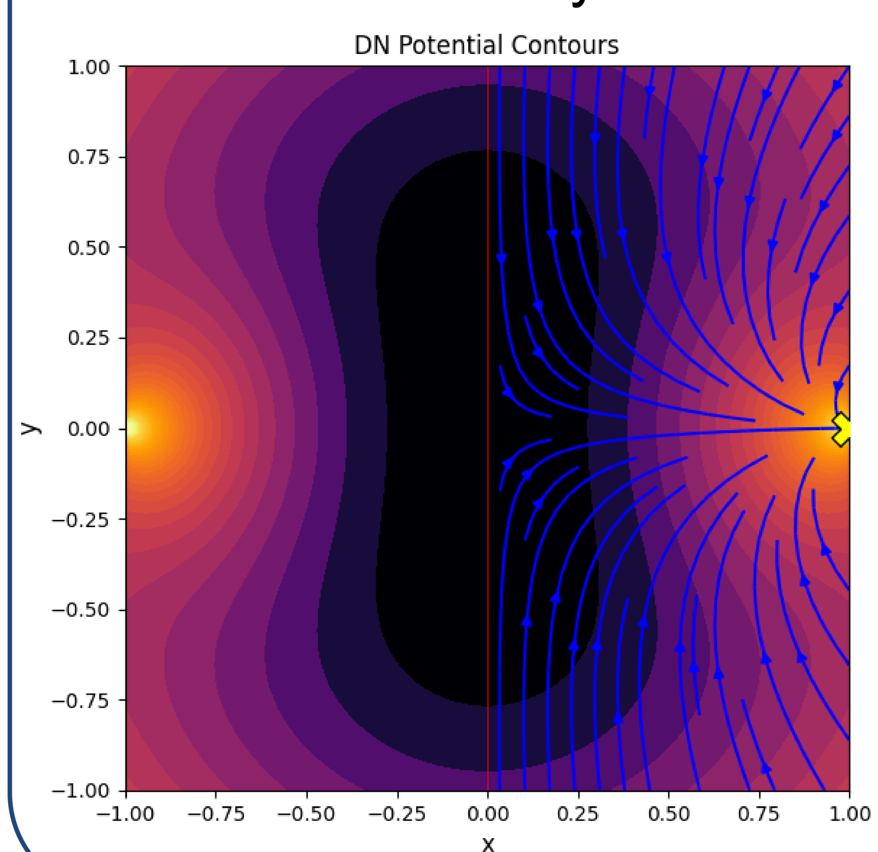
Experimental Measurements for Extracting Nonlinear Invariants in IOTA

IPAC24

J. N. Wieland, A. L. Romanov, A. Valishev, Fermilab; N. Kuklev, Argonne

Nonlinear integrable optics (NIO) combine strong amplitude dependent tune shift with bounded single particle trajectories.

A Hamiltonian system is integrable if it possesses as many independent invariants of motion as dimensions in configuration space. The Integrable Optics Test Accelerator (IOTA) was constructed to study the Danilov-Nagaitsev (DN) NIO system.



