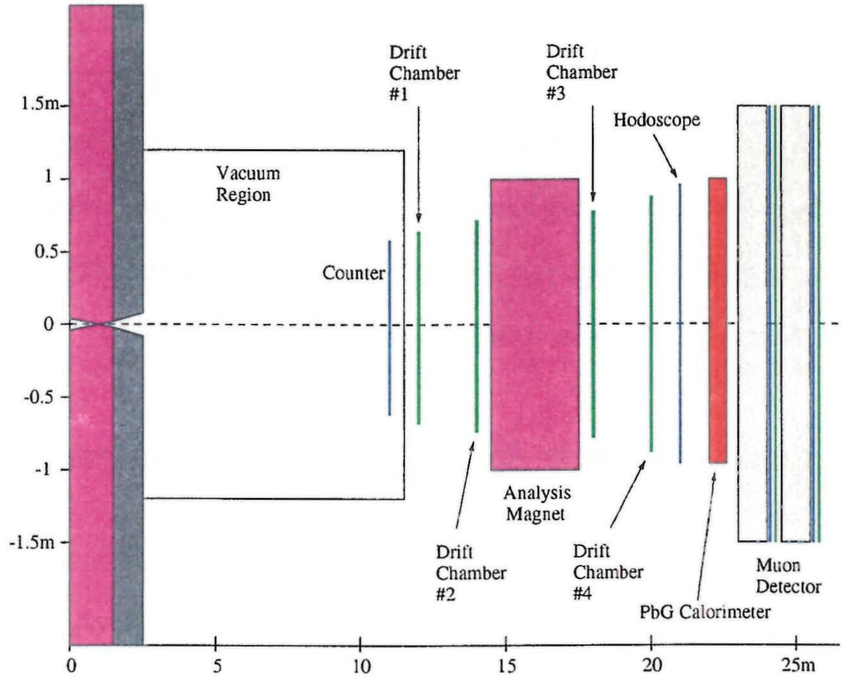

FIG. 1. Detector Layout

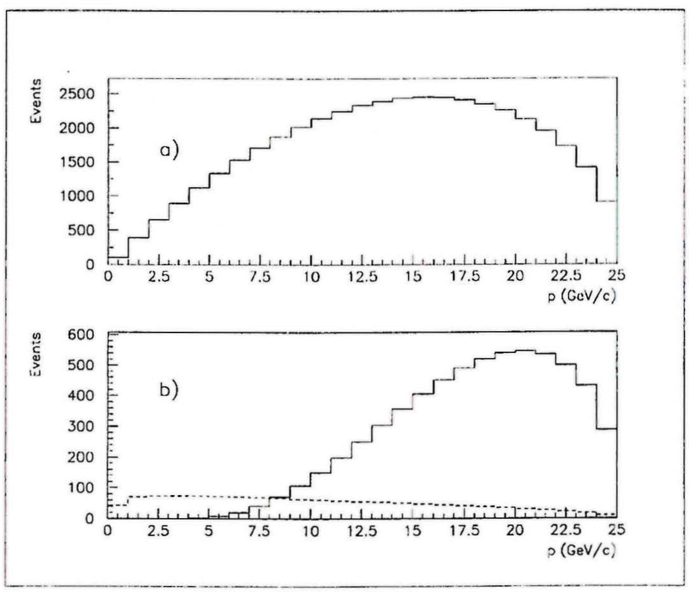

FIG. 2. Kaon Momentum Distributions. a) K^0's at the target. b) $K_S(K_L)$ solid (dashed) decays in the decay region. The flux from one spill of data is shown in this figure.

A. $\eta_{+-\gamma}$ and η_{+-}

The CP violation parameter for the decay $K_L \to \pi^+\pi^-\gamma$ is called $\eta_{+-\gamma}$. The contribution of direct CP violation in this decay is expected to be about 1%. The world's best measurement [25] of $\eta_{+-\gamma}$ was made in Fermilab experiment E773 of which three of us (S.R.S., S.V.S., and G.B.T.) took part. This result was $|\eta_{+-\gamma}| = [2.359 \pm 0.062(stat) \pm 0.040(syst)] \times 10^{-3}$, and it stands within about a factor of 3 of the expected difference from η_{+-}.

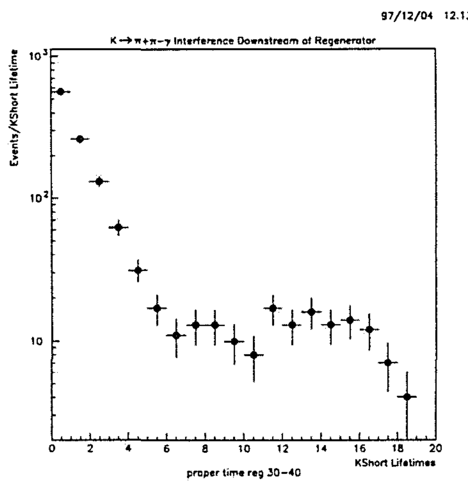

FIG. 3. Proper Time Distributions for $\pi^+\pi^-\gamma$ Decays from Fermilab Experiment E832. These events were in the "regenerator beam" and come from the decays of a mixture of K_S and K_L mesons. The oscillations visible come from $K_L - K_S$ interference.

Figure 3 shows the proper time dependence of $K_{L,S} \to \pi^+\pi^-\gamma$ decays downstream of the regenerator in the KTeV experiment. This is the complete data from the E832 1996 run, and comprises about 1/5 of the whole KTeV sample. In this figure the interference between K_L and K_S is seen. The KTeV experiment expects to measure the CP violation parameter $\eta_{+-\gamma}$ to better than 1% accuracy.

In the proposed experiment we will collect about 504 million $\pi^+\pi^-\gamma$ events and 20 billion $\pi^+\pi^-$ events and perform a simultaneous fit for $\eta_{+-\gamma}$ and η_{+-} to measure their difference. For the purpose of estimation we have performed a simpler fit, to only the $\pi^+\pi^-\gamma$ events, using as fitting parameters, $|\eta_{+-\gamma}|, \phi_{+-\gamma}, D$, and three parameters describing the normalization and shape of the kaon momentum spectrum. We fit the two-dimensional distribution of the events in momentum and proper time. The result of this fit is a measurement of $|\eta_{+-\gamma}|$ to $\pm 0.19\%$ accuracy, which will be a 5σ measurement of $\epsilon'_{+-\gamma}$ if it is of the expected size. Fig. 4 shows the proper time distribution of accepted $\pi^+\pi^-\gamma$ events.

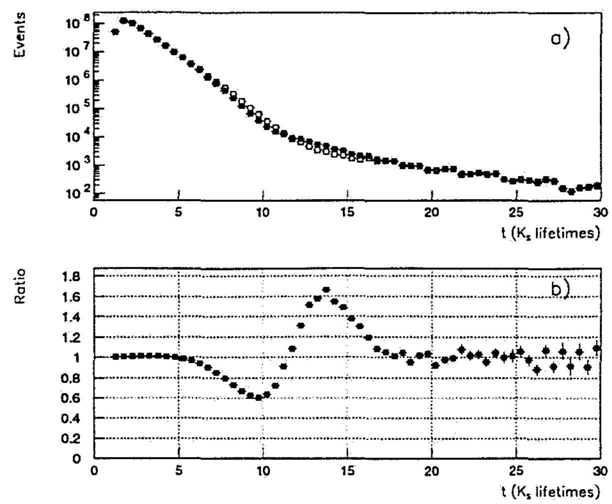

FIG. 4. Simulated Proper Time Distributions for $\pi^+\pi^-\gamma$ Decays. a) Distributions are shown both with interference (dark circles) and without (light circles). b) The ratio of the two distributions in part a) to show the size of the interference.

All previous experiments that measured $\eta_{+-\gamma}$ performed their fits in bins of (p,t) (or (p,z) which is equivalent). But since the electric dipole direct emission (E1DE) term which gives a direct CP violation contribution has a different k^* shape from that of the inner bremsstrahlung (IB) or magnetic dipole direct emission (M1DE) terms, we have investigated performing a three dimensional fit in p, t, and k^*.

The IB term falls a bit faster than $1/k^*$, dominates the spectrum at short proper times, and provides about half of the spectrum at long proper times. The M1DE term rises as k^{*3} then tapers off near the kinematic limit, and provides the other half of the spectrum at long proper times. The E1DE term has an intermediate behavior in both its k^* spectrum and its proper time behavior. It has a broad spectrum, and exhibits the damped oscillation of interference. Fig. 5 shows the three types of spectra.

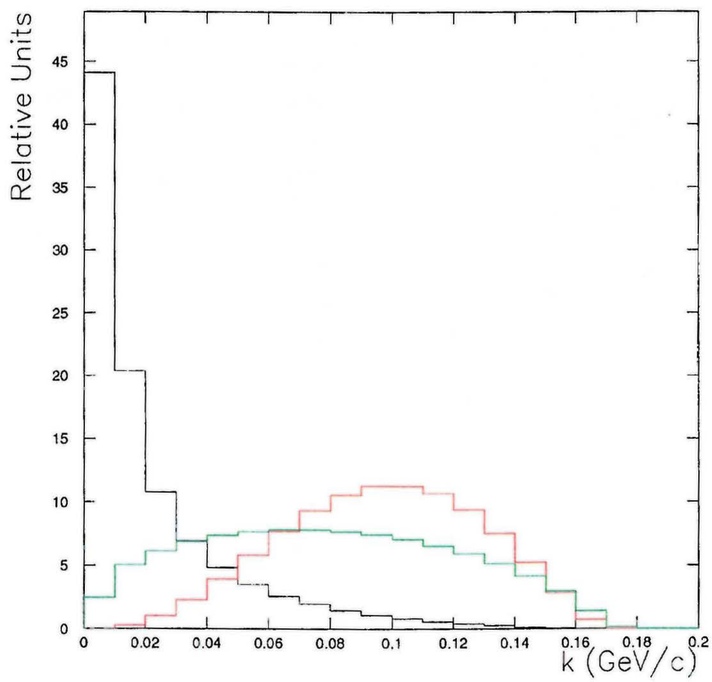

FIG. 5. Gamma ray spectra from the three terms of $\pi^+\pi^-\gamma$ decays. Inner bremsstrahlung is shown in black, magnetic dipole direct emission in red, and electric dipole direct emission in green.

We calculated the acceptance of our apparatus in the three dimensions, calculated what we will see in the data of the CP/T experiment in p, t, and k^*, and fit this data to the hypothesis that $\eta_{+-\gamma} = \eta_{+-} + \epsilon'_{+-\gamma}$, with each term carrying its characteristic k^* spectrum. The result was a factor of 2.3 improvement in the sensitivity of the experiment. Fig. 6 shows quantitatively how this improvement came about. The left half of this figure shows how the k^* spectrum changes with proper time: as proper time increases the IB/M1DE ratio decreases. The right half of this figure shows the ratio between a fit with $\epsilon'_{+-\gamma} = 0.01 \times \eta_{+-\gamma}$ and a fit with $\epsilon'_{+-\gamma} = 0.0$. One can see that the proper time range, $1\tau_S < t < 10\tau_S$ is where we are most sensitive to $\epsilon'_{+-\gamma}$.

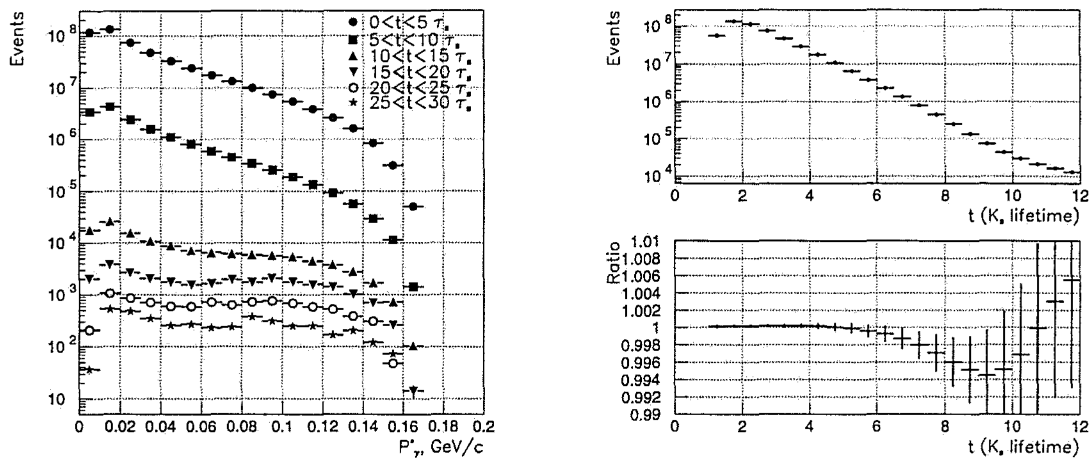

FIG. 6. k^* and proper time distributions for the three-dimensional fit. The left half of the figure shows the variation of the k^* spectrum with proper time. The right half shows the ratio of proper time distributions with and without direct CP violation.

In our fits we used the fact that the phase of $\epsilon'_{+-\gamma}$ is known to be $(45\pm5)^\circ$ [20]. The result for the magnitude is: $\delta|\epsilon'_{+-\gamma}| = 1.8 \times 10^{-6}$, or $\delta|\epsilon'_{+-\gamma}|/|\eta_{+-\gamma}| = 0.081\%$. If the prediction is true that direct CP violation is 1% of $\eta_{+-\gamma}$, this will be a measurement of 12σ significance.

1. Systematic Uncertainties in the $\eta_{+-\gamma}$ Measurement

We have investigated the two most important backgrounds to the $\pi^+\pi^-\gamma$ decay: $K_L \to \pi^+\pi^-\pi^0$ with one γ missing, and two charged particles from a kaon decay plus a random γ ray striking the lead glass electromagnetic calorimeter.

We eliminate the $\pi^+\pi^-\pi^0$ events by cuts on the invariant mass and transverse momentum of the $\pi^+\pi^-\gamma$, and by a cut in the $M_{\pi^+\pi^-} - k^*$ plane. The result is that the backgrounds are less than 1% in the worst momentum and proper time bin. This occurs at high proper times where the K_S $\pi^+\pi^-\gamma$ component has decayed away and only the K_L component is left. Here the $\pi^+\pi^-\pi^0$ decays outnumber the $\pi^+\pi^-\gamma$ decays by two orders of magnitude. The backgrounds can be subtracted in each bin in p and t to result in a negligible systematic uncertainty. Figure 7 shows the $M_{\pi^+\pi^-} - k^*$ plane in part a). The dark blob on the lower right is the $\pi^+\pi^-\pi^0$ background after invariant mass and transverse momentum cuts. The line shows the cut. Part c) below shows the signal and background as a function of proper time and illustrates the fact that the relative background is worst at high proper times. Parts b) and d) show the results after the cut. The background is all but eliminated. Figure 8 shows, after all cuts, the invariant mass of the $\pi^+\pi^-\gamma$ system, its p_T^2, and momentum spectrum as the histogram, with the remaining $\pi^+\pi^-\pi^0$ background as the hatched points.

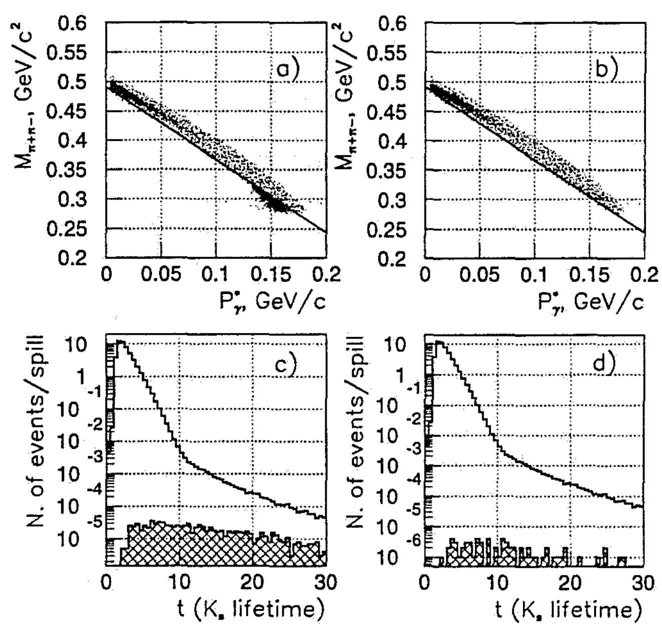

FIG. 7. Scatter plots of the $M_{\pi^+\pi^-} - k^*$ plane, and histograms in proper time. The line in part a) shows the cut described in the text. Parts a) and c) are before making this cut, and parts b) and d) are after the cut.

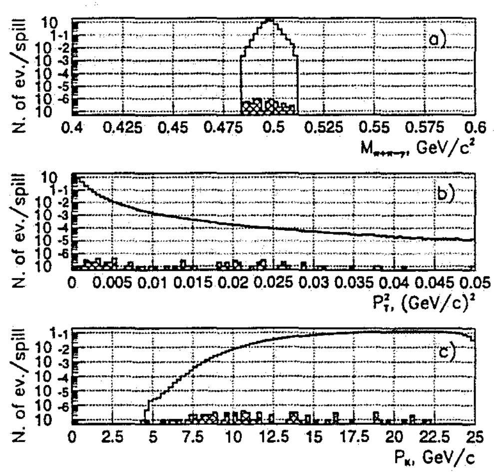

FIG. 8. $M_{\pi^+\pi^-\gamma}$, p_T^2, and kaon momentum. The $\pi^+\pi^-\gamma$ decays are the histogram and the $\pi^+\pi^-\pi^0$ background that remains after cuts are the hatched events.

The second background process that has been important in previous experiments is a kaon decay giving two charged particles that we reconstruct as pions, with a random gamma ray. We have used our GEANT simulation of the target, hyperon magnet collimator, and lead glass detector to generate a sample of events with random hits in the lead glass. In this simulation we made a library of events where one K^+ strikes the target and energy is deposited in the lead glass array. These are called "accidental events". We superimposed accidental events equivalent to the number of expected K^+ striking the target within a 100 nsec ADC gate used for the lead glass on top of other Monte Carlo decays to see if any then reconstruct as $\pi^+\pi^-\gamma$. Since ours is a low rate experiment we do not expect to see any significant backgrounds from accidental gamma rays, and this is the result of our simulation. Figure 9 shows the energy of accidental hits in the lead glass, and the number of blocks hit by accidentals. In these figures the histogram is the contribution from a single K^+ striking the target and the hatched area is the contribution of the K^+'s that strike the target during an ADC gate 100 ns long.

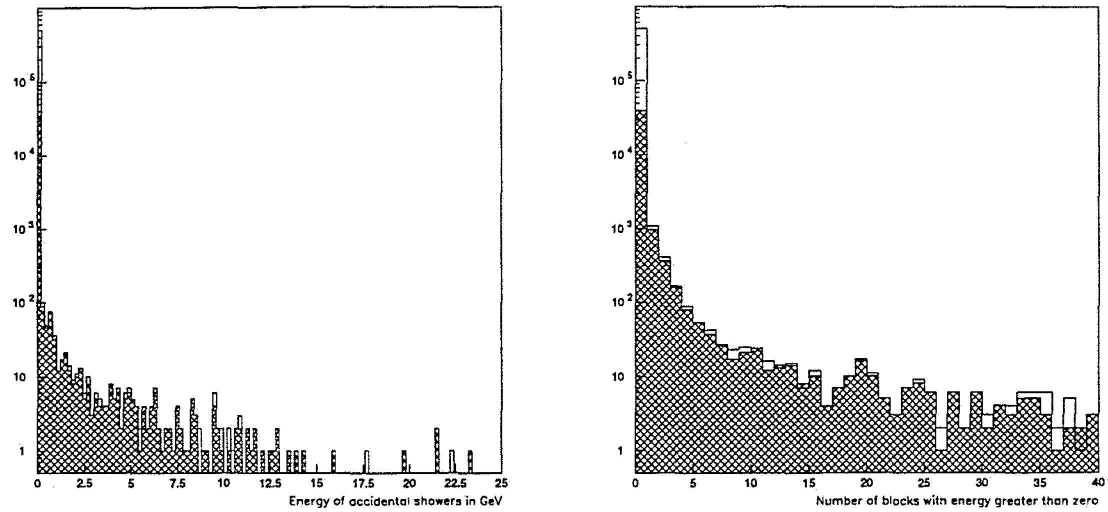

FIG. 9. Energy and number of blocks hit for accidental hits in the lead glass. The clear part of the histogram shows the contribution from one K^+ striking the target and the hatched area is the contribution of the K^+'s from 100 ns.

One decay mode that could give a background when an accidental hit in the lead glass is added is the K_{e3}. Figure 10 shows the invariant mass and p_T^2 of $\pi^+\pi^-\gamma$ decays. Figure 11 shows the invariant mass and p_T^2 of the K_{e3} - random γ combination. Figure 12 shows a scatter plot of the background events in the $P_K - \cos(\theta)$ plane. Here $\cos(\theta)$ is the cosine of the angle between the $\pi^+\pi^-$ momentum vector and the kaon momentum vector in the kaon center of mass system. Choosing events with $P_K > 10$ GeV/c and $\cos(\theta) > -0.6$ eliminates this background with a minimal loss of signal events.

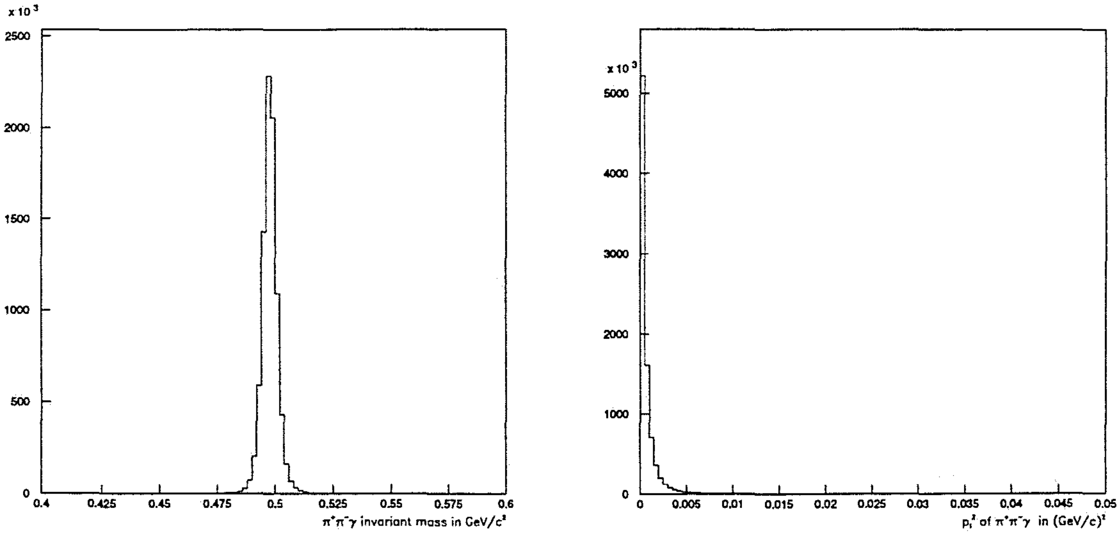

FIG. 10. $M_{\pi\pi\gamma}$ and p_T^2 for signal events.

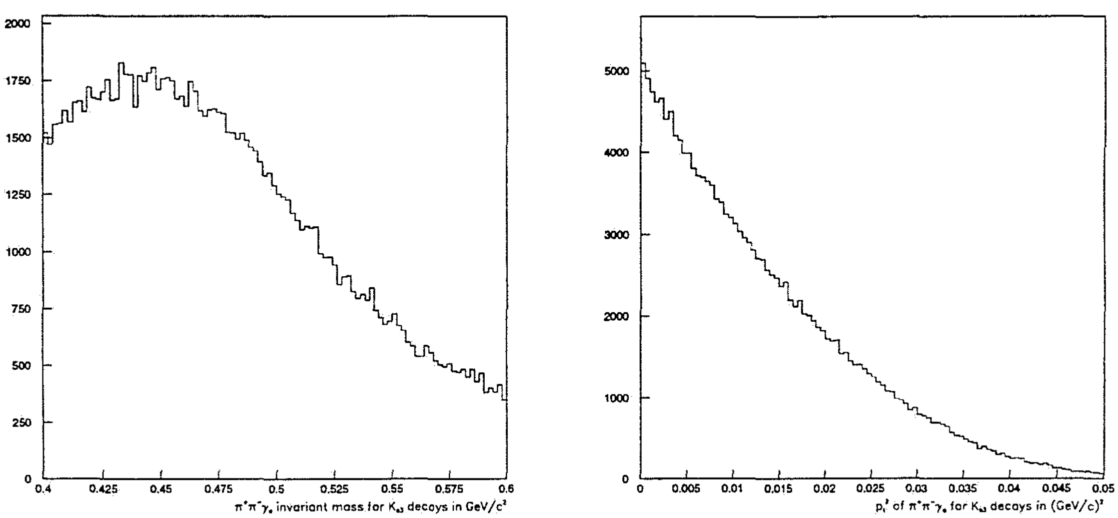

FIG. 11. $M_{\pi\pi\gamma}$ and p_T^2 for background events.

