national accelerator laboratory

TM-128
2050

SEMINAR

M. Goldberger
Princeton University

July 30, 1968

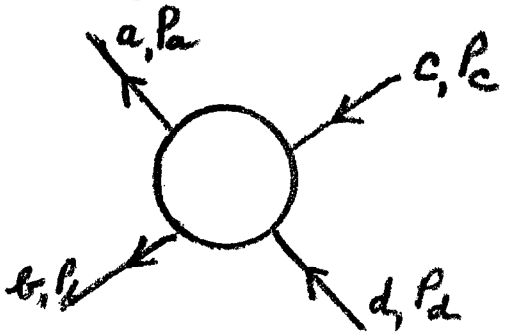
In a series of three talks aimed exclusively at experimentalists, I want to discuss high energy hadronic reactions and some aspects of Regge pole theory and phenomenology. There will necessarily be some mathematics, but nothing very fancy. I'll assume familiarity with collision theory but nothing more than can be obtained by reading the book by Watson and me.

Lets begin by briefly reviewing the general features of high energy elastic and quasi-elastic scattering. Geoff Chew will speak next week about multiparticle production. First as to elastic scattering: differential cross sections are sharply peaked in the forward direction; this peak is roughly energy independent. Regarded as a function of momentum transfer squared, $-t > 0$ one finds $d\sigma \sim \exp(t/10)\,(\mathrm{GeV}/c)^2$ and that the scattering amplitudes are nearly purely imaginary. There are essentially no indications of diffraction maxima and minima, however, and a number of other fine points to which we'll return that one can't take the pure diffraction theory too seriously. For example, the amplitudes are not purely imaginary--there are significant polarization effects, and backward peaks. For inelastic two body or quasi two body reactions one finds similarly roughly exponential forward peaking provided particles exist which can be exchanged ($K^- + p \to K^+ + \Xi^-$ - no peak), and the same in the backward direction. The cross sections for these inelastic

Operated by Universities Research Association Inc. Under Contract with the United States Atomic Energy Commission

processes mostly decrease with energy (like $P_{lab}^{-(1 \text{ to } 4)}$).

Before going any further into the experimental results and general theoretical framework, lets quickly go through some kinematics and scattering theory results.



General 2-body collision.

All masses different

$a + b \rightarrow c + d$

Define three scalar variables, s, t, u:

$$s = -(P_a + P_b)^2 = -(P_c + P_d)^2 \qquad \text{Metric: } a \cdot b = \vec{a} \cdot \vec{b} - a_o b_o$$

$$t = -(P_a - P_c)^2 = -(P_b - P_d)^2$$

$$u = -(P_b - P_c)^2 = -(P_d - P_a)^2$$

s is the total energy squared in the center of mass system; t is the negative of the square of the momentum transfer from a to c or from b to d; u is another kind of momentum transfer. By direct computation,

$$s + t + u = M_a^2 + M_b^2 + M_c^2 + M_d^2 \equiv \Sigma .$$

We shall later need another kinematic relation:

$$\frac{s - u}{2} \equiv \nu = s + \frac{t}{2} - \frac{\Sigma}{2} \qquad \begin{pmatrix} \text{as } \nu \rightarrow -\nu \\ s \rightleftarrows u \end{pmatrix}$$

or

$$-\nu = u + \frac{t}{2} - \frac{\Sigma}{2}$$

In the center of mass system, $\vec{P}_a + \vec{P}_b = 0 = \vec{P}_c + \vec{P}_d$

and we write $\vec{P}_a = \vec{P} = -\vec{P}_b$, $\vec{P}_c = \vec{P}^1 = \vec{P}_d$. We have then

$$|\vec{P}| = \frac{1}{2W} \sqrt{S(W^2, M_a^2, M_b^2)}$$

$$|\vec{P}| = \frac{1}{2W} \sqrt{S(W^2, M_c^2, M_d^2)}$$

$$\epsilon_a = W - \epsilon_b = \frac{1}{2W} (W^2 + M_a^2 - M_b^2)$$

$$\epsilon_c = W - \epsilon_d = \frac{1}{2W} (W^2 + M_c^2 - M_d^2)$$

where $s = W^2$

$$S(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 - 2x_1 x_2 - 2x_1 x_3 - 2x_2 x_3$$

e.g. $S(W^2, M_a^2, M_b^2) = [W^2 - (M_a + M_b)^2] [W^2 - (M_a - M_b)^2]$. The scattering angle θ in the center of mass system may be expressed in terms of s, t:

$$t = -(P_a - P_c)^2 = M_a^2 + M_c^2 + 2pp' \cos\theta - 2\epsilon_a \epsilon_c$$

It is conventional to express the differential cross-section for our reaction in terms of a scattering amplitude $f_{cd;ab}(\theta, \phi, W)$ such that

$$\frac{d\sigma}{d\Omega} = \frac{p'}{p} | f_{cd;ab} |^2$$

where in general the amplitude f will depend on spin labels c d a b; in the simplest case of no spins, $f = f(\cos\theta)$, independent of ϕ. Assuming that this is the case, or that we have summed and averaged over spins so that there is no ϕ dependence to $d\sigma/d\Omega$, we have

$$\frac{d\sigma}{d\Omega} = 2\pi \frac{d\sigma}{d\Omega} = 2\pi \frac{p'}{p} \mid f \mid^2$$

Noting $dt = 2\,p\,p' \cos\theta$

$$\frac{d\sigma}{dt} = \frac{\pi}{p^2} \mid f \mid^2$$; note that the factor which distinguishes inelastic from elastic scattering, p'/p, disappears when we use $\frac{d\sigma}{dt}$ rather than $d\sigma/d\Omega$.

Instead of the scattering amplitude, f , it is customary in relativistic quantum theory to use an invariant amplitude which I shall normalize as follows:

$$M = 8\pi W\ f$$ (note that M is dimensionless if $\pi = C = 1$)

so that

$$\frac{d}{dt} = \frac{1}{64\pi W^2 P^2} \mid M \mid^2$$

$$= \frac{1}{16\pi S(W^2, M_a^2, M_b^2} \mid M \mid^2$$

$$\longrightarrow \frac{1}{16\pi S^2} \mid M \mid^2$$

Next we recall the important relation

$$\sigma_{tot} = \frac{4\pi}{k} \text{ Im } f_{elastic} \ (\cos\theta = 1)$$

This result is a truth, independent of any restriction to two-body processes; f_{el} is the amplitude for $a + b \rightarrow a + b$. In terms of M,

$$\sigma_{tot} = \frac{1}{2PW} M_{ab;ab}\ (\theta = 0) \underset{s\rightarrow\infty}{\rightarrow} \frac{1}{S} \quad M$$

Note also that for elastic scattering $t = 2P^2(1 - \cos\theta)$ so $\theta = 0$ means $t = 0$. Finally I remind you of the partial wave expansion

$$f = \frac{1}{\sqrt{pp'}} \Sigma (2\ell + 1) f_{cd;ab}(\ell, W) P_\ell(\cos\theta)$$

and in particular for elastic scattering

$$f = \frac{1}{2iP} \Sigma (2\ell + 1) \left[\eta_\ell e^{2i\tilde{\delta}_\ell} - 1\right] P_\ell(\cos\theta)$$

where $0 \leq \eta_\ell \leq 1$ and $\tilde{\delta}_\ell$ is real. This form is a consequence of unitarity or conservation of probability, the same thing that gave us the optical theorem, and in fact we have

$$\sigma_{el} = \int d\Omega \, |f_{e\ell}|^2 = \frac{\pi}{P^2} \sum_\ell (2\ell + 1) \left| \eta_\ell e^{2i\tilde{\delta}_\ell} - 1 \right|^2$$

$$\sigma_{tot} = \frac{4\pi}{P} \operatorname{Im} f_{e\ell}(\theta = 0) = \frac{2\pi}{P^2} \Sigma (2\ell + 1)(1 - \eta_\ell \cos 2\tilde{\delta}_\ell)$$

$$\sigma_{in} = \sigma_+ - \sigma_{el} = \frac{\pi}{P^2} \sum_{\ell=0}^{\infty} (2\ell + 1)(1 - \eta_\ell^2)$$

