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Chromaticity Correction for the Collider at the SSC

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1 INTRODUCTION

Control of the chromaticity is important in order to avoid collective instabilities like the head-tail effect and to avoid crossing resonances. This is specially so for the collider where the linear chromaticity is spectacularly large, e.g reaching -470 units in the vertical plane at collision, for the design values of the luminosity at the four interaction points. Correcting the linear chromaticity is relatively simple however, and does not require much discussion. In this paper, we present the proposed scheme for reducing the non-linear chromaticity of the collider in the collision mode. Section 2 discusses briefly the optics of the collider lattice with special emphasis on the Interaction Regions (IRs) which are the main sources of chromaticity. Section 3 is devoted to a theoretical calculation of the second order chromaticity in a storage ring. Section 4 focuses on the second order tune shift due to the triplets, the dominant sources of chromaticity in the IRs. Section 5 discusses, mostly in pictures, the theory behind our proposed scheme for correcting higher order chromaticity. In Section 6 we evaluate the chromatic performance of the proposed scheme and in Section 7 we discuss the effect of the chromaticity correcting sextupoles on the dynamic aperture. We summarize our conclusions in Section 8.

2 THE COLLIDER LATTICE: SOURCES OF CHROMATICITY

The racetrack shaped collider lattice consists basically of 2 arcs located on the North and South sides and 2 clusters placed on the West and on the East. Each arc contains 196 identical FODO cells with the phase advance across a cell being 90 degrees and the length of each cell is 180 m. The lattice of each cluster includes 2 Interaction Regions, the utility section and the interconnect sections between them. It is intended that the IRs have similar configuration except for the free space length reserved for the detectors.

The arcs occupy about 81% of the lattice and therefore provide a significant contribution to the chromaticity of the machine. They mostly determine the collider chromaticity at injection conditions. However, in the collision mode, the IRs have a larger chromaticity than the arcs. This additional chromaticity comes from the final focussing quadrupoles which at collision are located in the region of extremely high values of the beta functions. The locality of the chromaticity sources in the IRs makes them more dangerous than the chromaticity contributed by the arcs.

The general scheme of an IR is shown on Figure 1. F and D quadrupoles are drawn above and under the beam lines, and the vertical bends are centered on the beam lines. The two rings are separated vertically by a distance of 90 cm everywhere except in the IRs. Within each IR, the beams are brought into collision at the Interaction Point (IP) in two steps by a set of vertical dipoles.

The optics of the IR consists of three main parts. The quadrupoles located in the region where the separation is 90cm form the tuning section. These quadrupoles are used to match the

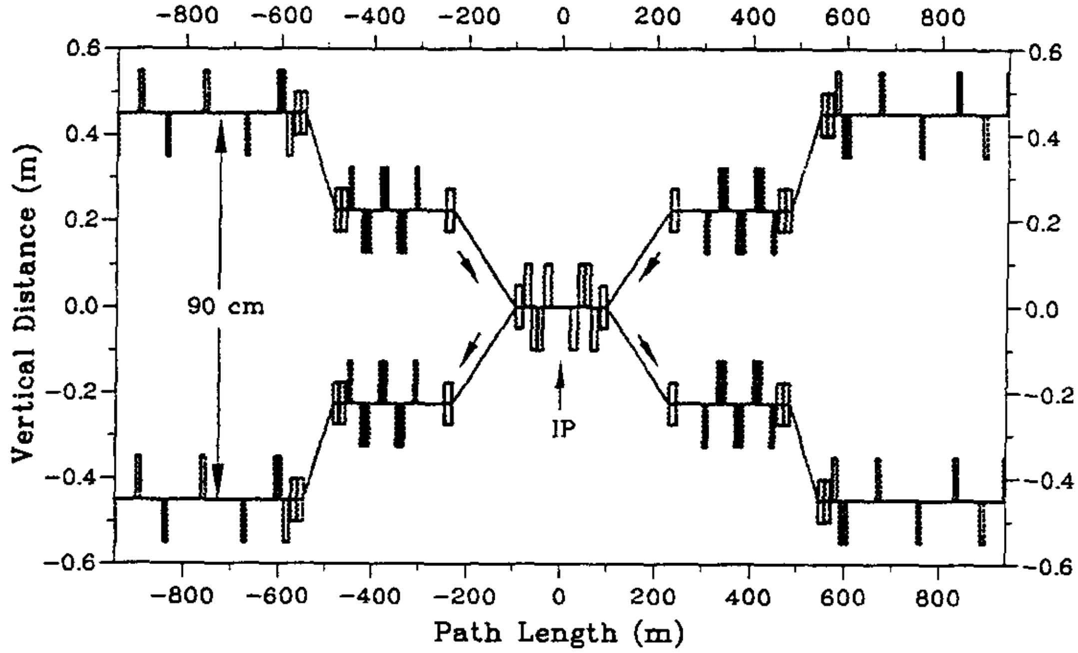

Figure 1: Elevation View of Low Beta IR.

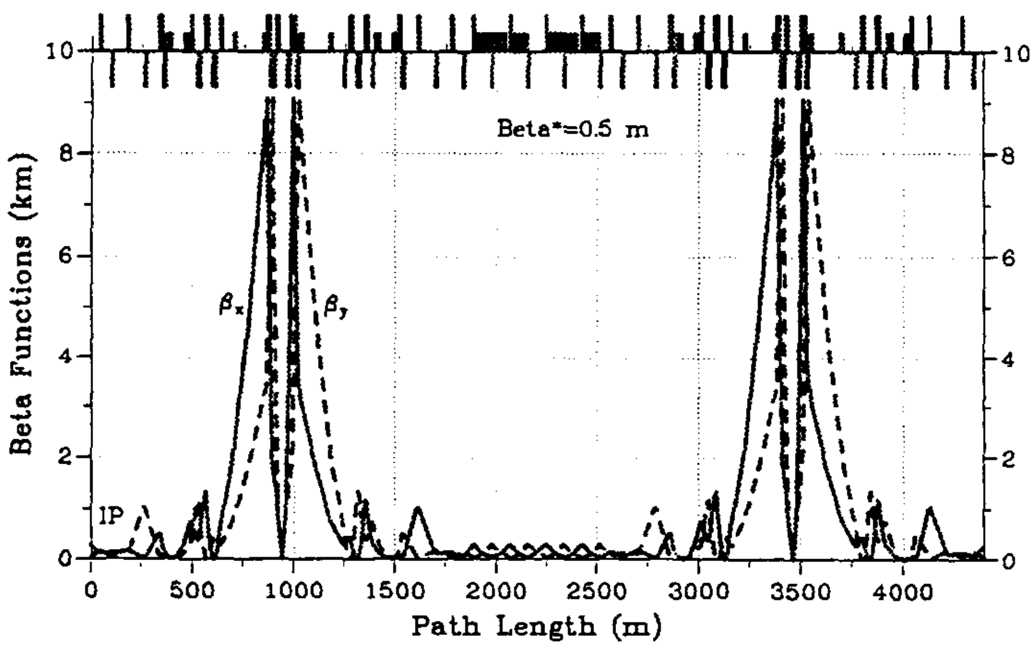

Figure 2: Beta Functions in the Low Beta IRs.

optics and vary the β^* at the IP. The second part consists of the vertical dipoles and the quadrupoles located in the 45 cm separation region. The dipoles are used to bring the beams into collision. The phase advance across the quadrupoles is π and collectively the quadrupoles form the M=−I section, whose role is to correct the vertical dispersion locally. The last part of the optics includes the final triplet quadrupoles which focus the beams to the IP. These quads are common to both rings and the beams share the same beam pipe inside them. At high luminosity conditions the beta functions go up rapidly inside the triplets which become a source of a large chromaticity. Figure 2 shows the particular beta functions in 2 IRs for the design value of β^*=0.5 m. Each IP is surrounded by 2 triplets where the beta function is 30 times larger than in the arcs. The contribution to the 1st order chromaticity from major sources is listed in Table 1. L* denotes the free space reserved on either side of each IP for the detectors.

SOURCE	horizontal chromaticity	vertical chromaticity
2 arcs	-124	-123
1 low β IR, L^*=20.5m, β^*=0.50m 1 low β IR, L^*=20.5m, β^*=0.25m	-51 -101	-51 -101
1 medium β IR, L^*=90m, β^*=1.95m	-45	-45
Maximum allowed sextupole component in dipoles ($b_2=0.8*10^{-4}$ m^{-2})	160	-136
Complete collider lattice: 2 low β IRs (β^*=0.50m) 2 medium β IRs (β^*=1.95m) $b_2=0.8*10^{-4}$ m^{-2} in dipoles	-171	-469

Table 1: Major sources of chromaticity in the collider lattice

3 TUNE SHIFT AND CHROMATICITY TO 2nd ORDER

Consider a storage ring and label two points on it as 1 and 2. Let μ_o be the global phase advance around the ring and (β_1, α_1, γ_1) the Twiss functions at point 1. The periodic transfer matrix at point 1 is

$$M_1 = \begin{bmatrix} \cos\mu_o + \alpha_1 \sin\mu_o & \beta_1 \sin\mu_o \\ -\gamma_1 \sin\mu_o & \cos\mu_o - \alpha_1 \sin\mu_o \end{bmatrix} \equiv M(2 \rightarrow 1) \times M(1 \rightarrow 2) \qquad (3.1)$$

where M (2 –> 1) is the transfer matrix from point 2 to 1 etc. Let μ_1 and μ_2 be the phase advances at points 1 and 2 respectively with respect to an arbitrary reference point and define $\mu_{21} = | \mu_2 - \mu_1 |$. We now introduce two infinitesimally thin quads of strengths $q_1 = k_1 \Delta s_1$ and $q_2 = k_2 \Delta s_2$ at points 1 and 2 respectively. Their perturbations to the transfer matrix are described by the matrices

$$P_1 = \begin{bmatrix} 1 & 0 \\ -k_1 \Delta s_1 & 1 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 1 & 0 \\ -k_2 \Delta s_2 & 1 \end{bmatrix} \tag{3.2}$$

These quad errors change the cyclic transfer matrix at point 1 to $\overline{M}_1$

$$\overline{M}_1 = M(2 \rightarrow 1) \times P_2 \times M(1 \rightarrow 2) \times P_1 \equiv M_1 + \Delta M_1 \tag{3.3}$$

The elements of ΔM_1 (the change in the transfer matrix) are found using this equality. Let $\Delta\mu$ be the change in the global phase advance around the ring. We scale the quad errors by an arbitrary parameter ε i.e.