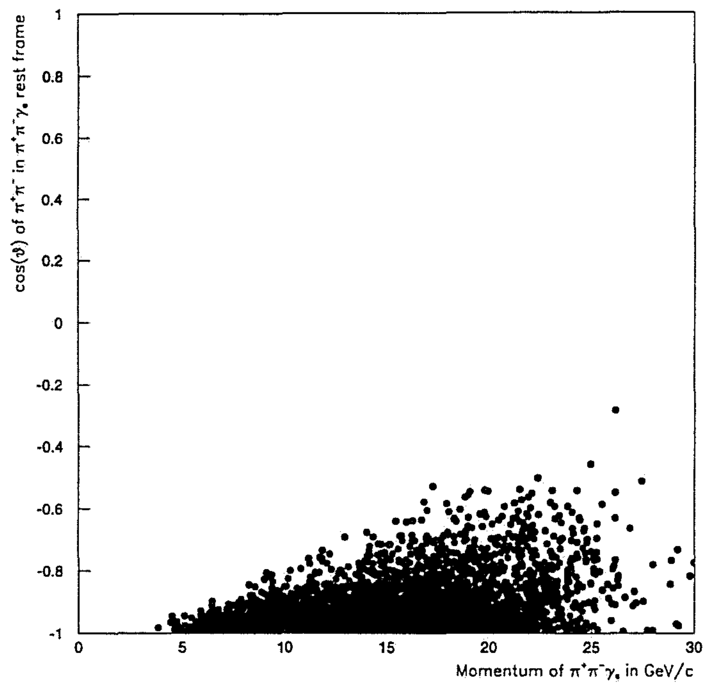

FIG. 12. P_K vs $\cos(\theta)$ scatter plot.

$\pi^+\pi^-$ decays with an accidental gamma ray can also satisfy our $\pi^+\pi^-\gamma$ cuts, and without a cut on the $\pi^+\pi^-$ invariant mass about 6% look like $\pi^+\pi^-\gamma$ events. With a relatively loose cut on the $\pi^+\pi^-$ invariant mass, $M_{+-} < 492MeV/c^2$, this is reduced to 1%.

There is another source of showers in the lead glass that is not random but is correlated with the charged pions. Charged pion showers in the lead glass can have tendrils sticking out from the main part of the shower that look like clusters of hit lead glass blocks made by neutral particles. This effect was seen in previous experiments: in E773 a cut of radius 0.2 m around the projection to the z of the lead glass of the charged pion track eliminated this source of background clusters. The GEANT simulation of the CP/T experiment lead glass electromagnetic calorimeter indicates that a similar cut would work here.

We assume that the total background is 1% of the signal, and that we can subtract it with accuracy of 1/10 of itself in every bin in momentum and proper time. This means that we have an additional 0.1% uncertainty in each bin to add in quadrature with the statistical uncertainty. The result of a fit including the background subtraction uncertainty is to move the central value of $\epsilon'_{+-\gamma}$ by 1×10^{-7}, which is less than 0.01% of $\eta_{+-\gamma}$.

We calculated the sensitivity of the $\epsilon'_{+-\gamma}$ fit to the uncertainty in the acceptance by using the same technique that is described in detail below in the section on systematic uncertainties in the ϕ_{+-} fit. The result of the calculation is that we would expect a systematic uncertinty in $\epsilon'_{+-\gamma}$ of 1×10^{-6} if there were a deviation of 0.004 over the 9.5 m long decay region between data and the Monte Carlo simulation. This is about 0.04% of $\eta_{+-\gamma}$. In the actual experiment we will have the ability to see a deviation about a factor of 10 smaller than this. We conservatively assign a systematic uncertainty of 0.02% of $\eta_{+-\gamma}$ to this effect.

Fig. 13 shows the effect on the fits for $|\eta_{+-}|$ and for $\epsilon'_{+-\gamma}$ when the z-dependence of the acceptance is changed. In the experiment we will be able to see a change which is 1/10 of the range shown in the figure. Hence the uncertainty in the values of $|\eta_{+-}|$ and $\epsilon'_{+-\gamma}$ would be much smaller than the variation of those quantities shown in the figure.

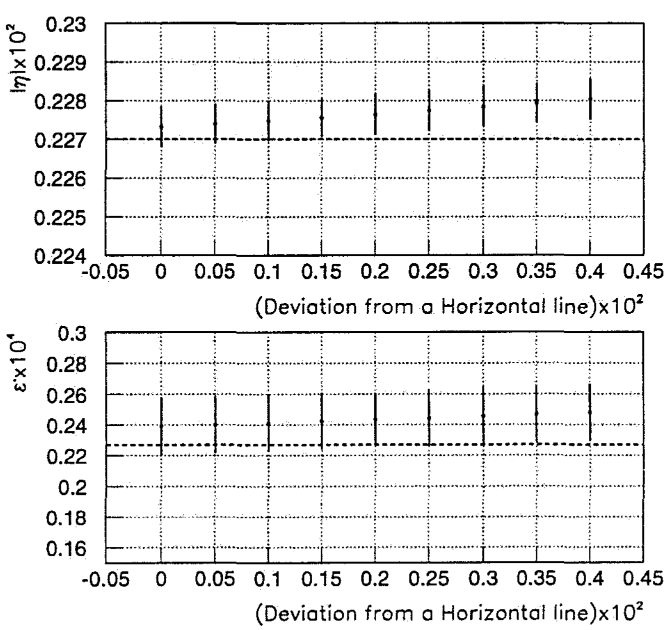

FIG. 13. Effects on $|\eta_{+-}|$ and $\epsilon'_{+-\gamma}$ of varying the z-dependence of the acceptance. The largest deviation shown is .004, and using the methods described in Section V.A.1 we would have 10 times better sensitivity.

The entire systematic uncertainty in the measurement of $\epsilon'_{+-\gamma}$ is 0.03% of $\eta_{+-\gamma}$. When we add this in quadrature with the statistical uncertainty of 0.081% the result is 0.086%.

B. η_{+-0} and η_{000}

The CP violation parameters η_{+-0} and η_{000} characterize CP violation in K_S decays to three pions. Every measured CP-violating decay has been of the K_L. Observation of CP-violation in these decays would be the first discovery of CP violation outside the K_L system. These decays are expected to have large contributions from direct CP violation (up to about 10%), and are the places where CP violating effects beyond-the-standard-model may be the largest. Since in the standard model ϵ'_{+-0} and ϵ'_{000} are both purely imaginary, fitting the proper time distribution of $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$ events to extract these parameters from $K_L - K_S$ interference should be relatively simple. We expect to collect about 230 M $\pi^+\pi^-\pi^0$ decays and measure $Im(\eta_{+-0})$ to $\pm 0.35 \times 10^{-3}$ accuracy. We will simultaneously collect about 70 M $\pi^0\pi^0\pi^0$ decays and measure $Im(\eta_{000})$ to $\pm 0.64 \times 10^{-3}$ accuracy (note: $\eta_{+-} = 2.27 \times 10^{-3}$).

The most recent experiment at Fermilab to search for CP violation in K_S decay was E621, of which two of us (TJD and GBT) were members. The proper time distribution of $\pi^+\pi^-\pi^0$ decays observed in E621 is shown in Figure 14. The result of that experiment [26] was $Im(\eta_{+-0}) = -0.015 \pm 0.017(\text{stat}) \pm 0.025(\text{syst})$. E621 was done with an 800 GeV/c proton beam striking a target at the entrance to the hyperon magnet in the Fermilab P-Center beam line. The apparatus consisted of a magnetic spectrometer and lead glass electromagnetic calorimeter.

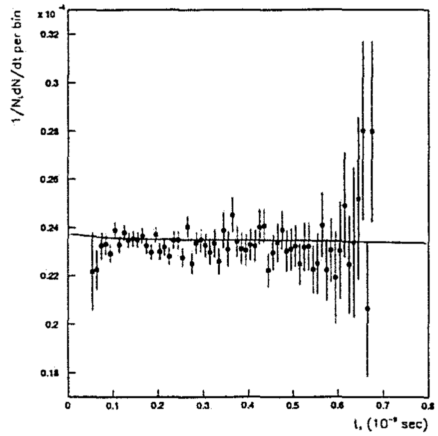

FIG. 14. Proper Time Distributions for $\pi^+\pi^-\pi^0$ Decays from Fermilab Experiment E621. The vertical axis is the absolutely normalized probability for the decay $K_L \rightarrow \pi^+\pi^-\pi^0$, and the horizontal axis is the proper lifetime of the kaons.

The design of the present experiment is vastly superior to that of E621: the K^+ beam yields a higher K^0 flux per incident beam particle, the showers from the charged beam particles are more easily contained and the rates in the detector are much lower, the K^0's are produced at much higher Feynman x, the size of the interference is much larger, the statistical accuracy will be much better, the acceptance will be larger, and the systematic uncertainties will be better.

The existing limit on η_{000} is $Im(\eta_{000})^2 < 0.1$, based on 632 events seen in a bubble chamber experiment [27]. The analysis of this experiment used the fact that ϵ'_{000} is imaginary, so that $Re(\eta_{000}) = Re(\epsilon)$, which equals $Re(\eta_{+-})$ to good accuracy.

The measurements of η_{+-0} and η_{000} are essential for a meaningful interpretation of tests of CPT symmetry conservation, including some tests already carried out. More details are given below in the context of CPT tests.

IV. CPT THEORY AND PHENOMENOLOGY

The CPT theorem [7] is based on the assumptions of locality, Lorentz invariance, the spin-statistics theorem, and asymptotically free wave functions. All quantum field theories (including the standard model of the elementary particles) assume CPT symmetry invariance.

There is a theoretical hint of the level at which CPT symmetry might be violated. This comes from the fact that gravity can't be consistently included in a quantum field theory, and the proof of the CPT theorem assumes Minkowski space [28,7]. To include gravity in a unified theory of all four forces of nature, many physicists think that a more general theory is needed, which would have quantum field theory embedded in it. In this more general theory the CPT theorem will be invalid.

One expects to see quantum effects of gravity at what is called the Planck scale: at energies of $M_{Planck}c^2 = \sqrt{\hbar c^5/G} = 1.2 \times 10^{19}$ GeV, or at distances of the order of 10^{-33} cm. The quantum effects of gravity are expected to be very small in ordinary processes. However, in a place where the standard model predicts a null result, like CPT violation, quantum effects of gravity would stand out. Therefore, it would be very interesting to test CPT symmetry conservation at the Planck scale.

One might think that string theory, as a candidate for the more general theory that has quantum field theory embedded in it, would give us guidance. CPT conservation is artificially built into string theories, first by G. Veneziano [29].

Kostelecky and Potting [11] suggested that spontaneous CPT violation might occur in string theory; i.e., they put the CPT violation in the solutions rather than in the equations of motion. They found that the mass difference between the K^0 and $\overline{K}^0$ is given by a coupling constant of order 1 times the ratio of the kaon mass to the Planck mass to some power which they don't know:

$$|M_{K^0} - M_{\overline{K^0}}| = O(1) M_{K^0} \left(\frac{M_{K^0}}{M_{Planck}}\right)^n \tag{4.1}$$

They comment that $n = 0$ is ruled out, and only $n = 1$ can ever be tested experimentally.

One of the problems with string theory in general is that it's not known how to relate string effects at the Planck scale to effects seen at current accelerator energies, and Kostelecky and Potting have the same difficulty. They have tried to remedy this by writing the most general additions to the Standard Model Lagrangian that maintain the SU(3) x SU(2) x U(1) effective structure of the theory but violate CPT symmetry. This allows them to classify the various types of CPT violation that might be seen (in the lepton sector, quark sector, etc.) and have a parameterization that includes all these effects. They find that the largest CPT violating effect is a change in quark propagators that has the opposite sign for antiquarks. This leads to a nonzero value of $|M_{K^0} - M_{\overline{K^0}}|$ coming from indirect CPT violation. This is much larger than any direct CPT violation effect. This is precisely the signature that this experiment would search for.

There have also been mechanisms proposed for the violation of quantum mechanics leading to CPT violating effects that would be seen in the $K^0 - \overline{K^0}$ system. J. Ellis **et al.** [9] introduced three new CPT violating parameters, α, β, and γ, describing this effect. In a recent analysis [30], Huet and Peskin pointed out that measurements of the magnitude and phase of η_{+-} and of the semileptonic charge asymmetry would be able to separately measure the three parameters. The experiment we will describe in this report would be the world's most sensitive [10] in this area.

The $K^0 - \overline{K}^0$ system provides us with an incredibly finely balanced interferometer that magnifies small perturbations such as CPT violating effects. It is a natural place to search for CPT symmetry violation since it exhibits C, P, and CP symmetry violation (and is the only place to date where CP violation has been seen). In the final analysis, the conservation or violation of CPT symmetry is an experimental question, and the search for this effect is of the utmost interest.

In K^0 physics, one can observe CPT violating effects through mixing or decays (called indirect or direct CPT violation respectively). In mixing, one introduces a parameter Δ which is both CP and CPT violating: All CP violation seen to date has been through the $(\epsilon - \Delta)$ term of the K_L. One can also have direct CPT violation, for example in semileptonic decays, where an amplitude y_l is introduced that is CPT violating [31]:

$$\begin{cases} \langle \pi^- l^+ \nu | T | K^0 \rangle = F_l(1 - y_l) \\ \langle \pi^+ l^- \overline{\nu} | T | \overline{K^0} \rangle = F_l^*(1 + y_l^*) \end{cases} \quad (4.2)$$

There are several measurements that would signify CPT violation: a difference between the phase of ϵ and the phase of η_{+-}, evidence for a non-zero Δ in the Bell-Steinberger relation, a difference between the phases of η_{+-} and η_{00}, or certain interference terms between K_L and K_S in semileptonic decays. In this report we will concentrate on the first two methods, measuring the phase of η_{+-} and comparing it to the calculated value of the phase of ϵ, and evaluating the Bell-Steinberger relation, since from them we can make the most accurate measurements.

We now consider the CPT test based on measuring the phase of η_{+-} and calculating the phase of ϵ. For what follows we adopt the Wu-Yang phase convention. Figure 15 shows the relationships between $\epsilon, \epsilon', \Delta$, and η_{+-}. ϵ' and Δ are shown greatly enlarged for clarity.

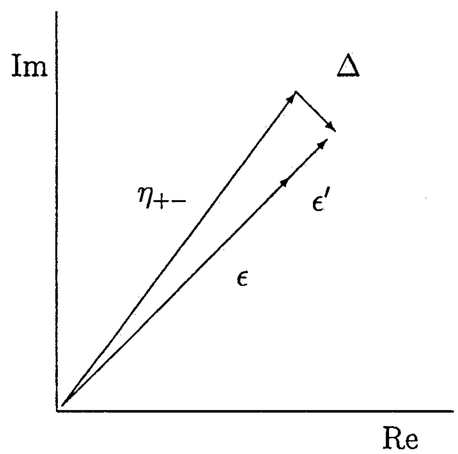

FIG. 15. The Wu-Yang Diagram

The size of $|\epsilon'/\epsilon|$ is of order 10^{-3}, and the phase of ϵ' is very close to that of ϵ, so the phase of the vector $\epsilon + \epsilon'$ is the same, to good accuracy, to the phase of ϵ (we show below that ϵ' is too small to have an affect on the calculation of the phase of ϵ at the level in which we are interested). We can see from the figure that the component of Δ perpendicular to ϵ, $\Delta_\perp$, is

$$\Delta_\perp = |\eta_{+-}|(\phi_{+-} - \phi_\epsilon) \tag{4.3}$$

where ϕ_{+-} (ϕ_ϵ) is the phase of $\eta_{+-}(\epsilon)$. In general, in terms of the elements of the kaon decay matrix Γ and mass matrix M, Δ is given by [32]:

$$\Delta = \frac{(\Gamma_{11} - \Gamma_{22}) + i(M_{11} - M_{22})}{(\Gamma_S - \Gamma_L) - 2i(M_L - M_S)} \tag{4.4}$$

The mass term has a phase perpendicular to ϕ_{SW}, the superweak phase, which is defined as $\tan\phi_{SW} = 2(M_L - M_S)/(\Gamma_S - \Gamma_L)$. ϕ_{SW} is approximately equal to ϕ_ϵ. The decay term is parallel to ϕ_{SW}. We can solve Eqns. 4.3 and 4.4 for $M_{11} - M_{22}$, which is the mass difference between the K^0 and $\overline{K^0}$ mesons, and get an equation which we can use to search for indirect CPT violation:

$$\frac{|M_{K^0} - M_{\overline{K^0}}|}{M_{K^0}} = \frac{2(M_L - M_S)}{M_{K^0}} \frac{|\eta_{+-}|}{\sin\phi_{SW}} |\phi_{+-} - \phi_\epsilon| \tag{4.5}$$

In Eqn. 4.5, Nature has been kind: the constant factors multiplying $|\phi_{+-} - \phi_\epsilon|$ are exceedingly small. $(M_L - M_S)$ is 10^{-6} eV, and when one divides by M_{K^0} the ratio is of order 10^{-15}. $|\eta_{+-}|$ is of order 10^{-3}. The product of all the factors multiplying $|\phi_{+-} - \phi_\epsilon|$ is 4×10^{-17}. By the Planck scale we mean

$$\frac{|M_{K^0} - M_{\overline{K^0}}|}{M_{K^0}} = \frac{M_{K^0}}{M_{Planck}} = 4.1 \times 10^{-20} \tag{4.6}$$

so a measurement of $|\phi_{+-} - \phi_\epsilon|$ accurate to 1 milliradian would test a CPT violating effect at the accuracy of the Planck scale.

Some of the present authors (SRS, SVS, and GBT) were part of Fermilab experiment E773. In this experiment we placed the limit (at 90% confidence level) [8],

$$\frac{|M_{K^0} - M_{\overline{K^0}}|}{M_{K^0}} < 1.3 \times 10^{-18} \tag{4.7}$$

so the result of Ref. [8] stands at 31 times the Planck scale.

That publication actually compared the phase of η_{+-} to the superweak phase (and stated clearly that in doing so the assumption was being made that CP violation would not be unexpectedly large in modes other than $\pi\pi$). In the calculation of the phase of ϵ, there are three corrections that should be made to the superweak phase: from $Im(x)$, the $\Delta S = \Delta Q$ rule violation parameter, from $Im(\eta_{+-0})$, and from $Im(\eta_{000})$. Together they have an uncertainty of 2.7 degrees which should be added in quadrature with the approximately 1 degree accuracy of Ref. [8].

The same three of us (plus D.B., H.W., T.A., and A.E.) are part of the KTeV experiment as well. There we expect to make an improvement of a factor of 3 to 5. Fig. 16 shows the z-dependence of $\pi^+\pi^-$ decays downstream of the regenerator in the KTeV experiment. The interference is seen very clearly. But the interference term from which ϕ_{+-} is measured, $2|\eta_{+-}||\rho|\cos(\Delta mt + \phi_\rho - \phi_{+-})\exp(-t/2\tau_s)$, is reduced by the regeneration amplitude $|\rho| \simeq 0.03$, and ϕ_{+-} and ϕ_ρ are hard to disentangle. Using the regeneration method will be difficult beyond the KTeV level [33].

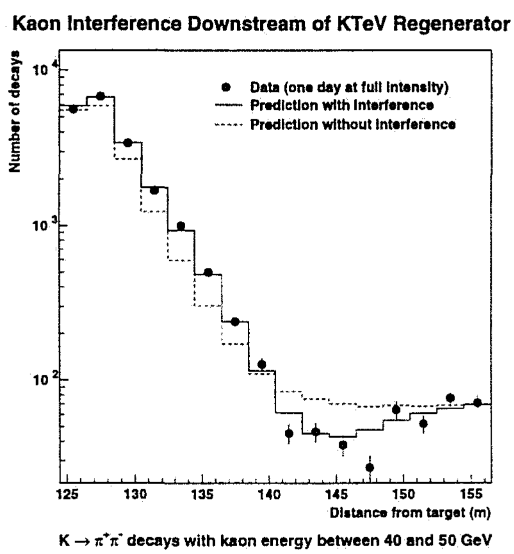

FIG. 16. $K_{L,S} \rightarrow \pi^+\pi^-$ Decays downstream from the regenerator in the KTeV Experiment. The longitudinal position of the kaon vertex is shown.

It should be understood clearly that measuring the phase of η_{+-} and comparing it to the superweak phase does not constitute a test of CPT symmetry conservation: the corrections to the superweak phase have larger uncertainties than existing experimental measurements of ϕ_{+-}. For example, if a significant difference between ϕ_{+-} and ϕ_{SW} were found in an experiment it would NOT prove that CPT symmetry was violated. More accurate measurements of $Im(x), Im(\eta_{+-0})$, and $Im(\eta_{000})$ must be made before this could be proved. An interference experiment located just downstream of the production target is needed for these measurements. In a regeneration experiment the interference in 3π decays is reduced in size by a factor of ρ, the regeneration amplitude, which is about 0.1 at most (at Main Injector energies), compared to an experiment near the production target, and it's extremely difficult for a regeneration experiment to measure $Im(x), Im(\eta_{+-0})$, and $Im(\eta_{000})$ to the required accuracy.

V. TWO TESTS OF CPT SYMMETRY CONSERVATION

A. The Phase Difference between η_{+-} and ϵ

After the KTeV experiment we expect to stand an order of magnitude above the Planck scale. To close that gap we will want to do an interference experiment near the kaon production target. The interference term is then $2D|\eta_{+-}|\cos(\Delta mt-\phi_{+-})\exp(-t/2\tau_s)$. Here ϕ_{+-} appears alone, and $|\rho|$ is replaced with the dilution factor, $D=(K^0-\overline{K}^0)/(K^0+\overline{K}^0)$ at the target. To maximize D and hence the interference, we choose to make our K^0 beam from a K^+ beam by charge exchange. Then at medium to high Feynman x, $D\simeq 1$. The charge exchange cross section is large, about 20% of the total cross section. To maximize the flux of K^+ made from the 120 GeV/c protons from the Fermilab Main Injector we choose a K^+ momentum of 25 GeV/c. We would use a hyperon magnet to define the K^0 beam, similar to the one in the Proton Center beam line. As in previous sections, in the calculations described below we assume the use of a Vee spectrometer, a lead glass electromagnetic calorimeter, and a muon detector.

In Ref. [8] ϕ_{+-} was measured to 1° accuracy. A CPT-violating mass difference exactly at the Planck scale would result in $|\phi_{+-} - \phi_\epsilon| = 0.06°$. We set ourselves the goal of measuring ϕ_{+-} and ϕ_ϵ to sufficient accuracy to see such a CPT-violating effect.

We have calculated the statistical sensitivity of the CPT measurements using the same assumptions and programs used above in describing the sensitivity of this experiment for CP violation measurements.

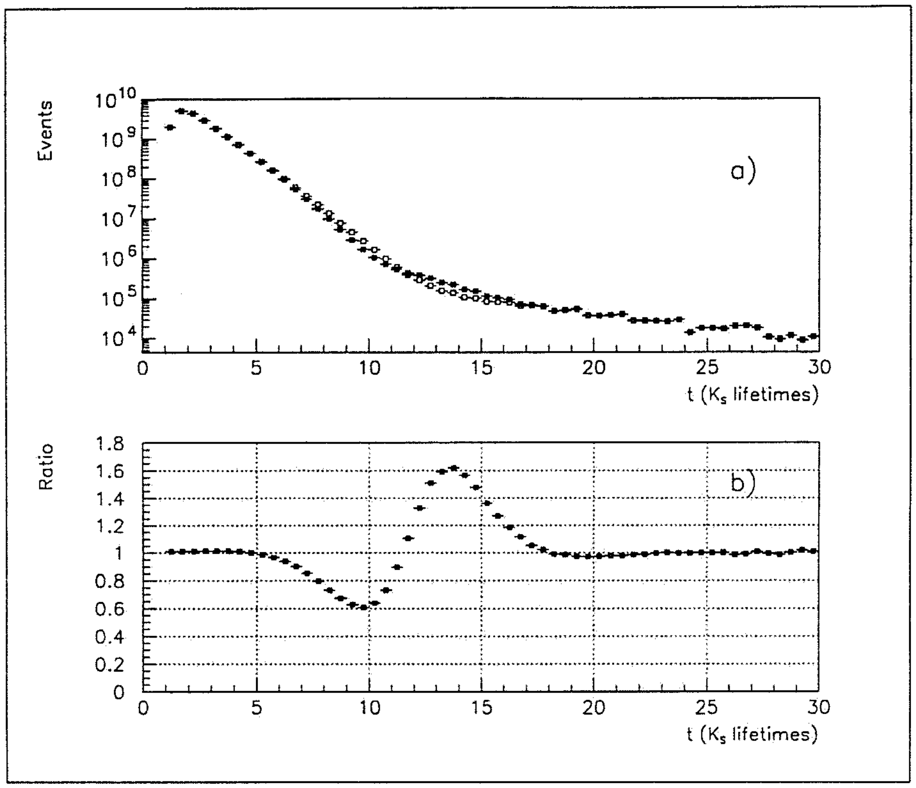

FIG. 17. Proper Time Distributions for $\pi^+\pi^-$ Decays a) Distributions are shown both with interference (dark squares) and without (light squares). b) The ratio of the two distributions in part a).