We have heard from Randy Durand about application of another representation of the scattering amplitude which has a certain intuitive appeal, the so-called impact parameter representation. One very simple way to obtain this representation is to imagine that so many ℓ's enter that one may replace in our partial curve expansions the sum over ℓ by an integral

$$f_{el} = -\frac{i}{P} \sum (\ell + 1/2)(e^{2i\delta_\ell} - 1) P_\ell \ (\cos\theta)$$

$$\rightarrow -\frac{i}{P} \int^\infty d\ell \ (\ell + 1/2)(e^{2i\delta_\ell} - 1) P_\ell \ (\cos\theta)$$

and for large ℓ, $P_\ell\,(\cos\theta) = J_o \quad 2(\ell + 1/2) \sin\frac{\theta}{2}$ and finally, introduce the impact parameter b by $\ell + 1/2 = Pb$, and regard $\delta_\ell(P)$ as $\delta(b, P)$, so that we have

$$f_{el} = -iP \int_o^\infty b\,db \left[e^{2i\delta(b,P)} - 1\right] J_o\,(2Pb \sin\theta/2)$$

$$= -iP \int_o^\infty b\,db \left[e^{2i\ (b,P)} - 1\right] J_o\,(b\sqrt{-t}\,)$$

where we have used (for elastic scattering)

$$-t = 2P^2 (1 - \cos\theta) = 4P^2 \sin^2\theta/2 \equiv \Delta^2$$

The phase $2\delta(b_\circ p)$ is given the interpretation of the phase change undergone by a partial passing thru the interaction region at impact parameter b.

Independent of the "derivation" given above of the impact parameter representation, one may write quite generally for the elastic scattering amplitude regarded as a function of s and t (or Δ^2)

$$f_{el}\,(s,t) = P \int_o^\infty dbb \quad H\,(b,s)\ J_o \ (b\sqrt{-t})$$

$$= P \int_o^\infty bdb \quad H\,(b,s)\ J_o \ (b\ \Delta)$$

$$\sigma_{el} = 2\pi \int_o^\infty bdb \ \left|H_{el}\,(b,s)\right|^2$$

$$\sigma_t = 4\pi \int_0^\infty bdb \ \mathrm{Im}\ H_{el}(b,s)$$

$$\sigma_{in} = 2\pi \int_0^\infty bdb \ \{2\ \mathrm{Im}\ H_{el} - |H_{el}|^2\}$$

The beauty of the partial wave expansion is that unitarity is easily expressible: $\mathrm{Im}\ f_\ell = |f_{e\ell}|^2 + \mathrm{Im}\ f_\ell^{\text{inelastic}}$ and $\therefore$ $\mathrm{Im}\ f_\ell > |f_\ell|^2$ or $|f_\ell - i/2|^2 \leqslant 1/4$. There is no correspondingly exact statement about $H(b,s)$ except at very high energies where $H(b,s) \cong \frac{1}{2}[\eta(b,s)\, e^{2i\tilde{\delta}} - 1]$.

With this background lets talk in a little more detail about high energy hadron scattering. The total cross-sections lie in the range of about 15-60 mb. at the highest energies, corresponding to $P_{LAB} \sim 25$ GeV/c, $\sigma_{\bar{p}p} \sim 50$, $\sigma_{pp} \cong \sigma_{pn} \sim 40$, $\sigma_{\pi-p} \sim \sigma_{\pi' p} \sim 25$ mb, $\sigma_{K^- -p} \sim 20$, $\sigma_{K+p} \sim 15$ mb. They seem to be approaching constants or decreasing very slightly. The constant behavior is consistent either with the view that the radius of interaction, whatever that means, is finite and $\sigma \sim$ to R^2 with $R \sim 1/m\pi = \sqrt{2} \times 10^{-13}$ cm. or with the idea that the radius increases with energy but the transparency decreases in such a way as to maintain a constant cross-section. There is no terribly convincing argument leading to the constancy of cross-section although as we will see there is a natural place for it within the framework of Regge pole theory and the following qualitative argument which almost leads to constant cross-sections due to Froissart and, independently, Feynman: One imagines the probability of interaction between particles at relatively large distances is expressed by $g\, e^{-r/a}$ where g may be a function of kinetic energy and that if r is so large that $g\, e^{-r/a} \ll 1$ there is essentially no interaction. Then for

$r \approx b$ such that $g\, e^{-b/a} \sim 1$, $b \sim a \ln g$, $\sigma \sim \pi a^2 (\ln g)^2$ and if g is at most a power of the energy,

$$\sigma \sim \pi a^2 (\ln s)^2$$

This result has also been obtained using the finest axioms of quantum field theory.

Another general feature of high energy total cross-sections that σ (particle - particle) $\rightarrow$ σ (particle - anti particle) at very high energies. That this should be true was first suggested by Pomeranchuk and whether this relation is in fact rigorously true is quite important to theorists. It is not trivial to test the Pomeranchuk experimentally as can be seen as follows:

$$\sigma_t (\pi^{\pm} p) = a + b_{\pm} \left(\frac{P_o}{P_{lab}} \right)^{m_{\pm}}$$

where the present data up to about $P_{lab} = 25$ BeV/c has been fitted with the parameters

$$a = 22.57 \text{ mb}, \quad b_+ = 24.51 \text{ mb}, \quad b_- = 19.55 \text{ mb}.$$

$$P_o = 1 \text{ BeV/c}, \quad m_+ = 1.02 \quad , \quad m_- = .664$$

$\sigma(\pi - 1)$ - $\sigma(\pi^+)$ becomes less than present errors for each $\sim .1$ mb at $P_{lab} = 4000$ BeV/c or $P_{cm} \sim 45$ BeV/c.

It has been suggested that cross-sections go to zero at high energies like P_{lab}^{-n} where $n \cong 0.07$ (Bond factor). Serpukov should show this because "a" should be down by about 2 mb from 20 to 70 BeV/c.

Lets talk a little more about the interaction range in terms of the impact parameter representation. We might say that this quantity is such that

$$H(b, s) = 0 \quad \text{for} \quad b > R$$

while R of course might depend on s in principle.

Now the crudest model of scattering is to assume that for all $b < R$, the scattering is completely inelastic, so that in terms of

$$H(b, s) = \frac{1}{i} \left[\eta(b, s)\, e^{2i\tilde{\delta}(b, s)} - 1 \right],$$

$\eta(b, s) = 0$ and

$$H(b, s) = +i \quad \text{for} \quad b < R$$

$$= o \quad \text{for} \quad b > R$$

Thus

$$f_{el}(s, t) = iP \int_o^R bdb \quad J_o(b\sqrt{-t})$$

$$_{tot} = \frac{4\pi}{P} \text{ Im } f_{el} \qquad _{tot} = \frac{4\pi}{P} \int_o^R bdb = 2\pi R^2.$$

This is an exceedingly crude model and doesn't fit the data very quantitatively.

