FLUCTUATIONS OF ELECTROMAGNETIC FIELDS IN HEAVY ION COLLISIONS

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We perform quantum calculations of fluctuations of the electromagnetic fields in AA collisions at RHIC and LHC energies. We find that in the quantum picture the field fluctuations are much smaller than predictions of the classical Monte-Carlo simulation with the Woods-Saxon nuclear density.

Non-central AA collisions at high energies can generate a very strong magnetic field perpendicular to the reaction plane [1,2]. In this talk I present results of quantum calculations of fluctuations of the electromagnetic fields in AA collisions at RHIC and LHC energies based on the fluctuation-dissipation theorem (FDT) [3]. This issue is very important in the context of the chiral magnetic effect and charge separation [1,4,5]in AA collisions because the fluctuations may partly destroy the correlation between the magnetic field direction and the reaction plane, and can lead to reduction of the $\mathbf{B}$-induced observables [6]. Previously the field fluctuations have been addressed by Monte-Carlo (MC) simulation with the Woods-Saxon (WS) nuclear distribution using the classical Lienard-Weichert potentials [7,8,6]. But the WS nuclear distribution ignores the collective quantum dynamics of the nuclear ground state. The classical treatment of the electromagnetic field may also be inadequate because, similarly to the van der Waals forces [9], it becomes invalid at large distances.

We consider the proper time region $\tau \sim 0.2-1$ fm which is of the most interest for the $\mathbf{B}$-induced effects in the quark-gluon plasma (QGP). We ignore the electromagnetic fields generated by the induced currents in the QGP fireball after interaction of the colliding nuclei [10]. We consider the right moving and left moving nuclei with velocities $\mathbf{V}_R = (0,0,V)$ and $\mathbf{V}_L = (0,0,-V)$, and with the impact parameters $\mathbf{b}_R = (-b/2,0,0)$ and $\mathbf{b}_L = (b/2,0,0)$. We take $z_{R,L} = \pm Vt$. For each nucleus the electromagnetic field is a sum of the mean field and the fluctuating field

$$F^{\mu\nu} = \langle F^{\mu\nu}\rangle + \delta F^{\mu\nu}\,. \tag{1}$$

The mean fields $\langle \mathbf{E}\rangle$ and $\langle \mathbf{B}\rangle$ are given by the Lorentz transformation of the Coulomb field in the nucleus rest frame. For two colliding nuclei the mean magnetic field at $\mathbf{r}=0$ has only y-component. At $t \gg R_A/\gamma$ (here $\gamma = 1/\sqrt{1-V^2}$ is the Lorentz factor, R_A is the nucleus radius) in the region $\rho \ll t\gamma$ $\langle B_y(t,\boldsymbol{\rho},z=0)\rangle$ takes a simple ρ-independent form

$$\langle B_y(t,\boldsymbol{\rho},z=0)\rangle \approx Zeb/\gamma^2 t^3\,. \tag{2}$$

The contribution of each nucleus to the correlators of the electromagnetic fields in the lab-frame may be expressed via the correlators in the nucleus rest frame. For $\gamma \gg 1$ the dominating fluctuations in the lab-frame are the ones of the transverse fields. The transverse components of the correlators of the electric and magnetic fields can be written as

$$\langle\delta E_i\delta E_k\rangle = \gamma^2\left[\langle\delta E_i\delta E_k\rangle + V^2 e_{3il}e_{3kj}\langle\delta B_l\delta B_j\rangle\right]_{rf}, \tag{3}$$

$$\langle\delta B_i\delta B_k\rangle = \gamma^2\left[\langle\delta B_i\delta B_k\rangle + V^2 e_{3il}e_{3kj}\langle\delta E_l\delta E_j\rangle\right]_{rf}, \tag{4}$$

where i,k are the transverse indices and the subscript rf on the right-hand side of (3), (4) indicates that the correlators are calculated in the nucleus rest frame.