Fig. 2 shows the momentum spectrum of kaons exiting from the target, and of K_S and K_L decaying in the decay region. Fig. 17 shows the proper time distribution of accepted events. The figure shows the actual proper time distribution and also what the distribution would look like if there were no interference. The second part of the figure shows the ratio of those two curves. Between 5 and 20 K_S lifetimes the interference is first a 40% destructive effect then is a 65% constructive effect.

We calculated the distribution of events in momentum and proper time for the resulting 20 billion events and fit this distribution using MINUIT, with fitting parameters $|\eta_{+-}|, \phi_{+-}, D$, and three parameters describing the normalization and shape of the kaon momentum spectrum. The interference term from which ϕ_{+-} is measured also depends on τ_S and Δm, the K_S lifetime and the $K_L - K_S$ mass difference. We include those parameters in the fit also. The latter parameters are strongly correlated with ϕ_{+-}. We can minimize the effects of these correlations by making a better measurement of Δm using our K_{e3} and $K_{\mu 3}$ data (this measurement of Δm will be about 20 times more accurate than the current PDG value), then use this result as a constraint in our ϕ_{+-} fit.

The uncertainty that results from this fit is 0.040 degrees. This will meet our goal of testing CPT symmetry conservation at the Planck scale. This number ± 0.040 degrees has another meaning: it is the statistical (including fitting) uncertainty of this measurement, and sets the scale against which all other aspects of the $|\phi_{+-} - \phi_\epsilon|$ measurement should be compared.

1. Systematic Uncertainties in the ϕ_{+-} Measurement

We have studied two kinds of systematic uncertainties for the ϕ_{+-} measurement: backgrounds under the signal events, and the uncertainty due to the level of knowledge of the acceptance.

We studied backgrounds to the $K_{\pi 2}$ decays coming from K_{e3} and $K_{\mu 3}$ decays. By making cuts on kinematics, E/p for electrons, and hits in the muon detectors to pick out the $K_{\pi 2}$'s and reject the semileptonic decays, we found that the backgrounds were about 1% in the worst case. The worse case, of course, is at high proper times, where the K_S component of the $K_{\pi 2}$ has died out and only the K_L remains, and where the branching ratios of semileptonic decays are two orders of magnitude higher.

We then tried subtracting the remaining background from the signal. Choosing the

$\pi^+\pi^-$ invariant mass as the fitting variable, we performed a fit of the mass distribution to the hypothesis of a signal shape (determined from a Monte Carlo distribution that was statistically independent and had more events) plus a polynomial background, which fit well.

We performed a study to indicate how this will work in the actual experiment. We generated approximately the correct statistics for the CP/T experiment for the bin in proper time of $10.0\tau_S < t < 10.5\tau_S$. We generated $K_{\pi 2}$, K_{e3}, and $K_{\mu 3}$ decays, made the cuts and added the resulting mass plots together. Then we made the subtraction in 1 GeV/c wide momentum bins. We ended up with no systematic bias at the level of about 0.1% of the signal. This is a very good result, and it shows the process that we will go through for the actual experiment.

Figures 18 shows the mass and transverse momentum squared of $K_{\pi 2}$ decays with the semileptonic background added. Figure 19 shows the E/p distribution for electrons and resolution of the muon chambers for muons that pass through the muon filter. Figure 20 illustrates the subtraction process and Figure 21 shows the difference between the number of $K_{\pi 2}$ decays determined from the fit and the number that went into the making of the plots.

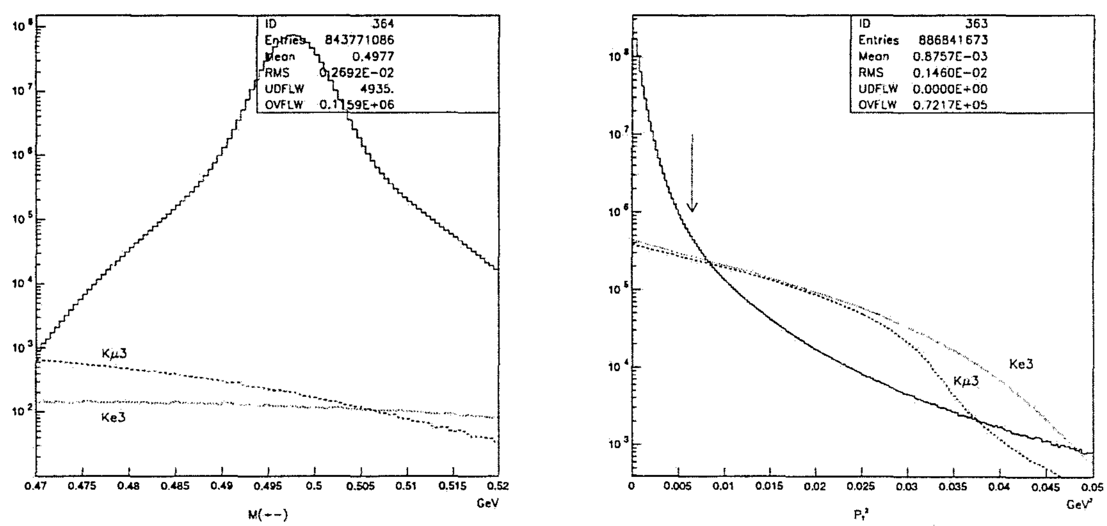

FIG. 18. $\pi^+\pi^-$ Invariant Mass and p_T^2 for $K_{\pi 2}$ decays with semileptonic background added.

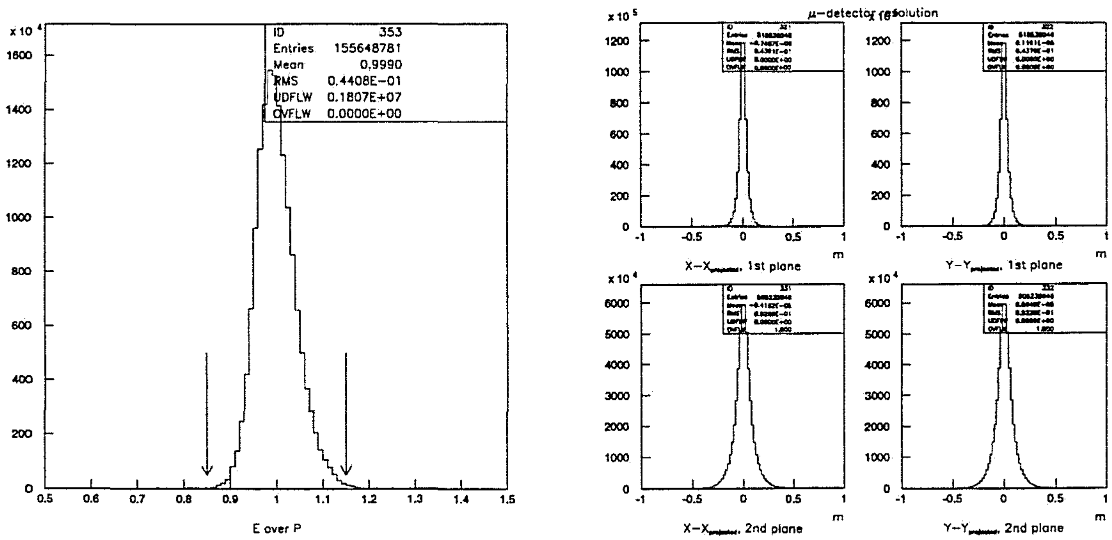

FIG. 19. E/p distribution for electrons from K_{e3} decays, and the muon position resolution of the muon chambers.

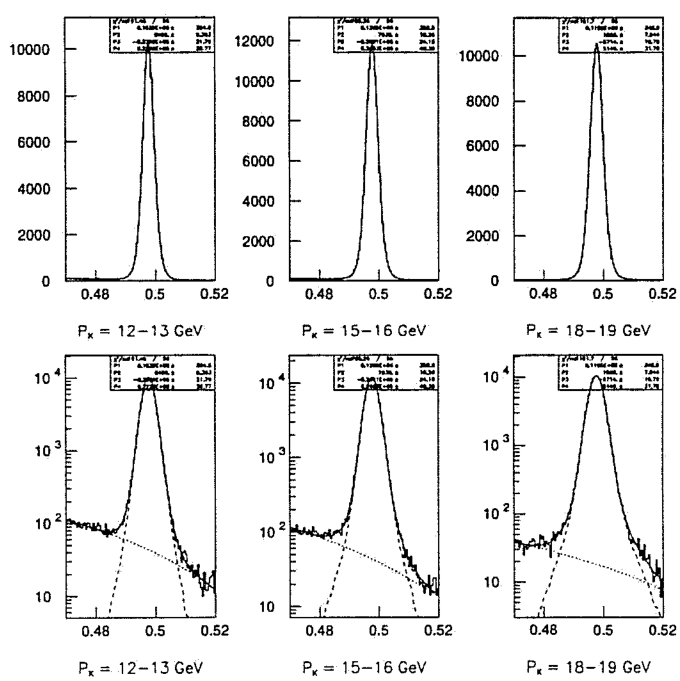

FIG. 20. Subtraction of the semileptonic Background. Data in the proper time bin $10.0 < t < 10.5$ for three momenta are shown.

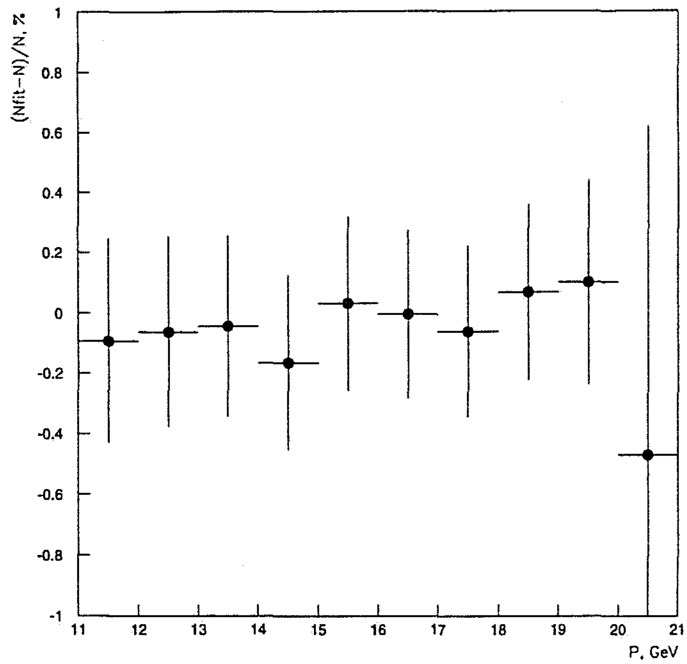

FIG. 21. Accuracy of the subtraction process. The relative difference between the fit and actual numbers of $K_{\pi 2}$ decays is shown.

To understand the effect of this source of uncertainty, in our fitting program we have added an uncertainty of 0.1% of the signal to the signal's statistical uncertainty and performed the fit as before. The fit is very robust. We found that ϕ_{+-} changes by 0.002°. Therefore we assign a systematic uncertainty to this process of 0.002°. We can understand this robustness by looking at Fig. 22, which shows the data as a function of proper time for various momentum bins. The proper time graph of Fig. 17 shows that the proper time range where we have the most sensitivity to the interference is from about 7 to 18 τ_S. The vertical lines in Fig. 22 are at $t = 7\tau_S$, and one can see that 10^6 events (which gives 0.1% statistical uncertainty) occurs very near $7\tau_S$.

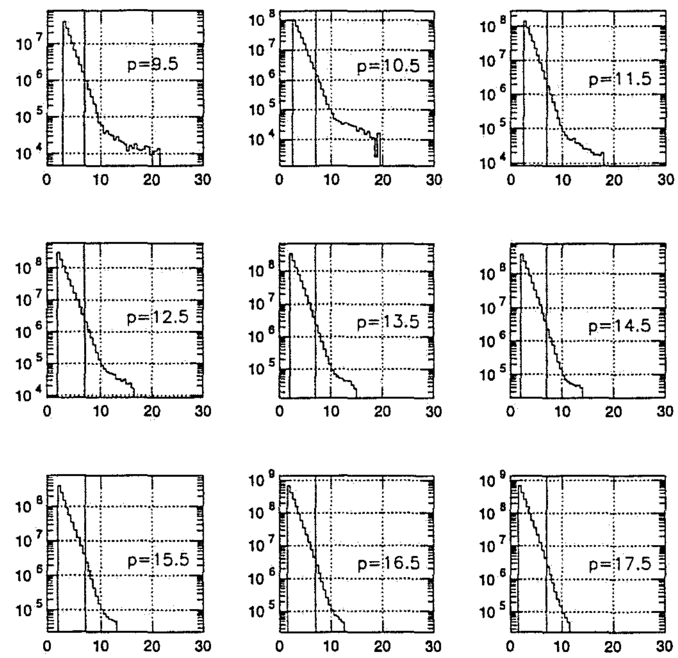

FIG. 22. Proper Time Distributions for $\pi^+\pi^-$ Decays for nine momentum bins

Incoherent scattering and production of kaons in the material of the hyperon magnet collimator is an effect included in GEANT, and in our GEANT simulation of the target and hyperon magnet collimator we find that after cuts these events number about 0.15% of the signal events. These are real $K_{\pi 2}$ decays and have approximately the same proper time dependence as the signal events. However their momentum spectrum is much softer than the signal and they are spread much wider than the signal events in their directions of flight and the transverse coordinates of their vertices.

Fig. 23 shows the x-y distribution of K_L mesons striking the face of the lead glass, and also the momentum spectrum of these events. Events produced in the target and those produced in the collimator are clearly seen. Of course these are kaons that did not decay in the decay region, but they exhibit the effect clearly. In Fig 23 part (b) the vertical line drawn at $r = 25$ cm divided the events clearly into those in the beam (to the left of the line) and those made in the collimator. The horizontal line drawn at $p = 10$ GeV/c represents a cut: only events above the line are accepted for the ϕ_{+-} anaysis. This removes most collimator production.

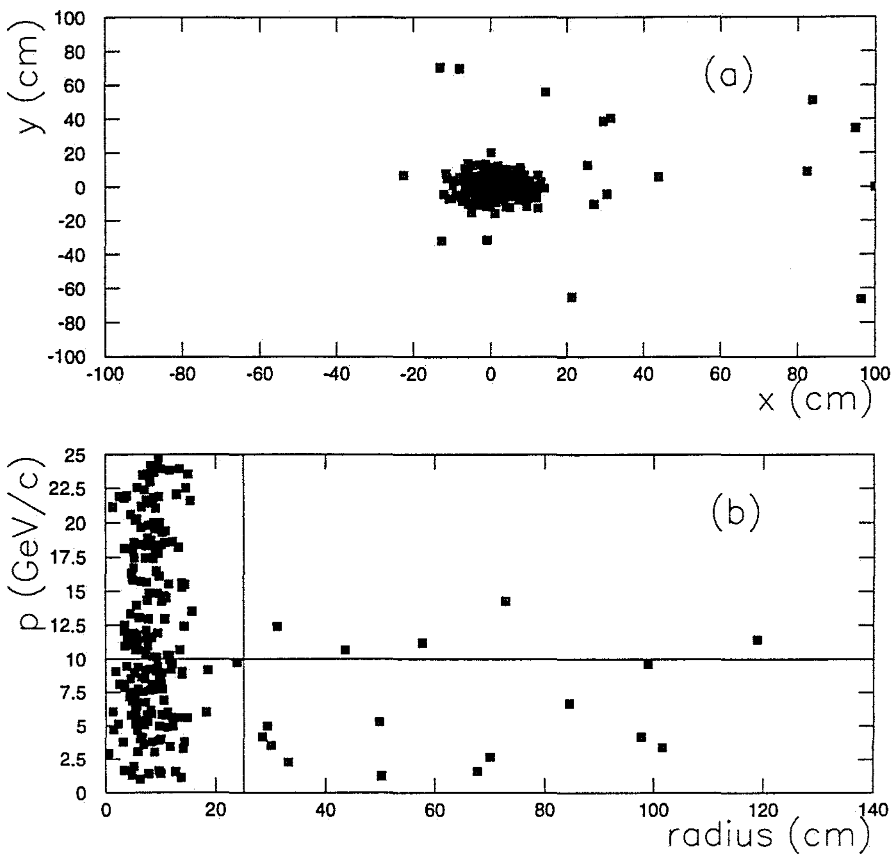

FIG. 23. Collimator production of K_L. Part (a) is the position where K_L mesons strike the lead glass, and shows the beam region and outlying events which come from collimator production. Part (b) shows the momentum vs. radius from the center of the lead glass of K_L mesons. The lines are the locations of cuts described in the text.

Collimator production background has been seen before in the E8 series experiments some of us have performed using hyperon magnets. In those experiments we have been able to understand collimator production background to better than 1/10 of itself. We expect this to be true for the CP/T experiment as well. We plan to make a subtraction of this background in every p and t bin in the experiment. Again the fit for ϕ_{+-} is robust against the resulting uncertainty in our fitting process. We expect a systematic uncertainty of less than 0.001° from this effect.

We put regeneration into the Monte Carlo program. Using the KTeV method of generating kaon decays, called the evolution method (which is very general and can handle kaons passing through matter as well as propagating through vacuum), we put in the geometry of the target and hyperon magnet collimator, and studied both coherent and diffractive regeneration processes. The largest background to the $K_{\pi 2}$ events comes from K_S events that have traversed the material of the collimator near the beam hole, and because of finite resolution are inseparable from the events that passed cleanly through the hole. These events have undergone coherent regeneration of K_L's and their phase has been changed in that process. Coherent events form a background of about 1.2% of signal events after cuts. These events move the average phase of the data by about $-0.027°$. This should be compared to the statistical uncertainty in our phase measurement of $\pm 0.04°$. Therefore we have to understand these events and make a correction to our data. If we know the correction to a quarter of itself the accuracy would be ample. We assign a systematic uncertainty for this effect of 0.01° in ϕ_{+-}.

Fig. 24 (upper left) shows the Monte Carlo simulation of the signal events plus the coherently regenerated background, and a polynomial showing the size of the GEANT simulation of the collimator production background. In this figure the vertex position of the $\pi^+\pi^-$ decay has been projected to $z = 2.6$ m, which is the average $K_{\pi 2}$ longitudinal decay position, and the distance from the beam center is plotted. Fig. 24 (lower left) shows the ratio between the model with coherent regenration and our model without coherent regeneration. Coherent regeneration changes the shape of this distribution by about 10% in the region $0.015 < r < 0.03$ m. We will be able to use this region to normalize a Monte Carlo study of K^+ interactions in the target and regeneration in the hyperon magnet collimator and calculate the coherent regeneration background.

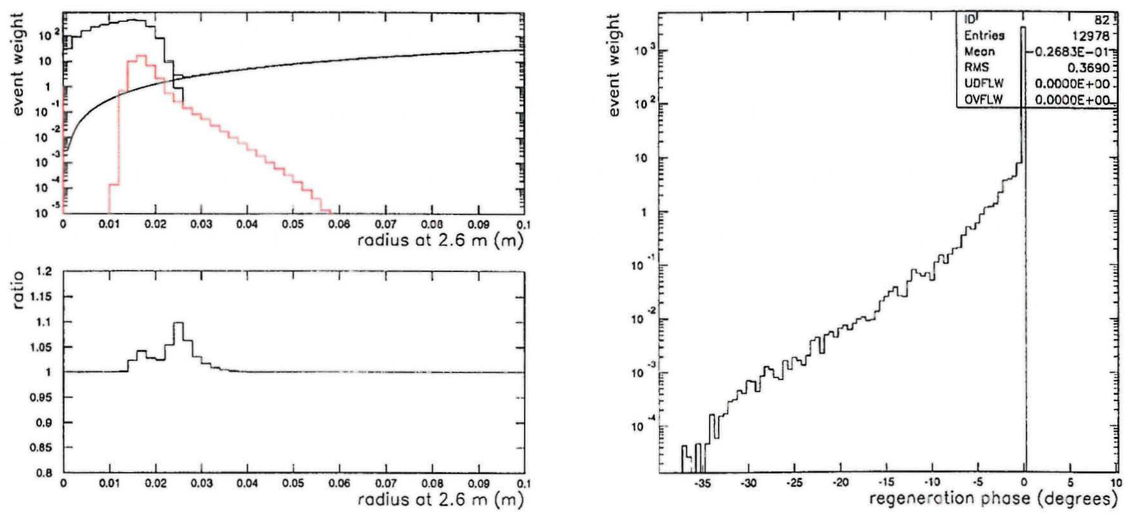

FIG. 24. Vertex radius projected to 2.6 m (upper left), for signal and coherently regenerated background events (red). Background from incoherent collimator production is also shown. The ratio between the model with and without coherent regeneration is shown at lower left. The right part of the figure shows the modification to the phase of $K_{\pi 2}$ events by coherent regenation.

The right hand part of Fig. 24 shows the coherent regeneration phase that is added to the $K_{\pi 2}$ events. The peak at the origin represents the events that have cleanly passed through the collimator, and the tail to the left represents the background. The average value of this phase, of $-.027°$, can be read from the figure. We put coherently regenerated events into our fitting program and reproduced that the phase is changed by this amount in the fit.

Next diffractive regeneration was studied. Here the cuts were more efficient, and after cuts there remained an 0.6% background. The shift in the phase from this background was $-0.016°$. This is negligible in itself, but it is in the same direction as the shift from coherent events. So the sum of the two processes must be corrected for in the actual experiment.

One crucial study for the experiment is the determination of the uncertainty in the acceptance calculation. In making this determination we can be guided by the way this has been done in previous kaon experiments at Fermilab, for example E773 and E832 (where some of us were collaborators). In E832 the signal events in the $\pi^+\pi^-$ mode are considerably fewer than K_{e3} decays, which number about 1 billion ($K_{\mu 3}$ are rejected at the trigger level in E832), so a high statistics study of the data/MC agreement is performed with K_{e3}'s. Then the results of this study are applied to the $\pi^+\pi^-$ mode in the fitting program. For example, if the histograms of the longitudinal (z) vertex position from the data and Monte Carlo are divided bin by bin, and the resulting ratio is fit to the hypothesis of a straight line, the slope of the fit can be used to estimate the systematic uncertainty of the acceptance, assuming of course that the fit is of good quality. If the slope of the data/MC ratio in z is measured to be a certain value, then in the fitting program such a slope is introduced into the Monte Carlo acceptance to see how it changes the value of the fit parameters. The change provides an estimate of the systematic error due to the acceptance.

For the February, 1999, first announcement of an E832 value for $Re(\epsilon'/\epsilon)$, this process was carried out. The E832 K_{e3} data/MC ratio slope in z was found to be $(-.53 \pm 0.34) \times 10^{-4}$ per meter. This slope is about 1.6 σ from zero. See Fig. 25. To test further the Monte Carlo simulation of the data the $\pi^+\pi^-$ slope in z was measured. It has a slope of $(-1.7 \pm 0.6) \times 10^{-4}$ per meter, about 2.8 σ from zero. See Fig. 26. These two results are not inconsistent with each other.

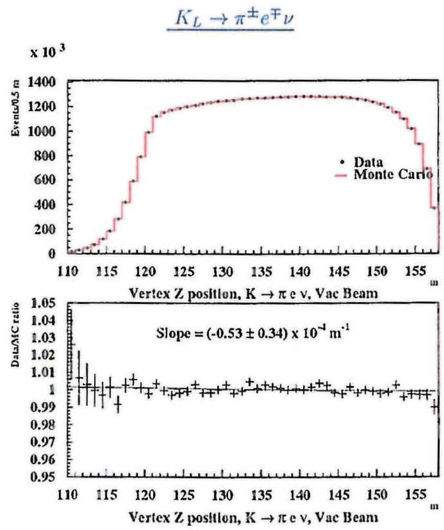

FIG. 25. KTeV Data vs. Monte Carlo comparison for the z vertex position of K_{e3} decays.

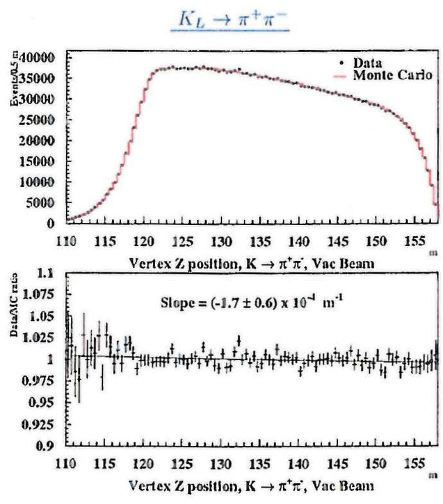

FIG. 26. KTeV Data vs. Monte Carlo comparison for the z vertex position of $K_{\pi 2}$ decays.