Lets turn now to a discussion of elastic scattering. Within the framework of our black disk model,

$$f_{el} = iP \int_o^R bdb \; J_o \; (b\Delta) = iP\, R^2 \; \frac{J_1(R\Delta)}{R\Delta}$$

$$\approx iP\, R^2 \; \frac{1}{2}\, e^{-\frac{1}{8} R^2 \Delta^2}, \text{ provided } \Delta R \ll 1.$$

$$\frac{d\sigma_{el}}{dt} = \pi R^4 \left(\frac{J_1(R\Delta)}{R\Delta}\right)^2 \approx \frac{\pi R^4}{4} e^{+\left(\frac{R}{2}\right)^2 t}$$

$$\sigma_{el} \cong \pi R^2 \text{ for large energies.}$$

Thus a pure imaginary amplitude which decreases exponentially for small t with loagrithemic slope $(R/2)^2$, $\sigma_{el}/\sigma_{tot} = 1/2$; should show dips and bumps - dip $\sim -t = 0.7\ (BeV/c)^2$ bump at $\sim 1\ (BeV/c)^2$ if $R = 10^{-13}$cm. These predictions of the black disk model are not borne out by experiment except in a very crude way. For π - N scattering there are indeed dips ($t \sim .8$) and bumps ($t \sim -1.4$) at rather low energies ($P_\ell \sim 2 - 4$ BeV/c) but at higher energies the structure disappears and for $P_\ell \gtrsim 5$ BeV/c,

$$\frac{d\sigma_{el}}{dt} \Big/ \frac{d\sigma_{el}}{dt}(t=0) = e^{(At + Bt^2)}$$

gives a very good fit for $|t| < 1 - 1.5\ (BeV/c)^2$.

A and B are approximately energy independent and are both negative.

$$A \sim 10\ (BeV/c)^{-2},\quad B/A^2 \lesssim .03.$$

$$\sigma_{el} = \int_{-4P_c^2}^{0} dt \frac{d\sigma_{el}}{dt} \approx \frac{d\sigma_{el}}{dt}(t=0) \cdot \frac{1}{A}$$

If we make the assumotion that the scattering amplitude is pure imaginary, from

$$\frac{d\sigma_{el}}{dt} = \frac{\pi}{P^2} \left| f_{el} \right|^2 \cong A\, \sigma_{el}\, e^{At}$$

we find

$$f_{el} = iP\sqrt{\frac{A\sigma_{el}}{\pi}}\; e^{\frac{1}{2}At}$$

and thus

$$\sigma_t = \frac{4\pi}{P}\;\mathrm{Im}\; f_{el} = \sqrt{16\pi A\sigma_{el}}$$

or

$$A = \frac{\sigma_t^2}{16\pi\sigma_{el}}\;;\quad f_{el} = 4iP\;A\;\frac{\sigma_{el}}{\sigma_t}\; e^{1/2\,At}$$

In terms of these parameters, the quantity H (b, s) which entered out impact paramater representation is

$$H(b,s) = 4\,i\,\frac{\sigma_{el}}{\sigma_t}\; e^{-b^2/2A}$$

It is clear that the wiggles in the simple diffraction model were caused by the sharp edge of our disk and these are quite absent in the present "gaussian" model.

By comparing the disk model with the parameterization we have adopted here, $A = \frac{R}{2}^{2}$, there is now, however, an additional parameter, namely $\sigma_{el}/\sigma_t \leq 1$ which is 1/2 for the disk but experimentally ranges from about 0.3 to 0.15. It is larger for those processes which have the fewst inelastic channels e.g. pp vs. $p\bar{p}$.

There are two important qualifications that must be made in connection with the description of elastic scattering we have given so far: the first is that the scattering amplitudes are not pure imaginary and the second is that spin effects are important and lead to measurable polarization. As is well known, the real part of scattering amplitudes has been measured by Lindenbaum, Yuan and co-workers. The ratio $\mathrm{Re}\, f_{el}(\theta = 0) / \mathrm{Im}\, f_{el} \equiv \gamma < 0.$ ranging from .3 to .1 for all processes; somewhat smaller for πN than for NN or $N\bar{N}$. In the former case the analysis is quite clean and in good agreement with the prediction from the forward dispersion relations physical model.

Polarization has been measured in πp and pp scattering; in the momentum transfer region $|t| < (\mathrm{BeV/c})^2$, they range up to about 20% for πp and to about 0.1 for pp and show considerable structure in the πp case.

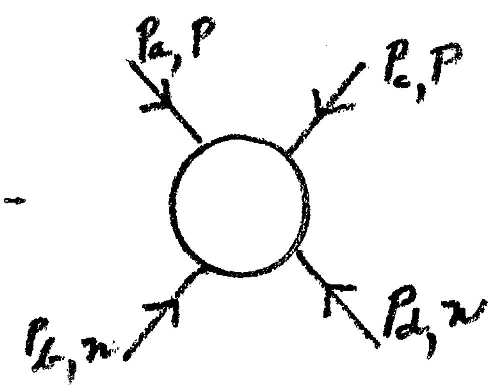
I shall not discuss at this time the inelastic or quasi-two-body reactions, but a table summarizing some of their important properties is included here.

To summarize: important problems to be settled in connection with elastic scattering and total cross-sections are: Do cross-sections tend to constants? Is Pomeranchuk theorem true? Do diffraction peaks shrink?

I want now to turn to the so-called Regge pole model of high energy processes. This treatment will necessarily be rather brief but will hopefully serve as an introduction to Geoff Chew's lectures of next week and remind you of some of the experimental and theoretical problems involved in the model.

We begin by recalling the kinematics of the two-body process

s →



$u + b \rightarrow c + d$

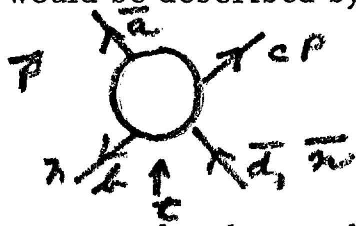
$p + n \rightarrow p + n$, say, which we choose for definiteness.

This is described by an invariant amplitude

$$M\,(P_c,\ P_d\ ;\ P_a,\ P_b) \;=\; M\,(s = (P_a + P_b)^2,\ t = -(P_a - P_c)^2$$

This is called the s-channel reaction. Now consider the process $\bar{n} + n \rightarrow p + \bar{p}$ which we write as $\bar{d} + b \rightarrow c + \bar{u}$ and the momentum conservation is $P_{\bar{d}} + P_b \rightarrow P_c + P_{\bar{a}}$; bars refer to anti-particles. This would be described by an amplitude $\bar{\mu}$ which we would write as

$$\overline{M}\,(P_c\, P_{\bar{a}}\, ;\ P_{\bar{a}},\ P_b)$$



Crossing symmetry says that these two functions are related according to

$$\overline{M}\,(P_c\, P_{\bar{a}}\ ;\ P_{\bar{d}}\, P_b) \;=\; M\,(P_c,\ -P_{\bar{d}}\ ;\ -P_{\bar{a}},\ P_b)$$

The process $\bar{n} + n \rightarrow p + \bar{p}$ is called the t-channel reaction. This is a somewhat subtle relation and is really defined by an analytic continuation in the following way: For the s-channel reaction

$$s \;=\; -(P_a + P_b)^2 \;>\; 4\,m^2$$

$$t \;=\; -2\,P^2\,(1 - Z), \quad -4P^2 < t < 0$$

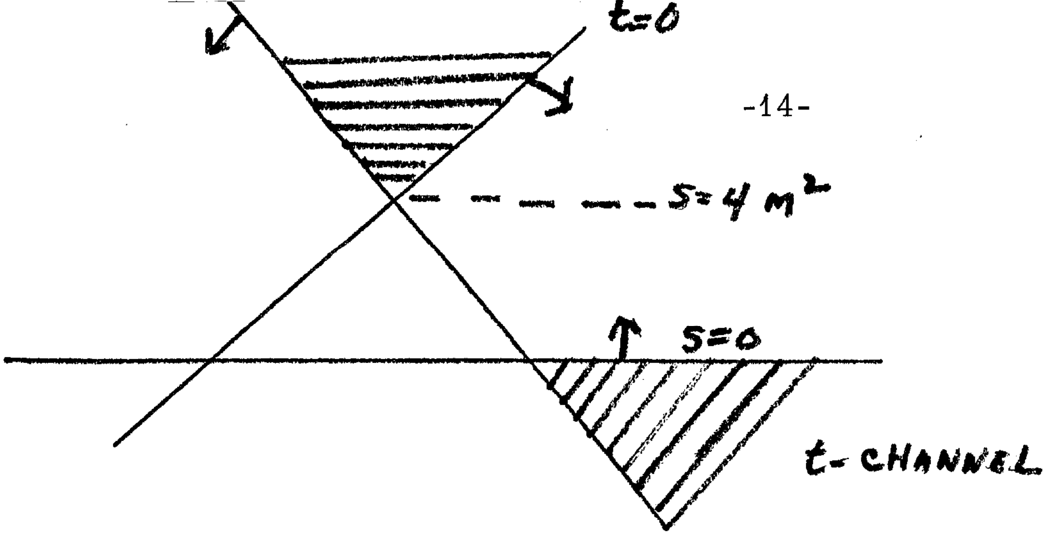
In the t-channel, $t \;=\; -(P_b - P_d)^2 \;\rightarrow\; -(P_b + P_{\bar{d}})^2$

$$= 4\,(P_t^{\,2} + m^2) \;>\; 4\,m^2$$

$$s \;=\; -(P_a + P_b) \;\rightarrow\; -(P_b - P_{\bar{a}})^2 < 4\,P_t^{\,2}$$

$$s \;=\; -(P_c + P_d) \;\rightarrow\; -(P_{\bar{d}} - P_c)^2$$

$$= -2\,P_t^{\,2}\,(1 - \cos\theta_t)$$



We have one function M (s, t) that describes both reactions and we can go from one to another by analytic continuation along some path.

Now in the s-channel region, we may imagine expanding M (s, t) as

$$M(s,t) = \sum (2\ell + 1) \quad f_\ell^{\,s}(s) \; P_\ell \; (\cos \theta)$$

or in the t region

$$M(s,t) = \sum (2\ell + 1) \quad f_\ell^{\,t}(t) \; P_\ell \; (\cos \theta)$$

where

$$f_\ell^{\,t} = \frac{8\pi .\, 2 \sqrt{M^2 + P_t^{\,2}}}{P_t} \qquad \frac{e^{2i\; e(t)} - 1}{2i}$$

In non-relativistic quantum theory it was shown by Regge that the $f_\ell^{\,t}(t)$ could be extended to a function of complex ℓ, $f(\ell, t)$, which coincides with $f_\ell(t)$ for integer ℓ and which has only poles in the complex ℓ - plane, $\ell = \alpha(t)$, which move with t.

i.e.

$$f(\ell, t) = \frac{\beta_i(t)}{\ell - \alpha_i(t)} \qquad + \ldots$$

In relativistic quantum theory there is good reason to believe that there are branch points in the ℓ - plane as well as poles and we'll return to this point later. Now if we expand Re α near a point $t = t_o$,

$$\alpha(t) = \operatorname{Re}\alpha(t_o) + (t - t_o)\operatorname{Re}\alpha'(0) + i \operatorname{Im}\alpha(t_o) + \ldots$$

$$f(\ell, t) = \frac{\beta(t)}{\ell - \operatorname{Re}\alpha(t_o) - \operatorname{Re}\alpha'(0)\left\{t - t_o + i \dfrac{\operatorname{Im}\alpha(t_o)}{\operatorname{Re}\alpha'(t_o)}\right\}}$$

Now if $\operatorname{Re}\alpha(t_o) = \text{integer} = \ell$,

$$f = \frac{\beta / - \operatorname{Re}\alpha'(0)}{t - t_o + i\,\Gamma/2}$$

where

$$\Gamma/2 \equiv \frac{\operatorname{Im}\alpha(t_o)}{\operatorname{Re}\alpha'(t_o)}$$

Breit-Wigner shape.

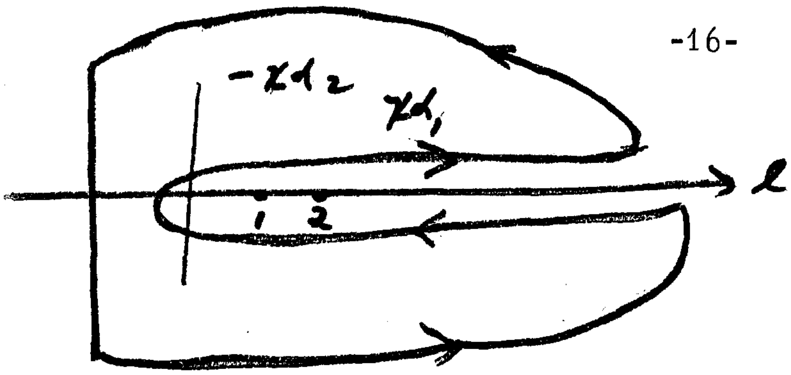
If $\operatorname{Im}\alpha(t_o) = 0$, stable particle or bound state.

It is an article of faith that all particles and resonances, stable or unstable, lie on Regge trajectories $\ell = \alpha_i(t)$.

Now the trick is to use the t-channel partial wave expansion transformed in such a way as to allow us to study, with the aid of the crossing relation, the behavior of the amplitude in the s-channel for large s. To do this we write

$$M(s,t) = \sum (2\ell + 1)\, f(\ell, t)\, P_\ell(\cos\theta)$$

$$= \frac{i}{2}\int d\ell\, \frac{(2\ell + 1)\, f(\ell, t)\, P_\ell(-\cos\theta_t)}{\sin\pi\ell}$$



$(-1)^{\lambda}(\ell-\lambda)$

Write $\cos\theta_t = Z_t$

$$= \frac{i}{2}\int_{-1/2 - i\infty}^{-1/2 + i\infty} d\ell \quad \frac{(2\ell + 1)\ f(\ell, t)}{\sin \pi \ell} \quad P_\ell\,(-Z_t)$$

$$- \sum_i \quad \frac{\pi\,(2\alpha_i + 1)\ \beta_i\,(t)\ \ P\alpha_i\,(-Z_t)}{\sin \pi\alpha_i\,(t)}$$

where we recall $Z_t = 1 + \frac{s}{2P_t^{\,2}}$

The utility of this represenration stems from the fact that the asymptotic form of $P_\alpha\,(Z)$ is $Z^\alpha\,(1 + 0\ 1/Z \sim\ )$ provided $\mathrm{Re}\ \alpha \geqslant -1/2$. It is reasonable to assume that the line integral goes like $Z^{-1/2}$ or like $s^{-1/2}$ for large s and thus will be smaller than the Regge pole contribution. Needless to say in the t channel, $|Z_t| \leqslant 1$ but we are contemplating an analytic continuation from the region $s < 0$ to $s > 4\,M^2$; the poles $\alpha_i\,(t)$ correspond to resonances and bound states in the t channel which thus relate such states to the large s regime. In the physical s-channel region $t < 0$ and $\alpha_i(t)$ is generally assumed to be real.