$k_1 \to \varepsilon k_1 \qquad\qquad k_2 \to \varepsilon k_2$

and expand $\Delta\mu$ as a power series in ε,

$$\Delta\mu = \varepsilon\Delta\mu_1 + \varepsilon^2\Delta\mu_2 + \ldots \tag{3.4}$$

In keeping with our declared aim, our calculation will be to second order in ε. The new global phase advance $\overline{\mu}_o = \mu_o + \Delta\mu$ is to be found from

$$\cos\overline{\mu}_o = \frac{1}{2} Tr \overline{M}_1 = \cos\mu_o + \frac{1}{2} Tr \Delta M_1 \tag{3.5}$$

We also have

$$\cos\overline{\mu}_o = \cos\mu_o \cos\Delta\mu - \sin\mu_o \sin\Delta\mu$$

Substituting Equation (3.4) into the above and equating it to the expression for $\cos\overline{\mu}_o$ given by Equation (3.5), we have

$$-\varepsilon\sin\mu_o\Delta\mu_1 - \varepsilon^2(\sin\mu_o\Delta\mu_2 + \frac{1}{2}\cos\mu_o(\Delta\mu_1)^2) + O(\varepsilon^3) = \frac{1}{2} Tr\Delta M_1 \tag{3.6}$$

To obtain the corrections to μ_o order by order, we equate the coefficients of like powers of ε on both sides of the above equation.

$$\Delta\mu_1 = \frac{1}{2}(k_1\Delta s_1\beta_1 + k_2\Delta s_2\beta_2)$$

$$\Delta\mu_2 = \frac{1}{4\sin\mu_o}k_1\Delta s_1\beta_1 k_2\Delta s_2\beta_2\,[\cos\mu_o - \cos(\mu_o - 2\mu_{21})] - \frac{\cot\mu_o}{2}(\Delta\mu_1)^2 \tag{3.7}$$

Generalizing to the case when there are N quad errors distributed around the ring,

$$\Delta\mu_1 = \frac{1}{2}\sum_{i=1}^{N} k_i\beta_i\Delta s_i$$

$$\Delta\mu_2 = \frac{1}{4\sin\mu_o}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(k_i\beta_i\Delta s_i)\,(k_j\beta_j\Delta s_j)\,[\cos\mu_o - \cos(\mu_o - 2\mu_{ji})]$$

$$-\frac{\cot\mu_o}{2}(\Delta\mu_1)^2 \tag{3.8}$$

We go now to the limit of infinitesimally thin quads distributed around the ring of circumference C. In this limit,

$$\Delta\mu_1 = \frac{1}{2}\int_0^C k(s)\,\beta_o(s)\,ds$$

$$\Delta\mu_2 = \frac{1}{4\sin\mu_o}\int_0^C k(s)\,\beta_o(s)\,ds\int_s^C k(s')\,\beta_o(s')\,[\cos\mu_o - \cos(\mu_o - 2|\mu(s') - \mu(s)|)]\,ds'$$

$$-\frac{\cot\mu_o}{2}(\Delta\mu_1)^2 \tag{3.9}$$

Here we have let $\beta_o(s)$ denote the unperturbed β function at the point s. In the equation for $\Delta\mu_2$ we convert the integral over part of the ring to one over the complete ring and obtain

$$\Delta\mu_2 = -\frac{1}{8\sin\mu_o}\int_0^C k(s)\,\beta_o(s)\,ds\int_s^{s+C} k(s')\,\beta_o(s')\cos[\mu_o - 2|\mu(s') - \mu(s)|]\,ds' \tag{3.10}$$

The gradient error changes not only the tune of the machine but also the β function around the ring. We recall that the change in β to first order in the gradient errors is given by [1]:

$$\frac{\Delta\beta_1(s)}{\beta_o(s)} = -\frac{1}{2\sin\mu_o}\int_s^{s+C} k(s')\,\beta_o(s')\cos[\mu_o - 2|\mu(s') - \mu(s)|]\,ds' \qquad (3.11)$$

Recognizing that the term on the right hand side occurs within Equation (3.10) we obtain the rather simple expression

$$\Delta\mu_2 = \frac{1}{4}\int_0^C k(s)\,\Delta\beta_1(s)\,ds \qquad (3.12)$$

This important relation tells us that the first order distortion in the β function propagating around the machine gives rise to the second order tune shift. The total phase shift to second order in the gradient errors is (after putting the arbitrary parameter ε to unity),

$$\Delta\mu = \frac{1}{2}\int_0^C k(s)\,\beta_o(s)\,ds + \frac{1}{4}\int_0^C k(s)\,\Delta\beta_1(s)\,ds + O(k^3) \qquad (3.13)$$

The gradient perturbations of interest here are those seen only by particles off the design momentum. The chromatic error introduced by the quads is corrected by placing sextupoles at places of non-zero dispersion. Assuming that only the horizontal dispersion D_x is non-zero, the effective quadrupole strength in the horizontal plane for a particle with relative momentum deviation $\delta = \Delta p/p_o$ is,

$$K^{eff} = K(s,\delta) + S(s,\delta)\,D(s,\delta)\,\delta \qquad (3.14)$$

As functions of δ,

$$K(s,\delta) = \frac{K_o(s)}{1+\delta} = K_o(s)\,[1 - \delta + \delta^2 + \ldots]$$

$$S(s,\delta) = \frac{S_o(s)}{1+\delta} = S_o(s)\,[1 - \delta + \delta^2 + \ldots] \qquad (3.15)$$

where K_o and S_o are the nominal quad and sextupole strengths experienced by a particle on momentum. The gradient errors also change the dispersion function around the ring, making it a function of δ as well. We expand D as a power series in δ,

$$D(s,\delta) = D_o(s) + \Delta D_1^C(s)\,\delta + \Delta D_2^C(s)\,\delta^2 + \ldots \qquad (3.16)$$

and similarly the β function,

$$\beta(s,\delta) = \beta_o(s) + \Delta\beta_1^C(s)\,\delta + \Delta\beta_2^C(s)\,\delta^2 + \ldots \tag{3.17}$$

where the superscript C denotes a chromatic expansion. Hence the gradient error in the horizontal plane for the off-momentum particles is

$$k(s) \equiv K^{eff} - K_o = [-K_o + S_o D_o]\,\delta + [K_o + S_o(-D_o + \Delta D_1^C)]\,\delta^2 + O(\delta^3) \tag{3.18}$$

Substituting into Equation (3.13) and writing the tune shift in terms of the first and second order chromaticity ξ_1 and ξ_2 respectively,

$$\Delta\nu \equiv \frac{1}{2\pi}\Delta\mu = \xi_1\delta + \xi_2\delta^2 + O(\delta^3) \tag{3.19}$$

we obtain

$$\xi_1 = \frac{1}{4\pi}\int_0^C \beta_o(s)\,[-K_o + S_o(s)\,D_o(s)]\,ds$$

$$\xi_2 = \frac{1}{8\pi}\int_0^C [-K_o + S_o(s)\,D_o(s)]\,\Delta\beta_1^C(s)\,ds + \frac{1}{4\pi}\int_0^C \beta_o(s)\,S_o(s)\,\Delta D_1^C(s)\,ds - \xi_1 \tag{3.20}$$

We recall that the first order changes in β and D are given by

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{-1}{2\sin\mu_o}\int_s^{s+C} \beta_o(s')\,[-K_o(s') + S_o(s')\,D_o(s')]\cos[\mu_o - 2|\mu(s') - \mu(s)|]\,ds'$$

$$\Delta D_1^C(s) = \frac{-\sqrt{\beta_o(s)}}{2\sin\frac{\mu_o}{2}}\int_s^{s+C} \sqrt{\beta_o(s')}\,D_o(s')\,[-K_o(s') + S_o(s')\,D_o(s')]\cos\left[\frac{\mu_o}{2} - |\mu(s') - \mu(s)|\right]ds' \tag{3.21}$$

Ignoring the phase factors for the moment, we see that $\Delta\beta_1{}^C$ which contains factors of $\beta_o(s)$ rather than $\beta_o(s)^{1/2}$ (as occurs in $\Delta D_1{}^C(s)$) will dominate the contribution to the second order chromaticity. This situation can change if we choose the phase advances between the major chromatic error sources appropriately. For example, two sources of equal strength $\pi/2$ apart in phase will produce β waves exactly out of phase so there will be no resultant β wave. The dispersion waves produced by the same two sources will add in quadrature. Alternatively, if we want to cancel the net dispersion wave, the two sources should be π apart in phase. In this case the β waves will add exactly in phase.

Returning to the issue of chromaticity correction, the sextupole strengths $S_o(s)$ are usually chosen to make the linear chromaticity ξ_1 vanish, i.e. $S_o(s)$ is obtained by solving

$$\int_0^C S_o(s)\,\beta_o(s)\,D_o(s)\,ds = \int_0^C \beta_o(s)\,K_0(s)\,ds \tag{3.22}$$

and ξ_2 is then given by

$$\xi_2 = \frac{1}{8\pi}\int_0^C [-K_o + S_o(s)\,D_o(s)]\,\Delta\beta_1^C(s)\,ds + \frac{1}{4\pi}\int_0^C \beta_o(s)\,S_o(s)\,\Delta D_1^C(s)\,ds \tag{3.23}$$

Hence to reduce the second order chromaticity, the first order changes in β and also in the dispersion D should be minimized. Conversely, the regions where $\Delta\beta_1{}^C$ is large (e.g. the triplets in the IRs) will contribute the most to the second order chromaticity. The above expression also exhibits the variation of ξ_2 with the global tune. Since the first order β wave diverges at integer and half-integer tunes, ξ_2 will be amplified as ν_o approaches 0 or 0.5 and will be a minimum at ν_o=0.25 .

4 BETA WAVE AND CHROMATICITY DUE TO IR TRIPLETS

The perturbation to the periodic β function from chromatic errors in the IR quads can be calculated as a power series in δ,

$$\frac{\Delta\beta(s)}{\beta_o(s)} = \frac{\Delta\beta_1^C(s)}{\beta_o(s)}\delta + \frac{\Delta\beta_2^C(s)}{\beta_o(s)}\delta^2 + O(\delta^3) \tag{4.1}$$

where $\beta_o(s)$ is the unperturbed β function. Let

$$Q_i\beta_i \equiv \int_{i^{th}\,triplet} K\beta ds$$

The first order change due solely to the IR triplets is,

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{1}{2\sin\mu_o}\sum_{i=1}^{4} Q_i\beta_i\cos[2\mu_{is}-\mu_o] \tag{4.2}$$

where μ_{is} is the phase advance from the point s to the *ith* triplet. i=1 labels the first triplet after the point of observation s when going along the ring in a specified direction.

From earlier considerations we have seen that the first order β wave (and hence the second order chromaticity) from the IR quads is minimized when the phase advance between the IPs is an odd multiple of $\pi/2$. In what follows we will assume this choice of phase advance.