In the CP/T experiment we will have about 1 billion K_{e3}'s, 1 billion $K_{\mu 3}$'s, and 20 billion $K_{\pi 2}$'s. We will use the semileptonic decays to measure the accuracy of our Monte Carlo simulation, and like the E832 experiment test this with the $K_{\pi 2}$ decays. We now describe our study of this process using our Monte Carlo and fitting programs.

First a Monte Carlo study was carried out to determine the statistical precision given by K_{e3} decays in the comparision of data to the Monte Carlo simulation. Two statistically independent K_{e3} samples were generated and treated as "data" and as Monte Carlo. Since the important criterion in performing a fit is how the z-dependence of the data determines the uncertainty of the slope we did not have to invest the week of 500 MHz DEC Alpha CPU time to generate a billion events. Instead we use smaller statistics and scale as Poisson statistics. The basic result is that 1 million K_{e3}'s gives a slope uncertainty of 0.39×10^{-3} per meter. For a 9.5 m long decay region this gives a total fractional deviation from a horizontal line of .0037. With 1 billion K_{e3} events divided into 15 momentum bins we can therefore achieve a slope uncertainty in each bin of 0.48×10^{-4}. In each bin we will be able to see a deviation from a horizontal line of fractional size $9.5 \times 0.48 \times 10^{-4} = .00046$. This result in one momentum bin is comparable to the accuracy quoted above for all momenta of the 20% sample of the E832 data.

In the CP/T experiment we will also have about 1 billion $K_{\mu 3}$ decays which can be used to reduce the observable deviation by about $\sqrt{2}$.

The second step we took was to introduce an error into the acceptance in our fitting program to see how sensitive ϕ_{+-} is to the acceptance uncertainty. In the fitting program the acceptance calculated from our Monte Carlo program is first used to calculate the number of "data" events we will see in the whole experiment in bins of momentum and proper time. Next a fitting function is calculated (that uses the acceptance) and MINUIT is allowed to vary the input parameters to determine the results of the fit. We introduced the acceptance error just before the call to MINUIT. Thus the fit was performed with an acceptance with a built-in systematic error.

Table I summarizes the results of a series of fits with different functional forms of the acceptance error. We introduced errors into the acceptance with the following functional forms: linear, quadratic, cubic, and exponential, both in z and in p, and a cubic error in p and z simultaneously. The exponential was used to simulate a "rolloff", which is a form often seen in data/MC comparisons where the agreement is good except at one end of the plot. As described above the fitting parameters were $|\eta_{+-}|, \phi_{+-}$, three parameters to fit the momentum spectrum (a normalization constant plus linear and quadratic correction factors), one parameter for the dilution factor, plus $\Delta m, \tau_S$, and τ_L.

We see from this table that the fit for ϕ_{+-} is robust. A deviation of a few parts in 10^3 is needed to change ϕ_{+-} by an amount comparable to its statistical uncertainty of .04°. The reason for this robustness is that the three fitting parameters describing the momentum spectrum change due to the deviation of the acceptance and ϕ_{+-} changes little. With our K_{e3} data we will have the ability to see deviations from a horizontal line about a factor of 5 - 10 smaller than this. We assign a systematic uncertainty due to acceptance in our ϕ_{+-} measurement of $\pm$.01°. This is a conservative estimate, and is considerably smaller than the statistical uncertainty of the experiment.

Functional Form	Total Deviation from a Horizontal Line	Change in ϕ_{+-}
Linear in z	.004	.03°
Quadratic in z	.004	.04°
Cubic in z	.0025	.04°
Low-z rolloff	.003	.05°
High-z rolloff	.004	.04°
Linear in p	.01	$<$ .01°
Quadratic in p	.01	$<$ .01°
Cubic in p	.01	$<$ .01°
High-p rolloff	.03	$<$ .01°
Cubic in p and z	.0022	.04°

TABLE I. Studies of Robustness of the ϕ_{+-} Fit

The E832 experiment has achieved an understanding of their data not unlike what we are discussing here. Since their (some of us would say "our") results are found using the same drift chamber system that appears in this proposal it is interesting to ask what uncertainty in ϕ_{+-} would come from the E832 slopes. For the E832 $\pi^+\pi^-$ acceptance uncertainty we would get ±0.012 degrees systematic error in ϕ_{+-}. With the E832 K_{e3} result the CP/T experiment would find a ϕ_{+-} systematic error of .004 degrees. This is to be compared to our expected statistical uncertainty of 0.04 degrees. So the E832 experiment, in its first announcement of an epsilon prime result from 20% of its data, is already achieving an accuracy that would suffice for the CP/T experiment.

If the phase of η_{+-} were to change by 0.04 degrees, the interference term would shift in space, for the average momentum kaon, by a distance of 1.3 mm. We must know the decay point and t=0 point (on the average) to better than this distance. For the decay point, we estimate that we can determine drift chamber wire positions to 0.2 mm, as well as measure the location of the center of the target to that accuracy. We assign a systematic uncertainty in ϕ_{+-} of 0.006° to this effect.

In the latter determination we are helped by the fact that the incoming K^+ and outgoing K^0 are being absorbed in the target with the same cross section, making the center of the target the average production point. To test whether scattering of the K^0's in the target could contribute to a change of the average production point we performed a Monte Carlo simulation that follows individual K^+ mesons through the target, produces K^0's through charge exchange, and allows the K^0's to scatter elastically. The outgoing K^0 then has a different energy than when it was produced, so when we observe it and calculate the production point as the center of the target we are making a small error. In performing this Monte Carlo calculation we learned that there is a solid angle effect which moves the average production point downstream by 0.9 mm, but that the scattering effect we were concerned with is only an 0.3 mm effect. It is clear that we must understand the size and divergence of the beam to good accuracy to be able to perform this experiment. Measurements of these quantities must be part of our experiment. Running with two different target thicknesses would provide a good test of our target-length correction calculation.

Table II lists all of the systematic uncertainties in the measurement of ϕ_{+-}. The total systematic uncertainty is 0.016°. Adding in quadrature the statistical uncertainty we obtain a total uncertainty of 0.043°. This should be compared to the effect of CPT violation at the Planck scale of 0.06°.

Source	Uncertainty (worst case)	ϕ_{+-} Uncertainty
Semileptonic Background	1%	0.002°
Collimator Production	0.15%	0.001°
Coherent Regeneration	1.2%	0.01°
Diffractive Regeneration	0.6%	0.005°
Acceptance Uncertainty	0.046%	0.01°
Target Center	0.2 mm	0.006°
Total Systematic Uncertainty		0.016°
Statistical Uncertainty		0.040°
Total Uncertainty		0.043°

TABLE II. Summary of uncertainties in the ϕ_{+-} measurement.

2. Measurement of ϕ_ϵ

In this experiment we measure ϕ_{+-}, but we must also determine ϕ_ϵ. The leading contribution to ϕ_ϵ is the superweak phase, ϕ_{SW}, given by $\tan(\phi_{SW}) = 2\Delta m/\Delta\Gamma$. The superweak phase will be measured by KTeV to accuracy sufficient for our purposes here. We next describe some corrections to this contribution.

For this experiment, ϵ' will have no meaningful effect. Assuming CPT invariance, the phase of ϵ' is known to be (48 ± 4) degrees [34]. Its magnitude is a bit controversial, but if we assume it to be the central value from E832 we find that the maximum possible difference it can provide between ϕ_{+-} and ϕ_ϵ is 0.012 degrees, a factor of 5 smaller than the contribution of CPT violation at the Planck scale.

The full formula for ϕ_ϵ is [35]

$$\tan\phi_\epsilon = \frac{2\Delta m}{\Gamma_S - \Gamma_L}\cos\xi + \frac{\sin\xi}{\delta} \tag{5.1}$$

where $\xi = \arg(\Gamma_{12} A_0 \overline{A}_0{}^*)$ and $\delta = 2Re(\epsilon)$. Here A_0 is the isospin 0 part of the $\pi^+\pi^-$ decay amplitude. In the Wu-Yang phase convention, A_0 is real, and Γ_{12} gives contributions from two sources: semileptonic decays through $Im(x)$, the $\Delta S = \Delta Q$ violation parameter, and 3π decays through $Im(\eta_{+-0})$ and $Im(\eta_{000})$.

In the standard model we expect $x \simeq 10^{-7}$, which is too small to affect this experiment, but $Im(x)$ is known experimentally only to an accuracy of ± 0.026. This results in an uncertainty in ϕ_ϵ of 1.7 degrees. To prove that an observed difference between ϕ_{+-} and ϕ_ϵ were due to CPT violation one would have to measure $Im(x)$ about 40 times more accurately than today's level. The way we will do this is described below.

The contribution to ϕ_ϵ from the 3π modes in the standard model is 0.017 degrees, which is smaller than the accuracy we are trying to obtain. But if one takes into account the current world's knowledge, the uncertainty these decay modes contribute is 2.2 degrees. So they also have to be measured better.

The best experimental approach to measuring these three quantities, x, η_{+-0}, and η_{000}, is the same: choose an experiment with high dilution factor and observe interference between K_L and K_S close to the target; i.e. the experiment described here. These measurements should be thought of as being an integral part of this experiment. We have performed a calculation of the sensitivity of this experiment for these quantities, and we estimate that we can reach at least the required sensitivity. We conclude that we can determine ϕ_ϵ to the required accuracy.

We used the same Monte Carlo and fitting programs to estimate the sensitivity of our experiment to the measurements necessary for the calculation of ϕ_ϵ, $Im(x), Im(\eta_{+-0})$, and $Im(\eta_{000})$, and conclude that we will have the required sensitivity. We find that the uncertainty in $Im(x)$ contributes much more than $Im(\eta_{+-0})$ and $Im(\eta_{000})$ to the uncertainty in ϕ_ϵ. The detailed results are as follows.

- In the same time we are collecting $K_{\pi 2}$ decays for the measurement of the phase of η_{+-}, we will also collect 980 M K_{e3} decays and 990 M $K_{\mu 3}$ decays with which to measure $Im(x)$.

Fig. 27 (left half) shows the momentum vs. proper time distribution of K_{e3} decays. The K_{e3} charge asymmetry is used to measure $Im(x)$, and its momentum and proper time dependence is shown in the right half of Fig. 27. As can be seen, the asymmetry is large, due to the fact that the experiment is performed near the kaon production target.

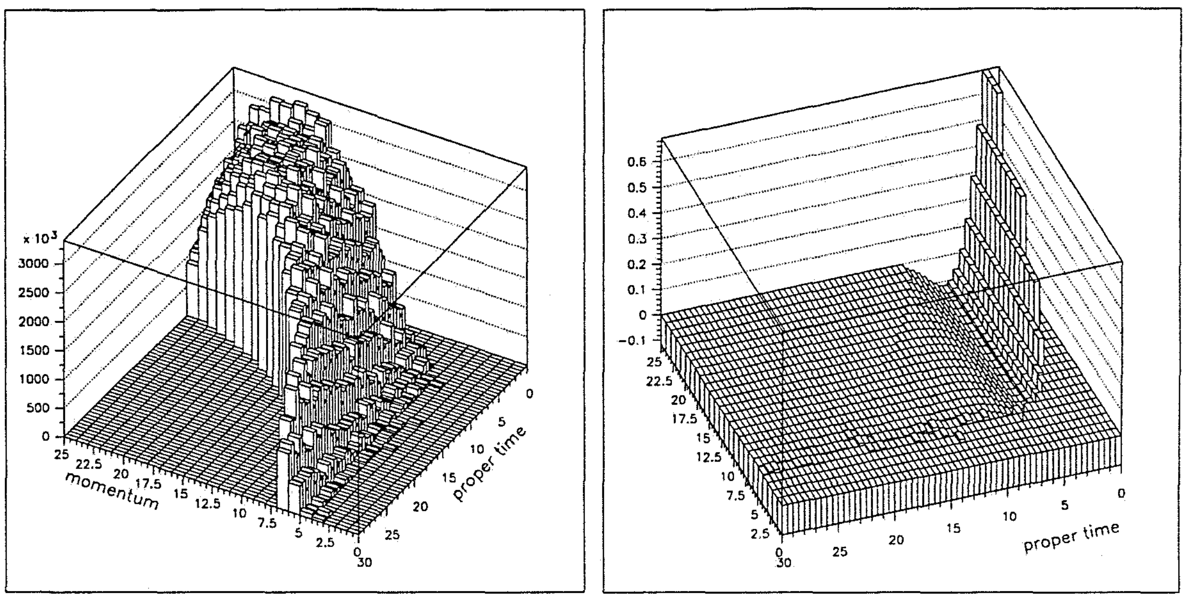

FIG. 27. Momentum vs. Proper Time for K_{e3} decays (left half). Charge Asymmetry vs. momentum and proper time for K_{e3} decays (right half).

The fit for $Im(x)$ uses p and t, the momentum and proper time distributions of the semileptonic decays, so we must reconstruct the parent kaon's momentum and decay time. Because there is a missing neutrino in each of these decays, there is a twofold ambiguity in the reconstruction of the the parent kaon's momentum. Using the spectrum of accepted K_{e3} events, we chose the more probable solution, which results in a successful choice 63% of the time. In addition there are two types of events where we can choose the correct solution with higher probability. About 1/3 of the events are "unambiguous", which means that the two solutions are close to each other. Also there are events at high momenta where one solution is physical and the other is above the K^+ beam momentum. When we take these events with more information into account, we choose the correct solution 79% of the time.

In our fitting program we must take into account the successes and failures of this method. When MINUIT is varying the physics parameters in performing the fit it uses the parent distribution in p and t. We must compare these distributions with the measured distributions. We do this by generating from our Monte Carlo simulation a four-dimensional map from parent to observed p vs. t distributions; i.e., for each bin in the parent distribution the map contains the probability of the events being measured to be in a different p and t bin. Fig. 28 shows an example of this map of measurement probabilities for the parent bin at 10 GeV/c and 10 τ_S. The tail on the right in this figure comes from the fact that these kaons are below the peak of the momentum distribution, so if we reconstruct their momentum wrong we will tend to err on the high side. When carrying out the fit using this method the possible systematic uncertainty coming from the K_{e3} momentum ambiguity becomes included in the statistical uncertainty.

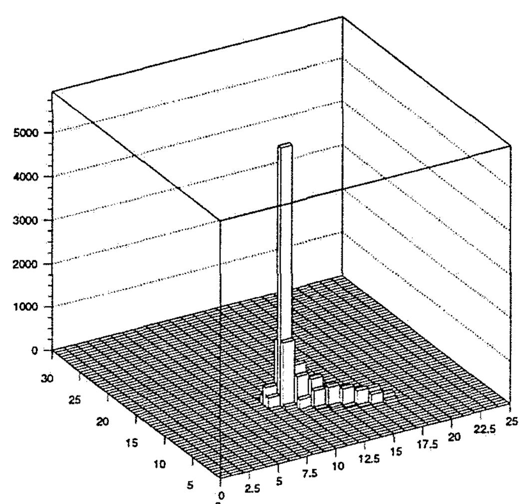

FIG. 28. Map of the reconstruction accuracy of K_{e3} decays for the p,t bin at 10 GeV/c and 10 τ_S.

We fit the resulting momentum vs. proper time distribution of the two semileptonic decays for $Re(x)$ and $Im(x)$. The resulting uncertainty in $Im(x)$ from the fit is $\pm 0.41 \times 10^{-3}$. This should be compared with $\pm 0.6 \times 10^{-3}$ which is needed to determine ϕ_ϵ to .04 degrees. Therefore the uncertainty in ϕ_ϵ from the determination of $Im(x)$ is 0.027°.

- We will collect about 230 M $\pi^+\pi^-\pi^0$ decays. These would allow us to measure $Im(\eta_{+-0})$ to $\pm 0.35 \times 10^{-3}$, which is smaller than the $\pm 4 \times 10^{-3}$ needed to determine ϕ_ϵ to ± 0.04 degrees. This will result in a 6.5σ measurement of η_{+-0}. Thus the uncertainty in ϕ_ϵ from the determination of $Im(\eta_{+-0})$ is 0.0035°.

Figure 29 (left half) shows the momentum spectrum and proper time distribution for $\pi^+\pi^-\pi^0$ decays. The right half of the plot shows the proper time distribution divided by a pure K_L distribution; i.e., the deviation from unity is the interference from which we plan to measure $Im(\eta_{+-0})$.

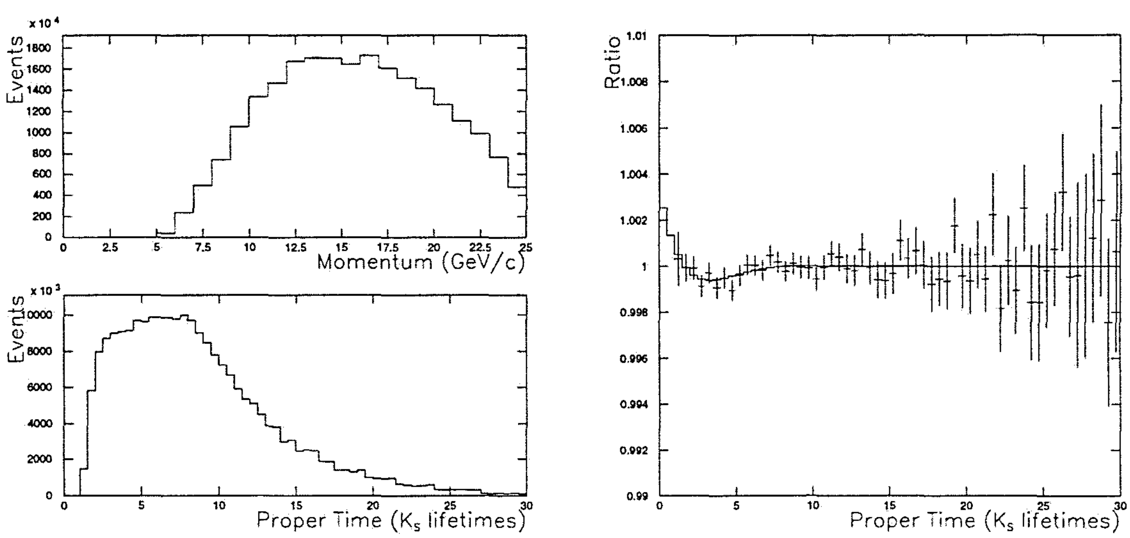

FIG. 29. Momentum Spectrum, Proper Time Distribution, and Relative Size of the Interference for $\pi^+\pi^-\pi^0$ Decays.

- We will collect about 70 M $\pi^0\pi^0\pi^0$ decays and measure $Im(\eta_{000})$ to $\pm 0.64 \times 10^{-3}$ accuracy. This is also better than the $\pm 4 \times 10^{-3}$ we need for the ϕ_ϵ determination, and will result in a 3.5σ measurement of η_{000}. This decay is technically the most difficult one to measure. It drives the design of our detector in several ways. Because the decay γ-rays spread out so much we need to have an electromagnetic calorimeter covering an area of about 4 square meters. In addition many of the gamma rays are concentrated near the center of the detector so no hole can be allowed in the center to let the beam pass through. Ours is a low-rate experiment so this criterion can be met without any problem. The uncertainty in ϕ_ϵ from the determination of $Im(\eta_{000})$ is 0.0064°.

  Figure 30 (left half) shows the momentum spectrum and proper time distribution for $\pi^0\pi^0\pi^0$ decays. The right half of the plot shows the proper time distribution divided by a pure K_L distribution; i.e., the deviation from unity is the interference from which we plan to measure $Im(\eta_{000})$.

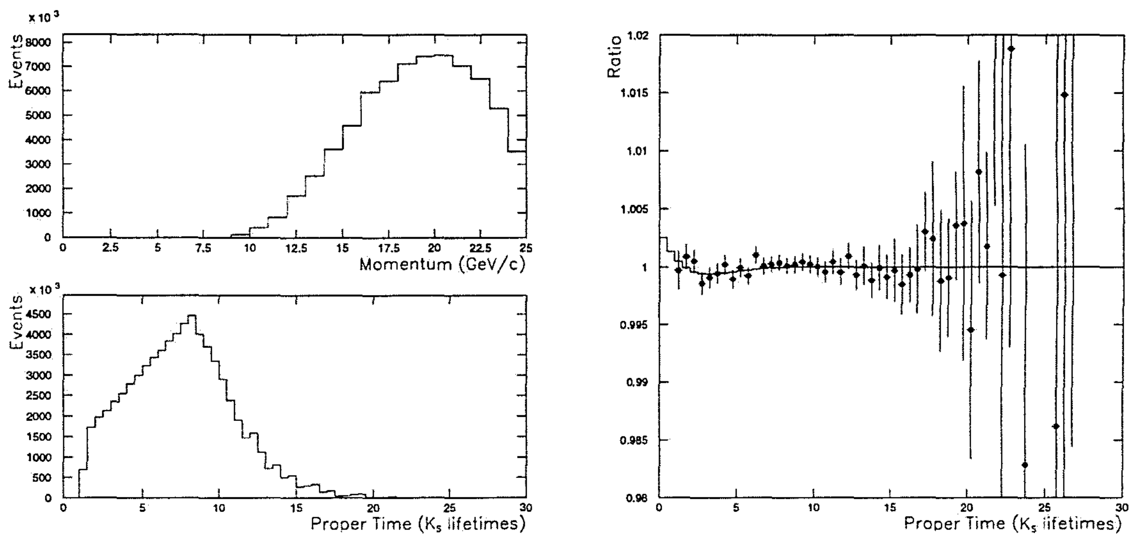

FIG. 30. Momentum Spectrum, Proper Time Distribution, and Relative Size of the Interference for $\pi^0\pi^0\pi^0$ Decays.

We studied the backgrounds under the K_{e3} signal events (for the determination of $Im(x)$) coming from $K_{\pi 2}$ decays. The left half of Figure 31 shows the momentum vs. proper time distribution of the $K_{\pi 2}$ background after cuts on mass and transverse momentum. This scatter plot is normalized to the number of good K_{e3} signal events. This background is quite low. The right half of Figure 31 shows the proper time distribution of the background coming from misidentified $K_{\mu 3}$ decays (where the muon has not been seen in the muon detector and the pion has been misidentified as an electron). This background is also very low.

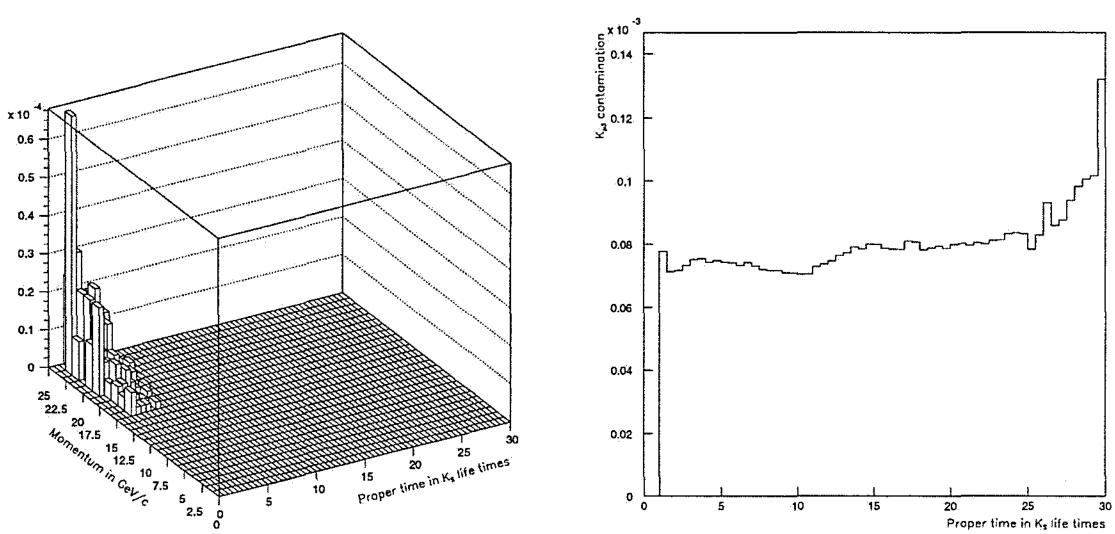

FIG. 31. Momentum vs. Proper Time scatter plots for $K_{\pi 2}$ decays and for background from $K_{\mu 3}$'s (after cuts).

We studied the effect of coherent regeneration on our fits for $Im(x)$ using K_{e3} decays. In the same way described above we calculated the amplitude and phase for kaons that traverse the collimator, and included both these effects in the fitting program where we calculated the statistics of the whole experiment and fit the K_{e3} charge asymmetry for $Im(x)$. Coherent regeneration tends to increase the charge asymmetry.