In calculations of the rest frame correlators $\langle\delta E_l\delta E_j\rangle$, $\langle\delta B_i\delta B_k\rangle$ (hereafter we drop the subscript rf) with the help of the FDT we follow the formalism of [11] (formulated in the gauge $\delta A^0=0$). It allows to relate the time Fourier component of the vector potential correlator

$$\langle\delta A_i(\mathbf{r}_1)\delta A_k(\mathbf{r}_2)\rangle_\omega = \frac{1}{2}\int dt e^{i\omega t}\langle\delta A_i(t,\mathbf{r}_1)\delta A_k(0,\mathbf{r}_2)+\delta A_k(0,\mathbf{r}_2)\delta A_i(t,\mathbf{r}_1)\rangle \tag{5}$$

and that of the retarded Green's function

$$D_{ik}(\omega,\mathbf{r}_1,\mathbf{r}_2) = -i\int dt e^{i\omega t}\theta(t)\langle\delta A_i(t,\mathbf{r}_1)\delta A_k(0,\mathbf{r}_2)-\delta A_k(0,\mathbf{r}_2)A_i(t,\mathbf{r}_1)\rangle\,. \tag{6}$$

In the zero temperature limit the FDT relation between (5) and (6) reads [11]

$$\langle\delta A_i(\mathbf{r}_1)\delta A_k(\mathbf{r}_2)\rangle_\omega = -\mathrm{sign}(\omega)\mathrm{Im}D_{ik}(\omega,\mathbf{r}_1,\mathbf{r}_2). \tag{7}$$

The time Fourier components of the electromagnetic field correlators in terms of that for the the vector potential correlator (5) are given by

$$\langle\delta E_i(\mathbf{r}_1)\delta E_k(\mathbf{r}_2)\rangle_\omega = \omega^2\langle\delta A_i(\mathbf{r}_1)\delta A_k(\mathbf{r}_2)\rangle_\omega\,, \tag{8}$$

$$\langle\delta B_i(\mathbf{r}_1)\delta B_k(\mathbf{r}_2)\rangle_\omega = \mathrm{rot}^{(1)}_{il}\mathrm{rot}^{(2)}_{kj}\langle\delta A_l(\mathbf{r}_1)\delta A_j(\mathbf{r}_2)\rangle_\omega. \tag{9}$$

In the time region of interest ($t \gtrsim 0.2$ fm in the lab-frame) for each nucleus the distance between the observation point and the center of the nucleus (in its rest frame) is much bigger than R_A. It allows one to treat each nucleus as a point like dipole described by the dipole polarizability $\alpha_{ik}(\omega)$. The field fluctuations are described by correction to the retarded Green's function proportional to the dipole polarizability [11]. The retarded Green's function coincides with the Green's function of Maxwell's equation [11]. For the point like dipole at $\mathbf{r}=\mathbf{r}_A$ the equation for the retarded Green's function reads

$$\left[\frac{\partial^2}{\partial x_i\partial_l} - \delta_{il}\triangle - \delta_{il}\omega^2 - 4\pi\omega^2\alpha_{il}(\omega)\delta(\mathbf{r}-\mathbf{r}_A)\right]D_{lk}(\omega,\mathbf{r},\mathbf{r}') = -4\pi\delta_{ik}\delta(\mathbf{r}-\mathbf{r}')\,. \tag{10}$$

The correction to D_{ik} due to α_{ik} reads [11]

$$\Delta D_{ik}(\omega,\mathbf{r}_1,\mathbf{r}_2) = -\omega^2 D^v_{il}(\omega,\mathbf{r}_1,\mathbf{r}_A)\alpha_{lm}(\omega)D^v_{mk}(\omega,\mathbf{r}_A,\mathbf{r}_2)\,, \tag{11}$$

where D^v_{ik} is the vacuum Green's function given by

$$D^v_{ik}(\omega,\mathbf{r}_1,\mathbf{r}_2) = \frac{e^{i\omega r}}{r}\left[-\delta_{ik}\left(1+\frac{i}{\omega r}-\frac{1}{\omega^2 r^2}\right)+\frac{x_i x_k}{r^2}\left(1+\frac{3i}{\omega r}-\frac{3}{\omega^2 r^2}\right)\right] \tag{12}$$

with $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$.

