Merging binary black holes in astrophysics

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Abstract

Many important advances in the understanding of black-hole physics took place after the numerical relativity breakthroughs of 2005 that allowed fully non-linear dynamical numerical simulations of the inspiral, merger and ringdown of black-hole binaries. We review recent exciting developments in the study of merging black-hole binaries and discuss future directions.

1 Introduction

According to the *no-hair theorem*, equilibrated black holes (BHs) in General Relativity are completely described by three quantities: the mass M, angular momentum J, and electric charge Q of the Kerr-Newman BH solution. However, the electric charge should be rapidly depleted by the surrounding plasma and astrophysical BHs can be completely described by their mass and spin. Although the interiors of BHs may be exotic objects where the spacetime curvature becomes singular, these regions are expected to be covered by a horizon and are thus invisible (*cosmic censorship conjecture*).

Kramer *et al.* [1] tested General Relativity to $\sim 0.05\,\%$ by calculating the inspiral rate of a double pulsar. However, there still are important open questions: Do BHs really exist in nature? Are they really represented by the Kerr solution? Are there naked singularities in the Universe? Is General Relativity the correct theory of gravity in the strong-field regime?

There is strong, but indirect, evidence that BHs exist in the Universe, with a vast range of masses from few tens to $10^9 M_\odot$. The stellar-mass BHs $(3-30M_\odot)$ should form from the collapse of massive stars, while intermediate-mass BHs (IMBHs, $10^2-10^4 M_\odot$) may assemble in globular clusters. Massive BHs (MBHs, $10^4-10^7 M_\odot$) and super-massive BHs (SMBHs, $10^7-10^9 M_\odot$) are *seen* in galactic cores by the motion of stars and/or gas surrounding them. These MBHs/SMBHs appear to be connected by the $M-\sigma$ relation [2, 3] to their host galaxies. From the electromagnetic observations, there is evidence that astrophysical BHs may be spinning relatively fast. Also note that since there is evidence that galaxies collide, it is plausible that their central BH's inspiral and collide through the stellar/gas dynamical friction, and the energy loss due to gravitational radiation below sub parsec scale (after *the final parsec problem* [4]).

We note that coalescing black-hole binaries (BHBs) are very loud gravitational-wave sources with the final merger event producing a strong burst at a luminosity of $L_{\rm GW} \sim 10^{22} L_\odot$ which makes them ideal targets for all gravitational-wave detectors (with different detectors sensitive to BHBs in different mass regimes). The gravitational waveforms from these mergers will inform us about the BH masses, (initial and final) spins, source locations, merger rates, and spacetime dynamics. These gravitational waveforms are also essential in matched filtering applications to assist gravitational-wave detection.

There are several past and ongoing ground-based gravitational-wave detector projects, including Initial LIGO (Laser Interferometer Gravitational wave Observatory, 2005 – 2010) [5], VIRGO [6], Advanced LIGO ($10\times$ increase in sensitivity from Initial LIGO, 2016+), LCGT (Large-scale Cryogenic Gravitational wave Telescope, $\sim$ 2017) [7], Einstein Telescope (ET, $10\times$ increase in sensitivity, 2027+) [8] and other third generation (3G) detectors. We expect that target sources for 3G detectors are not only the inspiral and merger of neutron star-neutron star binaries, neutron star-(stellar-mass) BH binaries, (stellar) BH-BH binaries, but also IMBHs at cosmological distances.

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There are also plans to go to space with LISA (Laser Interferometer Space Antenna, 2025+) [9] and DECIGO (DECi-hertz Interferometer Gravitational wave Observatory, $\sim$ 2027) [10]. LISA will measure gravitational waves at low frequency ($10^{-4} - 10^{-1}$ Hz), and DECIGO will be the "bridge" between LISA and ground-based detectors with the sensitivity range (30 mHz$-$30 Hz). MBH binary mergers are one of LISA's main target, and we will learn about merger rates, the history of hierarchical galaxy mergers, and the growth of MBHs over time from its detections. Extreme mass-ratio inspirals (EMRIs) which consist of a central MBH and a stellar/intermediate-mass compact object, are also important gravitational-wave sources. From these mergers we can obtain information about the mass and spin of central objects, and as a result, they will provide a census of MBHs in galaxies and the MBHs growth mechanisms (i.e. do these massive objects arise from the merger of comparable mass smaller objects or by accretion onto a large central object).