Fig. 32 part (a) shows the probability of having a kaon pass through some of the material of the collimator as a function of the change of its phase. The large peak at the origin comes from the events that have passed cleanly through the collimator hole. Parts (b) and (c) of this figure show the contours in χ^2 for the fits to $Re(x)$ and $Im(x)$. Part (b) is without coherent regeneration and part (c) is with that effect. The effects of coherent regeneration are to change the central fit values of $Re(x)$ and $Im(x)$ by 0.5 and 0.3 standard deviations respectively. Since we can measure the amount of coherent regeneration using $\pi^+\pi^-$ decays and correct for this effect to about 1/4 of itself, coherent regeration will have a negligible effect on our determination of $Im(x)$.

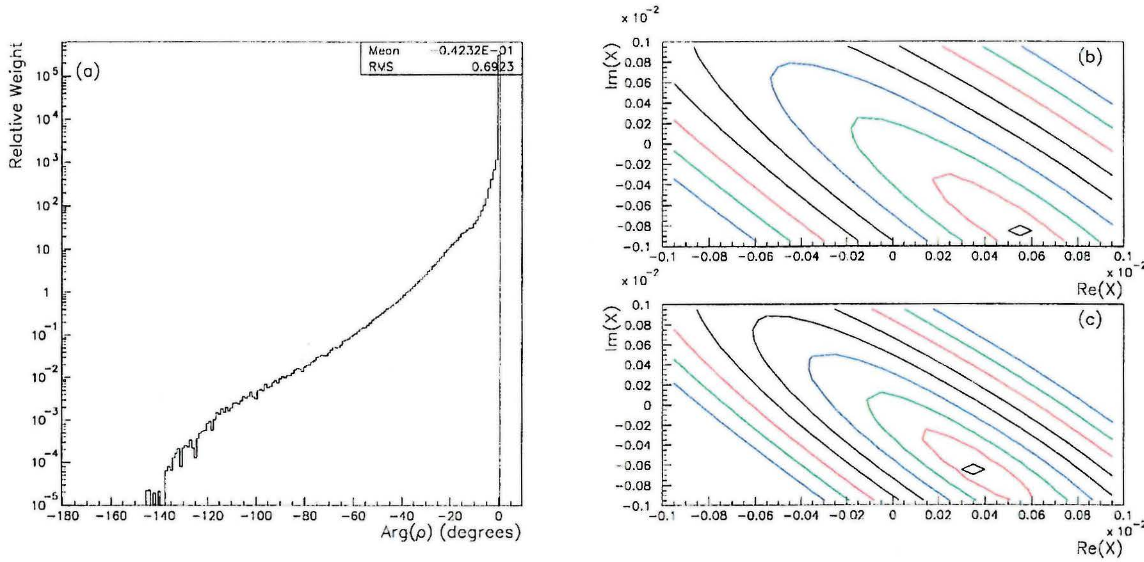

FIG. 32. Effects of coherent regeneration on K_{e3}'s. Part (a) shows the weight of events vs. their regeneration phase. The peak above the origin is events that cleanly passed through the collimator. Parts (b) and (c) show χ^2 contours for $Re(x)$ and $Im(x)$ before and after including coherent regeneration.

The fit for $Re(x)$ and $Im(x)$ uses the charge asymmetry in the two semileptonic decays. Experimental biases in measuring the charge asymmetry are very small. By frequently reversing the magnetic field of the spectrometer magnet we can cancel out geometrical biases. Previous experience in experiments of this kind has convinced us that this is a very effective technique. The interactions of the electrons and positrons in the material of the spectrometer are identical to sufficiently high accuracy that this introduces no appreciable bias.

The interactions of π^+ and π^- mesons with matter are not the same. But for matter of low atomic number, with approximately equal numbers of protons and neutrons, isospin symmetry makes the π^+ and π^- cross sections equal. A pion traversing our detector encounters only low Z materials (our drift chambers even have aluminum wires) until the lead glass electromagnetic calorimeter is encountered. Even lead glass is mostly silicon and oxygen. But a difference will exist in the number of π^+ and π^- mesons that have high E/p (i.e., within the range $0.85 < E/p < 1.15$ that would be used to collect 99% of electrons) . This difference will effect the particle identification that is necessary to determine the charge asymmetry.

We expect that, after cuts on E/p and shower shape, about 1% of pions will be indistinguishable from electrons. Our Geant calculation indicates that π^+ (π^-) will be 0.05% higher (lower) than this average. If we take this GEANT result to be approximately correct, and we choose the electron as the particle whose E/p is closest to 1 (if the pion has a flat E/p distribution and the electron has a Gaussian distribution then we choose correctly 69% of the time), then we would observe an experimental bias on the charge asymmetry of 0.00015. By looking at showers produced by pions (identified, for example, because they come from $K_{\pi 2}$ decays) we can measure this effect as a function of pion momentum and correct for it. If we assume that this correction has 100% uncertainty, then in our fitting program we have seen that it would increase the uncertainty on $Im(x)$ by only 5% of itself. This is a negligible effect.

The background to the decay modes $K_L \to \pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$ is very small. There are many powerful cuts for eliminating background for the $\pi^+\pi^-\pi^0$ decay; for example $\gamma - \gamma$ invariant mass, $\pi^+\pi^-\pi^0$ invariant mass, and p_T^2 each provide sharp peaks to cut around. In previous experiments we have seen much less than 0.1% background [26].

Fig. 33 shows four histograms from our Monte Carlo study of $\pi^+\pi^-\pi^0$ decays: the $\gamma - \gamma$ invariant mass, $\pi^+\pi^-\pi^0$ invariant mass, p_T^2, and the ratio of the momenta of the larger momentum pion to the smaller, and shows that the resolution of the detector is good.

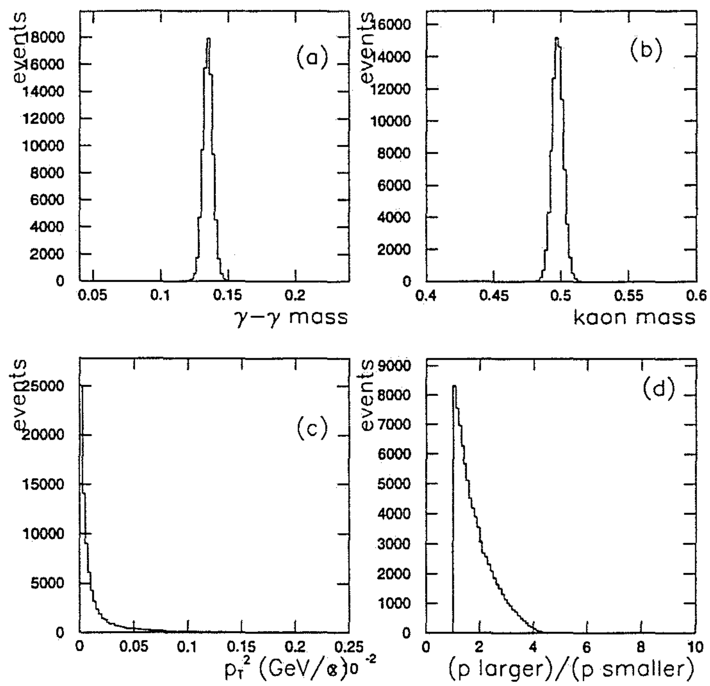

FIG. 33. Monte Carlo histograms for $K_L \to \pi^+\pi^-\pi^0$ decays. The $\gamma-\gamma$ invariant mass, $\pi^+\pi^-\pi^0$ invariant mass, p_T^2, and the ratio of the momenta of the larger momentum pion to the smaller are shown.

The $\pi^0\pi^0\pi^0$ has few background processes: we have seen in previous experiments that this decay is very clean. The measurement of the $3\pi^0$ Dalitz plot quadratic slope in E731 [36] was performed by one of us (SVS) with a very similar lead glass electromagnetic calorimeter. For the present experiment four Monte Carlo plots are shown in Fig. 34: the 6γ invariant mass ($\sigma = 3.8 MeV/c^2$), the γ ray energy, the reconstructed transverse decay position, and the resolution in the x-coordinate of the vertex ($\sigma = 6mm$).

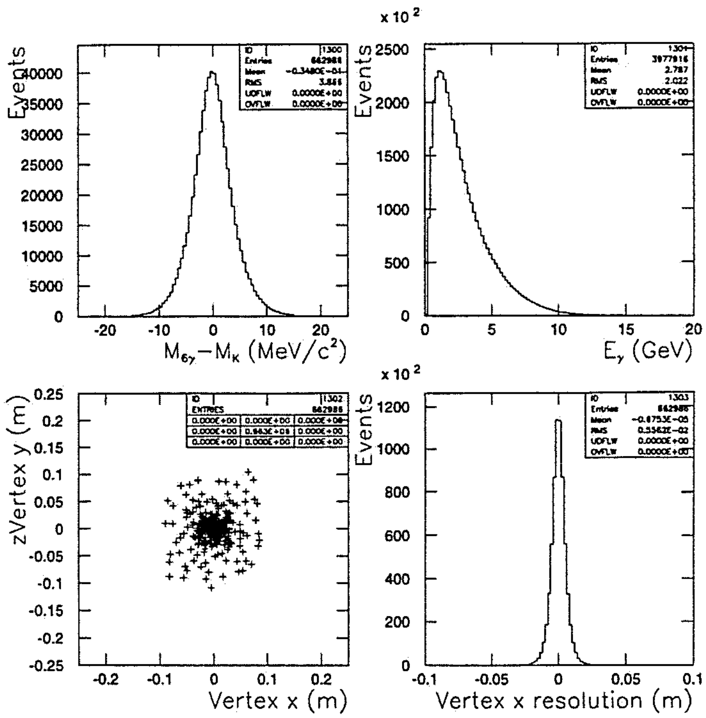

FIG. 34. Monte Carlo distributions for $\pi^0\pi^0\pi^0$ decays. The the 6γ invariant mass, γ ray energy, the reconstructed transverse decay position, and the resolution in the x-coordinate of the vertex are shown.

We have calculated the background caused by mispairing the gamma rays. We choose the gamma rays in pairs that come from individual π^0's using the fact that all three correct pairs will give the same vertex z. We throw out events where the two best pairings are ambiguous (where the best pairing has a z RMS less than 1.5 more than the second best pairing). This results in a background of 0.3%.

After calibrating the lead glass with electrons from K_{e3} decays, photons from $\pi^+\pi^-\pi^0$ decays will be used to measure the electron-photon energy scale shift on a block-by-block basis, if needed. Any residual nonlinearities after the $\pi^+\pi^-\pi^0$ calibration can be a source of systematic error. These nonlinearities typically obey a power law behaviour. A typical nonlinearity is a pedestal shift which has a power law coefficient of -1.0. While the effect can occur in the lead glass ADC system for several reasons (in E731, trigger related charge injection was responsible for pedestal shifts and was an important source of systematic uncertainty), the pedestal shift also represents the effect of accidental activity in the calorimeter in an average fashion. Therefore, we chose to investigate the effect of a pedestal shift the equivalent of 20 MeV. This has the effect of lowering $M_{6\gamma}$ by 0.75 MeV, and shifting the average vertez z 9.5 cm upstream. By making a gamma ray energy scale change of 0.7% (which will come from the $\pi^+\pi^-\pi^0$ mode) we can bring the average vertex back to the correct location. Part (a) of Fig. 35 shows the bin-by-bin ratio of the vertex z of the Monte Carlo run with the pedestal shift to the "standard" Monte Carlo. Part (b) shows the ratio for the corrected Monte Carlo, and has a slope an order of magnitude smaller.

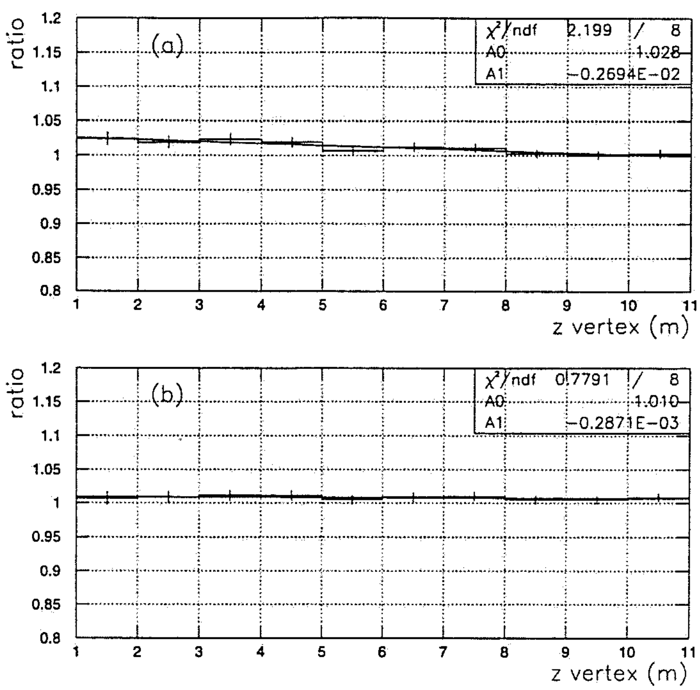

FIG. 35. Ratio plots of vertex z for $\pi^0\pi^0\pi^0$ decays. Part (a) shows the bias in z from a 20 MeV pedestal shift. Part (b) shows the bias eliminated by an energy scale correction.

Coherent regeneration will provide a background about 1% for the $\pi^+\pi^-\pi^0$ decay and slightly larger for $\pi^0\pi^0\pi^0$. But since the $K_S \to 3\pi$ branching ratios are of order 10^{-9} almost no regeneration occurs in these modes. Collimator production will give us backgrounds at the 1% level, but these are easily subtracted. We conclude that systematic uncertainties in the determination of η_{+-0} and η_{000} will be negligible.

The uncertainty in ϕ_{SW} is currently 0.080° [45]. The recent announcement of new results by the KTeV collaboration implied an uncertainty of 0.085°. The KTeV Δm result is preliminary (with systematic uncertainties about twice the statistical ones) so its uncertainty should be reduced by about a factor of 2. If this is true, when the remaining 80% of the KTeV data is included the result should be $\delta(\phi_{SW}) \sim 0.019°$. If the results of the 1999 run of the KTeV experiment is a doubling of their statistics (as expected) then ±0.013° should be achieved. In the CP/T experiment we expect to be able to measure ϕ_{SW} much better. Conservatively we assign an uncertainty to ϕ_{SW} of 0.010°.

Table III lists the uncertainties in the measurement of ϕ_ϵ. The total uncertainty is 0.030°. If we add in quadrature the uncertainty in ϕ_{+-} of 0.043°, shown in Table II, to the total uncertainty in ϕ_ϵ from Table III we get 0.052°. If the difference between the K^0 and $\overline{K}^0$ masses is given by CPT violation at the Planck scale, then $|\phi_{+-} - \phi_\epsilon| = 0.06°$. This measurement would see that difference.

Source	Uncertainty	ϕ_ϵ Uncertainty
ϕ_{SW}	0.010°	0.010°
$Im(x)$	0.41×10^{-3}	0.027°
$Im(\eta_{+-0})$	0.35×10^{-3}	0.0035°
$Im(\eta_{000})$	0.64×10^{-3}	0.0064°
Total Uncertainty		0.030°

TABLE III. Summary of uncertainties in the ϕ_ϵ measurement.

B. CPT Test via the Bell-Steinberger Relation

The next test of CPT symmetry conservation comes through an evaluation of the Bell-Steinberger relation. Our ability to measure CP violation parameters (and also $Im(x)$) very accurately will make it possible to reduce the uncertainties in the Bell-Steinberger relation by two orders of magnitude, which will make this CPT test be sensitive at the Planck scale also.

The Bell-Steinberger relation [12] is a statement of the conservation of probability in $K^0 - \overline{K}^0$ decays, in which, through Eq. 2.1, Δ appears. It is usually written as:

$$(1 + i\tan\phi_{SW})[Re(\epsilon) - iIm(\Delta)] = \sum_f \alpha_f \tag{5.2}$$

where the sum runs over all decay channels f, and $\alpha_f = \frac{1}{\Gamma_S} A^*(\mathrm{K_S} \to f) A(\mathrm{K_L} \to f)$. Table IV shows all of the decays that contribute to the states, f, and the formulas for each α_f. In addition the table lists $\delta Re(\alpha_f)$ and $\delta Im(\alpha_f)$, the uncertainties in the real and imaginary parts of each contribution. These uncertainties are taken from the 1996 Particle Data Group compilation [34]. The most recent published evaluation of the Bell-Steinberger relation is ref. [23].

Decay Mode	α_f	$10^6 \times \delta Re(\alpha_f)$	$10^6 \times \delta Im(\alpha_f)$
$\mathrm{K}_L \to \pi^+\pi^-$	$\alpha_{+-} = B^{(S)}_{+-}\eta_{+-}$	15.5	15.5
$\mathrm{K}_L \to \pi^0\pi^0$	$\alpha_{00} = B^{(S)}_{00}\eta_{00}$	10.7	10.8
$\mathrm{K}_L \to \pi^+\pi^-\gamma$	$\alpha_{+-\gamma} = B^{(S)}_{+-\gamma}\eta_{+-\gamma}$	0.64	0.66
$\mathrm{K}_L \to \pi e\nu$ and $\pi\mu\nu$	$\alpha_{l3} = \frac{\tau_S}{\tau_L}[B^{(L)}_{\pi e\nu} + B^{(L)}_{\pi\mu\nu}]$		
	$\times[\delta_l(1 + 2Re(x)) - 2iIm(x)]$	0.20	59.2
$\mathrm{K}_S \to \pi^+\pi^-\pi^0$	$\alpha_{+-0} = \frac{\tau_S}{\tau_L}B^{(L)}_{+-0}\eta^*_{+-0}$	0.01	6.51
$\mathrm{K}_S \to \pi^0\pi^0\pi^0$	$\alpha_{000} = \frac{\tau_S}{\tau_L}B^{(L)}_{000}\eta^*_{000}$	65.6	98.5

TABLE IV. Contributions of kaon decay modes to the Bell-Steinberger relation

The biggest uncertainties in the Bell-Steinberger relation at this time come from η_{000}, $Im(x)$, and δ_l (the charge asymmetry in K_L semileptonic decays). Although δ_l doesn't explicitly appear in Table IV, it is the best way of evaluating $Re(\epsilon)$. The proposed experiment will be able to make excellent measurements of the first two of these quantities, and KTeV will measure δ_l quite accurately. For the next level of accuracy in the Bell-Steinberger relation the uncertainties of the α_{+-} and α_{00} terms must be reduced. These uncertainties depend on those of $|\eta_{+-}|, Re(\epsilon'/\epsilon)$, and $\Delta\phi = \phi_{00} - \phi_{+-}$. The latter two quantities will be measured by the KTeV experiment to sufficient accuracy for our purposes here.

We will have good sensitivity for the $|\eta_{+-}|$ measurement. In our fits to the proper time dependence of $\pi^+\pi^-$ events we have excellent statistical sensitivity for measuring $|\eta_{+-}|$. In the interference term, however, it is highly correlated with D, the dilution factor. We will measure D using semileptonic decays. The semileptonic charge asymmetry at zero proper time equals D. We calculate that we will be able to measure D to better than 0.1% for momenta above 13 GeV/c. We should be able to measure $|\eta_{+-}|$ to 0.1% accuracy, about 10 times better than it is currently known.

The most accurate way to determine $|\eta_{00}|$ will be by using the KTeV value of ϵ'/ϵ and our measurement of $|\eta_{+-}|$. The most accurate way of determining ϕ_{00} will use the KTeV value of $\Delta\phi$ and our measurement of ϕ_{+-}.

We should be able to reduce the uncertainties in the Bell-Steinberger relation by about two orders of magnitude from their present values. The limit on $Re(\Delta)$ will be about 5×10^{-6}, about twice the contribution of CPT violation at the Planck scale, and will be set by the uncertainty in δ_l. For $Im(\Delta)$ the limit will be about 1×10^{-6}, dominated by the uncertainty in $Im(x)$, which would allow us to place a 2σ limit at the Planck scale. Since the Bell-Steinberger measurement is sensitive to $Re(\Delta)$ and $Im(\Delta)$ independently, these limits would be valid even if Δ is parallel to ϵ, in contrast to the CPT violation limits from $|\phi_{+-} - \phi_\epsilon|$, which are sensitive only to the component of Δ perpendicular to ϵ.

Item	Existing Uncertainty	Anticipated Measurements
ϕ_{+-}	$\pm 1^o$	$\pm 0.040^o$
$Im(x)$	± 0.026	± 0.0005
$Im(\eta_{+-0})$	± 0.017	± 0.0003
$Im(\eta_{000})$	± 0.3	± 0.0006
$\lvert\eta_{+-}\rvert$	$\pm 1\%$	$\pm 0.1\%$
ϕ_{SW}	$\pm 0.12^o$	$\pm 0.024^o$ (KTeV)
δ_l	$\pm 2.7\%$	$\pm 0.3\%$ (KTeV)
$Re(\epsilon'/\epsilon)$	$\pm 6 \times 10^{-4}$	$\pm 1 \times 10^{-4}$ (KTeV)
$\Delta\phi$	$\pm 1^o$	$\pm 0.25^o$ (KTeV)

TABLE V. Measurements for CPT Conservation Tests

Table V summarizes all of the quantities we would measure to make the two tests of CPT symmetry conservation. We also list the important contributions to be made to this experiment by measurements the KTeV experiment will make.

VI. STUDY OF RARE K_S DECAYS

A great deal of attention has been paid to searches for rare decays of the K_L meson, but little work has been done on the K_S. Some of the interesting K_S searches are for decays that test strangeness-changing neutral currents. One such decay is $K_S \to \pi^0 e^+ e^-$. This is a rare but CP conserving decay. Its branching ratio must be measured in order to unambiguously disentangle the indirect CP violating contribution to $K_L \to \pi^0 e^+ e^-$ decay. The latter decay is one of the main candidates for finding direct CP violation in a rare K^0 decay. It has contributions from direct and indirect CP violating processes, and from a CP conserving two-photon exchange diagram as well. An accurate theoretical calculation of the CP conserving rate can be made, but the uncertainties of the theoretical calculation of the indirect CP violating contribution are quite large. The branching ratio for the equivalent K_S decay can be used to calculate this indirect CP violating contribution. Once these two rates are known, the direct CP violating part can be extracted once the branching ratio of the K_L decay is measured.

The $K_S \to \pi^0 e^+ e^-$ branching ratio is expected to be between 5×10^{-10} and 5×10^{-9} [37]. Since our single event sensitivity for this decay will be 1.0×10^{-10}, we expect to collect between 5 and 50 of these events.

As an example of how this measurement could be used to contribute to the determination of the direct CP violating part of the $K_L \to \pi^0 e^+ e^-$ decay, let's assume that we have 16 K_S events and have thus measured the indirect CP violating part of the K_L to 25% of itself. Let's further assume that each of the three contributions to the K_L decay are equal, so the calculation of the CP conserving contribution and the measurement of the indirect CP violation part add up to 67% of the total, with uncertainty of $\pm 8\%$ for the indirect CP violating part (and much smaller uncertainty for the CP conserving part). Then this sum would be $(67 \pm 8)\%$, and a measurement of the K_L decay accurate to $\pm 5\%$ would determine the direct CP violating part to 3.5σ significance.

In this simple model the uncertainty in the K_S decay has reduced the significance of the measurement of the direct CP violating part of the K_L by a factor of 1.9. Since the measurement of the K_L branching ratio is so difficult one would not want to reduce the significance of a measurement in this way, hence a more accurate K_S measurement would be desirable. This could be made by using the pions in the secondary beam: by turning off the RF cavities and removing the stopper from the beam, an increase in beam intensity of a factor of 10 could be achieved. The dilution factor of these kaons would be decreased by a large factor, but that is not important for this measurement.

It is worth mentioning that an extension of this experiment with higher flux (but with high dilution factor) would be a good vehicle for finding direct CP violation in the $K_L \to \pi^0 e^+ e^-$ decay through $K_L - K_S$ interference. In this method the "Greenlee background" (the K_L radiative Dalitz decay) is not a problem as it is in a conventional K_L experiment.

One way to get this additional intensity would be to significantly increase the solid angle of the collimator in the hyperon magnet. We have examined this scenario only in a preliminary way, but it might be a possibility.

Another rare decay of interest is $K_S \to \gamma\gamma$. This decay exhibits an interesting set of interference terms, due to indirect and direct CP violation, that have never been seen. The K_S (K_L) branching ratio is 2×10^{-6} (6×10^{-4}). Our experimental design does not have the set of veto counters for photons that would be needed to make a clean identification of the $\gamma\gamma$ decay (due to the large background from $\pi^0\pi^0$ decays where two photons are lost), so we would concentrate on the Dalitz decay, $K_S \to e^+e^-\gamma$, which has not been observed previously. We would lose rate, but would still see several hundred K_S events, all at small proper times. We would see ten thousand K_L events, spread approximately evenly over the 30 τ_S proper time range of the experiment. The interference should stand out clearly.