β wave within the triplets

We split the region between the 1st and 4th triplets into 3 regions. Let s denote the point of observation and μ_{s1} be the phase advance from the 1st triplet to this point.

Region I : between the 1st and 2nd triplets ($0 < \mu_{s1} < \pi$)

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{1}{2\sin\mu_o}\{Q_1\beta_1\cos[2\mu_{s1}-\mu_o] + [Q_2\beta_2-(Q_3\beta_3+Q_4\beta_4)]\cos[2\mu_{s1}+\mu_o]\} \tag{4.3}$$

Region II : between the 2nd and 3rd triplets ($\pi < \mu_{s1} < 8.5\pi$)

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{1}{2\sin\mu_o}\{(Q_1\beta_1+Q_2\beta_2)\cos[2\mu_{s1}-\mu_o]-(Q_3\beta_3+Q_4\beta_4)\cos[2\mu_{s1}+\mu_o]\} \tag{4.4}$$

Region III : between the 3rd and 4th triplets ($8.5\pi < \mu_{s1} < 9.5\pi$)

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{1}{2\sin\mu_o}\{[(Q_1\beta_1+Q_2\beta_2)-Q_3\beta_3]\cos[2\mu_{s1}-\mu_o]-Q_4\beta_4\cos[2\mu_{s1}+\mu_o]\} \tag{4.5}$$

β wave outside the triplets

Let s be an arbitrary point outside the region bounded by the 4 triplets and μ_{1s} be the phase advance from this point to the 1st triplet. The change in β to first order in δ is

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{1}{2\sin\mu_o}[Q_1\beta_1+Q_2\beta_2-(Q_3\beta_3+Q_4\beta_4)]\cos[2\mu_{1s}-\mu_o] \tag{4.6}$$

Tune shift

Similarly we consider the tune shift due to the chromatic error of the 4 IR triplets only, ignoring the effect of other quadrupoles and sextupoles. Then to 2nd order in the momentum deviation δ, the phase shift due to these 4 triplets is

$$\Delta\mu = \Delta\mu_1^C\delta+\Delta\mu_2^C\delta^2+O(\delta^3) \tag{4.7}$$

where

$$\Delta\mu_1^C = -\frac{1}{2}\sum_{i=1}^{4}Q_i\beta_i$$

$$\Delta\mu_2^C = \frac{1}{4\sin\mu_o}\sum_{i=1}^{3}\sum_{j=i+1}^{4}Q_i\beta_iQ_j\beta_j[\cos\mu_o-\cos(2\mu_{ji}-\mu_o)]$$

$$-\Delta\mu_1^C-\frac{1}{2}\cot\mu_o(\Delta\mu_1^C)^2 \tag{4.8}$$

μ_{ji} is the phase advance from the ith triplet to the jth triplet and $\nu_o = \mu_o/2\pi$ is the global tune of the ring. The first order chromaticity is independent of phase advances between the triplets. However the second order chromaticity depends crucially on the relative phase advances between the triplets. If the phase advance between the IPs is $(2n+1)\pi/2$, then the relative phase advances have the following values,

$$\mu_{21} = \pi, \qquad \mu_{31} = (2n+1)\pi/2 \qquad \mu_{41} = (2n+3)\pi/2$$
$$\mu_{32} = (2n-1)\pi/2 \qquad \mu_{42} = (2n+1)\pi/2$$
$$\mu_{43} = \pi$$

With these values, the second order contribution reduces to

$$\Delta\mu_2^C = \Delta\mu_{2Q}^C + \frac{1}{2}(Q_1\beta_1 + Q_2\beta_2 + Q_3\beta_3 + Q_4\beta_4) \tag{4.9}$$

where $\Delta\mu_{2Q}{}^C$ is the contribution from terms second order in the quad strengths,

$$\Delta\mu_{2Q}^C = \frac{\cot\mu_o}{2}[(Q_1\beta_1 + Q_2\beta_2)(Q_3\beta_3 + Q_4\beta_4) - \frac{1}{4}(Q_1\beta_1 + Q_2\beta_2 + Q_3\beta_3 + Q_4\beta_4)^2] \tag{4.10}$$

The large β functions in the triplets ensures that $\Delta\mu_{2Q}{}^C$ completely dominates the contribution to $\Delta\mu_2{}^C$.

4.1 Different configurations of IPs

A) Two IPs with equal β^*

In the chosen design we have repetitive symmetry across the two IRs. Here this symmetry implies

$$Q_3\,\beta_3 = Q_1\,\beta_1 \text{ and } Q_4\,\beta_4 = Q_2\,\beta_2$$

In this configuration, the β wave in the three regions within the triplets is

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = Q_1\beta_1 \sin 2\mu_{s1} \qquad \text{I}$$

$$= (Q_1\beta_1 + Q_2\beta_2)\sin 2\mu_{s1} \qquad \text{II}$$

$$= Q_2\beta_2 \sin 2\mu_{s1} \qquad \text{III} \tag{4.11}$$

and the beta wave outside the triplets is

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = 0 \tag{4.12}$$

Figure 3 shows the first order chromatic beta wave from the triplets in this configuration.

The different contributions to the phase shift are

$$\Delta\mu_1^C = -(Q_1\beta_1 + Q_2\beta_2)$$

$$\Delta\mu_{2Q}^C = 0 \tag{4.13}$$

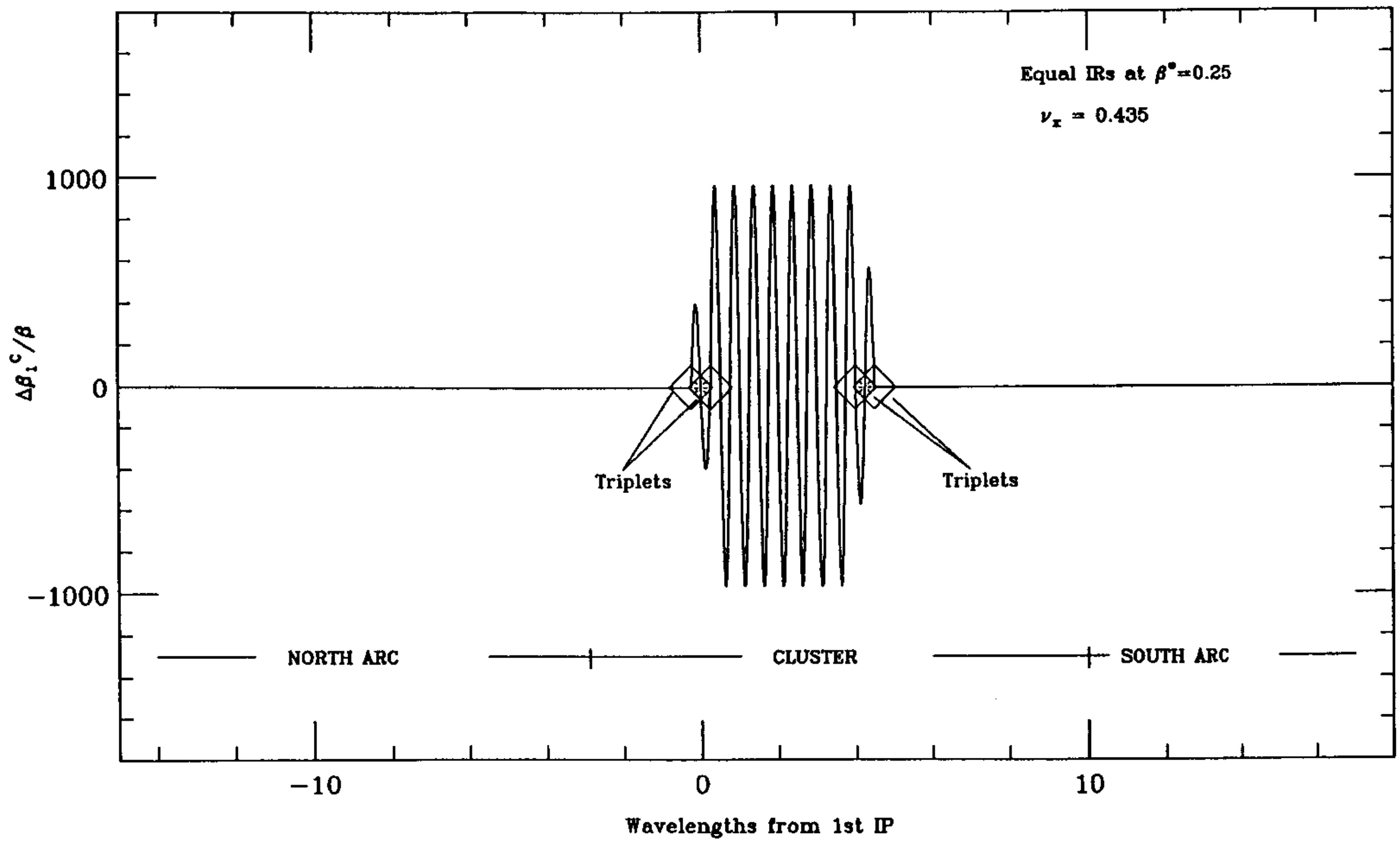

Figure 3 : 1st order Chromatic β beat from the triplets (Case A)

The vanishing of $\Delta\mu^C_{2Q}$ is a consequence of the fact that the β wave is zero outside the triplets. Hence the entire second order phase shift arises from the first order phase shift.

$$\Delta\mu_2^C = (Q_1\beta_1 + Q_2\beta_2) \qquad (4.14)$$

For this case $\Delta\mu_2{}^C$ is independent of the global tune ν_o.