Merging MBH/IMBH binaries will not only be observed through gravitational-waves, but also likely to be accompanied by electromagnetic counterparts. From correlations between the electromagnetic and gravitational-wave spectra, we will obtain important astrophysical information. This includes an improvement in the sky localization of the source and identification of the host galaxy morphology, tests of galaxy merger scenarios, and detection rates for gravitational-wave sources. Importantly, these observation will provide a novel measurements of the luminosity distance (from the gravitational waveform) to redshift relation out to cosmological distances (*cosmological standard sirens*), as well as provide tests of the fundamental principles of General Relativity (e.g. graviton's speed). They will also improve our understanding of BH accretion physics and magneto-hydrodynamics, circumbinary disks, as well as gravitational kicks. Consequently, these studies are very important to our understanding of the dynamics of our Universe (see Astro2010 Decadal Survey White papers [11, 12]).

2 Simulation of Black-Hole Binaries

In Numerical Relativity, we solve numerically General Relativity's field equations for a dynamical spacetime. The goals are to understand gravity at its strongest manifestation, to inform gravitational-wave detection, and to determine characteristics of compact objects. To simulate black-hole binaries in Numerical Relativity, we need tackle many challenges, 1) several scales required for the physics that arise from the mass of the smallest black hole, black hole's spins, and the wavelength of the emitted gravitational waves in the wave zone, 2) long waveforms matching the early post-Newtonian inspiral phase, 3) large parameter space of black-hole binaries, mass ratio, individual spins, eccentricity, etc.

The first simulation in Numerical Relativity was done by Hahn and Lindquist [13] in 1964 for a head-on collision of two equal-mass black holes in two dimensional (2D) space. In 1990s, there was a big effort by the Binary Black Hole Grand Challenge Alliance (e.g. [14]) to solve the BBH problem. We then saw various development in the evolution system (NOK-BSSN [15–17]), initial data ("puncture" initial data [18]), and gauge conditions ("fixed puncture" evolutions [19]) in black-hole binary simulations. In the Lazarus project (see [20]), the final moments of the merger of black-hole binaries in the three dimensional (3D) simulations has been modeled through the identification of perturbations at late times. In 2004, [21] presented a simulation of black-hole binaries for about one orbital period.

After 40+ years of hard work, the black-hole binary problem in full General Relativity has been solved with the breakthroughs of 2005 by the "generalized harmonic" [22] and the "moving punctures" [23, 24] methods. In these works, the first successful fully non-linear dynamical numerical simulations were done for the inspiral, merger, and ringdown of orbiting black-hole binary systems. In particular, the moving punctures approach, developed independently by the Numerical Relativity groups at UTB (Now RIT) and NASA/GSFC, has become the most widely used method in the field and was successfully applied to evolve generic black-hole binaries. In this approach, a singular term in the spacetime metric is numerically regularized and the black holes move across the computational domain. The generalized harmonic approach has also been successfully applied to accurately evolve generic BHBs for tens of orbits with the use of pseudospectral codes [25, 26].

There have been since 2005 many important advances in the understanding of black-hole physics: studies of the orbital dynamics of spinning black-hole binaries [27–33], calculations of recoil velocities from the merger of unequal mass black-hole binaries [34–36], and the surprising discovery that very large recoils can be acquired by the remnant of the merger of two spinning black holes [30, 37–52], empirical

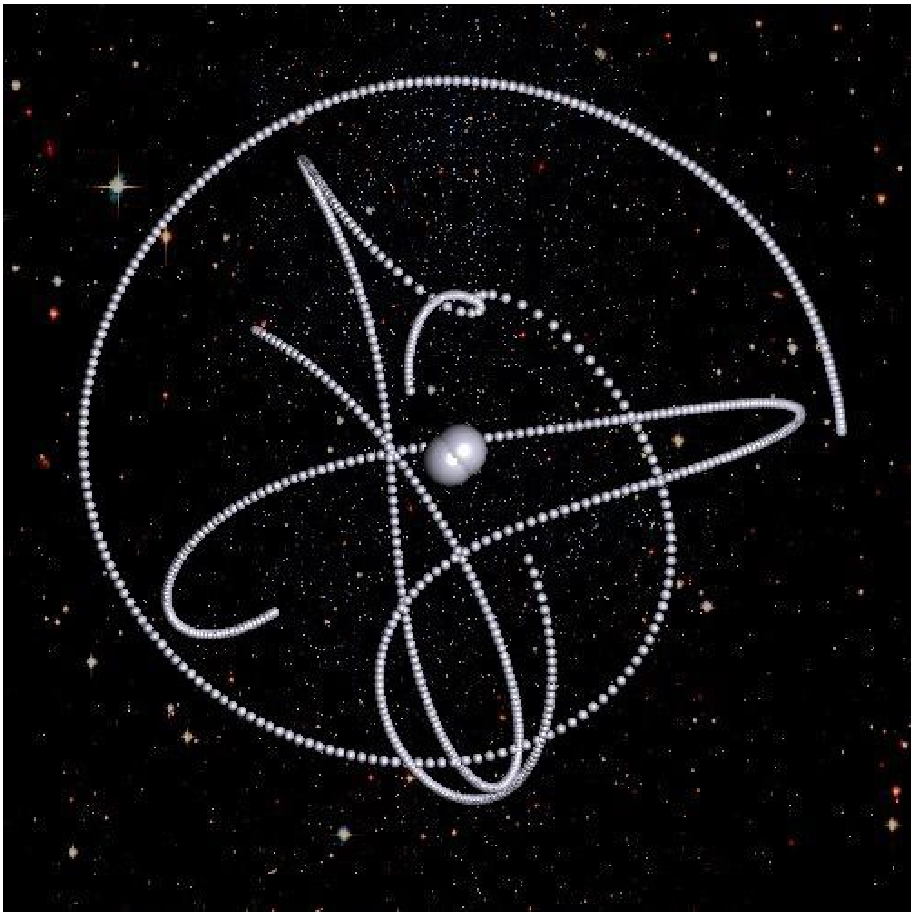

Figure 1: [**Simulation: Manuela Campanelli, Carlos Lousto, Yosef Zlochower, Visualization: Hans-Peter Bischof**] Inspiral and merger of three black holes. The small dots show their trajectories. This movie is downloadable from http://ccrg.rit.edu/movies.

models relating the final mass and spin of the remnant with the spins of the individual black holes [53–60], and comparisons of waveforms and orbital dynamics of black-hole binary inspirals with post-Newtonian predictions [61–68].

3 *LazEv* code results

We can accurately and stably evolve black-hole binaries (and multiplets, e.g. [69–71] and Figure 1) for a vast range of mass-ratios and spins, and compute the gravitational-wave radiation, black-hole remnant and spacetime dynamics. In the following, we summarize some work using the *LazEv* [72] implementation of the moving puncture approach.

3.1 Merger of Spinning Black Holes: Hang-Up Orbits

To understand the dynamics of highly spinning black-hole binaries, we accurately simulated the inspiral orbit of black-hole binaries with two equal mass m and individual spins with equal amplitude $S/m^2 = 0.757$ parallel to the orbital angular momentum in [27]. The simulations start from the orbital frequency $\Omega = 0.05/M_{\text{ADM}}$ where M_{ADM} denotes the ADM mass of the system.