Another Dalitz decay with interesting CP properties is $K_S \to \pi^+\pi^-e^+e^-$. In addition to time-dependent interference between K_S and K_L, there is a CP-violating asymmetry in the angle between the decay planes of the two pions and two electrons. This decay-angle asymmetry has been discovered by the KTeV experiment. Our experiment would be the first observation of the K_S decay.

The decay $K_S \to \pi^+\pi^-\pi^0$ has a CP conserving part as well as a CP violating part. The final state isospin of the three pions must be 0, 1, 2, or 3, and the $I = 0$ and 3 states are strongly suppressed. The CP of the final state is $(-1)^I$ so the $I = 1$ state is CP violating for K_S decays and the $I = 2$ state is CP conserving. These two final states are easily separated since the Dalitz plot distribution is symmetric (antisymmetric) under the exchange of the π^+ and π^- for the CP violating (conserving) decay. For a detector which is charge symmetric, simply integrating over the Dalitz plot picks out the CP violating decay very precisely. Forming the asymmetry between the decays where the π^+ has more energy in the center of mass system, and where the π^- is more energetic, picks out the CP conserving decay. Then one can observe the $K_L - K_S$ interference as a function of proper time, fit for the K_S decay amplitude, and compute the branching ratio. This technique was used in

Fermilab experiment E621, which discovered [39] this decay. They measured the branching ratio to be $(4.8^{+2.2}_{-1.6}(\text{stat})\pm1.1(\text{syst}))\times10^{-7}$. Figure 36 shows the proper time graph from which this branching ratio was computed. The decay amplitude can be predicted by chiral perturbation theory, and with better experimental precision an interesting test can be made of that theory.

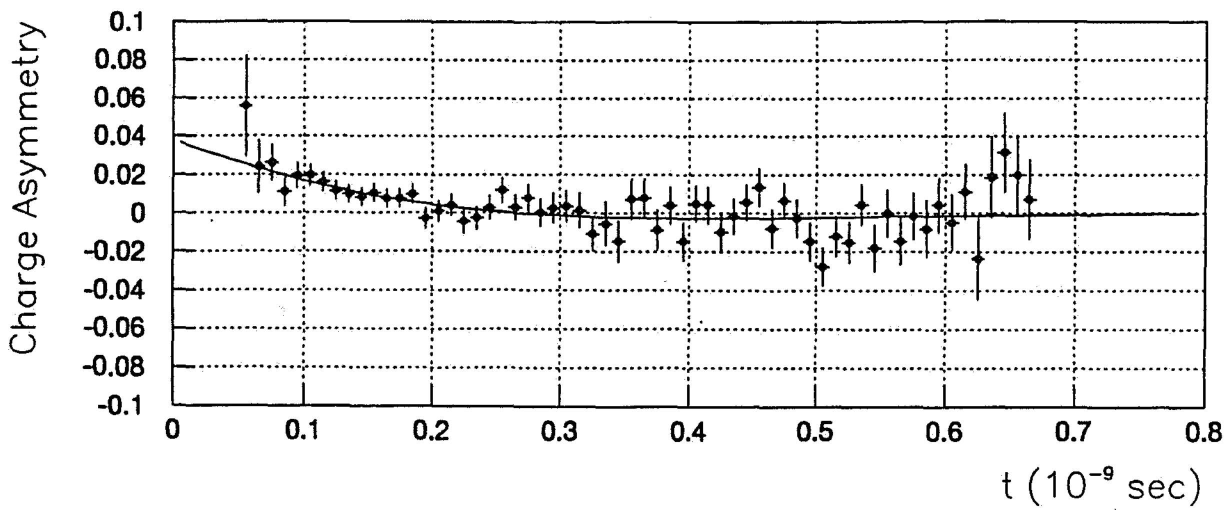

FIG. 36. Proper Time Distributions for $K_{L,S}\to\pi^+\pi^-\pi^0$ CP-conserving Decays from Fermilab Experiment E621. The asymmetry between events where the π^+ is more energetic in the center of mass, and events where the π^- is more energetic, is shown, as a function of the proper time of the decay.

VII. K^+ BEAM DESIGN

Our experiment requires an RF-separated K^+ beam. There are two technical issues to be addressed before such a beam can be made: how to design the optics of the beam, and how to build the necessary RF cavities. Our progress in tackling these issues is outlined below.

One of us (J. D.) designed the optics of an RF separated K^+ beam for the TRIUMF KAON accelerator [38], and has modified his design for the present experiment's needs. Our design has superconducting C-band RF cavities, operating at 5.79710 GHz to perform the separation. We also made a design using cavities operating at 3.9 GHz, which may be easier to construct. In the past a separated K^+ beam was built at CERN using superconducting S-band (2.79 GHz) cavities, and normal S-band cavities have been used to make K^+ beams at the Brookhaven AGS and at the Argonne ZGS. The superconducting RF cavities used for the CERN beam have been loaned to the Serpukhov Accelerator Laboratory, where they will be used in the near future to make an RF separated K^+ beam.

In general, the way the RF separation works is that the first cavity imposes a transverse momentum kick on the beam of 10 MeV/c per meter of cavity length, creating a waving-fan of particles exiting from it. The second RF cavity is located 75 m downstream, and in between is a system of quadrupoles with a transformation matrix of (-1). The phase of the second cavity is tuned so π^+ mesons arrive with the same phase that they had in the first cavity. Since the quadrupoles have reversed the π^+ momenta they get the same kick from the second cavity and the π^+ end up with no net kick. With this design choice of RF frequency and distance between the cavities, the π^+ and protons are 360 degrees out of phase at the second cavity, so protons also receive no net kick. K^+ mesons are 90 degrees out of phase from the π^+ so they get a net bend of 1.7 mrad in our design. A beam plug downstream of the second cavity destroys the π^+ and protons, and passes most of the K^+. So our design separates K^+ from both π^+ and protons. We choose the direction of the RF cavities' kick to be vertical to decouple from the momentum selection, which is in the horizontal direction.

Fig. 37 shows a plan view of the beam line, and indicates how the CPT and CKM experiments could share the same K^+ beam. Fig. 38 shows the beam line optical elements and beam size in x and y, as a function of z. The first quadrupoles collect the K^+'s and focus them in x at a focus between bending magnets B2 and B3, where a momentum slit selects the central momentum and momentum width of the beam.

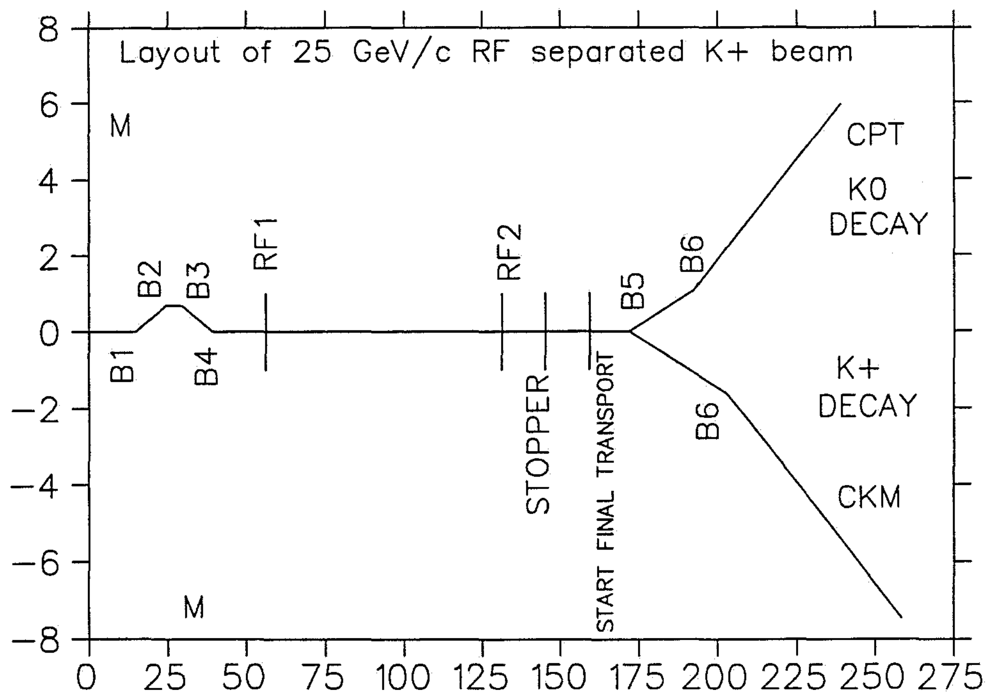

FIG. 37. Plan View of the K^+ Beam

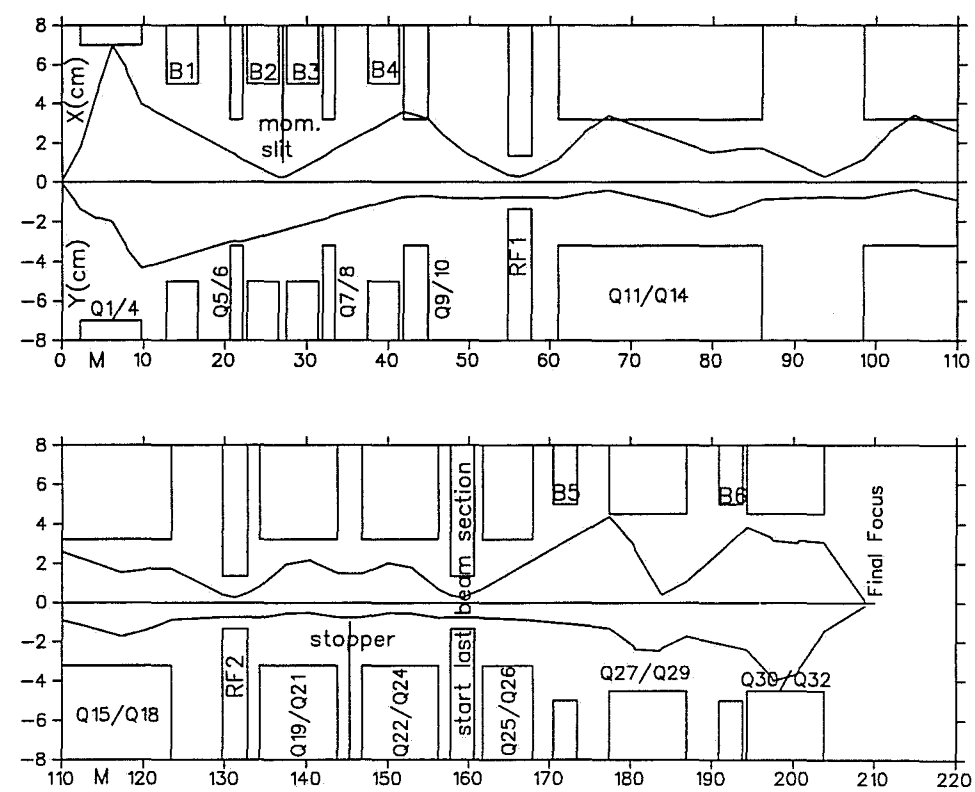

FIG. 38. K^+ Beam Line Elements and Beam Sizes

The proton dump is located after the first bending magnet. The quadrupoles before the first RF cavity transform the large y' acceptance into a small divergence (0.6 mrad) so the 1.2 mrad kick of the first RF cavity dominates the particles' angles. Two (-1) matrix quadrupole strings follow which give a net (+1) transformation, and the second RF cavity (RF2) is run at opposite phase (for pions) from the first. The next quadrupoles transform y' into y at the stopper. After that the beam is cleaned up with bending magnets and focused on the K^0 production target. The spot size at the target is about 5 mm in diameter.

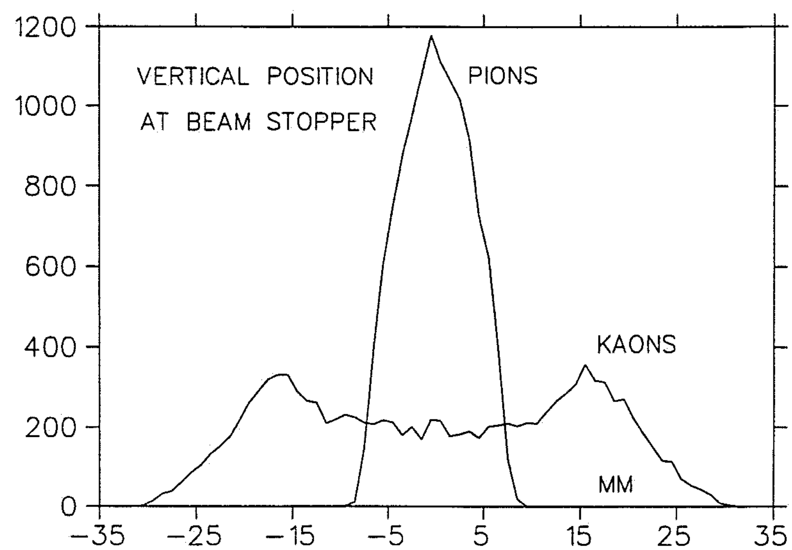

FIG. 39. y Coordinate of Pions and Kaons at the Stopper

This is a highly symmetric design which minimizes higher order aberrations. Using the TRIUMF Monte Carlo program REVMOC, which has been tested to give identical results to DECAY TURTLE, we calculated the acceptance of the beam, taking into account kaon decays. Fig. 39 shows the y coordinate of pions and kaons at the location of the stopper. It can be seen from the figure that the physical size of the stopper should be about 16 mm (±8 mm), and that about half of the kaons are transported past it. With 3×10^{12} protons per pulse at 120 GeV/c striking the target at 0 mrad, we got a flux of 1.2×10^8 kaons per pulse, which is what we used in our statistical significance calculations.

The prediction of TRANSPORT and REVMOC is that the beam will be very clean. One might expect that the main source of backgrounds would be interactions in the stopper. To test this we put the stopper and the magnets following it in GEANT, and used the phase space of pions, protons, and kaons determined from REVMOC as the input parameters of beam particles. We found that there was very little background from pions hitting the stopper, probably because they are heading into the material, but kaons that just clip the edge did produce pions of only slightly lower energy that were closer to the beam phase space. They can be removed by collimators just before the B5 bending magnet, however. The largest source of background that we have found so far comes from scattering in the vicinity of the first RF cavity. This produces about 2% background in the beam, which is well within the cleanliness we need.

The second technical question is how to build the RF cavities. We need two cavities each 3 m long, which produce a deflecting field of about 7 MV/m. They must be superconducting, since the Main Injector spill length would require enormous RF power otherwise. The superconducting cavities built at CERN in 1977 had a field strength of 1.3 MV/m, so are too weak for our application. The state of the art in achieving field strength has advanced considerably since then, and cavities reaching 25 MV/m are regularly produced at DESY for the TESLA project.

A group in the Fermilab Beams Division, headed by Helen Edwards, is working on developing the lab's capability for building superconducting RF cavities. They are interested in carrying out the R&D, and the construction, for the cavities that we need, and three of us (T.K., G.B.T. and H.B.W.) have joined Dr. Edwards' group to work on this project. Al Moretti of Fermilab has estimated that an initial period of R&D would be required that would last a year, and would cost about $750k. Building the actual cavities would take about 1 1/2 years and would cost about $1800k. The time scale of this work is reasonably matched to the Main Injector schedule, and the costs are reasonable for a facility that would initially serve two experiments and perhaps others in the future. For the review of the superconducting RF cavity R&D program, described below, a cost and time estimate was made where the cost was higher and the time longer. These estimates were for producing more cavities than are needed for this experiment, however.

The following steps have been made in the R&D on the superconducting cavities.

- The first cavity prototype that we built was designed by Mark Champion and Mike McAshan of Fermilab, and was based on the design that has been used for the TESLA prototype cavities. It has a 3 cm iris, and is designed to operate in the $TM_{110} - \pi$ mode at 3.9 GHz (exactly three times higher frequency than TESLA cavities). The first cavity, of two cells with cylindrical end pieces, was made by the Rutgers Physics Department Machine Shop, and was of copper. It was tested at Dr. Edwards' lab at A0, and was found to perform satisfactorily. The $TM_{110} - \pi$ mode has two polarizations and both were seen at frequencies that indicated that the cavities were constructed within 0.001 inch tolerance on the outer diameter.

- The cavity parts are made by deep drawing, and we have tested the dies made for the copper prototype on niobium as well. The intercell area (called the iris) has a curve of radius 0.13 inches but because of springback of the material the radius turned out to be 0.16 inches. We now have embarked on a study of springback in deep drawing of niobium at small radii to design a die that will give us the correct shape after springback occurs.

- A design of a complete cavity was made, including the cryogenic, electrical, and mechanical parts. The cavity structure would consist of 13 cells, of which 11 would be identical and the two end cells would have a different shape. Such a structure would be 0.5 m long, and two of them would be housed in a single cryostat.

- At the request of Steve Holmes, the head of the Fermilab Beams Division, there occurred in October, 1998, a review of the superconducting RF cavity (SCRF) project. The review committee was headed by Maury Tigner and included many experts in the SCRF area. They gave the project high marks.

- One of the review committee members has since loaned to us a two-cell niobium cavity complete with all the fittings to test it in a liquid helium cryostat. This is an S-band cavity which has its $TM_{110} - \pi$ mode at 3.0 GHz. It is under test and we expect to learn a lot from it.

- The committee suggested that before we build a 13 cell niobium cavity we first build a model of a 13 cell cavity machined from aluminum (constructed of individual cells that fit together) to test field flatness. The flatness of the field is very sensitive to the shape of the end cells of the cavity. With the aluminum model we can modify the end cells' shape easily and optimize our design. Construction of this cavity model is underway.

- The SCRF R&D project has been funded at the level of $500,000 this fiscal year.

Although it is not a big project to construct superconducting RF cavities that deflect rather than accelerate a beam, and achieve the modest fields that we require at a somewhat higher frequency than people have used previously, it is the largest R&D project necessary for this experiment. The CP/T collaboration recognizes the importance of this project, and is investing appropriate person power in this project.

VIII. HYPERON MAGNET DESIGN

For the hyperon magnet, which cleans up the region where the K^+ beam strikes the K^0 production target, we have chosen a design similar to that of the working hyperon magnet now in the Fermilab Proton Center beam. This conventional magnet uses tapered pole pieces to make a 35 kGauss field in its central region. A plan view of the collimator in this magnet is shown in Fig. 40. The K^+ beam strikes a 10 cm tungsten target at 9 mrad, and is bent to the side to be destroyed in the collimator. Neutral particles follow a straight path through the waist of the collimator to form the beam. The magnetic field region is 1.5 m long, and the collimator shown is made of tungsten. After the magnet is a further meter of tungsten absorber which absorbs particles which punch through the collimator in the magnet. The jaws of this absorber point toward the edges of the first drift chamber, so they provide the maximum of absorption power without reducing the acceptance of the detector.

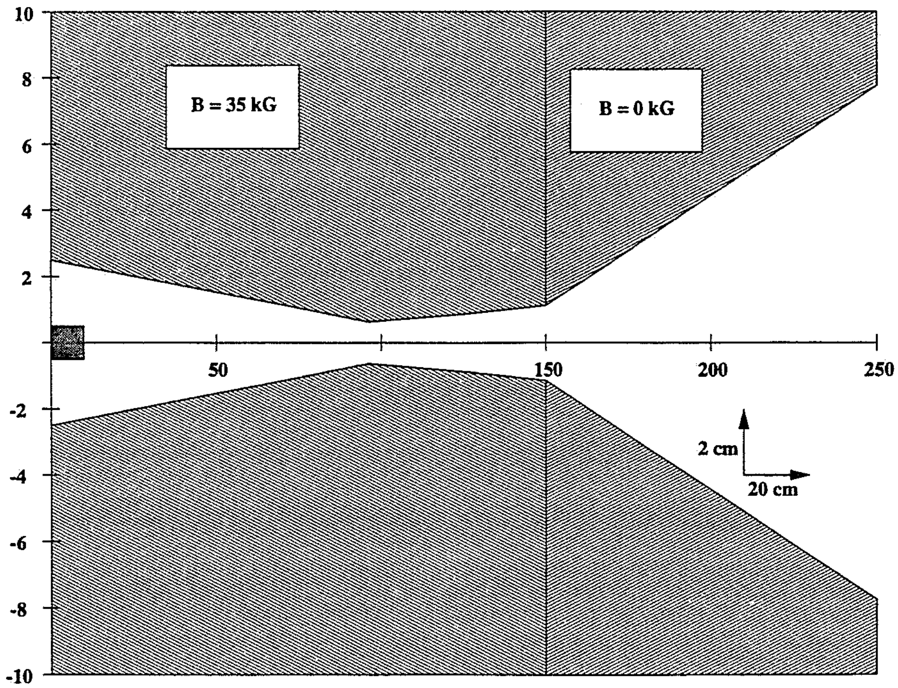

FIG. 40. Plan View of the Collimator in the Hyperon Magnet

IX. DETECTOR APPARATUS

Members of our collaboration already own many of the detectors that we will need for the experiment, like the lead glass electromagnetic calorimeter and the muon drift tube chambers. We are looking into the question of borrowing other detectors we need. A system of drift chambers for our charged particle spectrometer is the main item in question. There are several analysis magnets at Fermilab that would work for our experiment, for example the Jolly Green Giant.

A. Drift Chambers

There are several drift chamber systems that will be available at the time of the first 120 GeV/c Main Injector fixed target run. One that many of us in the CPT collaboration are familiar with is the KTeV system. KTeV will metamorphose into KAMI on this time scale, and build a new tracking system in the process. The KTeV chamber frames are owned by the University of Chicago, and the wires and electronics are owned by Fermilab. The electronics was built by the Rutgers group, and one of us (SVS) was responsible for the entire drift chamber system management for the 1996-7 run of KTeV. As these are the chambers we know best, it is they who are in our Monte Carlo program, and their sizes and resolution have gone into all the calculations of acceptances and resolutions for the present proposal.

In a discussion with Bruce Winstein of the University of Chicago and Greg Bock of Fermilab it was arranged that we would borrow the chambers once the KTeV and KAMI experiments were finished with them.

The chambers have a sense wire spacing of 1/4 inch, or .635 cm. The cells have a cylindrical geometry with six high voltage wires surrounding each sense wire. Each coordinate consists of two views offset by 1/2 wire spacing, as is shown in Fig. 41.

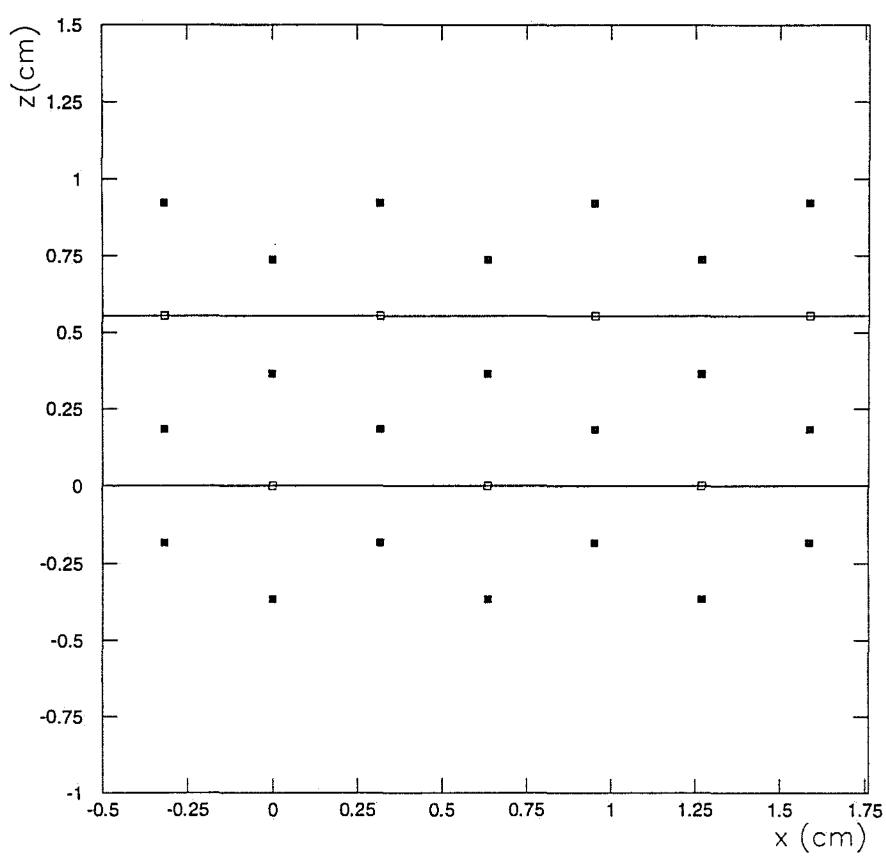

FIG. 41. Arrangement of drift chamber wires. The solid (hollow) squares represent high voltage (sense) wires. The horizontal lines are drawn along the sense-wire planes to guide the eye.