For spherical nuclei the polarizability tensor can be written as $\alpha_{ik}(\omega)=\delta_{ik}\alpha(\omega)$. $\alpha(\omega)$ is an analytical function of ω in the upper half-plane [9]. It satisfies the relation $\alpha^*(-\omega^*)=\alpha(\omega)$ [9]. It means that on the upper imaginary axis $\alpha(\omega)$ is real. Using this fact, one can express the rest frame field correlators $\langle \delta E_i(t,\mathbf{r}) \delta E_k(t,\mathbf{r}) \rangle$, $\langle \delta B_i(t,\mathbf{r}) \delta B_k(t,\mathbf{r}) \rangle$ via integrals of the type $I_n = \int_0^\infty d\xi \xi^n e^{-\xi} \alpha\left(\frac{i\xi}{2r}\right)$ with $n = 0-4$ [12].

The function $\alpha(\omega)$ reads [9]

$$\alpha(\omega) = \frac{1}{3} \sum_s \left[\frac{|\langle 0|\mathbf{d}|s\rangle|^2}{\omega_{s0} - \omega - i\delta} + \frac{|\langle 0|\mathbf{d}|s\rangle|^2}{\omega_{s0} + \omega + i\delta} \right] , \tag{13}$$

where $\mathbf{d} = \left(eN \sum_p \mathbf{r}_p - eZ \sum_n \mathbf{r}_n \right) / A$ is the dipole operator. At $\omega > 0$ the imaginary part of $\alpha(\omega)$ is connected with the dipole photoabsorption cross section

$$\sigma_{abs}(\omega) = 4\pi\omega \mathrm{Im}\alpha(\omega) . \tag{14}$$

For heavy nuclei the dipole strength is dominated by the giant dipole resonance (GDR) [13]. It appears as a broad peak in σ_{abs} at $\omega \sim 14$ MeV. We parametrize the dipole polarizability for ^{197}Au and ^{208}Pb nuclei by a single GDR state

$$\alpha(\omega) = c \left[\frac{1}{\omega_{10} - \omega - i\Gamma/2} + \frac{1}{\omega_{10} + \omega + i\Gamma/2} \right] . \tag{15}$$

By fitting the data on the photoabsorption cross section from [14] for ^{197}Au and from [15] for ^{208}Pb we obtained the following values of the parameters: $\omega_{10} \approx 13.6$ MeV, $\Gamma \approx 4.38$ MeV, $c \approx 18.2$ GeV^{-2} for ^{197}Au, and $\omega_{10} \approx 13.3$ MeV, $\Gamma \approx 3.72$ MeV, $c \approx 18.93$ Gev^{-2} for ^{208}Pb. Fig. 1 illustrates the quality of our fit. Using these parameters we calculated the fluctuations of the nuclear dipole moment. From (13), (15) one can obtain

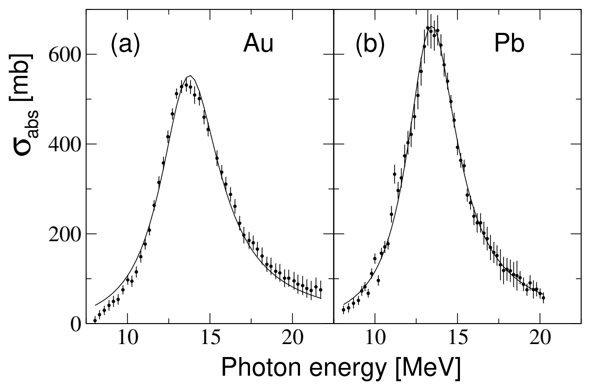

Figure 1 – Fit of the photoabsorption cross section in the GDR region to the experimental data for ^{197}Au and ^{208}Pb targets. The data are from Refs.[14] and [15], respectively.