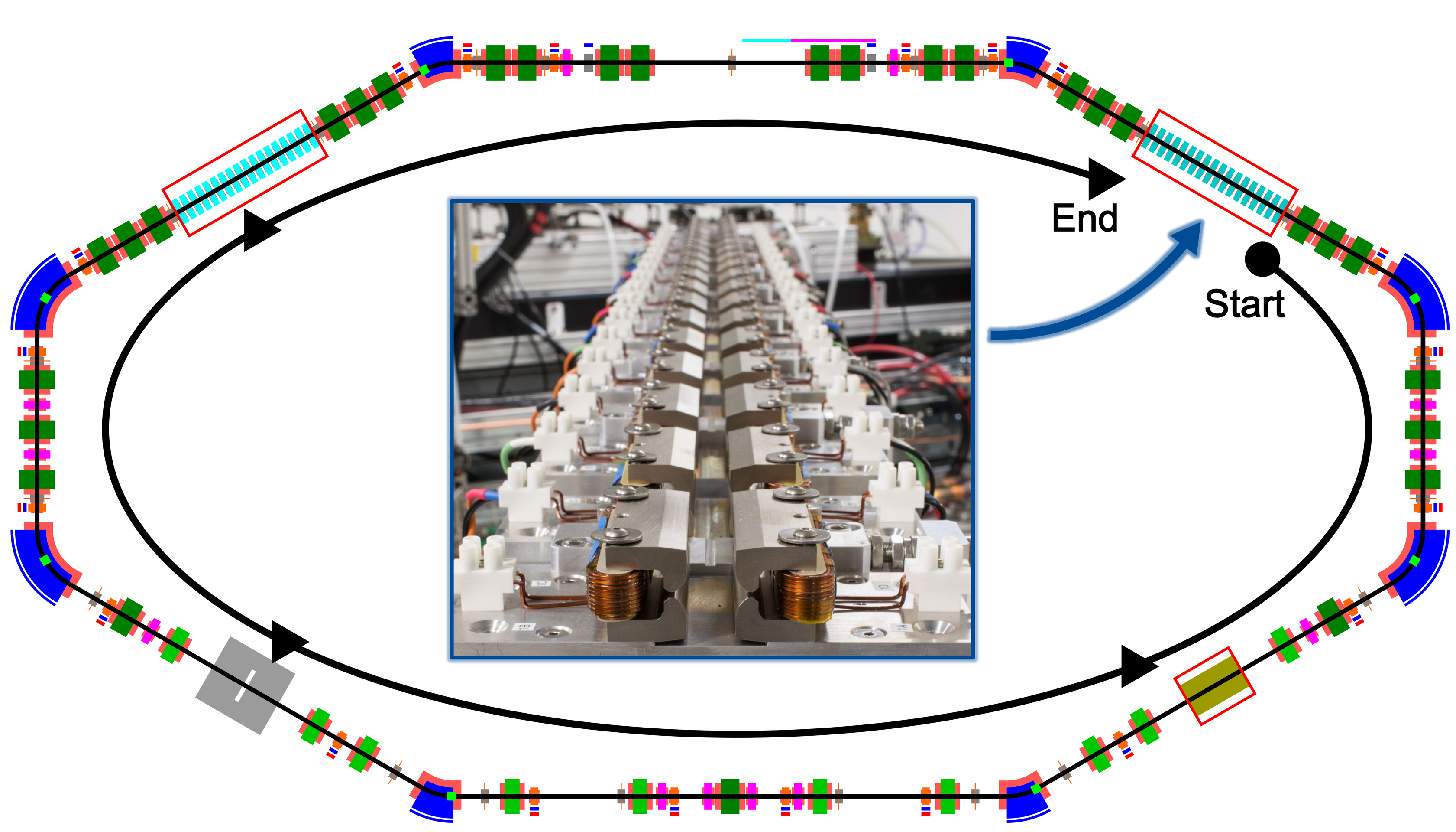
$$H = \frac{1}{2}(x^2 + y^2 + p_x^2 + p_y^2) - t\mathrm{Re}\left(\frac{x+iy}{\sqrt{1-(x+iy)^2}}\arcsin(x+iy)\right)$$

$$I = (xp_y - yp_x)^2 + p_x^2 + x^2 - t\mathrm{Re}\left(\frac{2x}{\sqrt{1-(x+iy)^2}}\arcsin(x+iy)\right)$$