We write the asymptotic form

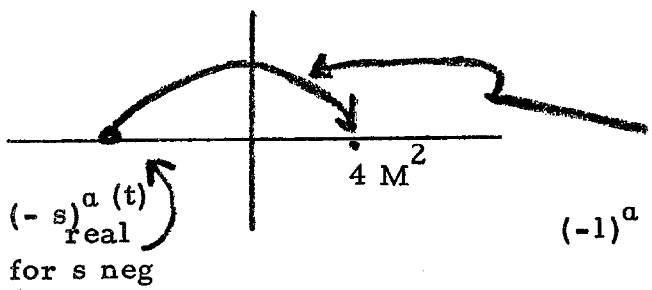
$$M(s,t) \rightarrow \pi \sum \left(\frac{s}{s_o}\right)^{\alpha_i(t)} \frac{\gamma_i(t)}{\sin \pi \alpha_i(t)}$$

where $$\gamma \,\cdot\, (t) = \beta_i(t) \,\cdot \left(\frac{s_o}{2P_t^{\,2}}\right)^{\alpha_i(t)}$$

The reduced residue $\gamma_i(t)$ shares the analyticity of $\alpha_i(t)$ namely of having only a right hand cut.

$$A(s,t) \rightarrow \pi \sum \left(\frac{s}{s_o}\right)^{\alpha_i(t)} e^{-i\pi\alpha_i(t)} \frac{\gamma_i(t)}{\sin \pi \alpha_i(t)}$$

s-plane.



phase decreases by π, hence

$$(-1)^{\alpha} = e^{-i\pi\alpha}$$

There is a refinement of these equations necessitated by the inevitable presence in relativistic quantum theory of what are essentially exchange potentials. One must treat even and odd angular momenta differently:

$$M(s,t) = \sum (2\ell + 1) \left\{ f(\ell,t) \frac{P_\ell(Z) + P_\ell(-Z)}{2} + f(\ell,t) \frac{P_\ell(Z) - P_\ell(-Z)}{2} \right\}$$

The potentials being different for odd and even ℓ means that the Regge poles associated with them are different and we have for the asymptotic form

$$M(s,t) \rightarrow -\pi \sum \frac{\gamma_i^+(t)\,(s/s_o)^{\alpha_i^+}}{\sin \pi \alpha_i^+(t)} \left\{ \frac{e^{-i\pi\alpha_i^+(t)}+1}{2} \right\}$$

$$-\pi \sum \frac{\gamma_i^-(s/s_o)^{\alpha_i^-}}{\sin \pi \alpha_i^-} \left\{ \frac{e^{-i\pi\alpha_i^-(t)}-1}{2} \right\}$$

We refer to these as positive and negative signature (sig $= (-1)^J$ for boson, $(-1)^{J-1/2}$ for fermion).

If $\alpha_i^+ \rightarrow$ odd integer, no pole in amplitude, hence no particle.

$\alpha_i^- \rightarrow$ even integer, no pole in amplitude, hence no particle.

Now suppose the highest trajectory, namely the one which has the largest Re α and hence the one which dominates as $s \rightarrow \alpha$ has positive signature:

$$M(s,t) \rightarrow -\gamma(t)\,\pi \left(\frac{s}{s_o}\right)^{\alpha(t)} \frac{e^{-i\pi\alpha}+1}{2 \sin \pi \alpha(t)}$$

$$= -\frac{\pi\gamma(t)}{2} \left(\frac{s}{s_o}\right)^{\alpha(t)} \left\{ \frac{1+\cos \pi\alpha}{\sin \pi\alpha} - i \right\}$$

$$= -\frac{\pi\gamma(t)}{2} \left(\frac{s}{s_o}\right)^{\alpha(t)} \left\{ \text{tn}\,\frac{\pi\alpha}{2} - i \right\}$$

$$\text{Im } M(s,t) = \frac{\pi\gamma(t)}{2} \left(\frac{s}{s_o}\right)^{\alpha(t)}, \quad \text{Re } M = -\frac{\pi\gamma(t)}{2} \left(\frac{s}{s_o}\right)^{\alpha} \text{tn}\,\frac{\pi\alpha}{2}$$

$$\sigma_t \rightarrow \frac{1}{s}\,\text{Im } M(s,0) = \frac{\pi\gamma(0)}{2 s_o^{\alpha(0)}} \left(\frac{s}{s_o}\right)^{\alpha(0)-1}$$

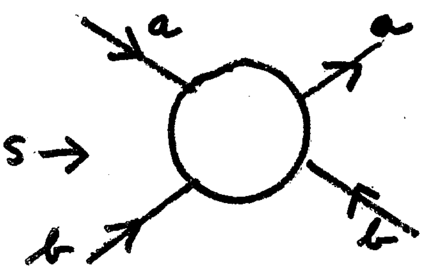
Now to get a total constant cross- section, this highest trajectory must pass through 1 at T = 0, which implies Re M → 0. This is the so-called Pomeranchuk trajectory and is assumed to carry the quantum numbers of the vacuum. The fact that such a trajectory can be exchanged between any two systems will automatically yield constant total cross-sections. Iwe had conjectured odd signature,

$$\frac{e^{-i\pi\alpha} - 1}{\sin \pi\alpha} = \tan \frac{\pi\alpha}{2} - i$$

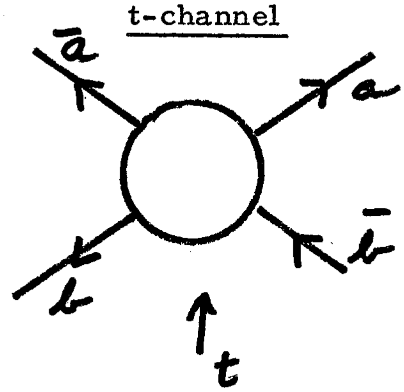
We would have had Re M (s, o) = ∞ if α (0) = 1 which is unacceptable.

Another argument in favor of the even signature for this highest (Pomeranchuk) pole is the following. Consider the two reactions

a + b → a + b

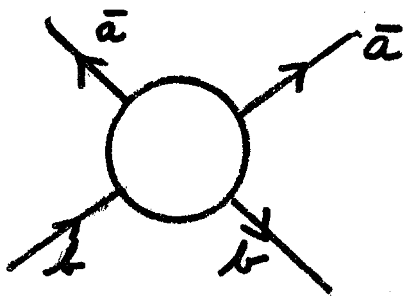


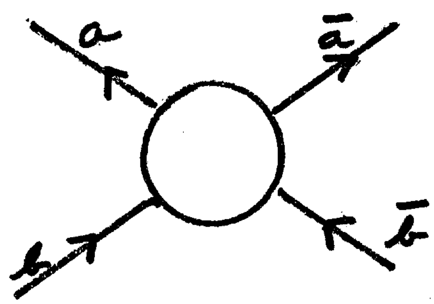
t-channel



$\cos\theta_t \quad \vec{P}_a \cdot \vec{P}_{\bar{b}}$

ā + b → ā + b





$\cos\theta_t = \vec{P}_{\bar{a}} \cdot \vec{P}_{\bar{b}}$

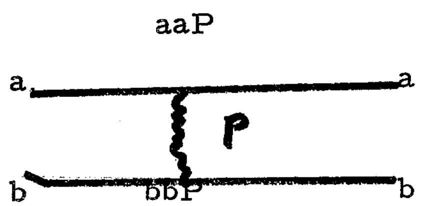
In the second reaction, cos θ_t has the opposite sign from the first; if however the Pomeranchuk trajectory has positive signature, the contribution of such a trajectory to either process will be the same, so that a b and $\bar{a}$ b will be the same since we have $P_\alpha(-\cos\theta_t) + P_\alpha(\cos\theta_t)$, unaffected by $\cos\theta_t \rightarrow \cos\theta_t$.

Notice, incidentally, that a negative signature trajectory makes a contribution equal in magnitude but opposite in sign to a b → a b and $\bar{a}$ b → $\bar{a}$ b. The same is true in any two body process:

$$a + b \rightarrow c + d$$

$$\bar{c} + b \rightarrow \bar{a} + d$$

If we imagine that the asymptotic region is completely described by the exchange of a single Regge pole, then because the residue of a pole must barring accident factor which means that in a process like a + b → a + b we may think of it as



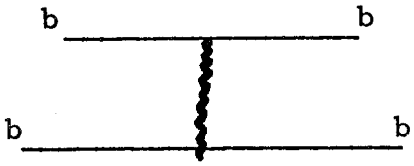
$$\beta_{ab;ab} = \xi_{aaP}\, \xi_{bbP}$$

described by $-\dfrac{\pi\beta_{ab;ab}}{\sin\pi\alpha}\,\dfrac{(e^{-i\pi\alpha}-1)}{2}\left(\dfrac{s}{2P_t q_t}\right)^{\alpha(t)}$

$$P_t^2 = \frac{t}{4} - M_a^2$$

$$q_t^2 = \frac{t}{4} - M_b^2$$

$$\beta_{aa;aa}\left(\frac{s}{P_t^2}\right)^{\alpha(t)}$$



$$\beta_{bb;\,bb}\left(\frac{s}{q_t^2}\right)^{\alpha(t)}$$

$$\frac{\beta_{aa;\,aa}}{P_t^2}\quad\frac{\beta_{bb;\,bb}}{q_t^2}=\left(\frac{\beta_{ab;\,ab}}{P_t q_t}\right)^2$$

$$\sigma\sim\left(\frac{\beta_a}{P^2}\right)_{t=0}\ ,\quad \sigma_{\pi\pi}\ \sigma_{pp}=(\sigma_{\pi P})^2\Rightarrow\sigma_{\pi\pi}\sim 15\text{ mb}$$

Now ask about $\frac{d\sigma_{el}}{dt}$ according to the Regge model:

$$\frac{d\sigma}{dt}=\frac{1}{16\pi s^2}\quad|M|^2$$

$$=g(t)\left(\frac{s}{s_o}\right)^{2(\alpha(t)-1)}$$

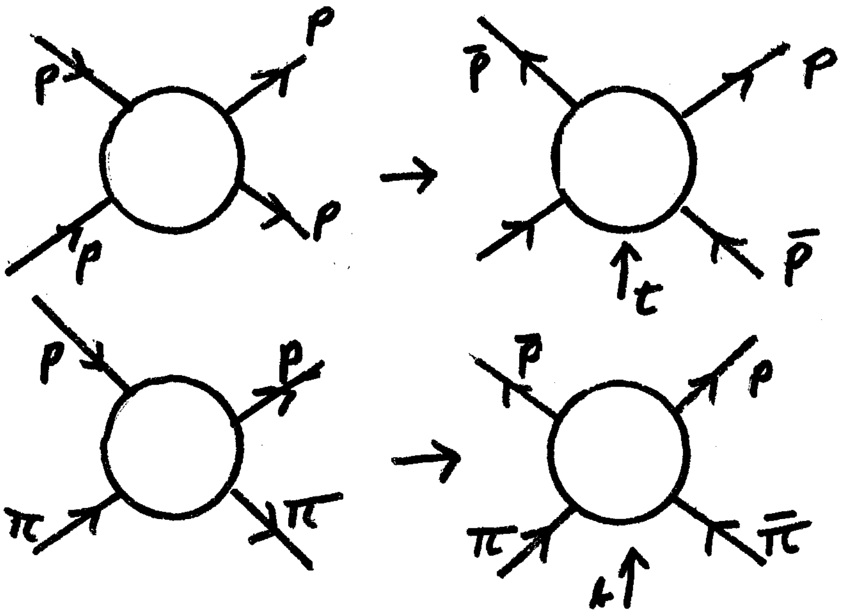
where g (t) is made up of constants and the reduced residue. If $\alpha(t)=1+\alpha'(0)\,t$, we get the famous shrinkage of the diffraction peak:

$$\frac{d\sigma_{el}}{dt}=g(t)\quad e^{2\alpha'(0)\ell n\,(s/s_o)\ .\ t}$$

$$\sigma_{el}=\frac{g(0)}{2\alpha'(0)\,\ell n\,(s/s_o)}$$

Thus another interesting quantity will be $\frac{d\sigma_{el}}{\sigma_+}$ = 1/2 in optical model, .15 to .3 in practice and should slowly approach zero in Regge theory.

The final topic we will consider today is that of how one goes about isolating the Regge trajectories which can contribute to a given process. These depend upon the quantum numbers which can occur in the t-channel. Examples



t channel in $p\bar{p}$

$s = 0, \; B = 0, \; I = 0, 1$

$P, \; \omega \; \pi \; \rho \; \eta \; A_2 \ldots$

t channel has $\pi\pi$

$s = 0, \; B = 0, \; I = 0, \; 1, \; 2$

$G = +1 \quad I = \underline{2}$ is out for $p\bar{p}$

$G = +1$ means only ρ, η, P

Consider states of definite isotopic spin $\mid I, I_3 \rangle$

$$e^{+i\pi I_2} \mid I, I_3 \rangle = e^{-i\pi(I + I_3)} \mid I, -I_3 \rangle$$

Rotation about "2" axis

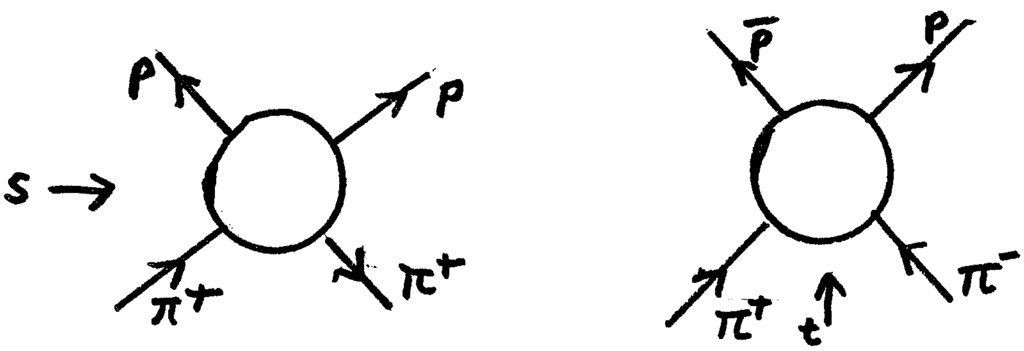
Recall $Q = I_3 + \frac{1}{2}(B + S)$

so if $Q = B = S = 0, \; I_3 = 0.$

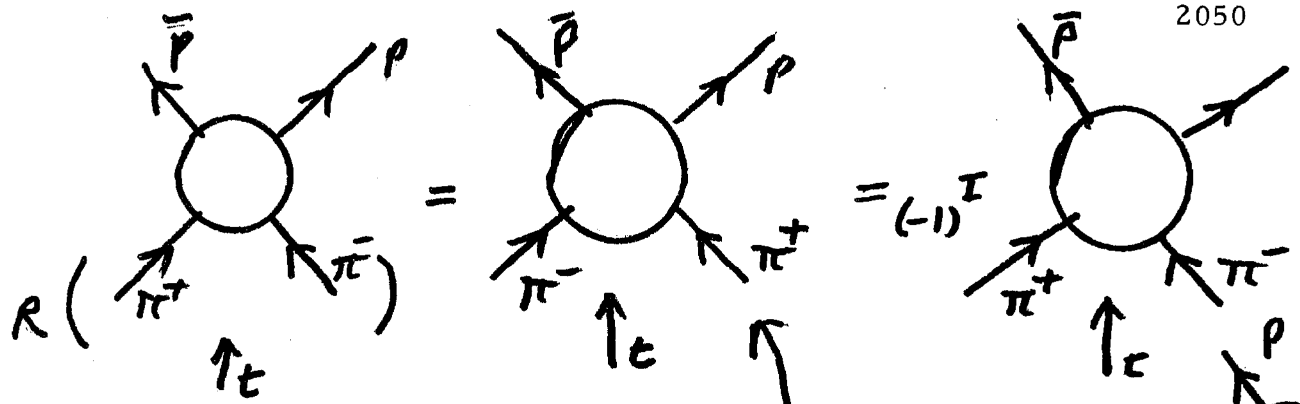
$$R \mid I, 0 \rangle = e^{-i\pi I} \mid I, 0 \rangle$$

$$= (-1)^I \mid I, 0 \rangle .$$

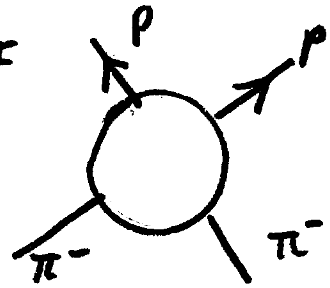
Consider $\pi^+ + p$:



$$R \mid \pi^+ \pi^- \rangle = \mid \pi^- \pi^+ \rangle = (-1)^I \mid \pi^+ \pi^- \rangle$$



From standpoint of s channel, this channel process is



in s-channel.

$$M(\pi^+ p) + M(\pi^-) = (1 + (-1)^I)$$

only I = 0 survives.

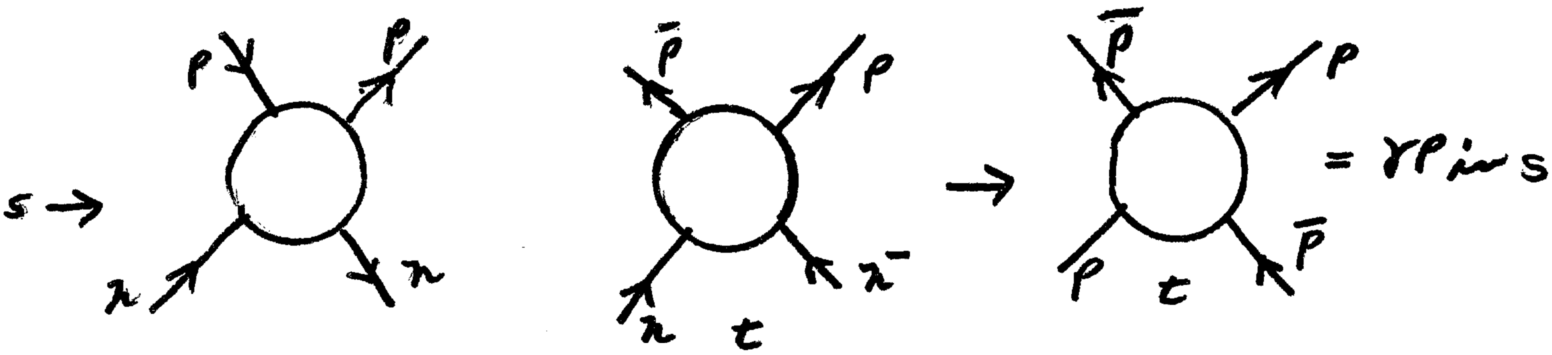
$$M(\pi^+ p) - M(\pi^- p) = (1 - (-1)^I$$

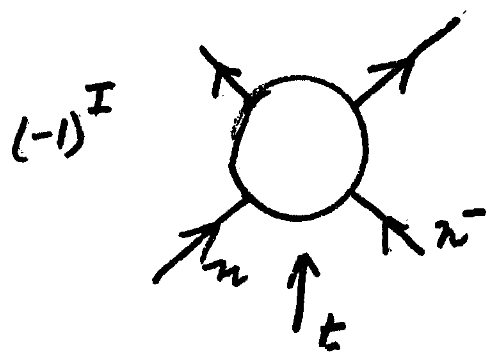
only I = 1 survives.

$$\sigma(\pi^- p \to \pi N) \sim \sigma(\pi^+ p) + \sigma(\pi^- p) = b_P\ S^{a_P - 1} + b_P\ S^{a_{P1} - 1}$$

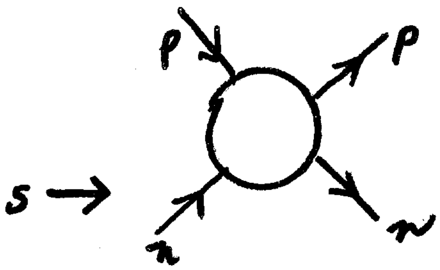
$$\sigma(\pi^+ p) \quad \sigma(\pi^- p) = b_P\ S^{a_P - 1}$$

$\sigma(\pi^{\pm} p)$ have ρ contribution with opposite sign. Similarly considering R on the t channel states of

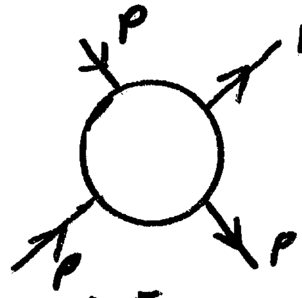


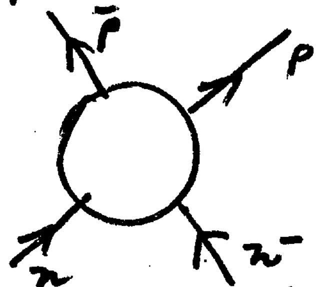


We find



$\pm$





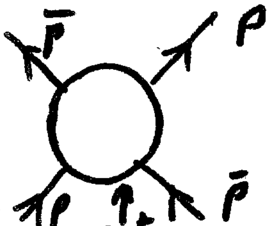
$= [1 \pm (-1)^I]$

Thus

$\sigma(\bar{P}n) - \sigma(\bar{p}p) \sim s^{\alpha_P(0) - 1}$

Use G parity. $R \begin{pmatrix} p \\ n \\ \bar{n} \\ \bar{p} \end{pmatrix} = \begin{pmatrix} -n \\ p \\ \bar{p} \\ -\bar{n} \end{pmatrix}$ and $G = C\,R = \begin{pmatrix} -\bar{n} \\ p \\ p \\ -n \end{pmatrix}$

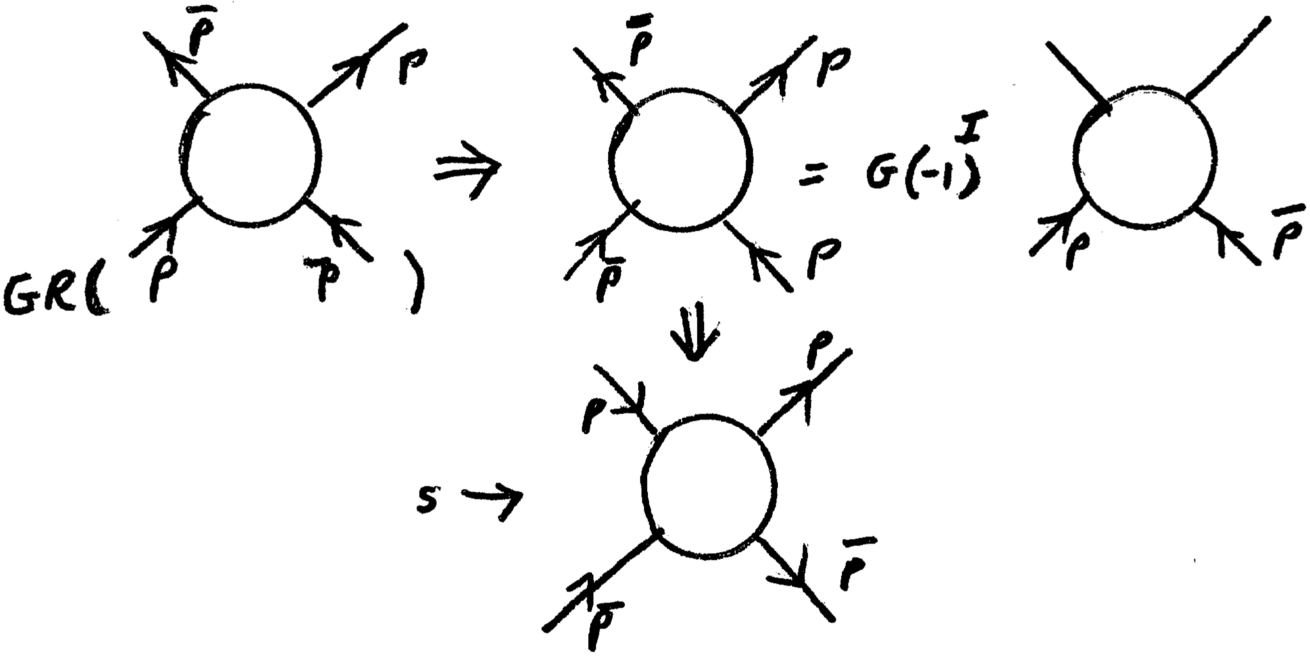
Consider $\bar{p}$ p:



G = $\pm$ 1

$GR\,|\,\bar{p}\,p\, > \; = \; G\,(\bar{n}\,n > \; = \; |\,p\,\bar{p}\; >$

& $GR\,|\,\bar{p}\,p\, > \; = \; G\,(-1)^I\,|\,\bar{p}\,p\; >$



$\therefore$ σ (p p) $\pm$ σ (p p) contains $(1 \pm G(-1)^I)$

t- channel things.

Take σ (p p) - σ (p p) $\therefore$ $G(-1)^I$

G = - 1
I = 0 ω

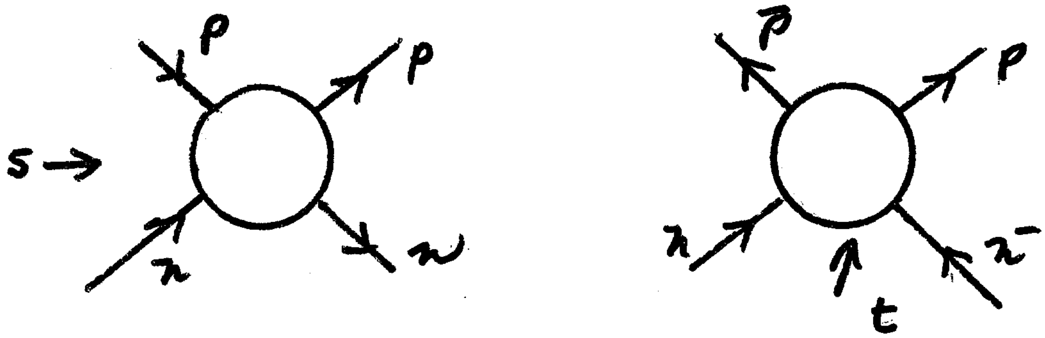
G = + 1
I = 1 ρ ρ

For run $G(-1)^I = +1$

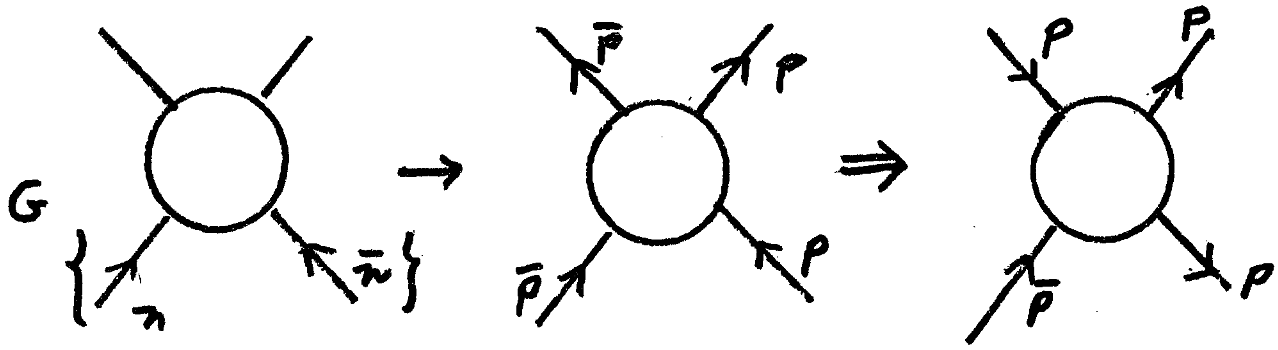
G = + 1, I = 0 pomeron

G = 1, I = 1 A_2

Use just G:



$$G \,|\, n\bar{n} \rangle = |\, \bar{p} p \rangle$$
$$= G \,|\, n\bar{n} \rangle$$



$\therefore$ σ (n p) - σ (p $\bar{p}$) has (1 - G) - t channel things

sum has ρ . ω, π, A_2

(doesn't contribute)

In practice:

$$\sigma(\bar{p}\,p) - \sigma(p\,p) = 2\ \mathrm{Im}\ \omega$$

Thus forward amplitudes:

$$M_{p\,p} = P + P' - \omega - \rho + A_2$$

$$M_{p\,\bar{p}} = P + P' + \omega + \rho + A_2$$

$$M_{p\,n} = P + P' - \omega + p - A_2$$

$$M_{\bar{p}\,n} = P + P' + \omega - p - A_2$$

$$M_{p\,n} \to n\,p = -2\rho + 2A_2$$

$$M_{\bar{p}\,p} \to p\,\bar{p} = 2\rho + 2A_2$$

$\pi^- p \to \rho^o N$

A_2 not good.

Cuts.