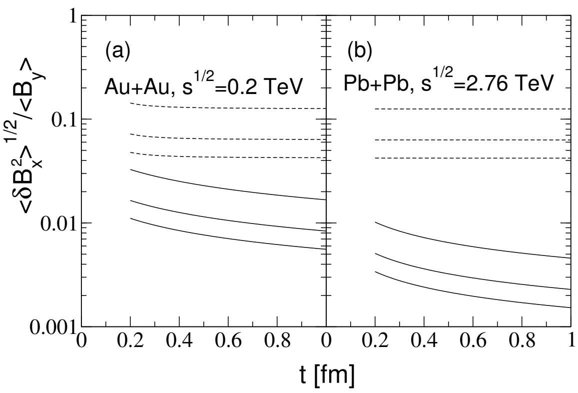

Figure 2 – The t-dependence of the ratio $\langle \delta B_x^2 \rangle^{1/2} / \langle B_y \rangle$ at $\mathbf{r} = 0$ for Au+Au collisions at $\sqrt{s} = 0.2$ TeV (left) and for Pb+Pb collisions at $\sqrt{s} = 2.76$ TeV for the impact parameters $b = 3$, 6 and 9 fm (from top to bottom). Solid lines are for quantum calculations, dashed lines for classical MC calculations with the WS nuclear density.

$$\langle 0|\mathbf{d}^2|0\rangle = \frac{3}{\pi} \int_0^\infty d\omega \mathrm{Im}\alpha(\omega) = \frac{6c}{\pi} \mathrm{arctg}\left(2\omega_{10}/\Gamma\right) . \tag{16}$$

This formula gives $\langle 0|\mathbf{d}^2|0\rangle \approx 1.91$ fm^2 and $\langle 0|\mathbf{d}^2|0\rangle \approx 2.02$ fm^2 for ^{197}Au and ^{208}Pb, respectively. The classical MC calculation with the WS nuclear density gives for these nuclei the values $\langle \mathbf{d}^2 \rangle \approx 9.89$ fm^2 and $\langle \mathbf{d}^2 \rangle \approx 10.39$ fm^2. Thus, we see that the classical treatment overestimates the dipole moment squared by a factor of ~ 5.

At the center of the plasma fireball the fluctuations of the direction of the magnetic field are dominated by the fluctuations of the component B_x that vanishes without fluctuations. In Fig. 2 we show our quantum and classical results for t-dependence of the ratio $\langle \delta B_x^2 \rangle^{1/2} / \langle B_y \rangle$

at $x = y = 0$ for several impact parameters for Au+Au collisions at $\sqrt{s} = 0.2$ TeV and Pb+Pb collisions at $\sqrt{s} = 2.76$ TeV. This figure shows that the quantum treatment gives $\langle \delta B_x^2 \rangle^{1/2} / \langle B_y \rangle$ smaller than the classical one by a factor of $\sim 5 - 8$ for RHIC and by a factor of $\sim 13 - 27$ for LHC. Thus, we see that in the quantum picture both for RHIC and LHC fluctuations of the direction of the magnetic field relative to the reaction plane should be very small. Of course, experimentally the reaction plane itself cannot be determined exactly. In the event-by-event measurements the orientation of the reaction plane is extracted from the elliptic flow in the particle distribution [16,17] (it is often called the participant plane), and it fluctuates around the real reaction plane. Calculations of the fluctuations of the direction of the magnetic field relative to the participant plane require a joint analysis of the field fluctuations and of the fluctuations of the initial entropy deposition that control the fluctuations of the orientation of the participant plane in the hydrodynamical simulations of AA collisions. The initial entropy distribution is sensitive to the long range fluctuations of the nuclear density. Besides the nuclear fluctuations related to the GDR there are other collective nuclear modes [13] such as the giant monopole resonance and the giant quadrupole resonance that may also be important for the participant plane fluctuations. It would be of great interest to clarify the situation with the MC simulation with the WS nuclear density for these collective modes. This is of great interest for the event-by-event hydrodynamic simulations of AA collision.

In summary, we have performed a quantum analysis of fluctuations of the electromagnetic field in AA collisions at RHIC and LHC energies. Our quantum calculations show that the field fluctuations are very small. We have demonstrated that the classical picture overestimates strongly the field fluctuations.

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