Above: DN Analytical Invariants

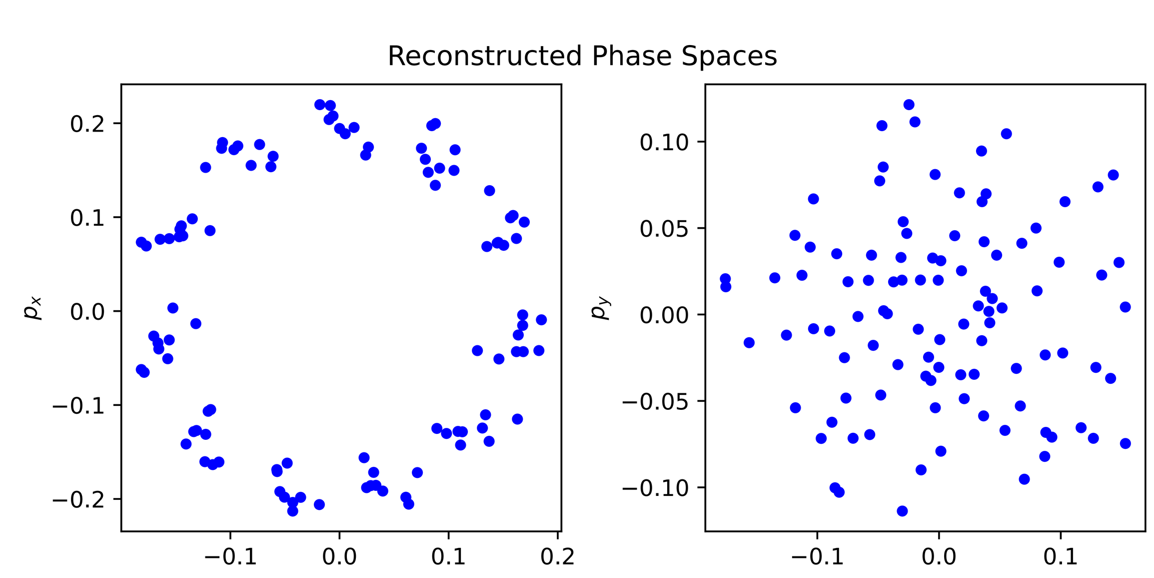
Left: DN Potential Contours, right half illustrates effects of nonlinear terms. Blue lines are gradient of nonlinear component, proportional to "t" parameter. Yellow marker is location of singularity

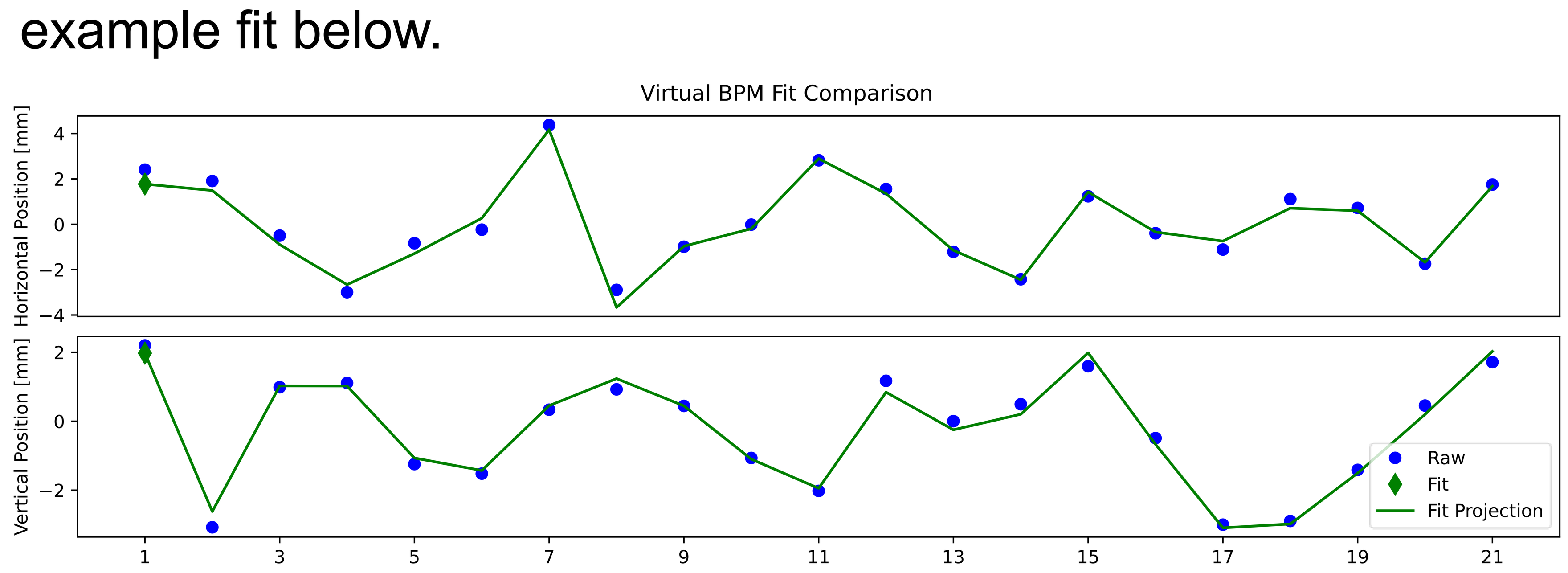
Below: Location of nonlinear element in IOTA