B) Two unequal IPs with different β^*

Consider the first IP at β^*=0.25 and the second IP at β^*=0.50. With repetitive symmetry, this implies

$$Q_3\,\beta_3 = 1/2\,Q_1\,\beta_1 \text{ and } Q_4\,\beta_4 = 1/2\,Q_2\,\beta_2$$

Within the triplets, the chromatic β wave is

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{1}{2\sin\mu_o}\left\{Q_1\beta_1\left[\cos(2\mu_{s1}-\mu_o) - \frac{1}{2}\cos(2\mu_{s1}+\mu_o)\right] + \frac{Q_2\beta_2}{2}\cos(2\mu_{s1}+\mu_o)\right\}$$

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{1}{2\sin\mu_o}(Q_1\beta_1 + Q_2\beta_2)\left\{\cos(2\mu_{s1}-\mu_o) - \frac{1}{2}\cos(2\mu_{s1}+\mu_o)\right\}$$

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{1}{2\sin\mu_o}\left\{\frac{Q_1\beta_1}{2}\cos(2\mu_{s1}-\mu_o) + Q_2\beta_2\left[\cos(2\mu_{s1}-\mu_o) - \frac{1}{2}\cos(2\mu_{s1}+\mu_o)\right]\right\}$$

(4.15)

for regions I, II and III respectively. The first order change in β at an arbitrary point s outside the triplets is

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{1}{4\sin\mu_o}(Q_1\beta_1 + Q_2\beta_2)\cos(2\mu_{1s}-\mu_o) \qquad (4.16)$$

The propagation of the first order chromatic beta wave from the triplets into the arcs is shown in Figure 4. The phase shift due to the triplets is, up to second order,

$$\Delta\mu_1^C = -\frac{3}{4}(Q_1\beta_1 + Q_2\beta_2)$$

$$\Delta\mu_2^C = \frac{-\cot\mu_o}{32}(Q_1\beta_1 + Q_2\beta_2)^2 + \frac{3}{4}(Q_1\beta_1 + Q_2\beta_2) \qquad (4.17)$$

C) Only one IP (effectively)

Let β^* = 0.25m at the first IP and β^*= 8 m at the second IP . Then

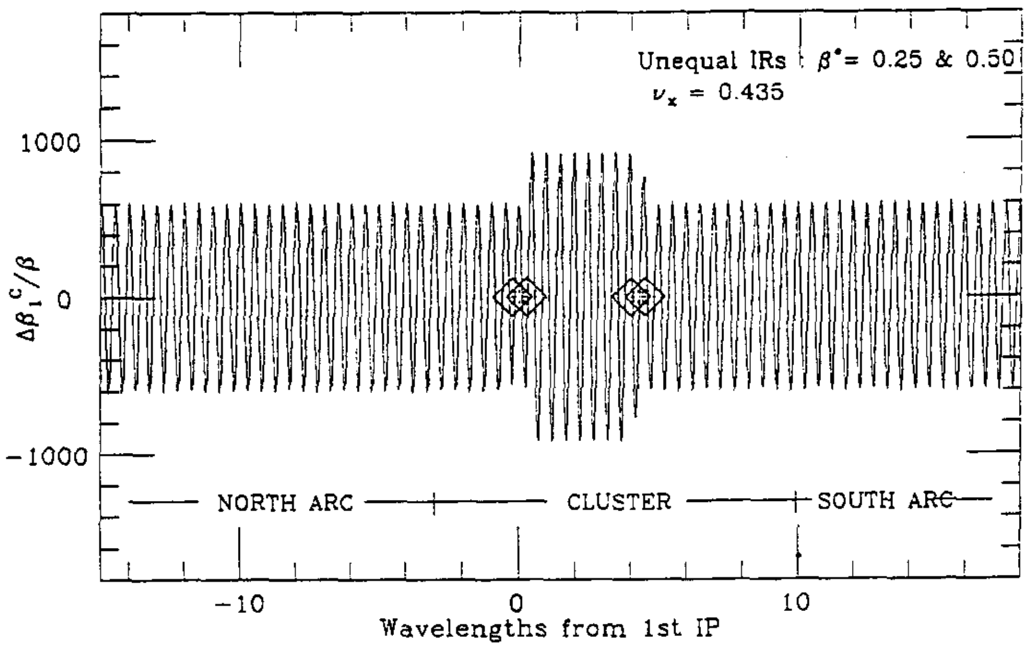

Figure 4 : 1st order Chromatic β beat from the triplets (Case B)

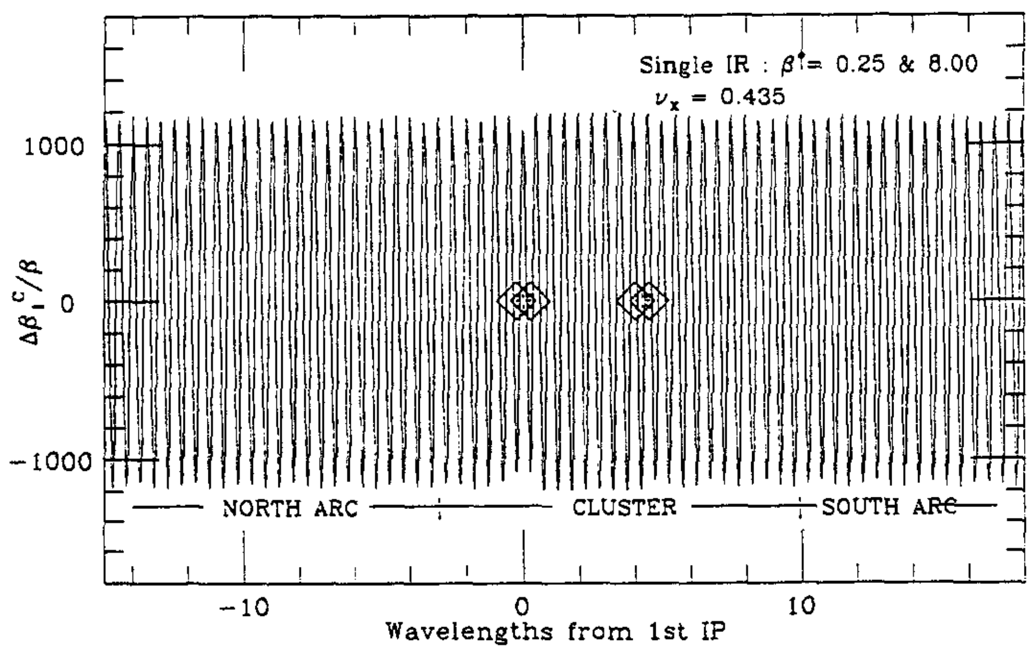

Figure 5 : 1st order Chromatic β beat from the triplets (Case C)

$$Q_3\,\beta_3 = (1/32)\,Q_1\,\beta_1 \text{ and } Q_4\,\beta_4 = (1/32)\,Q_2\,\beta_2$$

Inside the region bounded by the triplets, the chromatic β wave (for regions I, II and III) is

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{1}{2\sin\mu_o}\{Q_1\beta_1\left[\cos(2\mu_{s1}-\mu_o)-\frac{1}{32}\cos(2\mu_{s1}+\mu_o)\right]+\frac{31}{32}Q_2\beta_2\cos(2\mu_{s1}+\mu_o)\}$$

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{1}{2\sin\mu_o}(Q_1\beta_1+Q_2\beta_2)\{\cos(2\mu_{s1}-\mu_o)-\frac{1}{32}\cos(2\mu_{s1}+\mu_o)\}$$

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{1}{2\sin\mu_o}\{\frac{31}{32}Q_1\beta_1\cos(2\mu_{s1}-\mu_o)+Q_2\beta_2\left[\cos(2\mu_{s1}-\mu_o)-\frac{1}{32}\cos(2\mu_{s1}+\mu_o)\right]\} \tag{4.18}$$

and outside the triplets

$$\frac{\Delta\beta_1^C(s)}{\beta_o(s)} = \frac{31}{64\sin\mu_o}(Q_1\beta_1+Q_2\beta_2)\cos(2\mu_{1s}-\mu_o) \tag{4.19}$$

which is nearly twice as large as the β wave at the same point with the two unequal IPs considered in Case B. This is evident in Figure 5.

The phase shifts are

$$\Delta\mu_1^C = -\frac{33}{64}(Q_1\beta_1+Q_2\beta_2)$$

$$\Delta\mu_2^C = -\left[\frac{1}{2}\left(\frac{33}{64}\right)^2-\frac{1}{64}\right]\cot\mu_o(Q_1\beta_1+Q_2\beta_2)^2+\frac{33}{64}(Q_1\beta_1+Q_2\beta_2) \tag{4.20}$$

The contribution $\Delta\mu^C_{2Q}$, of second order in the quad strengths, in this case is about 3.75 times larger than in the case with two unequal IPs.

These three cases show that if the IPs are $(2n+1)\pi/2$ apart in phase advance, then the triplets at one IP wholly or partially cancel the contribution to the second order chromaticity from the triplets at the other IP. For all cases, except the first with equally balanced IPs, the second order chromaticity will be significantly amplified as $\nu_o \rightarrow 0.5$.

In the following table, we evaluate the tune shift due to the triplets in the three configurations and at two choices of tunes. The tune shift is,

$$\Delta\nu = \xi_1\delta+\xi_2\delta^2 \equiv \frac{1}{2\pi}(\Delta\mu_1^C\delta+\Delta\mu_2^C\delta^2)$$

CASE	ξ_1	ξ_2	ξ_2	$\Delta\nu$ at $\delta=5*10^{-4}$	$\Delta\nu$ at $\delta=5*10^{-4}$
		$\nu_o=0.285$	$\nu_o=0.4$	$\nu_o=0.285$	$\nu_o=0.4$
A) equal IPs $\beta^*=0.25$ $\beta^*=0.50$	-154.0 -77.0	+154.0 +77.0	+154.0 +77.0	-0.07696 -0.03848	-0.07696 -0.03848
B) unequal IPs $\beta^*=0.25$, $\beta^*=0.5$	-115.5	+1156.4	+6524.8	-0.05746	-0.05612
C) one IP $\beta^*=0.25$, $\beta^*=8.0$	-79.4	+3977.5	+24132.6	-0.03871	-0.03367

Table 2: Tune shift due to the triplets

The total tune shift is dominated by the linear contribution in any configuration. Removing the linear tune shift is relatively simple, so our concern is with minimizing the higher order contributions. Clearly the largest 2nd order chromaticity occurs for case C at $\nu_0=0.4$.

The total chromaticity of an IR includes contributions from the triplets, the quadrupoles in the M= -I section and the variable strength quadrupoles in the tuning section. We concentrated on the triplets in this section because at collision the contribution of the triplets is about 76%, that of the M= -I section about 19% and the remaining is due to the tuning section. At injection, where higher order chromaticity is not an issue, all three sections contribute about equally to the chromaticity which is now dominated by the contributions from the arc quadrupoles.

5 THEORY OF 2nd ORDER CHROMATICITY CORRECTION

Our aim is to correct the 2nd order chromaticity of the Interaction Regions. We assume that the linear chromaticity is corrected for. We start with two facts derived in the previous sections.

a) The 2nd order chromaticity is driven by the 1st order chromatic beta wave, and

b) The beta wave propagates at twice the betatron frequency.

The 4 triplets at the two IPs contribute the most to the 2nd order chromaticity.

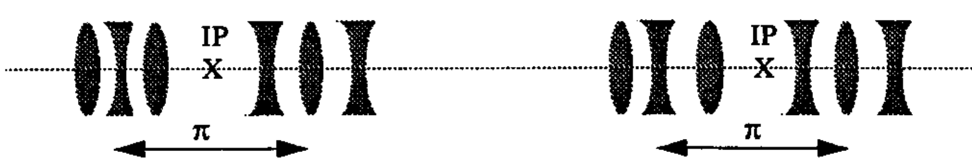

Figure 6 : The triplets at the two IPs

Since the triplets on either side of an IP are π apart in phase, the chromatic beta waves produced by them add in phase. Hence we can combine the two triplets into a composite lens at each IP.