We found that the *orbital hang up* effect (spin-orbit coupling) delays the onset of the plunge phase (compared to the non-spinning case) when the spins are aligned with the orbital angular momentum, while in the anti-aligned case the plunge phase is hastened. This effect can be considered as the leading

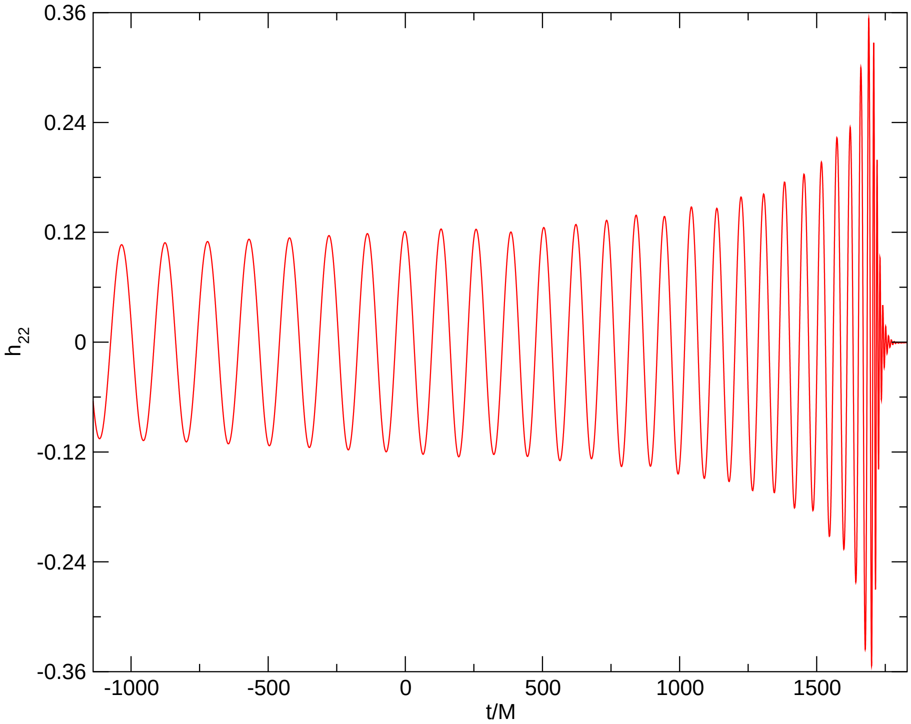

Figure 2: The real part of the $\ell = 2$, $m = 2$ mode of the hybrid gravitational waveform for a precessing black-hole binary [75]. This is created by matching the NR waveform to the waveform derived from the 3.5 post-Newtonian order equations of motion, and the matching starts around $t = 226M_{\rm ADM}$.

order spin-orbit coupling (the 1.5 post-Newtonian order interaction) in the post-Newtonian equations of motion. The total radiated energy $E_{\rm rad}/M_{\rm ADM}$ is $(6.7 \pm 0.2)\,\%$ for the aligned case, while for the anti-aligned case it is only $(2.2 \pm 0.1)\,\%$ (compared to the non-spinning case $(3.5 \pm 0.1)\,\%$). And in all cases the black holes merged to form a single final Kerr black hole with sub-maximal spin (the non-dimensional spin $\chi < 1$), i.e., the *cosmic censorship conjecture* holds in our all cases.

3.2 Merger of Generic, Precessing Black-Holes

In [73], we compared Numerical Relativity and post-Newtonian waveforms of a generic black-hole binary, i.e., a binary with unequal masses, with mass ratio $q = m_1/m_2 = 0.8$ and unequal, non-aligned, precessing spins of magnitude, $S_1/m_1^2 = 0.4$ and $S_2/m_2^2 = 0.6$. The numerical simulation starts with an initial separation of $r \sim 11M_{\rm ADM}$, and has 9 orbits prior to the merger. To obtain the initial data (the positions, momenta, and spins of each black hole), we considered purely post-Newtonian evolutions of a nearly quasi-circular binary with initial orbital separation $r \sim 50M_{\rm ADM}$ by the procedure in [74], extended to spinning particles.

Comparison of numerical simulations with post-Newtonian ones have several benefits aside from the theoretical verification of the post-Newtonian calculations. From a practical point of view, one can directly propose a phenomenological description and thus make predictions in regions of the parameter space still not explored by numerical simulations. From the theoretical point of view, an important application is to have a calibration of the post-Newtonian error (and fitting parameters in the Effective-One-Body approach) in the last stages of the binary merger. Also, combining the post-Newtonian waveform from large separations and smoothly attaching this waveform to the corresponding Numerical Relativity waveform produced by the binary during the late-inspiral, we can provide a hybrid waveform [75]. The waveform in Figure 2 is available for download from http://ccrg.rit.edu/downloads/waveforms[5].

[5] We would like to introduce an interesting website: http://www.black-holes.org/. Some numerically-generated gravitational waveforms are also publicly available from this website.

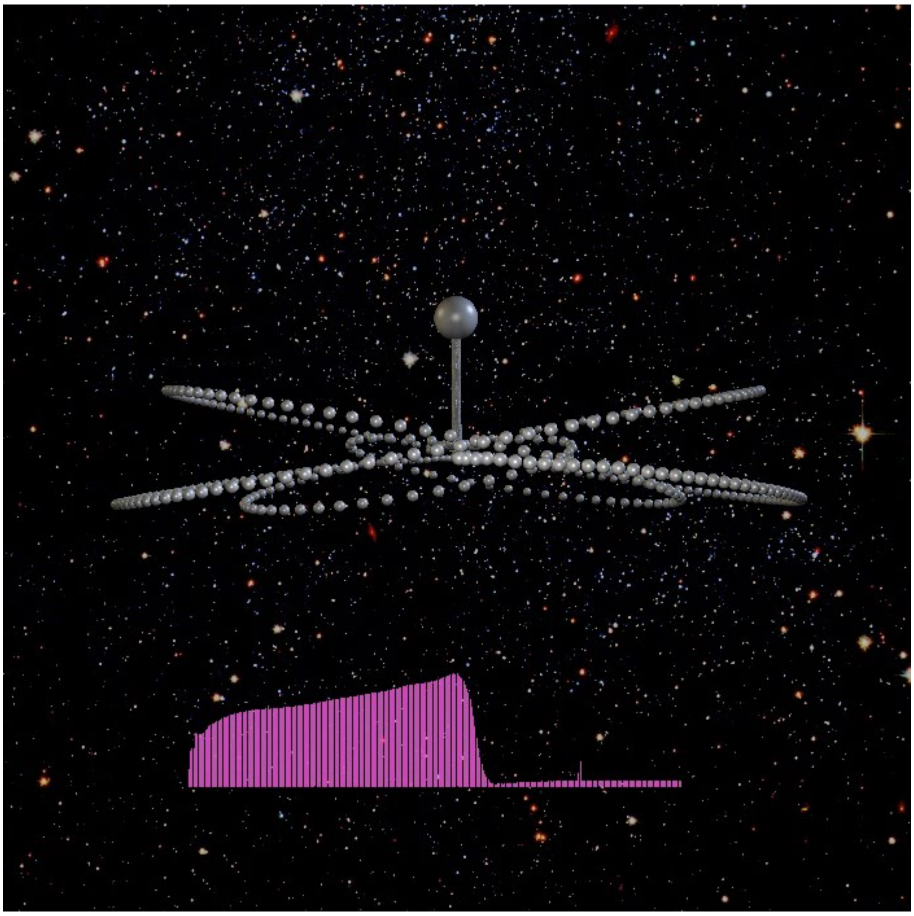

Figure 3: [**Simulation: Manuela Campanelli, Carlos Lousto, Yosef Zlochower, Visualization: Hans-Peter Bischof**] Remnant gravitational recoil of the merger of a black-hole binary. We see the bobbling and merger of two black hole and the resulting merger superkick. The bar at the bottom indicates the speed of the black holes. This movie is downloadable from http://ccrg.rit.edu/movies.

3.3 Gravitational Recoiling Black-Holes

We have modeled the remnant gravitational kick of the merger of black-hole binaries with very large recoil velocities due to gravitational radiation since the discovery [38, 39] (see Figure 3) in the numerical simulations of generic spinning binaries. The spins of the black holes play a crucial role in producing recoils of up to 4000 km/s which allow remnant black holes to escape from major galaxies. The large gravitational recoils found had a significant impact on astrophysics since, the gravitational kicks affect the SMBH retention rates in galaxies, the IMBH retention rates in globular clusters, galactic core dynamics and accretion disk dynamics. Direct observation of them can lead to the first confirmation of a prediction of General Relativity in the highly-dynamical, strong-field regime.

Based on the notion [40, 76] that the leading description of the recoil can be modeled by the post-Newtonian dependence [77, 78], empirical formulae for the final remnant black-hole recoil velocity (also mass and spin) from merging black-hole binaries were obtained in [60] (and references therein). In [79], considering cubic and possible fifth-order corrections [54], we obtained enhanced recoil formulae for the "maximum kick" configurations, and have predicted that the maximum recoil will be 3680 ± 130 km/s. In [80] we confirmed that the recoil formula is accurate to within a few km/s in the comparable mass-ratio regime for the out-of-plane recoil by using a new set of 20 numerical simulations.

In [81], we studied the statistical distributions of the spins of generic black-hole binaries during the 'dry' inspiral (i.e. gravitational radiation driven) and merger, as well as the distributions of the remnant mass, spin, and recoil velocity. In the statistical results, we found a small bias towards counter-alignment of the vectors $\vec{\Delta}$ and $\vec{S}$ with respect to the orbital angular momentum $\vec{L}$ just prior to merger, where $\vec{S} = \vec{S}_1 + \vec{S}_2$, $\vec{\Delta} = (m_1 + m_2)(\vec{S}_2/m_2 - \vec{S}_1/m_1)$. This effect essentially takes place at close separations and can be studied analytically at low post-Newtonian orders. The anti-alignment effect is associated with the late-time precession of the orbital plane due to gravitational radiation reaction. This effect for dry mergers seems to oppose the alignment mechanism observed in 'wet' mergers [82, 83]. After the initial inspiral regime, we studied the merger of black-hole binaries using full numerical simulations. We found that the merged black holes have a considerable probability (23 %) to reach recoil velocities above 1000 km/s and the distribution is highly peaked along the orbital angular momentum (see Table 1).

Table 1: The probability to obtain large recoil velocities, and large recoil velocities along the line of sight assuming a uniform distribution of mass ratios [81]. Large recoil magnitudes are highly probable, but less observable.

v [km/s] $\geq$	500	1000	2000	2500
Recoil	50.2 %	23.2 %	2.2 %	0.24 %
Observer	22.6 %	6.4 %	0.22 %	0.01 %

3.4 Simulations of Small Mass-Ratio Black-Hole Binaries

According to [84], small mass-ratios black-hole binaries in the range $0.01 < q = m_1/m_2 < 0.1$ are most likely to occur. Current Numerical Relativity simulations have focused on the comparable mass-ratio regime, and black-hole binaries in the very small mass-ratio regime can, in principle, be modeled accurately with the black hole perturbation approach. The small mass-ratio regime is hard to model with Numerical Relativity simulations.

In [85] we introduced a new technique that makes use of the trajectories obtained in the Numerical Relativity simulations and efficient perturbative evolutions to compute waveforms at large radii for the leading and non-leading modes in the black hole perturbation approach, i.e., the Regge-Wheeler-Zerilli (RWZ) formalism [86, 87]. As a proof-of-concept, we computed waveforms for a relatively close binary with mass ratio $q = 1/10$.

In the next paper [88], we reached smaller mass ratios, to the $q = 1/15$ case, and extended the RWZ formalism for the Schwarzschild perturbations by including, perturbatively, a term linear in the spin of the larger black hole (SRWZ formulation). For intermediate mass-ratio black-hole binaries with the mass ratio $q = 1/10$ and $1/15$, we have found good agreement in the Numerical Relativity and SRWZ waveforms which include the late inspiral, plunge, merger and ringdown phases (see Figure 4).

Recently, the merger of a mass ratio 100 : 1 black-hole binary has been simulated in [89] using an optimal choice of the mesh refinement structure around the smaller black hole. Using the techniques presented in this paper, we can simulate even smaller mass-ratios q and initially spinning black holes. It is also important to choose quasi-circular orbital parameters (see [90]) and to evolve initial data with lower spurious radiation content to obtain realistic true inspiral wave information [91, 92].

4 Community-Wide Collaborations

The *NINJA* (Numerical INJection Analysis, https://www.ninja-project.org/doku.php?id=ninja:home) is a collaborative effort between members of the numerical relativity and gravitational-wave data analysis communities. In *NINJA-1* [93, 94] with ten numerical relativity groups providing gravitational waveforms and data analysis contributions from nine different groups, short Numerical Relativity waveforms from the merger of black-hole binaries without any hybrids, i.e., attaching post-Newtonian inspiral waveforms, have been used for the injections. The data set did not include the type of non-Gaussian transients seen in real gravitational-wave detector data. In *NINJA-2* with more groups, we are now attempting to use various (including different mass-ratios and spins) hybrid waveforms for LIGO and VIRGO data analysis.

To develop accurate analytical gravitational-wave template banks of black-hole binaries for the data analysis, a larger region of the parameter space including spin precessing systems, has been simulated

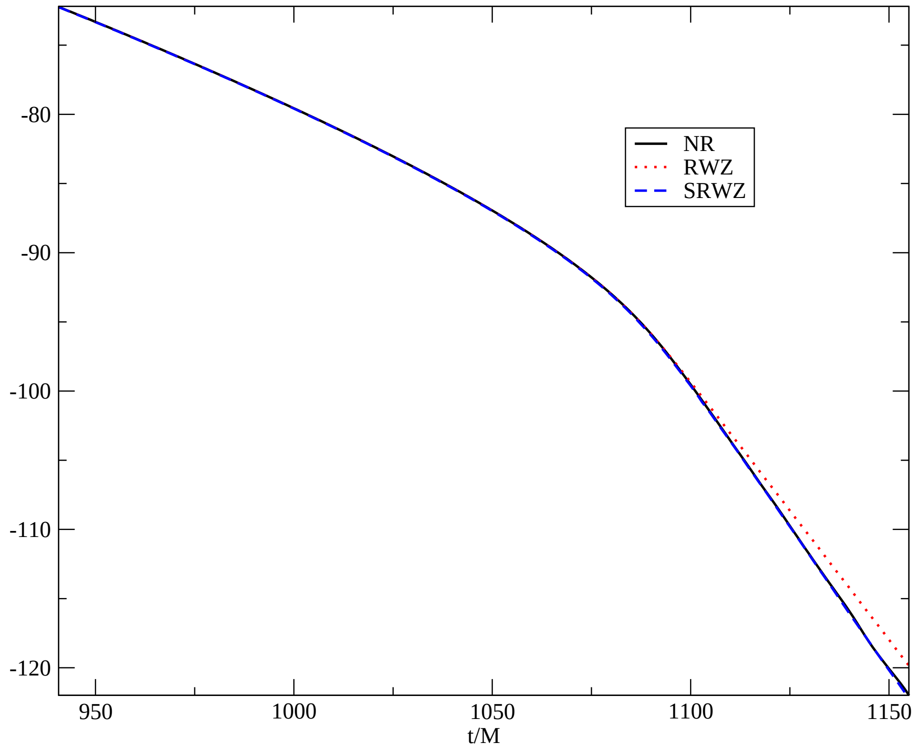

Figure 4: The phase evolution of the ($\ell = 2$, $m = 2$) wave for the $q = 1/10$ case [88]. The (black) solid, (red) dotted, and (blue) dashed curves show the Numerical Relativity, RWZ (spin-off), and SRWZ (spin-on) calculations, respectively. Note that although the initial part of the three curves show almost the same evolution, we can see differences after the merger.

on the TeraGrid machine Kraken, and the precise full numerical gravitational waveforms have been computed in the *NRAR* collaboration (Numerical-Relativity and Analytical-Relativity, https://www.ninja-project.org/doku.php?id=nrar:home).

The *Einstein Toolkit* (http://einsteintoolkit.org/) is a state of the art, open, community developed software infrastructure for relativistic astrophysics. The targets are black holes, neutron stars, core-collapse supernovae, etc. This type of a collaborative work will be needed in the future directions discussed next.

5 Future Directions

As mentioned in the Introduction, coincident detections of gravitational-wave and electromagnetic signatures from merging black-hole binaries give us varied astrophysical information. Observable consequences of large gravitational recoils include effects of a SMBH passing through an accretion disk. The disk around the kicked black hole will have broader spectrum and be shifted with respect to the host disk. The sudden loss of mass of the central black hole will perturb the disk, MBHs will be displaced or wandering from the core of the host structure, and the kicks lead the reorientation of jets, e.g. X-shaped radio-morphologies.

Studies of electromagnetic counterparts of merging black-hole binaries should include the gaseous environment. The scale problem, from 10^5 pc to 10^{-5} pcs, represents a huge computational challenge. One should split the problem into stages, the capture, pre-merger, merger and post-merger (e.g. kicked) stages. Simulations of the merger require full General Relativity-magneto-hydrodynamics (GR-MHD) and radiation physics modeling.

Regarding the gaseous environment close to the merging black-hole binaries, (at scales < 0.01 pc) there are some initial explorations about the electromagnetic fields [95] and radiatively inefficient hot gas clouds [96] around them. One may also consider alternative models, e.g. thin circumbinary disk. The most important point is to create reliable GR-MHD codes which properly account for accretion rates, electromagnetic fields, radiation transport, and the required numerical resolution. To complete the above studies we will need large collaborative efforts.

6 Conclusions

Exciting new astrophysical phenomena have been found thanks to Numerical Relativity's black-hole binary simulations. We can accurately and stably evolve almost arbitrary black-hole binaries. The gravitational waveforms are accurately produced for use in gravitational wave data analysis for ground and space-based detectors. The results can be compared with the post-Newtonian (including Effective-One-Body) and black hole perturbation theories, and be used to calibrate them. The remnant mass-loss reaches up to 10% of the total mass of the system, and depending on the spins magnitude and orientation we can observe the gravitational superkicks.

On the other hand, we still have some open challenges for the numerical simulations of black-hole binaries, 1) extreme black-hole binaries, nearly maximal spins and extreme mass-ratios are still very hard to accurately simulate, 2) long, accurate simulations and multiple physical scales require more efficient evolution codes, and 3) better initial data are needed especially for highly spinning black-hole binaries and for the inclusion of matter.

The possibility of observing electromagnetic counterparts from binary black-hole mergers is a very exciting topic. This study requires 1) careful pre-merger modeling efforts, 2) a comprehensive approach and reliable GR-MHD codes, and 3) open source codes for general relativistic astrophysics, such as (http://einsteintoolkit.org/).

Astrophysicists can now test General Relativity in the highly-nonlinear regime. The simulations of black-hole binaries can soon be used to look for gravitational waves reaching earth opening thus up a brand new window onto the Universe.

Acknowledgments

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Finally, it is our pleasure to celebrate Takashi Nakamura and Kei-ichi Maeda's sixtieth birthday.

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