4D Position Reconstruction

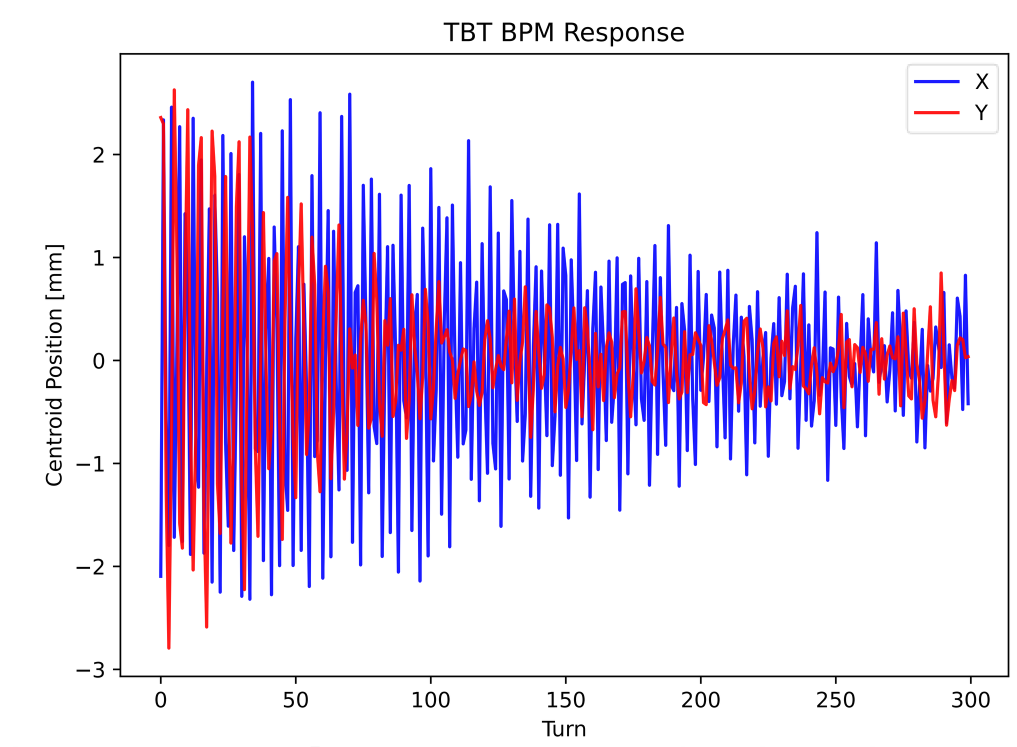
IOTA is mostly linear outside of the insert. Reconstructed 4D position at virtual BPM using design lattice transfer matrices in all 21 BPMs. Fit channel in IOTA figure above, example fit below.





Kicked Beam Data Collection

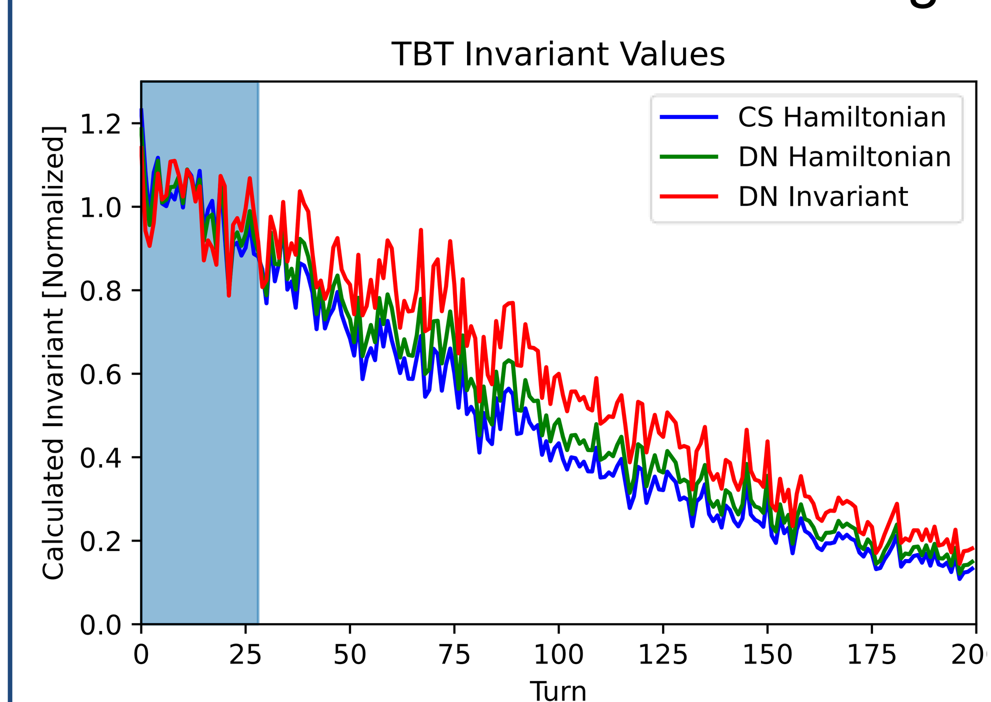
1. Inject single bunch and scrape to BPM sensitive current
2. Excite betatron oscillations with stripline kickers
3. Measure BPM response TBT