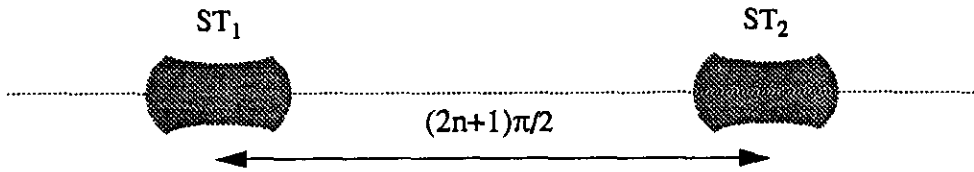

Figure 7 : Effective Super triplets at each IP

The beta waves produced by the two super triplets are exactly out of phase in the region outside the triplets. If the two super triplets have the same strength i.e. $ST_1 = ST_2$,

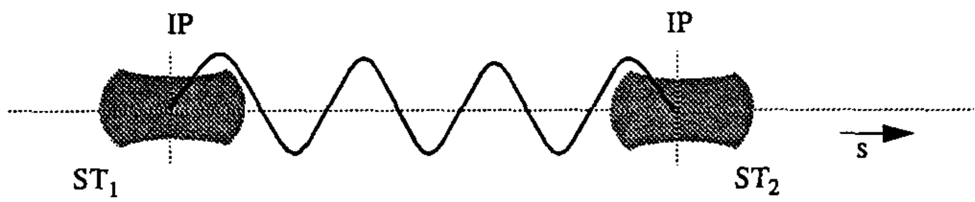

Figure 8 : Exact Cancellation with Equal IRs

then no beta wave gushes out from the IRs and the 2nd order chromaticity is a minimum.

Now turn off one of the IRs (with linear chromaticity still corrected). The chromatic beta wave from the remaining IR now flows uncorrected into the arcs.

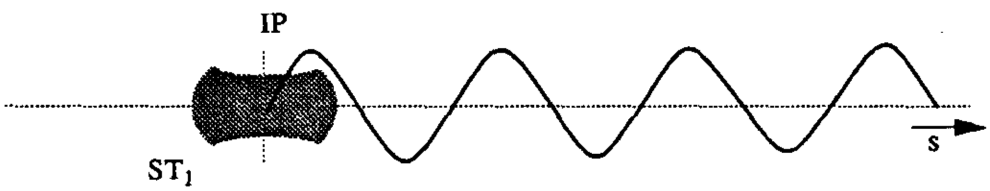

Figure 9 : Beta wave with 1 IR

Here we have drawn only the beta wave propagating out from the supertriplet ST_1 and not the periodic 1st order beta wave in the ring. How do we stop this beta wave from ST_1 from going all around the ring ?

Put another source of chromatic beta waves (e.g. a sextupole) $\pi/2$ from the IP to interfere destructively with the waves produced by ST_1 (i.e to do the same as the missing ST_2)

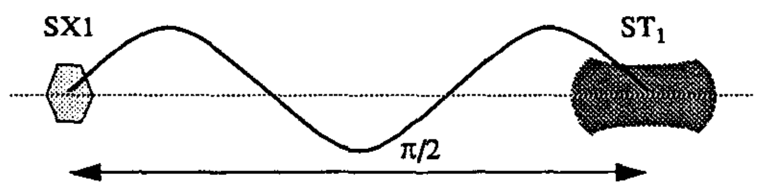

Figure 10 : Cancelling the beta wave from 1 IR

The SX1 sextupole produces a beta wave with the opposite phase. Hence the 2nd order chromaticity should be a minimum. However there is an unwanted side effect. The SX1 sextupole introduces linear chromaticity into the ring.

Put another sextupole SX2 to cancel the linear chromaticity due to SX1.

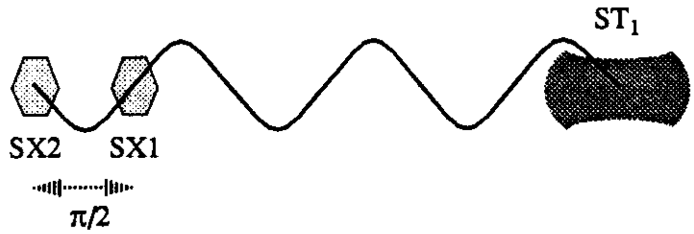

Figure 11 : Correction of 1st and 2nd order chromaticity with 2 sextupoles

The beta functions and the dispersion at the locations of SX1 and SX2 are the same.

Zero linear chromaticity –> SX1 + SX2 = 0. Since these two sextupoles are $\pi/2$ apart and their strengths have the opposite signs, their beta waves add in phase.

Other details:

The beta wave has to be corrected in each plane. This requires 2 sextupoles SX1F and SX2F to correct for the 1st and 2nd order chromaticity in the horizontal plane (primarily) and sextupoles SX1D and SX2D to correct for the effects in the vertical plane (primarily). For each sextupole to be at the proper phase with respect to the IP requires a specific choice of phase advance across each IR.

Only 1 sextupole in each family would require too large a strength for each sextupole. With 24 members in a family, the required strength would be under the maximum allowed strength for each sextupole. 12 of these are placed at the edge of the North Arc adjacent to the cluster and the other 12 at the edge of the South Arc.

To have each member in a family produce a beta wave in phase with the other members of the family, there must be a phase advance of π between the members of a family. This has the additional advantage of removing the second order geometrical aberrations[2]. This is easily achieved if the phase advance across each cell is 90°.

The sextupole distribution over 1 betatron wavelength looks as follows.

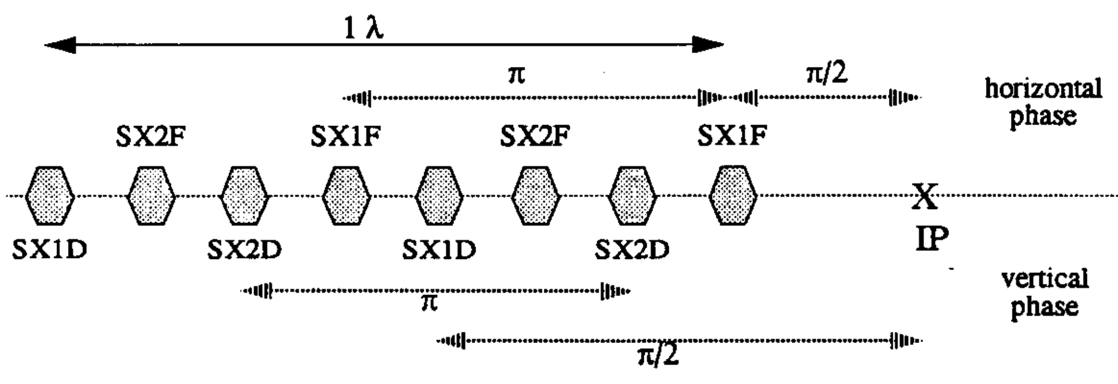

Figure 12 : Sextupole Distribution

The entire distribution spans 6 wavelengths on either side of the cluster.

Imperfections :

The contribution to the beta wave in the x plane from the sextupoles SX1D and SX2D is not cancelled and similarly for the y plane contributions from SX1F and SX2F. To see this, we represent the beta waves from each source in a phasor diagram. The angle between any two vectors in this diagram will be twice the phase advance between the corresponding sources. We will refer to this as a 2ψ diagram. Let the phase be measured from an arbitrary point in the lattice.

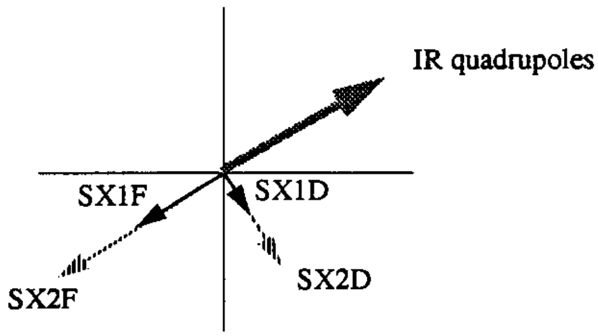

Figure 13 : 2ψ Phasor Diagram for the beta waves

SX1F is at $\pi/2$ phase from ST_1 while SX2F is at $\pi/2$ phase from SX1F but has the opposite sign to SX1F. Hence the vectors representing 1F and 2F are parallel and combine to cancel the vector from the IR quads. However SX1D is at $\pi/4$ phase from SX1F, hence its vector is orthogonal to SX1F. SX2D is $\pi/2$ away from SX1D but with the opposite strength so its vector adds in phase to that of SX1D. The orthogonal contributions from SX1D and SX2D are not removed by any source and remain as a residual beta wave. The relative amplitude of the beta wave in the horizontal plane from the D sextupoles is $\beta_{min}/\beta_{max} \sim 1/6$ (for a FODO cell with 90 degrees phase advance per cell). With the scheme outlined above, the second order chromaticity is thus reduced by a factor of 6.

6 PERFORMANCE OF THE CHROMATICITY CORRECTION SYSTEM

We correct for the chromaticity of the collider ring by placing sextupoles next to each quadrupole in the two arcs of the collider. This amounts to 392 D and 392 F sextupoles. Of these, 96 sextupoles of each type are placed in the 24 cells at both ends of each arc for correcting the non-linear chromaticity of both the interaction regions. In all, eight families of sextupoles labelled SX1F, SX1D, ..., SX4F, SX4D are available for the non-linear correction. These will be referred to as the *local sextupoles*. The families (SX1F, SX4F) are $\pi/2$ (mod 2π) in horizontal phase away from the North IP and (SX1D, SX4D) are $\pi/2$ (mod 2π) in vertical phase from this IP. The same statement can be made for the families (SX2F,SX3F) and (SX2D, SX3D) with respect to the South IP. The 296 sextupoles in each of two families SXF, SXD correct for the linear chromaticity of the total ring including the interaction regions. These will be called the *global sextupoles*.

The collider has two low-β (or high luminosity) interaction regions on the east side of the ring and two medium-β IRs on the west side. In what follows, the chromaticity of only the low-β IRs has been corrected for. Consequently, for this report, we use 2x48 local sextupoles and 2x344 global sextupoles. The optimization of the non-linear correction included the minimization of the tune shift as a function of momentum to the 2nd and 3rd order. It was done using the module HARMON in MAD [3].

We have seen in earlier sections that the second order chromaticity is largest when the fractional part of the global tune is close to 0.5 and one of the interaction regions in a cluster is tuned to collision optics while the other is tuned to injection optics. For brevity, we will present results for this "worst" case only. The tunes in the two planes are chosen to be (0.435,0.415). An important advantage of correcting this configuration is that the phase advances between the IPs becomes irrelevant and we do not need to rely on the chromatic cancellation of one IR by another. This is important in practice since the detectors at the two IPs will possibly be operating at different luminosities and the phase advance between IPs may not be an odd multiple of $\pi/2$.

First we look at the chromatic behaviour with only the linear chromaticity corrected. This will show if a) nonlinear chromaticity correction is needed and b) serve as a benchmark by which to compare the improvement due to the nonlinear correction. All 392 sextupoles in each of the D and F families are used for the correction in this case. Figure 14 shows the variation of the tune shift with the relative momentum deviation δ and Figure 15 shows how the relative β at the IP varies with δ. The standard deviation σ_p for the relative momentum spread in the beam is approximately $6\ \mathrm{x}10^{-5}$ at 20 TeV. For stable operation of the beam we require the tune shift $\Delta\nu < 0.002$. We find that the linear correction provides us with a momentum aperture of approximately 2.5 σ_p which is inadequate. The relative variation in β^* is also large, reaching 10% at $1\sigma_p$. Clearly, higher order chromaticity correction is needed under these conditions.- Now we examine the performance of the nonlinear chromaticity correction system. The local sextupoles strengths were limited to be less than 0.25 T-m at 1cm. The following tables compare the strengths of the sextupoles in the two cases. (β^* = 0.25 and 8m, ν_x=123.435, ν_y=122.415)

Sextupole name	Integrated Field Strength T-m at 1cm
Global F	0.068
Global D	-0.136

Table 3: Linear chromaticity correction only ("global")

Sextupole name	Integrated Field Strength T-m at 1cm
Global F	0.078
Global D	-0.155
Local F1, F4	0.168
Local D1, D4	-0.250
Local F2, F3	-0.164
Local D2, D3	0.247

Table 4: Linear and nonlinear chromaticity correction ("local")

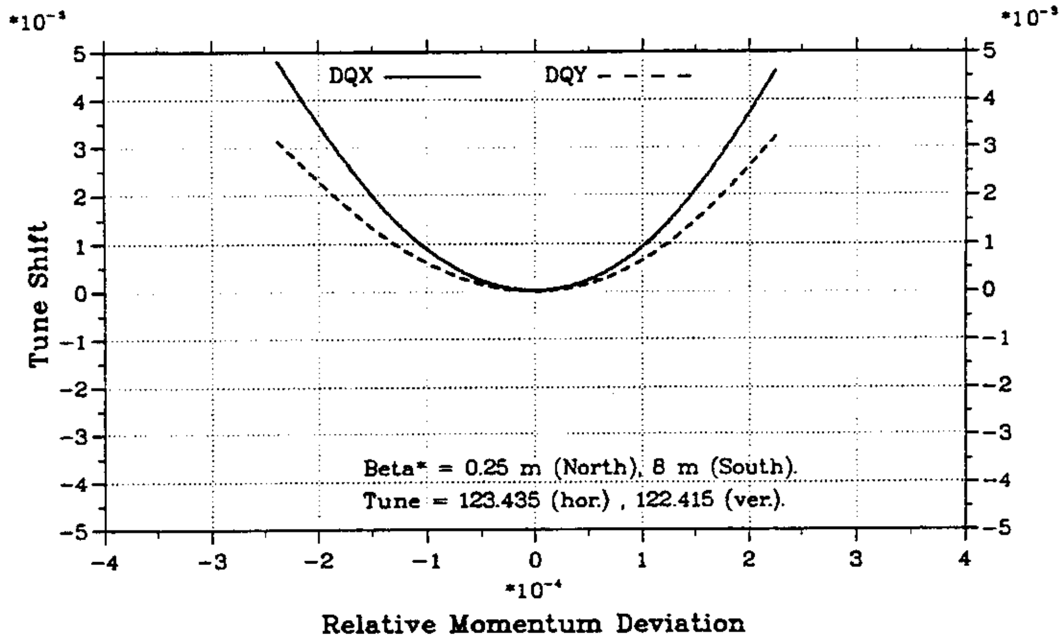

Figure 14: Tune Variation with Momentum: Global Linear Correction.

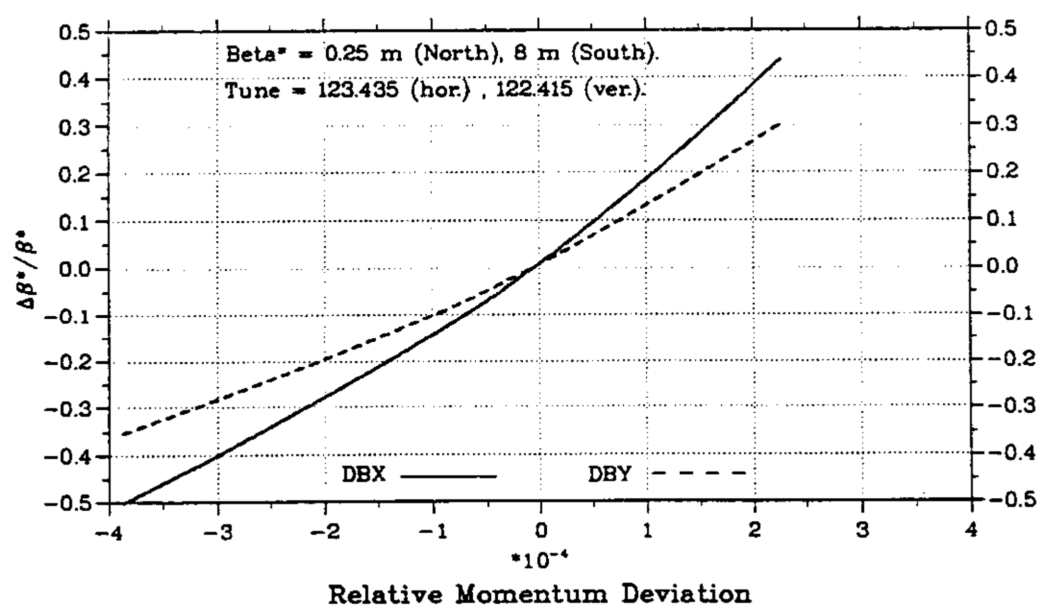

Figure 15: Variation of Beta* with Momentum: Global Linear Correction.

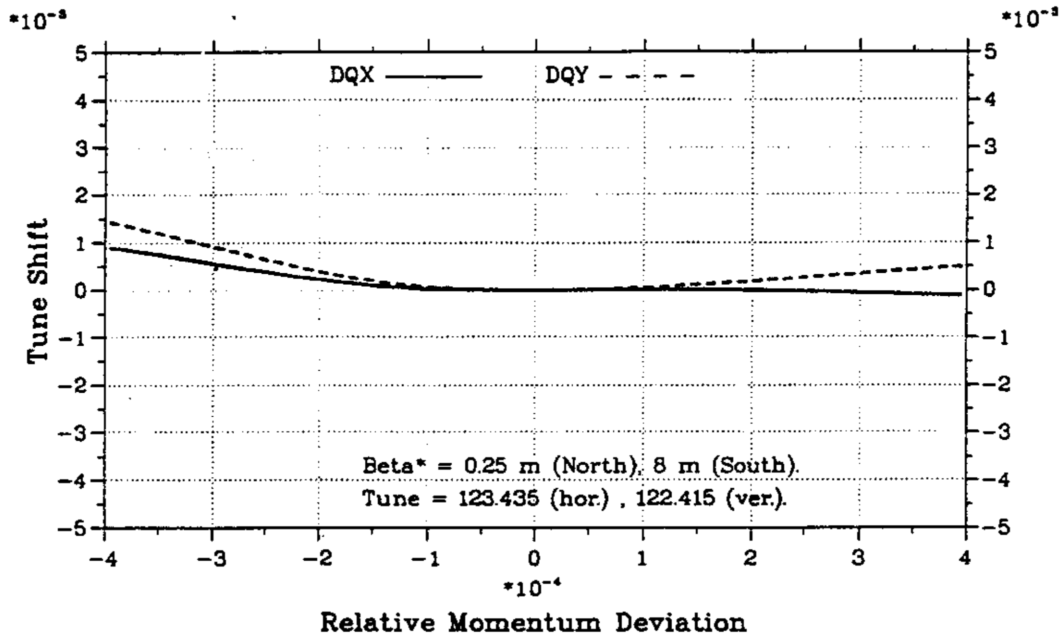

Figure 16: Tune Variation with Momentum: Local Nonlinear Correction.

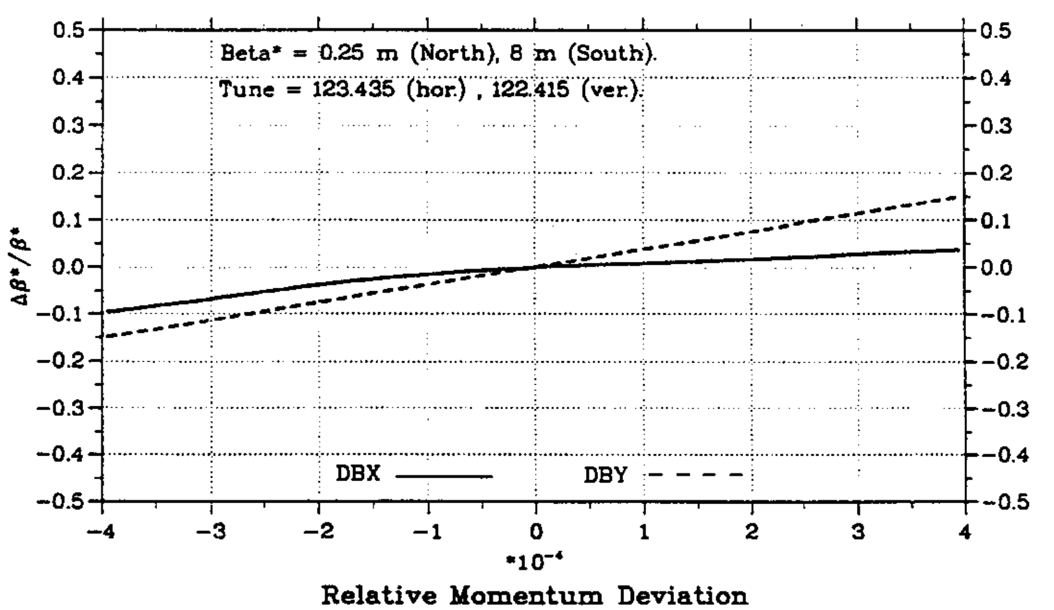

Figure 17: Variation of Beta* with Momentum: Local Nonlinear Correction.

We note that the strengths of F1 and F2 are approximately equal and opposite and similarly for D1 and D2. This is required to cancel the net linear chromaticity of the local sextupoles. For the correction of a single IR, only 4 families (F1,D1,F2,D2) of sextupoles are required. When both IRs are tuned to collision optics F1 and F4 are allowed to have different strengths and so too for the other pairs of sextupoles which have been constrained to be equal above.

Figure 16 shows the variation of tune with δ in the presence of the local sextupoles. The tune variation is flat over 2 σ_p and the momentum aperture is increased to approximately $8\sigma_p$ for $\Delta\nu < 0.002$. The relative β^* variation (shown in Figure 17) is limited to 2.5% at $1\sigma_p$. These are definite improvements in the chromatic behaviour and may be adequate for the stable operation of the collider. The final check of the chromaticity correction system is to examine the effect of the local sextupoles on the dynamic aperture.

7 DYNAMIC AND MOMENTUM APERTURE

The dynamic and momentum aperture of the collider ring in the presence of the local sextupole scheme has been studied in detail in order to establish the performance as well as the feasibility of the scheme itself. Sextupoles, as nonlinear elements, can potentially reduce the dynamic aperture of the machine, so the behaviour of the collider lattice has been checked with a realistic simulation model. For every configuration of the optics and setting of the local sextupoles correction scheme we determined the dynamic aperture, identified with the largest amplitude surviving 1024 turns. The choice of the short term dynamic aperture as a figure of merit is justified by the fact that the main aim is to compare different machine configurations and not to investigate its long term stability.

The behaviour of different configurations of the Interaction Region optics and of the sextupole scheme has been studied first for the ideal lattice, i.e. the first order lattice plus the sextupoles as the only source of nonlinearity. The ideal lattice has then been compared with a realistic model of the lattice where the effect of errors and their operational corrections is taken into consideration.

The initial conditions for the increasing amplitudes of the particles tracked have been selected as follows:

$x_n = n * \sigma_x$ where $\sigma_x = (\beta_x \varepsilon_x)^{1/2}$ and $\beta\gamma\varepsilon_x = 1$ mm mrad
$y_n = n * \sigma_y$ where $\sigma_y = (\beta_y \varepsilon_y)^{1/2}$ and $\beta\gamma\varepsilon_y = 1$ mm mrad

At the beginning of the lattice $\beta_x \sim \beta_y \sim 460$m, $\alpha_x \sim \alpha_y \sim 0$ and $D_x = 0$, so that $\sigma_x \sim \sigma_y \sim 1.47 * 10^{-4}$m at 20 TeV.

7.1 Dynamic and momentum aperture of the ideal lattice

The β^* at the collision point of the collider low-beta interaction regions can be tuned between 0.25m and 8m, the latter value corresponding to the injection optics; the nominal value at collision is 0.5m. Each interaction region, 2 in the East Cluster and 2 in the West Cluster, can be tuned independently to a value of β^* in this range. The optical configurations simulated have the injection optics in the West Cluster and several combinations of β^* in the East Cluster, i.e. in the north low beta IR (ENLB) and in the south low beta IR (ESLB). The configuration studied are labeled as follows:

N50-S50: baseline symmetric configuration, β^*_{ENLB}=0.50m, β^*_{ESLB}=0.50m
N25-S25: low β^*symmetric configuration, β^*_{ENLB}=0.25m, β^*_{ESLB}=0.25m
N25-S50: low β^*asymmetric configuration, β^*_{ENLB}=0.25m, β^*_{ESLB}=0.50m
N25-S800: asymmetric configuration, β^*_{ENLB}=0.25m, β^*_{ESLB}=8m

As previously discussed, the last configuration where one east IR is tuned for maximum luminosity and the other is tuned to the injection optics, is expected to be the most sensitive to chromatic effects. The first configuration is the baseline optics for the Collider and it is the least sensitive to chromatic effects. In order to enhance the effect of higher order chromaticity, the fractional tune of the lattice for this simulation has been chosen reasonably close to the half integer (ν_x=123.435, ν_y=122.415). Studies of beam-beam effects also suggest a working point close to 0.4.

$\Delta p/p$	N50-S50	N50-S50	N25-S25	N25-S25	N25-S50	N25-S50	N25-S800	N25-S800
	global	local	global	local	global	local	global	local
0.0000	100	70	100	40	100	50	100	50
0.0001	100	70	100	40	100	50	100	50
0.0002	100	70	100	40	100	50	100	50
0.0003	100	70	100	40	100	50	90	50
0.0004	100	70	100	40	100	40	50	50
0.0005	100	70	100	40	80	40	20	50
0.0006	100	70	80	30	70	40	0	50
0.0007	100	70	70	30	50	40	0	50
0.0008	100	70	50	30	40	40	0	50
0.0009	100	70	0	30	20	40	0	50
0.0010	100	70	0	30	0	40	0	50
0.0011	100	70	0	30	0	40	0	40
0.0012	100	70	0	30	0	40	0	30
0.0013	100	70	0	30	0	40	0	30
0.0014	100	70	0	30	0	40	0	30

Table 5: Dynamic and momentum aperture (σ) for the ideal lattice

Table 5 summarizes the results for the ideal lattice: for each configuration described above, the dynamic and momentum aperture for the lattice with the total linear chromaticity compensated by the sextupoles in the arcs only (global), is compared to one (local) where the linear and second order chromatic effects arising from the IRs are corrected by the local sextupole scheme and the linear chromaticity from the rest of the machine is corrected by the arc sextupoles.

The local sextupoles cause a significant improvement of the momentum aperture, in particular for the asymmetric optics configurations. This confirms the improvement in machine performance expected, given the better tune versus amplitude and beta beat in the presence of the local scheme. The strong local sextupoles cause however a reduction of the dynamic aperture on momentum. This effect, together with the strength requirement on the local sextupoles, led us to limit the use of the local system only to compensating the higher order chromaticity of the IRs and to correct the linear chromaticity, caused by both the arcs and the IRs, with the arc sextupoles.

7.2 Dynamic and momentum aperture of the lattice with errors

The investigation of performance of the local sextupole scheme done for the ideal lattice has been repeated and extended to a realistic model of the machine where the effect of errors and their corrections are accurately simulated. This study allows us to establish whether the benefits of the local scheme demonstrated for the ideal lattice still holds in the presence of errors that could potentially mask the effectiveness of sextupoles, and to investigate more thoroughly the issue of loss of dynamic aperture on momentum.

The model used for the simulation and implemented in the code TEAPOT [4] describes realistically the single particle dynamics of the Collider as far as errors and corrections are concerned. Collective and beam beam effects are not included in the model. Every relevant element in the lattice such as a bend, quadrupole, sextupole, beam position monitor, etc., is assigned random alignment errors and roll errors; main dipoles and quadrupoles also have systematic and random field errors associated with them, where normal and skew multipoles are specified up to the order 9. The issue of the error specifications for the Collider is a matter of continuing study and will not be discussed here in detail: except where otherwise specified, the assumptions for the alignment and field errors reflect the so called Collider 3B specifications document [5]. We did not include alignment errors in the IR triplets: the triplets are extremely sensitive to these errors and the correction of their effects on the collider dynamics is the topic of an ongoing independent study. Also, the effect of the crossing angle at the interaction point is not generally included in the results that follow. Preliminary results on the effect of the crossing angle on the baseline collider optics will however be discussed at the end.

The operational corrections necessary to operate the machine with imperfections are also accurately described in the model: the closed orbit is found by a steering algorithm, the lattice is retuned to the original fractional tune by means of trim quadrupoles and the local compensation of coupling is achieved by a set of 44 skew quadrupoles, 24 of them placed in the clusters and 20 in the arcs.

Several configurations have been studied with the above described set of errors and corrections: N25-S800, N50-S800 and the baseline collider optics N50-S50. We will limit the detailed discussion to the former one.

7.3 N25-S800

As already remarked, this optical setting has been studied in more detail since it represents a worst case scenario as far as chromatic effects from the IRs are concerned. The low beta IR tuned at 0.25m contributes about 100 units of chromaticity. We compared the following sextupole correction schemes:

global	Linear chromaticity ξ from arcs *and* IRs corrected with the arc sextupoles
local_100	100 units of linear ξ corrected by the local system, the rest by the arc sextupoles. The local system minimizes the 2[nd] and 3[rd] order tune shift with momentum.
local_50	50 units of linear ξ corrected by the local system, the rest by the arc sextupoles. The local system minimizes the 2[nd] and 3[rd] order tune shift with momentum.
local_0	All the linear chromaticity ξ is corrected with the arc sextupoles. The local system minimizes the 2[nd] and 3[rd] order tune shift with momentum.

For every correction scheme the dynamic aperture as a function of momentum has been determined for different error sets.The results are summarized in Figure 18.a-d..

Figure 18.a describes the ideal lattice, while in Figure 18.b-d are summarized the results for the lattice with errors: they have the same set of alignment errors but differ in the assignment of field errors. When field errors are added to the arc dipoles and quadrupoles, the dynamic aperture obviously decreases, but there is a clear improvement in the momentum aperture with the local schemes compared to the global scheme.

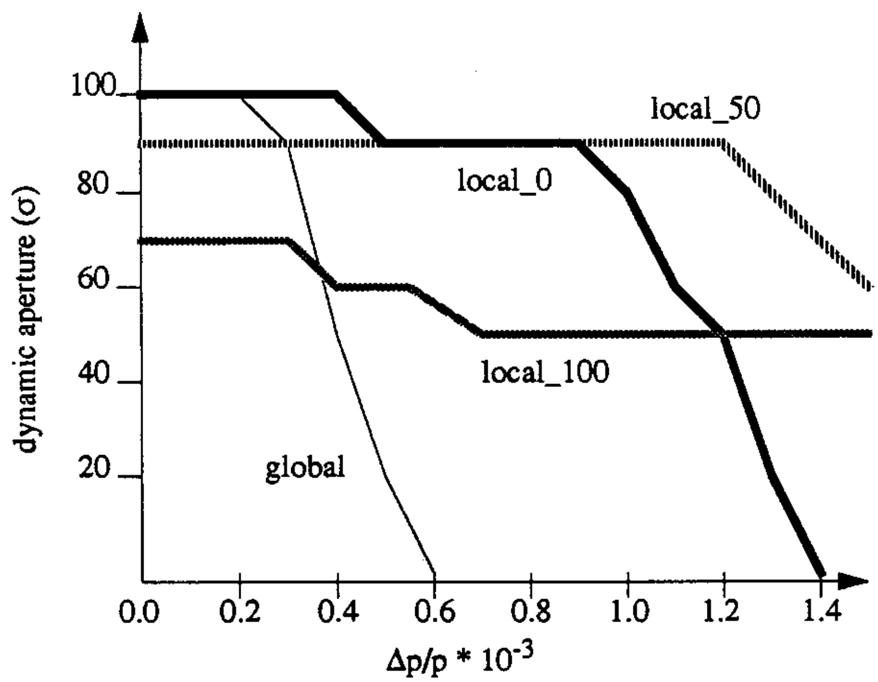

Figure 18.a N25-S800: ideal lattice

The dynamic aperture of the global scheme on momentum is still larger than for the local schemes. The assignment of field errors to the IR quadrupoles (Figure 18.c) and successively to the IR triplets (Figure 18.d) further reduces the dynamic aperture of the machine as expected but the increase in momentum aperture over the global scheme is verified.