The chambers have been operated successfully in the high rate KTeV experiment (run at about 3 MHz in the first drift chamber) and will work even better in the low rate CP/T experiment. Typically the resolution of one cell is better than 100 μm. For one part of the KTeV data that was run at relatively low rates, comparable to the expected 120 kHz in the CP/T experiment, the resolution is shown in Fig. 42. A track perpendicular to the plane of a drift chamber has the property that the sum of distances to the two sense wires that detect it is a constant equal to the wire spacing. The sum of distances (corrected for track angles) is shown in this figure.

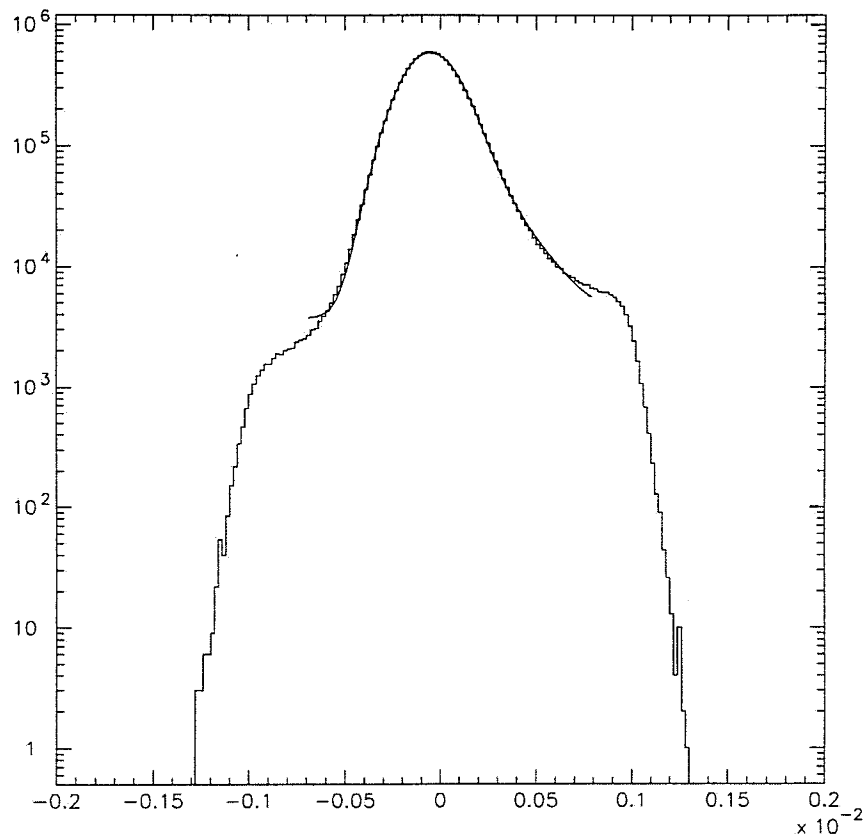

FIG. 42. Sum of distances plot for lower rate KTeV data. The histogram is the data and the line is the Monte Carlo simulation.

The shoulder on the left of the peak is due to δ-rays, and that on the right is due to the Poisson statistics of ionization electrons drifting to the sense wires. Both of these effects are included in the Monte Carlo simulation.

B. Lead Glass Electromagnetic Calorimeter

The electromagnetic Calorimeter of the CP/T experiment is owned partly by IHEP, Protvino and partly by Fermilab. It consists of 2916 total absorption counters made of transparent lead-glass. The detector is a matrix of 54 x 54 such counters. This calorimeter makes it possible to measure simultaneously the coordinates (with an accuracy of 1.5 mm), and energy (to a few percent) of a large number of gammas and to reconstruct the mass and momentum of the particles decaying into gammas.

The counters are inside a steel cassette. The material in front of calorimeter surface is an opaque plastic curtain with a patch in the center that can be removed for curing the central lead glass blocks with UV light. The cassette is placed on a mechanical platform which can be moved perpendicular to the beam axis in the horizontal direction a distance of 1 meter to the left or right for the purposes of initial calibration using electrons and positrons. No movement in the vertical direction is foreseen.

Three types of the lead glass counters with slightly different physical and chemical properties are used in the calorimeter (see Table VI). The most radiative resistant lead glass TF101 is placed in the central part of the detector.

Glass Type	Density (g/cm^2)	Radiation Length (cm)	Refractive Index
F8-00	3.61	3.09	1.62
TF1-000	3.8	2.8	1.65
TF101	3.8	2.8	1.65

TABLE VI. Properties of the lead glass counters.

The dimensions of the counters are 38 x 38 x 450 mm^3. The lateral dimensions are compared to the width of electromagnetic showers in this medium. The longitudinal dimension corresponds to 16 radiation lengths. In order to improve the light collection and to screen the radiators from each other, each cell is wrapped in aluminized Mylar foil. The Cherenkov radiation is detected by Russian photomultipliers FEU-84-3, with a 24 mm diameter photocathode, one photomultiplier for each cell.

The spatial resolution of the calorimeter is about 2.5 mm in the center of the cell and 0.5 mm at the boundary of two cells. Mean space resolution is about 1.5 mm. The energy resolution is $\sigma(E)/E = 1.5\% + 5\%/\sqrt{E}$, where E is in GeV . No deviations from linearity were observed in the energy interval from 2 to 40 Gev taking into account Cherenkov light absorption. See Ref. [40]. To insure a large linearity range, the last four dynodes of the photomultipliers are fed in parallel by four boosters. The gaps between counters are very narrow (0.04 mm), making the calorimeter very efficient because practically no photons will hit a radiator perpendicularly to the surface.

The detector is calibrated initially by using a beam of electrons and positrons made by conversion of gamma rays exiting from the hyperon magnet collimator. In order to expose all counters of the calorimeter to electrons of several energies it is necessary to deflect the electrons in the vertical plane by a special magnet placed just downstream of the hyperon magnet, and move the cassette in the horizontal plane. After the calorimeter is calibrated initially, the calibration can be checked and maintained using electrons from K_{e3} decays.

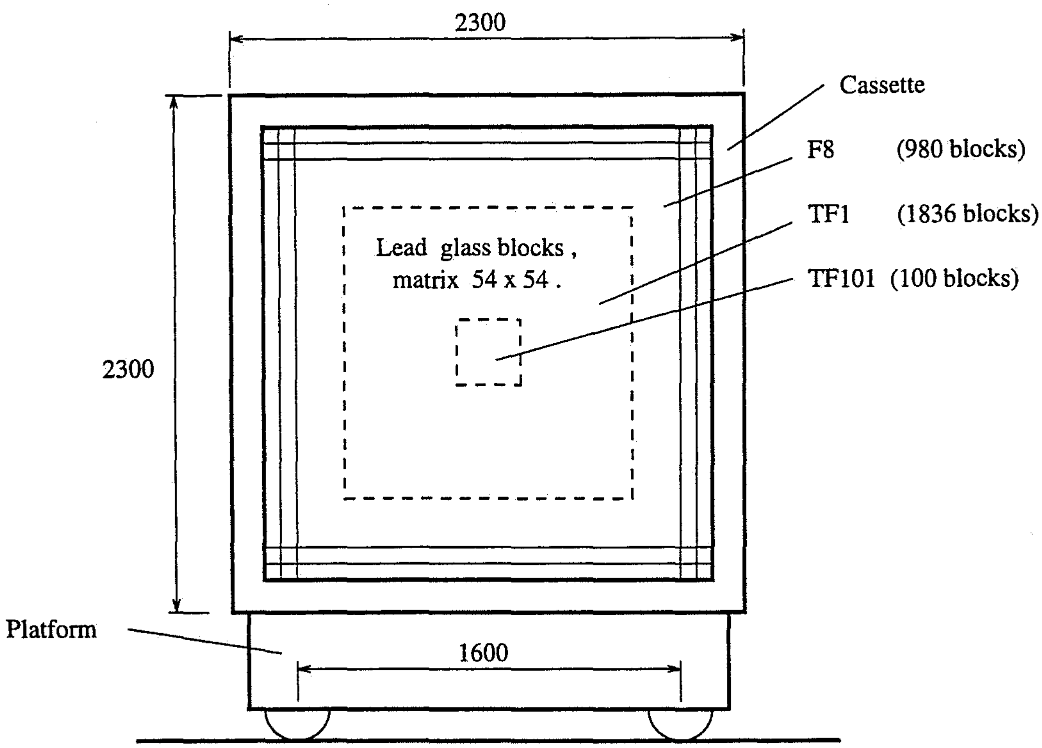

FIG. 43. Front View of the lead glass detector in its casette.

All counters of the calorimeter are monitored, once each accelerator cycle, with light pulses from a set of light-emitting diodes (LED and/or laser). The light is delivered at the forward end of each radiator cell by a flexible fiberglass light guide with small light absorption.

We plan to stack the calorimeter as a 54 block x 54 block array. The beam will strike the center and will be about 30 cm in diameter. Many experiments of this sort leave a hole in their electromagnetic calorimeter for the beam to pass through. We do not plan to do this. The reason is that the rates are too low. We have looked into two ways that the beam striking the calorimeter will effect us, radiation damage to the lead glass blocks and random hits spoiling the event by either increasing the resolution or adding an extra cluster that causes the event to be lost. Neither of these effects is significant.

The optical effect of radiation damage has been measured for our lead glass blocks, and has been published in reference [40]. In one year of running we expect there to be a net decrease in light transmission by 20% if we do not cure the blocks with ultraviolet light. This number is composed of a 30% decrease in transmission plus a 10% spontaneous improvement (as has been observed previously with our blocks). Since we do plan to cure the lead glass array regularly, radiation damage will be a minimal effect.

We have predicted the effect of random hits in the lead glass using a GEANT simulation of K^+ mesons striking the target in the hyperon magnet, and observing the resulting secondary particles that hit the lead glass array. In particular we studied the rates in the central four blocks of the calorimeter where the beam is most intense. The average K^+ beam particle produces a gamma ray hitting these central four blocks 0.13% of the time, and a hadron .007% of the time. In an ADC gate time of 100 ns there are five Fermilab RF buckets containing 11 K^+ on the average, so when a trigger occurs we expect that 1.4% of the time there will be random activity in these central four blocks. The random activity decreases as one moves away from the center of the lead glass array, so that 15 cm away the rates are down by a factor of 30.

The random gamma rays have an energy spectrum falling like $E^{-1.3}$ with an mean of 32 MeV and RMS of 36 MeV. The average energy scale shift they produce is 0.7 MeV. For a 200 MeV gamma ray, the lowest energy that we accept, the RMS due to resolution of 25 MeV would change to 26 MeV due to random gamma rays. So the effect on the resolution is minimal.

About 0.15% of ADC gates will have a random gamma ray of energy 200 MeV or higher hitting the central four blocks. For most decay modes of interest this will have no effect on the kaon decay products. For some $K_{S,L} \to \pi^+\pi^-\gamma$ decays or $3\pi^0$ decays the random hit might interfere with the event's reconstruction. This would happen at a rate low enough that it wouldn't have an effect on the experiment.

We conclude that there will be no adverse effect of not having a hole in the center of our lead glass array. The acceptance for decays with gamma rays is increased greatly by having the unimpeded lead glass coverage.

Having a lead glass calorimeter that is one detector without a hole in it is very important for the experiment. This fact enters into our plans in many ways, some of them unexpected.

- We can increase the solid angle of the hole in the hyperon magnet collimator which defines the beam size without losing coverage of photons and electrons in the lead glass. If we needed a hole in the lead glass then we would have to increase the size of the hole if we open up the hyperon magnet collimator, with deliterious effects.

- Increasing the solid angle of the beam allows us to reduce the fraction of background from collimator production and from regeneration. In forming the background fraction we are dividing by a larger number of kaons that cleanly go through the hole. In addition we can cut out the edges of the beam where the regenerated events are located if we need to do so.

- Increasing the solid angle of the beam allows us to use more K^0's per incident K^+ striking our target. This means that random rates in our detector coming from the destruction of the K^+ beam in the hyperon magnet collimator and absorber following the hyperon magnet is reduced relative to the K^0 rates.

- Increasing the K^0/K^+ ratio means that the design of the optics of the K^+ beam is simpler: we can close collimators defining the momentum spread and angular acceptance of the K^+ beam, which gives us a smaller spot size at the hyperon magnet target, and also at the first RF cavity. The spot size at the target is very important to the experiment.

- Having a smaller spot size at the first RF cavity will allow us to reduce the iris radius of the RF cavities, which will give us higher and more uniform fields in the cavities. This could result in as much as twice the field produced per cavity, and a considerable simplification (and cost reduction) of the superconducting RF cavity system.

We plan to perform a test of several aspects of our plans for the lead glass electromagnetic calorimeter in a run at the Serpukhov accelerator. First we want to know if there is a difference between the energy resolution of the lead glass blocks of TF1-000 and TF101 types. The TF101 blocks are much more resistant to radiation damage than the TF1-000 blocks, and if we were to use TF101 blocks in the center of the calorimeter we calculate that the loss of light transmission in a year of running would be only 1%. Previous use of these blocks has been in high radiation environments where making a precise test of their resolution has been difficult. The second test is of the resolution of the lead glass blocks for the lowest energy photons we plan to accept; i.e., at 200 MeV. In a recent Serpukhov run electronic noise was seen to degrade the blocks' resolution at these low energies, and we want to see whether we will need amplifiers on the phototubes, and test one amplifier design.

C. Muon Filter and Muon Chambers

We made a preliminary design of a muon detector using GEANT. It consists of two modules, each consisting of an iron filter 3 m vertically × 3 m horizontally × 1 m thick followed by a hodoscope of scintillation counters and a drift tube chamber. This array would identify 98% of muons above 1 GeV/c and reject 99.8% of pions. We will use the drift tube chambers built by the IHEP group as the D0 small angle muon detectors, which are being replaced in the ongoing D0 upgrade. We will build new muon detector hodoscopes.

D. Scintillation Counters

The counter in the vacuum at the end of the decay region will be built new. In previous K_S and hyperon experiments we have used scintillation counters 1 mm thick in this type of situation.

The two hodoscopes of counters for triggering would be built new. They would be similar to the trigger counters of the KTeV experiment.

X. RATES AND TRIGGERING

Using the hyperon magnet design shown above we have performed a GEANT calculation of the resulting beam and particle intensities. GEANT reproduces the kaon flux described above in our subsection on the sensitivity of the experiment. We calculated the rates in the first drift chamber, which is the detector that will be exposed to the largest particle flux. With 1.2×10^8 K^+/sec the first drift chamber will see 120 kHz of charged particles, approximately evenly distributed over its face (which adds up to about 8 Hz per centimeter of sense wire). This rate is considerably lower than that seen in KTeV. In addition there is 1.4 MHz of γ-rays, most less than 10 MeV, striking it in the beam region. These photons are the remnants of electromagnetic showers in the high-Z target. Taking account of the material in the front end of the spectrometer, this leads to 4 kHz of conversions, or 50 Hz per centimeter of sense wire. This is much less than the space charge limit of 20 kHz per centimeter of sense wire.

The neutron/K_L ratio is 2.0 (a little better than KTeV), with average neutron momentum about 1 GeV/c. The Λ/K_S ratio is 0.1 (with average Λ momentum of 3 GeV/c), which is much lower than previous K_S experiments at Fermilab.

At these low rates the correct strategy would be to trigger on all 2-track events. This would result in 4 kHz of $K_{\pi 2}$ events per spill, and a data volume of about 2 MBytes/second. Again using KTeV for comparison, this is 1/10 of its capabilities. To estimate the total trigger rate we must add about 10% for Λ's, 20% for K_L decays, and perhaps 30% more for triggers that do not reconstruct into good Vees. We will also need a trigger for decays to $\pi^0\pi^0\pi^0$ final states. Here we will use similar strategies to our previous experiments and have a hardware trigger processor that counts clusters of hit blocks in the lead glass array, and another one that measures the total energy in the array. The Rutgers Physics Department Electronics Shop has made preliminary designs for both of these trigger processors.

These are quite low rates, easily handled by detectors and a data acquisition system. This comes from the fact that we are working in a tertiary K^0 beam. If we were to increase the proton beam intensity to 2×10^{13} protons per pulse (the Main Injector is expected to produce 3×10^{13}), all rates would still be less than those expected for KTeV, and the 12 months of data collection time would become only 3 months. This would be an important running consideration.

XI. SITING THE EXPERIMENT IN THE MESON LAB

Tom Kobilarcik of the Fermilab Beams Division has looked into how to fit our beam and experiment into the existing beam lines of the Meson Lab. He has found that our very straight K^+ beam must be modified to fit the gently curving tunnels. This can be done without compromising the beam's quality by placing bending magnets at the locations of the RF cavities. There are two beam lines that seem appropriate for our K^+ beam, M-East and M-Test. There is an existing target pile in M-Test, and the beam is almost the perfect length if we use C-band RF cavities. For the M-East beam line, there is a target hall, currently empty, called the pion target hall, about 850 feet from the entrance to the Meson Detector Building. The currently existing Meson East target pile could be mined for most of the elements we would need. The experiment would go into the east end of the Meson Detector Building. There is room in this beam line and experimental hall for the beam, the experiment, plus the CKM experiment, with whom we would share the beam.

The M-East beam line is about 200 feet longer than the optimal length if we were to use C-band RF cavities at 5.79 GHz. However, it may turn out that the cheapest cavities to build would use the somewhat lower RF frequency of 3.9 GHz. If this is the case we may need this extra 200 feet.

All of the magnets needed for this beam exist at Fermilab, and have been "tagged" for our experiment.

XII. COSTS

Table VII lists all of the items needed for our beam line and detector, the lead time needed to produce them, and their costs. The net cost of new items is \$4160k. The longest lead-time items are the RF cavities for the K^+ beam, which will take 2.5 years for R&D and construction.

Location	Item	Lead Time	New Costs
Beam Line	Target Pile	0.5 yr	\$1000k
	Magnets for K^+ Beam	exist	0
	RF Cavities	2.5 yrs	\$2550k
	Hyperon Magnet	1 yr	\$250k
Detector	Counter in vacuum	0.5 y	\$10k
	Drift Chambers	exist	0
	Analysis Magnet	exist	0
	Trigger Hodoscope	0.5 yr	\$50k
	Lead Glass Calorimeter	exist	\$150k
	Muon Detector	0.5 yr	\$150k
Total		2.5 yr	\$4160k

TABLE VII. Costs and Leadtimes

XIII. OTHER EXPERIMENTS AROUND THE WORLD

Another experiment that aims to search for CPT symmetry violation is the KLOE experiment at the Frascati DAΦNE Φ factory. This is an "open geometry" experiment limited by e^+e^- beam luminosity. They expect to achieve a sensitivity to the fractional $K^0 - \overline{K}^0$ mass difference of about 1×10^{-18} [41], by studying interference in K_{e3} decays. This is 25 times worse than what we propose here.

The KLOE experiment expects to produce $15 \times 10^9 K_L + K_S$ events per year if the DAΦNE luminosity were $10^{33} cm^{-2} sec^{-1}$. For the decay $K_S \to \pi^0\pi^0\pi^0$, with a branching ratio expected to be 2×10^{-9}, this would amount to 30 events produced per year. Their best way to measure $|\eta_{000}|$ is by tagging K_S decays by observing an unambiguous K_L decay, since their $K_L - K_S$ interference is very small in this channel. If they achieve that luminosity, their acceptance were 100%, there were no tagging inefficiencies or backgrounds, and their poor resolution allowed them to reconstruct the 6γ decays properly, this would result in a measurement of $|\eta_{000}|$ accurate to about 10%, which would be excellent. However, the maximum luminosity of the DAΦNE storage ring is expected to be a factor of 2 lower, about half of the K_L decays will occur outside their fiducial region, and inefficiencies of all kinds will result in their finding many fewer events. Using their method for η_{+-0} is harder because the branching ratio is half as big. So the KLOE experiment will not be competitive to the present proposal.

The CPLEAR experiment has concluded its data collection, and their results will not be competitive.

The KAMI experiment at Fermilab, as proposed, does not have a regenerator with which to measure ϕ_{+-}. However the KTeV collaboration does own a beautiful regenerator, which was built by a collaborator on the present proposal (SVS), and that could be put into service. The $K_L - K_S$ interference term for decays downstream of the regenerator is smaller than that in the present proposal. In that term there is a factor $\cos(\Delta m t + \phi_\rho - \phi_{+-})$, from which ϕ_{+-} can be measured. Unfortunately ϕ_{+-} is 100% correlated with ϕ_ρ, which has been measured previously to only 1 degree accuracy. In KTeV it is hoped to measure ϕ_ρ to about 1/4 degree accuracy using K_{e3} decays, but an additional factor of 17 is needed to make a competitive measurement. This will be extremely difficult.

The importance of measuring in the same experiment the phase of ϵ and the phase of η_{+-} cannot be stressed too heavily. In an experiment with a regenerator x, η_{+-0}, and η_{000} cannot be measured to the required accuracy, therefore neither can the phase of ϵ.

The tests of the standard model using η_{+-0}, and η_{000} cannot be performed in a regenerator experiment. Studies of rare K_S decays are hard to perform in a regenerator experiment.

Due to the excellent photon veto system that the KAMI collaboration plans to build, there is one CPT conservation test that can be performed better there than with our detector, namely the measurement of $\Delta\phi = \phi_{00} - \phi_{+-}$. In the standard model this quantity is zero to the precision of a few tenths of a degree, and might deviate from zero at this level due to uncertainties in final state interaction phase shifts. A nonzero value of $\Delta\phi$ which is larger than this would be a signal of direct CPT violation. In our chapter on CPT phenomenology we pointed out the way that Nature has been incredibly kind in the search for indirect CPT violation by providing a free 17 order of magnitude factor in Eqn. 4.5. However, no such enhancement factor exists for $\Delta\phi$ and direct CPT violation. For example, in the model of Ref. [11] direct CPT violation is much too small for the KAMI experiment to see.

Encouraged by the work of A. Kostelecky [11], the B-factory experiments, Belle and BaBar, intend to perform CPT symmetry conservation tests similar to the $|\phi_{+-} - \phi_\epsilon|$ one described here [42]. The $B^0/\overline{B}^0$ system does not provide as sensitive a test as the kaon system, however.

At JHF, the 50 GeV/c proton accelerator proposed to be built at KEK in Japan, there is a proposal in preparation for an experiment very similar to ours to perform CP and CPT symmetry tests and measurements [43]. They plan to use the higher intensity of their proton beam to make a more intense K^+ beam and collect higher statistics than can be amassed at Fermilab.

Appendix B includes more details of comparisons between this experiment and others around the world.

XIV. CONCLUSION

We have described an experiment to carry out a systematic program of measurements in $K_S - K_L$ interference physics. We will search for CPT symmetry violation in the decays of

K^0 mesons with the sensitivity to reach the Planck scale, measure CP violation parameters to test the detailed predictions of the Standard Model, and study rare kaon decays.

Our design uses protons from the Fermilab Main Injector to make an RF separated K^+ beam. With this we make a tertiary neutral kaon beam created in just the way to maximize the interference between K_S and K_L while maintaining high flux. We use a "closed geometry" hyperon magnet for beam definition. A standard Vee spectrometer, with drift chambers, an electromagnetic calorimeter, and a muon detector, is used to make the measurement.

APPENDIX A: THE CP/T MONTE CARLO PROGRAM

One important part of all of our calculations is the acceptance calculated by our Monte Carlo program. This program includes the geometry of the target, hyperon magnet collimator, decay region, and detector apparatus. It has the same drift chamber simulation that is used in the KTeV experiment (which is appropriate since we plan to borrow those drift chambers for this expriment) modified for the much lower beam rates we expect in CP/T. The Monte Carlo program uses a library of GEANT electromagnetic showers for electrons and photons. In the Monte Carlo is a muon detector which was designed using a GEANT simulation. In the following paragraphs each of these elements is described more fully.

We put the drift chamber resolution simulation from the KTeV Monte Carlo program into our Monte Carlo program. This seems appropriate because the KTeV drift chambers are the ones in our Monte Carlo. The simulation takes into account the intrinsic resolution of the wires, the effect of delta rays, and the effect of the finite number of drift electrons. It reproduces the data seen in the (high rate) KTeV experiment, and will give even better results in the (low rate) CP/T experiment. Fig. 42 shows the resolution of the drift chambers for a low rate part of the KTeV 1997 run. Included on the figure is a Monte Carlo simulation of this run that includes the intrinsic chamber resolution, chamber wire inefficiencies, delta rays, and the Poisson statistics of drift electrons.

We generated a GEANT library of electromagnetic showers in our lead glass calorimeter. We used the GEANT facility for generating Cerenkov light, put in absorption of the light and the efficiency of the photocathode as a function of wavelength, and tuned the parameters until the result reproduced the measured energy resolution of our lead glass, $\sigma/E = .015 + .05/\sqrt{E}$, and the measured position resolution of about 1.5 mm. Fig. 19 shows the energy and position resolution as a function of the gamma ray energy.

Then choosing bins in position within a block of size 1 mm square and twelve energy bins chosen logarithmically from 0.1 GeV to 12 GeV, we generated 20 events per (x,y,E) bin for the shower library. We then analyzed this data and wrote reconstruction programs for energy and position that are now in the Monte Carlo program. In our subsequent analysis efforts we've seen that this process has worked very well.

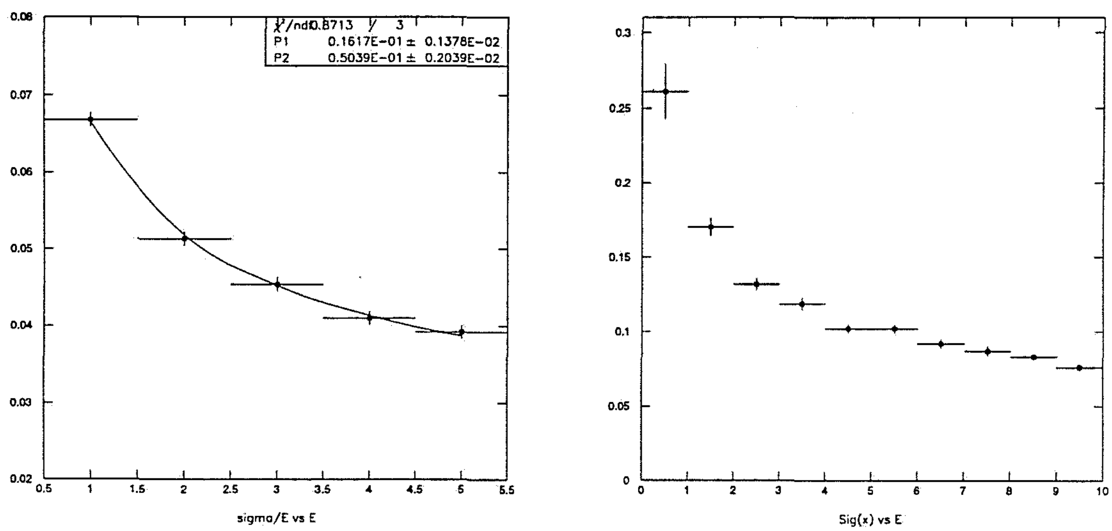

FIG. 44. Energy and position resolution of the GEANT Simulation

We put the muon filter into the Monte Carlo program, and included the efficiency of detecting muons as a function of momentum as calculated in a GEANT model. In addition we put pion decay into the Monte Carlo. The spatial resolution of the muon detector is shown in Figure 19.

The design of the muon filter resulted from a GEANT study of muons and pions traversing various thicknesses of steel. 1.0, 1.5, and 2.0 meter thicknesses were considered. Whenever a charged particle exited the steel a cut was made on the difference of the position of the particle from the projected position of the track entering the steel. The entering particle was called a muon if that difference was less than the cut value.

In the GEANT simulation of the muon filter a steel box surrounded by vacuum was created. The momentum spectrum of the incident particles, muons and pions, were obtained from the momentum spectra of $K_{\mu 3}$ decay products as generated by the CPT Monte Carlo simulation (see Figure 45).

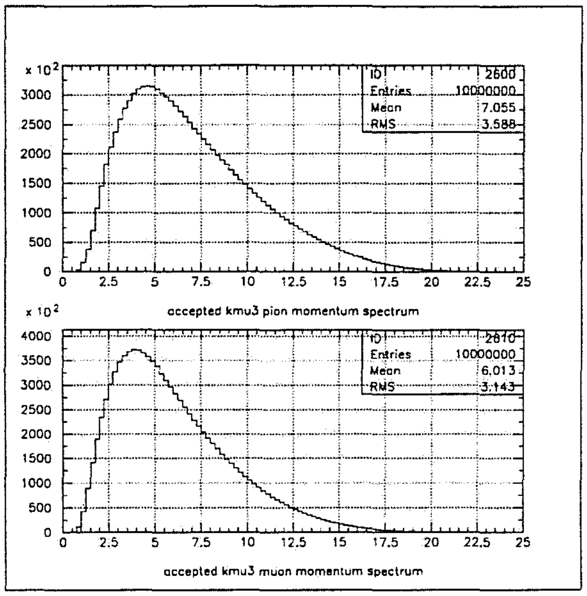

FIG. 45. Momentum spectra of muons and pions from $K_{\mu 3}$ decays.

Using the momentum spectrum appropriate for each particle, one million of each type were randomly applied to the surface of the steel barrier in a 5cm x 5cm square at the center of the front face with all momentum in the z direction. A uniform distribution in x and y was used to place the particle in the square and momenta were randomly assigned using the appropriate momentum spectrum. If a charged particle exited the rear face of the barrier the following information was recorded in an NTUPLE: x, y position and p_z entering the barrier, x, y position exiting the barrier, p_x, p_y, p_z momentum components. Three thicknesses of steel material were tested in the above manner: 1, 1.5, and 2 meters.

GEANT's results for muons were compared to a model of small angle scattering described in the Particle Data Group Full Review, Ref. [34], pp. 146-147. The distance between the x position entering the barrier and the x position exiting the barrier was then computed for all particles in 1 GeV/c momentum ranges. The same was done for the y coordinate. The standard deviation of these distributions for the GEANT and PDG data were then averaged together to create the basis for cuts in radii of particles that exit the steel barrier. Fig. 46 shows the standard deviation in position for both the GEANT simulation and the PDG formula for a 1.0 meter thick steel filter.

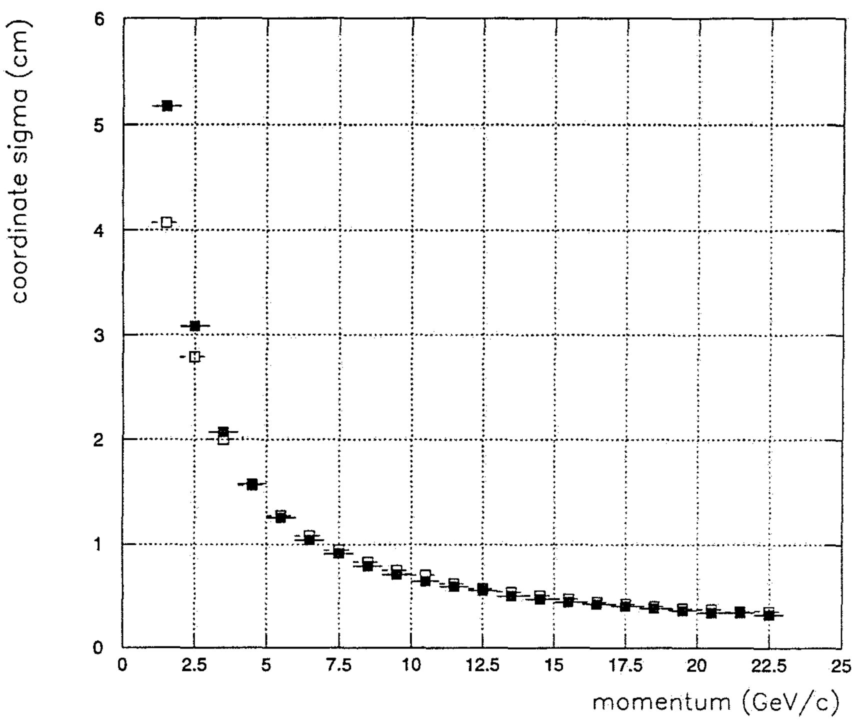

FIG. 46. Sigma of x-coordinate for particles exiting from the muon filter. The solid squares are the GEANT simulation and the open squares are the PDG formula.

The momentum dependent cuts were then applied to the GEANT events. A $3\sigma_{\text{avg}}$ cut was determined to be the most useful.

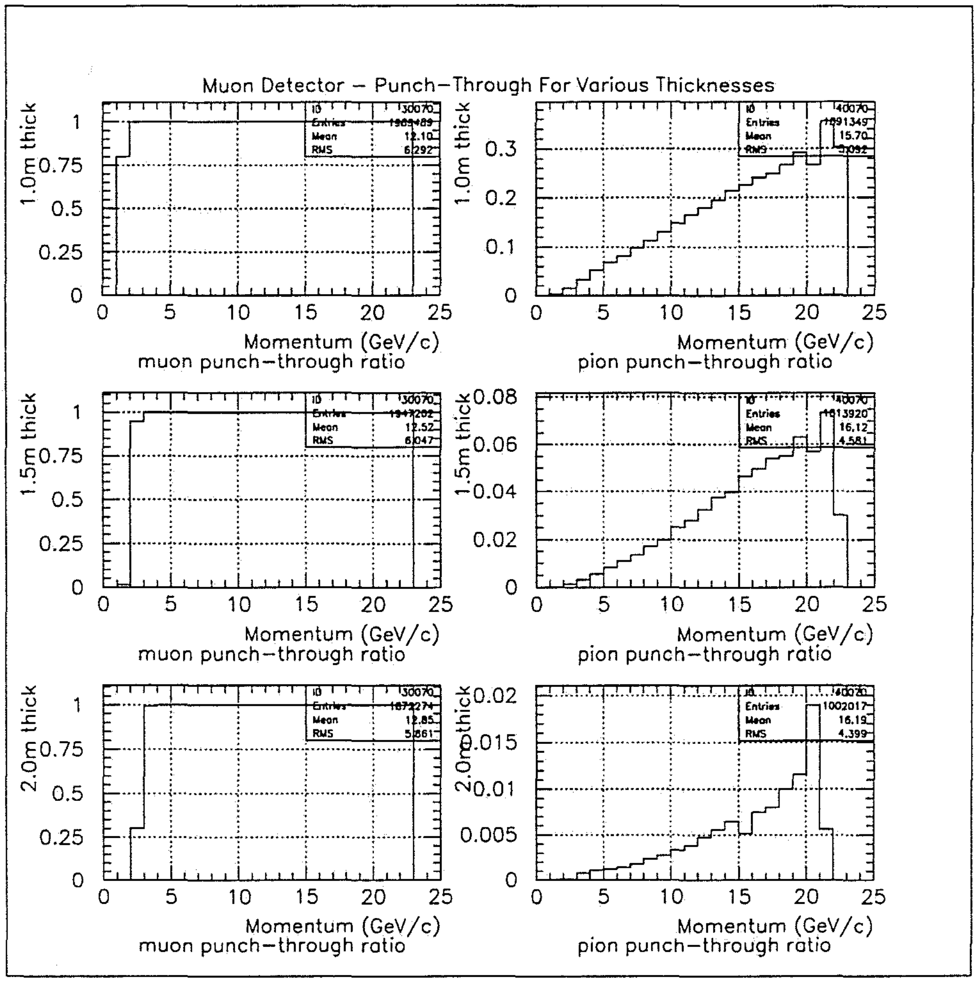

FIG. 47. Ratios of numbers of particles that traverse the steel barrier.

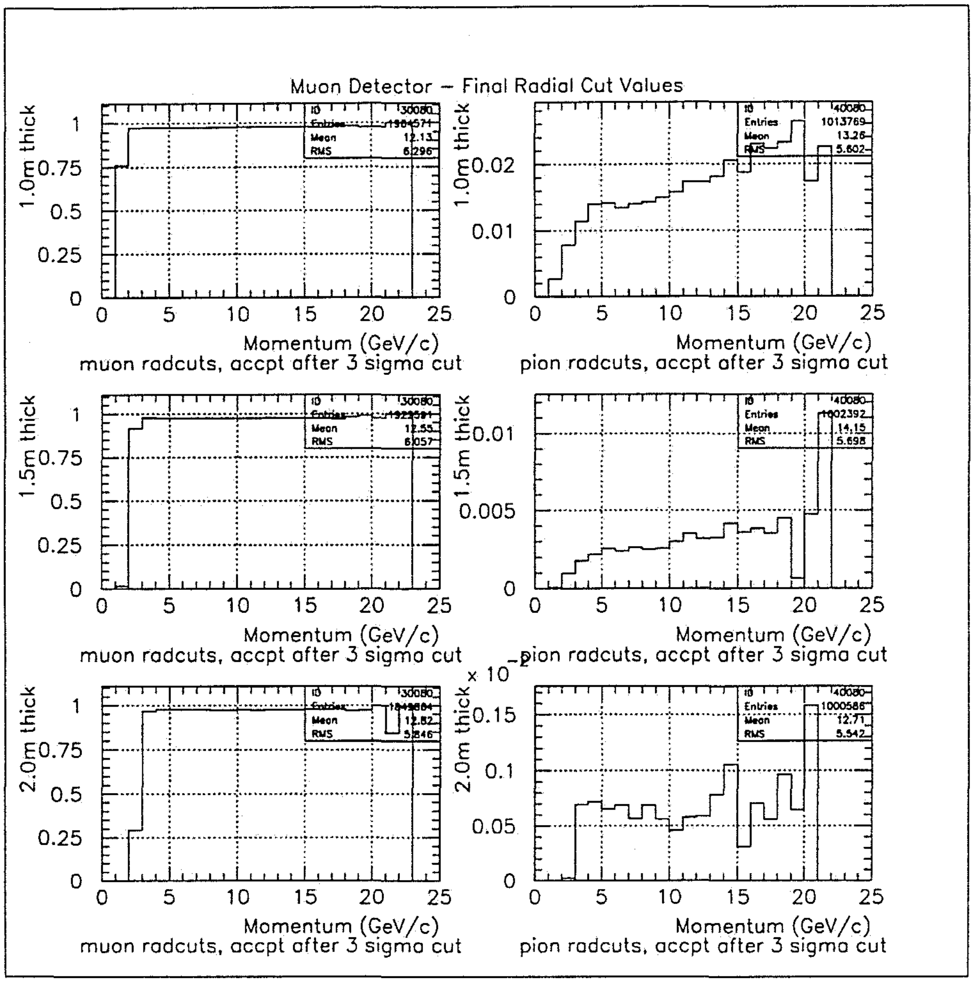

FIG. 48. Muon detector efficiency.

Figures 47 and 48 show how the cuts reduce the number of pions that could be counted as a muon. Figure 47 shows the ratio of the number of particles that fully penetrate the barrier to the number of particles thrown as a function of momentum. No cuts were made on position for this figure. Figure 48 shows the results of the $3\sigma_{avg}$ cut on the radial distance between the point of entry on the front face and point of exit on the back face of the barrier.

The thickness chosen for the muon detector was two 1 meter thick slabs separated by 0.5 meters. Hodoscopes of scintillation counters and drift tube chambers would be placed behind each barrier to detect the passage of charged particles. The drift tube chambers built by the IHEP Provino group originally for the D0 experiment, now released by D0 as part of their upgrade process, will be used. Since there are about a dozen of these chambers we would use four per 1 meter thick slab of steel in our muon filter: three interspersed in the steel and one at the end of the steel with a hodoscope of scintillation counters. With this arrangement we will be able to track a muon's multiple scattering in its traversal of the steel.

To determine the transverse dimensions needed for the steel and detectors, the CP/T Monte Carlo program was used. For $K_{\mu 3}$ decays the distribution of muons and pions for accepted events were projected to z = 24.5 meters, the location of the front face of the second barrier, to see how wide in x and y the distribution was spread.

APPENDIX B: DETAILS ON OTHER EXPERIMENTS

The comparison of the expected CP/T results with possible data of several other experiments (DAΦNE(KLOE)), CPLEAR, KTeV) is presented in the following four tables. It is clear from the tables that all these experiments compliment each other and provide the possibility of obtaining together a great advance in studying CP violation in several processes and in searches for possible CPT nonconservation.

These experiments are characterized by different approaches and by different systematic uncertainties which is very important in precise measurements of small effects and rare processes. It is clear also that the CP/T experiment has many advantages in studying CP violation effects in K_S decays ($K_S \to \pi^+\pi^-\gamma$, $\pi^+\pi^-\pi^0$, $3\pi^0$) and is the only experiment with the possibility of performing a search for CPT nonconservation effects at the gravitational level.

Process	CP/T	DAΦNE [44]	CP LEAR	KTeV	Other experiments
$N(K_S^0 \text{ decays})$	$3.4 \cdot 10^{10}$	$7.5 \cdot 10^9$ ($1.6 \cdot 10^9$ tag.)	$\sim 10^8$		
$N(K_L^0 \text{ decays})$	$7.1 \cdot 10^9$	$7.5 \cdot 10^9$ ($2 \cdot 10^9$ tag.)	$\sim 5 \cdot 10^6$		
$N(K_S^0 \to \pi^+\pi^-\gamma)$	$5.0 \cdot 10^8$ decays	$3 \cdot 10^6$ ($E_\gamma^* > 50$ MeV)			
$N(K_L^0 \to \pi^+\pi^-\gamma)$	($E_\gamma^* > 5$ MeV)	$1.4 \cdot 10^4$ ($E_\gamma^* > 20$ MeV)			
$\eta_{+-\gamma}$					$(2.359 \pm 0.062 \pm 0.04) \cdot 10^{-3}$
$\varphi_{+-\gamma}$					$43.8° \pm 3.5° \pm 1.9°$(E773) [25]
$\delta(\eta_{+-\gamma})/\eta_{+-\gamma}$	$\pm 0.86 \cdot 10^{-3}$	$< \pm 3 \cdot 10^{-2}$		$\pm 10^{-2}$	$\pm 3.1 \cdot 10^{-2}$ (E773) [25] $\pm 15 \cdot 10^{-2}$ (E731) [2]
η_{+-}					$(2.285 \pm 0.019 \pm 0.019) \cdot 10^{-3}$;
φ_{+-}					$(43.5° \pm 0.6)°$ (PDG)[4]
$\delta(\eta_{+-})/\eta_{+-}$	$\pm 10^{-3}$				$\pm 1 \cdot 10^{-2}$
$\delta(\phi_{+-})$	$\pm 0.04°$				$\pm 0.6°$ (PDG)[4]
Upper limit for direct CP violation $\lvert\varepsilon'_{\pi^+\pi^-\gamma}/\varepsilon\rvert$ (95% c.l.)	$< 1 \cdot 10^{-3}$			$< 2 \cdot 10^{-2}$	< 0.3 (E731) [2] < 0.06 (E773) [25]

Process	CP/T	DAΦNE	CPLEAR	Other experiments
$N(K^0) \to \pi^+\pi^-\pi^0$ $N(K^0) \to 3\pi^0$	$2.3 \cdot 10^8$ $0.7 \cdot 10^8$		$5.08 \cdot 10^5\ (K^0 + \bar{K}^0)$ $1.7 \cdot 10^3 (K^0 + \bar{K}^0)$	
$BR(K_S^0 \to \pi^+\pi^-\pi^0)$			$\left(2.5^{+1.3+0.5}_{-1.0-0.6}\right) \cdot 10^{-7}$	$\left(4.8^{+2.2}_{-1.6} \pm 1.1\right) \cdot 10^{-7}$ [8]
$BR(K_S^0 \to 3\pi^0)$			$< 1.9 \cdot 10^{-5}$ (90% c.l.) $BR(K_S^0 \to 3\pi^0)\vert_{\text{theor}} \simeq$ $\simeq 2 \cdot 10^{-9}$	
$K_S^0 \to \pi^+\pi^-\pi^0$		$N = (3.4 \div 6.8) \cdot 10^2$		
$K_S^0 \to 3\pi^0$		$N = 3 - 15$? tagged events		
Re η_{+-0}	$\delta(Re\eta) = \pm 0.35 \cdot 10^{-3}$		$(-2 \pm 7^{+4}_{-1}) \cdot 10^{-3}$	
Im η_{+-0}	$\delta(Im\eta) = \pm 0.35 \cdot 10^{-3}$	$\delta(\eta_{3\pi}) < (7 \div 8) \cdot 10^{-3}$	$(-2 \pm 9^{+2}_{-1}) \cdot 10^{-3}$	$(-15 \pm 17 \pm 25) \cdot 10^{-3}$ (E621) [26]
Re (λ)			$(+28 \pm 7 \pm 3) \cdot 10^{-3}$	
Im (λ)			$(-10 \pm 8 \pm 2) \cdot 10^{-3}$	
Re η_{000}			$(0.18 \pm 0.14 \pm 0.06)$	
Im η_{000}	$\delta(Im\eta) = \pm 0.64 \cdot 10^{-3}$		$(0.15 \pm 0.20 \pm 0.03)$	$(Im\ \eta_{000})^2 < 0.1$ (90% c.l.) [45]

Process	CP/T	DAΦNE [44,46]	CPLEAR [47]	KTeV	Other experiments
$N(K^0 \to l^\pm \pi^\mp \nu)$	$\sim 2 \cdot 10^9$		$1 \cdot 10^6$		
$\delta(Im\ x)$	$\pm 0.5 \cdot 10^{-3}$		$Im\ x = (1.2 \pm 2.2 \pm 0.3) \cdot 10^{-2}$		$Im\ x = -0.003 \pm 0.026$ [45]
$\delta(Re\ x)$		$\pm 3 \cdot 10^{-3}$	$Re\ x = (0.2 \pm 1.3 \pm 0.3) \cdot 10^{-2}$		$Re\ x = 0.006 \pm 0.018$ [45]
$\delta(A_L)$		$\pm 4 \cdot 10^{-5}$		$\pm 1 \cdot 10^{-5}$	$A_L = (3.27 \pm 0.12) \cdot 10^{-3}$ [45]
$\delta(A_S)$		$\pm(9 \div 2.5) \cdot 10^{-5}$			
$y = B_l/A_l$		$\delta(y) = \pm 2 \cdot 10^{-4}$	$(-0.1 \pm 1.4) \cdot 10^{-3}$		
$Re\ \Delta$	$\delta(Re\ \Delta) = \pm 5 \cdot 10^{-6}$	$\delta(Re\ \Delta) = \pm 4 \cdot 10^{-4}$ ($\pm 6 \cdot 10^{-5}$ if $x = 0$)	$(2.0 \pm 3.3 \pm 0.6) \cdot 10^{-4}$		0.018 ± 0.020 [45]
$Im\ \Delta$	$\delta(Im\ \Delta) = \pm 1 \cdot 10^{-6}$		$(-1.5 \pm 2.3 \pm 0.3) \cdot 10^{-2}$		0.021 ± 0.037 [45]
$(\frac{ReB_0}{ReA_0})$		$\delta \frac{ReB_0}{ReA_0} = 4 \cdot 10^{-4}$	$(-0.1 \pm 9.9) \cdot 10^{-4}$(*)		
$\delta(\varphi_{+-})$	$\pm 0.040°$		$\varphi_{+-} = (43.5 \pm 0.6)°$		$\varphi_{+-} = (43.5 \pm 0.6)°$ [45]
$\delta(\varphi_\varepsilon)$	$\pm 0.030°$				
$\delta(\varphi_{SW})$				$\pm 0.024°$	$\pm 0.12°$
$\varphi_{+-} - \varphi_{00} = \Delta\varphi$				$0.25°$	$(-0.1 \pm 0.8)°$ [45]
$\delta\left[\frac{M_{K^0} - M_{\bar{K}^0}}{M_K}\right]$	$\pm 3 \cdot 10^{-20}$(**) $\pm 2 \cdot 10^{-20}$(***a) $\pm 3 \cdot 10^{-20}$(***b)	$\pm 1 \cdot 10^{-18}$	$(0.04 \pm 6.91) \cdot 10^{-18}$;	$\pm 4 \cdot 10^{-19}$	$\pm 1.3 \cdot 10^{-18}$(****) (E773 [13])

Note.(*) Preliminary data.

(**) From the data on $(\varphi_{+-} - \varphi_\varepsilon)$ value in the assumption $ReB_0 = 0$.

(***) From the study of the Bell-Steinberger relation: a) with the assumption $\Gamma(K^0) = \Gamma(\bar{K}^0)$;

b) in a general case.

(****) With theor. assumptions for $Im\ x$, $Im(\eta_{3\pi})$. Without these assumptions $\delta = \pm 4 \cdot 10^{-18}$

Process	CP/T	DAΦNE [2]
$K_S^0 \to \pi^0 e^+ e^-$ $BR_{\text{theor}} \sim 5 \cdot 10^{-9} \div 5 \cdot 10^{-10}$	$N = 50 \div 5$ events	$N = 10 \div 1$ events
$K_S^0 \to \pi^0 \mu^+ \mu^-$ $BR_{\text{theor}} \sim 10^{-9} \div 510^{-10}$	$N \simeq 10 \div 1$ events	$N \simeq 2 \div 0$ events
$K_S^0 \to e^+ e^- \gamma$ $BR_{\text{theor}} \simeq 3.4 \cdot 10^{-8}$	$N \sim 5 \cdot 10^2$ events	$N \sim 60$ events
$K_L \to e^+ e^- \gamma$ $BR_{\text{theor}} \simeq 9 \cdot 10^{-6}$	$N \sim 4 \cdot 10^4$ events	$N \sim 10^4$ events
$K_S \to \pi^+ \pi^- e^+ e^- \gamma$ $BR \simeq 10^{-5}$	$N \sim 10^5$ events	$N \sim 10^4$ events

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this paper of why their data is wrong. We do not mean to take the limits they quote as a benchmark against which the CP/T experiment should be compared, but the CP/T experiment will be able to place limits two orders of magnitude better than those quoted by CPLEAR.

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