Strong nonlinear detuning means fast asymmetric decoherence. Varied kick amplitudes to probe full configuration space.

Invariant Calculations

Substitute reconstructed position data into invariant expressions. TBT decoherence means that calculated value decreases quickly. To evaluate quality of conservation, look at first few turns before amplitude decoheres. Compared with Courant Snyder invariant as baseline. Calculated noise larger than simulated difference.

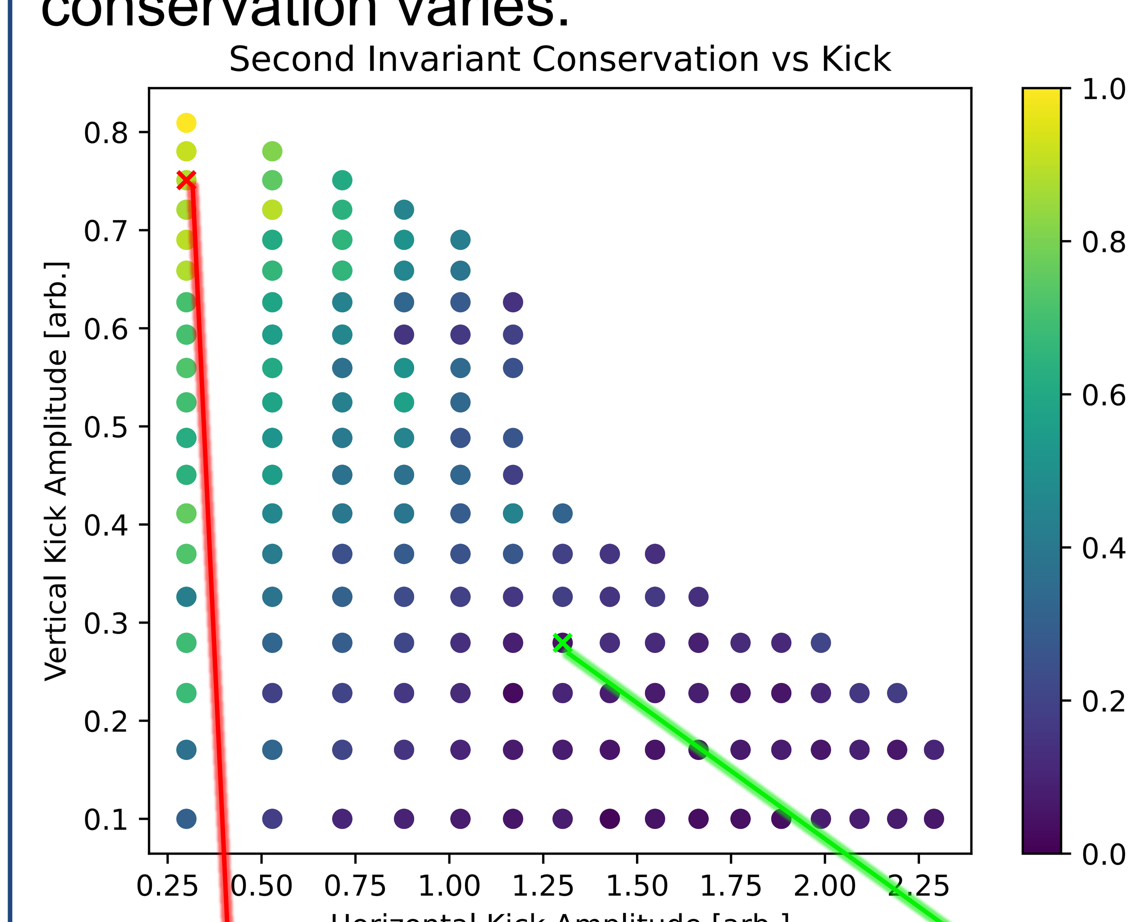


Left: Minimum conserved invariant measurements, multiple kicks sampled at this amplitude.

Below: Conservation comparison