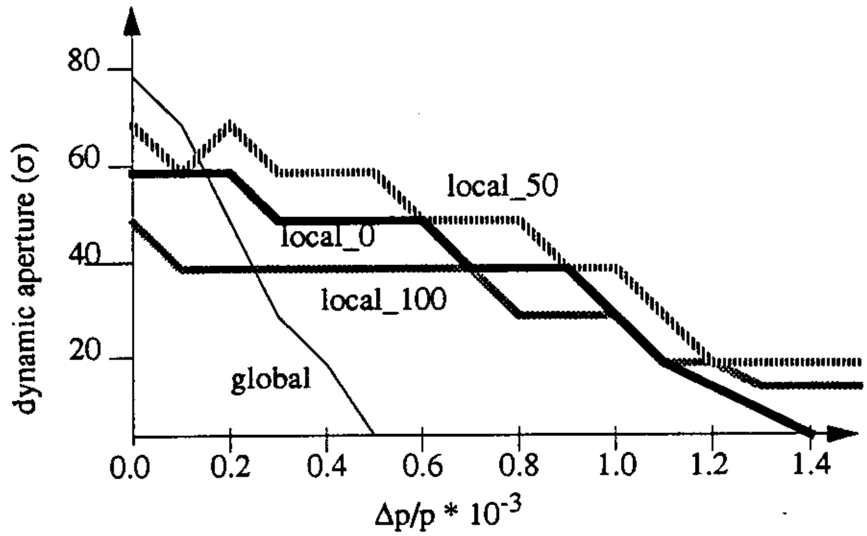

Figure 18.b N25-S800: field errors in arcs

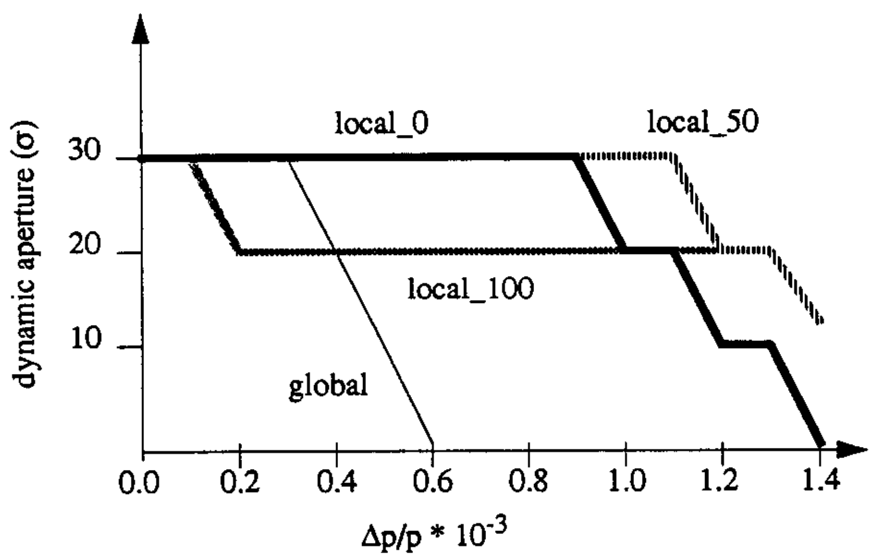

Figure 18.c N25-S800: field errors in arc and IR quads (not in triplets)

Furthermore, the reduction of aperture for $\Delta p/p=0$ of the local schemes versus the global one is no longer present, because the effect of the field errors in the IR quadrupoles dominates the dynamics on momentum. Lattice performance in the presence of errors led us to select the local_0 scheme as the most effective way of correcting the chromatic effects of the IR. The local_0 scheme is also preferred because of the minimum strength of the local sextupoles.

The dynamic aperture for this optics, without crossing angle and assuming the standard 3B specifications for the IR quadrupoles, is described in Fig.19. The global sextupole correction scheme is used here since the optical symmetry makes this optics less sensitive to chromatic effects.

A horizontal (vertical) crossing angle of 135 μrad between the two beams at interaction points is achieved with a system of 4 horizontal (vertical) kickers per IP. The residual horizontal (vertical) dispersion produced by the system is matched with a set of 6 normal (skew) quadrupoles per IP. The effect of the crossing angle is to make the beam pass off axis through the triplets, increasing the effect of the higher order multipoles in the quadrupoles. For a crossing angle of 135 μrad the maximum closed orbit offset in the triplets is 5 mm: this effect has been simulated and the reduction of the aperture at collision found to be at the 1-2 sigma level.

The multipoles assumed so far for the triplets have been derived from the specifications for the 40mm aperture arc quadrupoles by appropriately rescaling the values to an aperture of 50mm in the IR quadrupoles. A study is now in progress towards the exact determination of the field quality required for the IR triplets, in particular the higher order multipoles responsible for aperture reductions. Preliminary results show that the systematic b_5 multipole in the triplets, the first multipole allowed by symmetry in a quadrupole, has a significant effect on the aperture. Lowering b_5 from $0.534 * 10^{-4}$ (at 1cm) to $0.1 * 10^{-4}$ increases the dynamic aperture by 3-4 sigma. A typical value for the dynamic aperture at collision, taking into consideration the crossing angle and the b_5 multipole is 12 sigma.

8 Summary

Our scheme for correcting the nonlinear chromaticity of each IR consists of placing sextupoles in 4 families in the regular cells adjacent to the IRs and spread out over 6 betatron wavelengths into the arcs on each side of a cluster. These 'local' sextupoles correct primarily for the second and to a lesser extent the third order chromaticity of the IRs while contributing net zero linear chromaticity. The linear chromaticity of the entire collider ring is removed by two families of sextupoles in the remaining cells in the arcs.

We have tested the above scheme with different configurations of IRs. It improves the chromatic and dynamic behaviour for every configuration studied. Even for the worst case with one IP at β^*=0.25m and the other at β^*=8m, the tune shift with momentum data show that the nonlinear correction scheme increase the momentum aperture more than three times. This increased momentum aperture is obtained at the expense of a slight reduction in the dynamic aperture for particles on momentum, when no field errors in the magnets are included. When we add a realistic set of errors, specially the field errors in the IR triplets, the local sextupoles do not affect the dynamic aperture on momentum.

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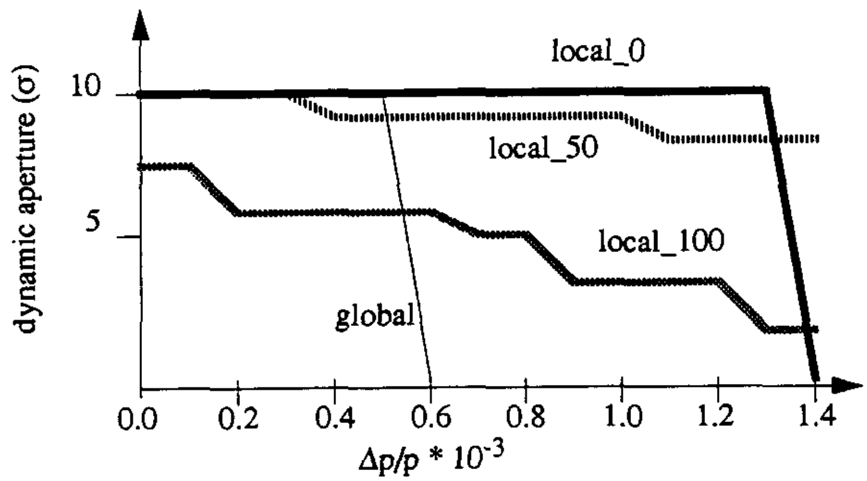

Figure 18.d N25-S800: field errors in arcs, IR quads and triplets

7.4 Effect of the crossing angle and field quality in the IR triplets.

As previously remarked, the former results about the collider dynamic aperture at top energy do not take into consideration the effect of the crossing angle and assume the 3B specifications for the field quality in the IR quadrupoles. Both assumptions have important consequences as far as the effect of the IR triplets quadrupoles on the dynamics is concerned. Work is presently in progress that specifically addresses IR triplet issues: some preliminary results will be summarized here for the N50-S50 baseline optics configuration.

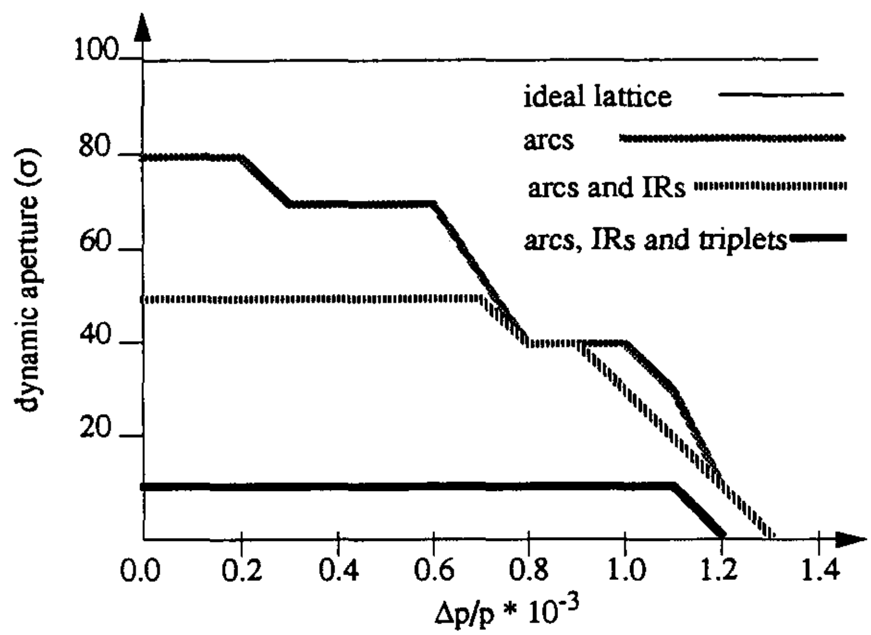

Figure 19 Baseline configuration N50-S50