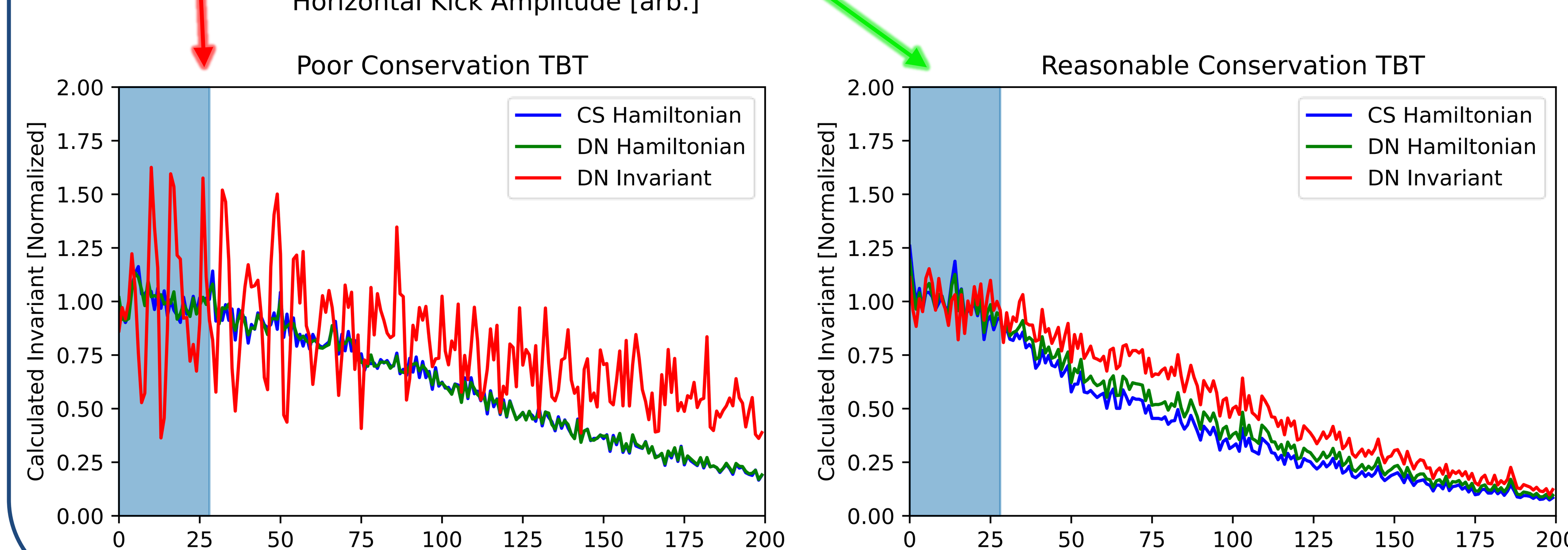
Invariant	Simulation	Experiment
CS Hamiltonian	1.9%	6.9%
DN Hamiltonian	0.3%	5.6%
DN Invariant	0.3%	6.9%

At different amplitudes for the same t, second invariant conservation varies.



Left: DN Invariant conservation metric for different amplitudes, a poorly conserved kick and reasonably conserved kick sampled.

Below: TBT invariant quantities sampled from range. Poorly conserved plot displays oscillations consistent with a nearly conserved quantity



Fermi National Accelerator Laboratory

Fermilab

U.S. DEPARTMENT OF ENERGY

This manuscript has been authored by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics