PHYSICS

Violation of Bell inequality with unentangled photons

Kai Wang[1]†, Zhaohua Hou[1]†, Kaiyi Qian[1], Leizhen Chen[1], Mario Krenn[2,3]*, Markus Aspelmeyer[4,5]*, Anton Zeilinger[4,5]*, Shining Zhu[1]*, Xiao-Song Ma[1,6,7]*

Violation of local realism via Bell inequality—a profound and counterintuitive manifestation of quantum theory that conflicts with the prediction of local realism—is viewed to be intimately linked with quantum entanglement. Experimental demonstrations of such a phenomenon using quantum entangled states are among the landmark experiments of modern physics and paved the way for quantum technology. Here, we report the violation of the Bell inequality that cannot be described by quantum entanglement in the system but arises from quantum indistinguishability by path identity, shown by the multiphoton frustrated interference. By analyzing the measurement of four-photon frustrated interference within the standard Bell-test formalism, we find a violation of Bell inequality by more than four SDs. Our work establishes a connection between quantum correlation and quantum indistinguishability, providing insights into the fundamental origin of the counterintuitive characteristics observed in quantum physics.

INTRODUCTION

Interference of single quanta epitomizes the counterintuitive character of quantum physics. As Feynman *et al.* (*1*) said, the interference of the single quantum "has in it the heart of quantum mechanics." In all interference phenomena, there is more than one possibility giving rise to an event (such as detecting a photon at the outputs of an interferometer). Only when these possibilities are indistinguishable, their probability amplitudes are summed up, which leads to quantum interference (*2*–*5*).

By extending interference phenomena from single to multiple quanta, one creates entanglement (*6*–*8*), the coherent superposition of states shared by several quanta. Entanglement allows quantum correlations between many quanta that are stronger than classically possible, which is manifested by multiquantum interference. Quantum correlations cannot be explained or reproduced by classical physics, as proven by Bell's theorem (*9*, *10*). Entanglement is one of the most profound traits of quantum mechanics and is a key resource for quantum information technology.

In 1991, Zou, Wang, and Mandel (*11*, *12*) realized a novel type of quantum interference and explored the concept of quantum indistinguishability in a mind-boggling way. In their work and the later work by Herzog *et al.* (*13*), two processes of photon-pair generation were arranged such that the paths of the emitted photons are indistinguishable. The path information never exists in the first place due to this indistinguishability of the generation processes. The detection of individual photons carries no information about where they are generated. Consequently, Herzog *et al.* observed so-called frustrated interference (FI), in which the photon-pair–generation processes can be suppressed or promoted by varying the interferometric phases. This kind of quantum interference due to indistinguishability has been applied to such fields as quantum imaging (*14*, *15*), spectroscopy (*16*), optical coherence tomography (*17*), quantum state generation and analysis (*18*–*20*), microscopy (*21*, *22*), and quantum holography (*23*). Related quantum interference has also been observed with two weak fields from a local oscillator (*24*).

However, two-photon FI is an intrinsically local phenomenon (*25*). This is due to two key factors: First, individual photon counts exhibit interference based on the combined phases of both photons, leading to interference in joint outcomes. This is fundamentally different from entanglement, in which joint outcomes interfere as well, but individual photon counts exhibit no interference. Second, the experimental settings, such as those determining the phases of each photon, are decided inside the backward light cones of the detection events.

Here, we show that much richer phenomena appear with correlations between more than two particles, by demonstrating experimental violation of Bell inequality through quantum indistinguishability by path identity rather than quantum entanglement. The key to these experiments is multiphoton FI, which was recently proposed (*26*) and demonstrated (*27*, *28*). In the present work, we demonstrate the violation of a Bell inequality by more than four SDs and establish that the observed effect cannot be explained by entanglement, neither at the outcome nor anywhere within the experimental setup. Our result has intriguing consequences for the foundations of quantum information science and opens up a different perspective on the question regarding the fundamental origin of conflict between quantum mechanics and local realism.

RESULTS

The scheme of our experiment is shown in Fig. 1A. There are four probabilistic two-photon sources, which are coherently pumped by classical lasers and are labeled as sources I to IV (represented by white squares in Fig. 1A). Each source generates a pair of photons (signal and idler photons) in a polarization product state ($|HV\rangle$; H/V represents horizontal/vertical polarization). The spatial modes of signal photons (H-polarized) from sources I and III and II and IV

[1]National Laboratory of Solid State Microstructures, School of Physics, Collaborative Innovation Center of Advanced Microstructures, Nanjing University, Nanjing 210093, China. [2]Max Planck Institute for the Science of Light (MPL), Erlangen, Germany. [3]Machine Learning in Science, Department for Computer Science, Faculty of Science, University of Tuebingen, Tübingen, Germany. [4]Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, Boltzmanngasse 3, Vienna A-1090, Austria. [5]Faculty of Physics, Vienna Center for Quantum Science and Technology (VCQ), University of Vienna, Boltzmanngasse 5, Vienna A-1090, Austria. [6]Hefei National Laboratory, Hefei 230088, China. [7]Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, Anhui 230026, China.
*Corresponding author. Email: mario.krenn@mpl.mpg.de (M.K.); markus.aspelmeyer@univie.ac.at (M.A.); anton.zeilinger@univie.ac.at (A.Z.); zhusn@nju.edu.cn (S.Z.); xiaosong.ma@nju.edu.cn (X.-S.M.)
†These authors contributed equally to this work.

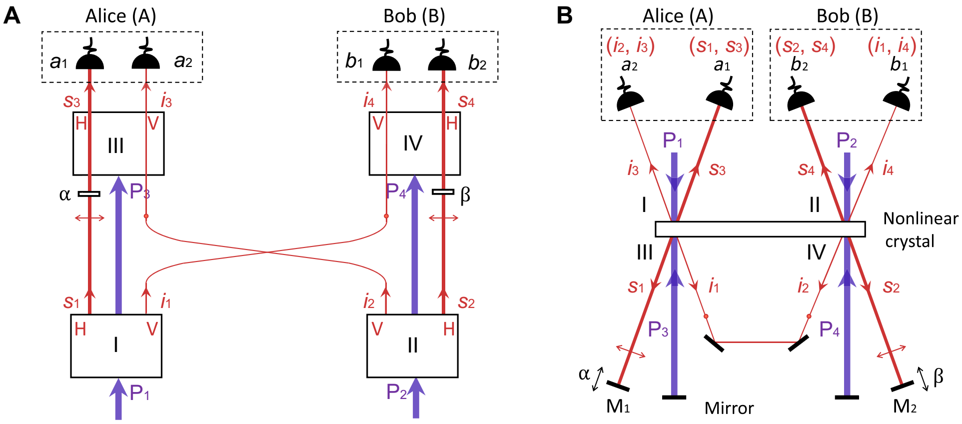

Fig. 1. Four-photon FI. (**A**) Every white square (I, II, III, and IV) represents a two-photon source that can generate polarization product states in $|HV\rangle$. The four-photon state we postselected can either be generated from sources I and II (s_1, i_1, i_2, and s_2) or sources III and IV (s_3, i_4, i_3, and s_4). When the photons on each individual path are identical in every df, they interfere with each other. (**B**) Simplified scheme of the experimental setup. Pumps P_1 and P_2 provide the possibility to generate two pairs of photons $(s_1, i_1)_{I}$ and $(s_2, i_2)_{II}$ in the form of polarization product states from a nonlinear crystal. The pump beams are then reflected as P_3 and P_4, providing the possibility to generate another two pairs of photons $(s_3, i_3)_{III}$ and $(s_4, i_4)_{IV}$, respectively. The signal photons (horizontally polarized) s_1 and s_2 are reflected back to the crystal by two mirrors M_1 and M_2, respectively, and are aligned with s_3 and s_4, respectively, to ensure path indistinguishability. Photons s_1 and s_3 are then in the mode a_1, and photons s_2 and s_4 in the mode b_2. The idler photons i_1 and i_2 (vertically polarized) propagate along the same path but in opposite directions. This is equivalent to swapping their paths in (A). Alice (Bob) detects photons on the modes a_1 and a_2 (modes b_1 and b_2). By moving M_1 (M_2), we change the measurement settings by controlling the phase α (β). The phase stability, including the relative phases of four pump beams, signal photons, and idler photons, in this four-photon frustrated interferometer is crucial and is passively maintained experimentally.

are aligned identically to the paths a_1 and b_2, shown with thicker red lines in Fig. 1A. The spatial modes of idler photons (V-polarized) from sources I and II are swapped and then aligned with those of the idler photons from sources III and IV, whose paths are identical to a_2 and b_1, shown with thin red lines in Fig. 1A. The phases of the signal photons at Alice and Bob's sides, α and β, are set by the respective phase shifters. The final state of the photons occupying the modes a_1, a_2, b_1, and b_2 by postselecting the four-photon term is

$$|\psi_f\rangle_{a_1a_2b_1b_2} = g^2\left[e^{i(\alpha+\beta)}\,|1111\rangle + |1111\rangle\right] \tag{1}$$

where the numbers represent the photon-number states in the respective modes a_1, a_2, b_1, and b_2. For the complete form of the output state to the second-order approximation, see Materials and Methods. The parameter g is the efficiency of the spontaneous parametric down-conversion (SPDC) process, which depends on the pump power and the nonlinearity of the nonlinear crystal. The generated rate of the four-photon coincidence events by simultaneously detecting a_1, a_2, b_1, and b_2 (Eq. 1) is a function of both phases α and β

$$\mathcal{N}(\alpha,\beta) = g^4 N_0\left[2 + 2\cos(\alpha+\beta)\right] \tag{2}$$

where N_0 is the repetition rate of the classical pump light with high intensity.

A simplified schematic of our experimental setup is shown in Fig. 1B. See fig. S2 in the Supplementary Materials for the detailed setup. Two parallel and mutually coherent pump beams P_1 and P_2 (violet arrows) provide the possibility to generate two pairs of photons from crystals I (s_1 and i_1) and II (s_2 and i_2) in the form of a polarization product state from the nonlinear crystal. The paths of idlers from sources I (i_1) and II (i_2) are swapped by routing the same path reversely. The signal photons s_1 and s_2 are reflected by two mirrors (M_1 and M_2), with which Alice and Bob can change their measurement settings of phases α and β by moving their respective mirrors. The propagating pumps are reflected back to the crystal (as beams P_3 and P_4). They provide the possibility to generate another two photon pairs III (s_3 and i_3) and IV (s_4 and i_4). On the basis of the scheme shown in Fig. 1A, the modes of photons from sources III and IV are aligned with the photons from sources I and II: s_1 and s_3 occupying mode a_1, i_1 and i_4 occupying mode b_1, i_2 and i_3 occupying mode a_2, and s_2 and s_4 occupying mode b_2. We emphasize that photons on the same path are indistinguishable in every degree of freedom (df). Therefore, when we detect four photons simultaneously on modes a_1, a_2, b_1, and b_2, we cannot determine whether they are from sources I and II or sources III and IV. Then we obtain the interference of the two possible processes and the phase-dependent fourfold coincidence counts (C. C.), as given by Eq. 2.

Initially, the two photons from the sources in our experiment are entangled in momentum and frequency. We then destroy their momentum entanglement using single-mode fiber coupling, which performs strong projective measurements on the momentum of the photons. Using band-pass filters (~3-nm bandwidth), we also destroy the frequency entanglement between them. Therefore, we cannot use these internal entangled df in the Bell-inequality test presented below.

In our analysis, we represent the simultaneous detection of the two photons (a_1 and a_2 for Alice, and b_1 and b_2 for Bob; see Fig. 1) as outcome +1. To get outcome −1 in the phase setting α, Alice sets the phase to an orthogonal base α + π and then detects two photons simultaneously. Similarly, Bob sets the phase β + π to get his −1 outcome. This is based on the cosine-phase dependence with period 2π in the FI of our work; see fig. S3 in the Supplementary Materials. We therefore make the following assumptions about the count rate $N(a,b|\alpha,\beta)$ to construct the joint probability: $N(+1,-1|\alpha,\beta) = N(+1,+1|\alpha,\beta+\pi)$; $N(-1,+1|\alpha,\beta) = N(+1,+1|\alpha+\pi,\beta)$; $N(-1,-1|\alpha,\beta) = N(+1,+1|\alpha+\pi,\beta+\pi)$. For details, see Materials and Methods. Using this procedure, the probabilities of a (+1, +1) outcome for the setting (α, β) are

$$p(+1,+1|\alpha,\beta) = \frac{\mathcal{N}(\alpha,\beta)}{\mathcal{N}(\alpha,\beta)+\mathcal{N}(\alpha+\pi,\beta)+\mathcal{N}(\alpha,\beta+\pi)+\mathcal{N}(\alpha+\pi,\beta+\pi)} = \frac{1}{4}+\frac{1}{4}\cos(\alpha+\beta) \quad (3)$$

where $N(\alpha, \beta)$ is the fourfold C. C. rate [Eq. 2; $N(+1,+1|\alpha,\beta)$] of a_1, a_2, b_1, and b_2 for the setting (α, β) of Alice and Bob. The probabilities of the other possible outcomes are

$$p(-1,+1|\alpha,\beta) = \frac{\mathcal{N}(\alpha+\pi,\beta)}{\mathcal{N}(\alpha,\beta)+\mathcal{N}(\alpha+\pi,\beta)+\mathcal{N}(\alpha,\beta+\pi)+\mathcal{N}(\alpha+\pi,\beta+\pi)} = \frac{1}{4}-\frac{1}{4}\cos(\alpha+\beta) \quad (4)$$

$$p(+1,-1|\alpha,\beta) = \frac{\mathcal{N}(\alpha,\beta+\pi)}{\mathcal{N}(\alpha,\beta)+\mathcal{N}(\alpha+\pi,\beta)+\mathcal{N}(\alpha,\beta+\pi)+\mathcal{N}(\alpha+\pi,\beta+\pi)} = \frac{1}{4}-\frac{1}{4}\cos(\alpha+\beta) \quad (5)$$

$$p(-1,-1|\alpha,\beta) = \frac{\mathcal{N}(\alpha+\pi,\beta+\pi)}{\mathcal{N}(\alpha,\beta)+\mathcal{N}(\alpha+\pi,\beta)+\mathcal{N}(\alpha,\beta+\pi)+\mathcal{N}(\alpha+\pi,\beta+\pi)} = \frac{1}{4}+\frac{1}{4}\cos(\alpha+\beta) \quad (6)$$

This method can be viewed as an analogy to Bell experiments using polarizers (which exhibit π-period polarization rotation). In Bell experiments based on polarizers (*29*, *30*), when the polarizer is set to 0°, only the +1 (H) outcome is detected, while the −1 (V) outcome is filtered out; conversely, when set to 90°, only the +1 (V) outcome is observed, while the −1 (H) outcome is filtered out. For a detailed comparison of constructing joint probabilities between two entangled photons [as in the standard Bell experiment (*29*, *30*)] and unentangled four photons by postselection (as shown in this work), see the Supplementary Materials. Therefore, once Alice and Bob choose their specific settings of phase α and β, the normalized expectation value of their result is

$$E(\alpha,\beta) = p(+1,+1|\alpha,\beta) - p(-1,+1|\alpha,\beta) - p(+1,-1|\alpha,\beta) + p(-1,-1|\alpha,\beta) = \cos(\alpha+\beta) \quad (7)$$

With Eq. 7, we construct the Clauser-Horne-Shimony-Holt (CHSH) form of the Bell inequality (*31*) for the four-photon FI

$$S = |-E(\alpha_1,\beta_1) + E(\alpha_1,\beta_2) + E(\alpha_2,\beta_1) + E(\alpha_2,\beta_2)| \quad (8)$$

Therefore, we established a connection between our experimental configuration and the CHSH inequality through the joint probability distribution. In our scheme, the single observer measures the twofold C. C. rather than single counts as in the standard CHSH-Bell experiment. When we set $\alpha_1 = 0$, $\alpha_2 = \pi/2$, $\beta_1 = \pi/4$, and $\beta_2 = 3\pi/4$, the Bell parameter S reaches the maximum value of $2\sqrt{2}$. The classical bound of the S parameter is 2.

Experimentally, we measure the quantum correlation between Alice and Bob by fixing α at $\alpha_1 = 0, \alpha_1^{\perp} = \pi, \alpha_2 = \pi/2$, and $\alpha_2^{\perp} = 3\pi/2$ and sweeping the phase β. The results of the fourfold C. C. are shown in Fig. 2. The fourfold C. C. for two orthogonal bases with phases $\alpha = 0$ and $\alpha = \pi$, respectively, show complementary results (Fig. 2A). The other two orthogonal bases with $\alpha = \pi/2$ and $\alpha = 3\pi/2$ also yield complementary results (Fig. 2B). For the systematic analysis of the complete data, see figs. S3 and S4 in the Supplementary Materials.

The correlation functions $E(\alpha, \beta)$ (Eq. 7) of Alice and Bob are shown in Fig. 3A, which are calculated from the data presented in Fig. 2. The maximum values of the correlation functions are 0.766 ± 0.029 ($\alpha = 0$) and 0.784 ± 0.028 ($\alpha = \pi/2$), respectively, both of which are above the threshold $\left(V = 1/\sqrt{2}\right)$ as required for violating the CHSH inequality (*31*). We calculate the CHSH inequality from the C. C. (Fig. 2), which are listed in table S1 in the Supplementary Materials. We obtain a Bell parameter S of 2.275 ± 0.057, which means that the Bell inequality is violated by more than four SDs. In our previous work (*28*), the visibility and its statistical significance were not high enough for achieving confident violations of the Bell inequality. We have further systematically measured the fourfold FI over more periods. The detailed results are shown in fig. S4 in the Supplementary Materials. The visibilities are all above the threshold $\left(V = 1/\sqrt{2}\right)$. The observed violation of the CHSH inequality implies that the interference phenomenon we have witnessed cannot be accounted for by local realism. One may think that the entanglement between vacuum and four-photon product states leads to the violation of the CHSH inequality. In section S2 of the Supplementary Materials, we show that the generated state from pair-creation processes contains a substantially smaller amount of entanglement than necessary for the observed Bell-inequality violation.

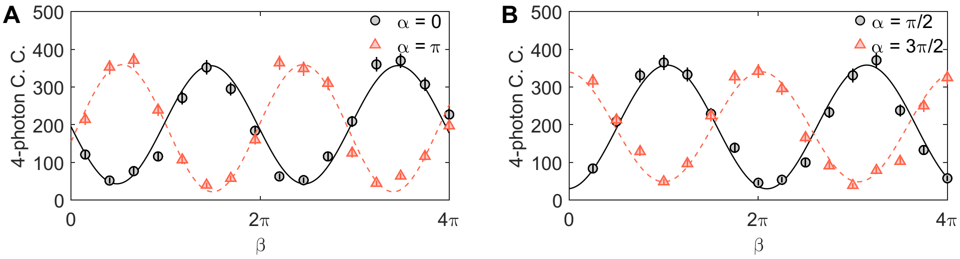

Fig. 2. Correlation results of four-photon coincidence counts *N*(α, β). (**A**) The coincidence of Alice and Bob in fixed settings α = 0 (black circles, fitted with black solid curve) and α = π (red triangles, fitted with red dashed curve). The visibilities of the two curves are 0.754 ± 0.021 (α = 0) and 0.805 ± 0.019 (α = π), respectively. (**B**) The coincidence of Alice and Bob in fixed settings α = π/2 (black circles, fitted with black solid curve) and α = 3π/2 (red triangles, fitted with red dashed curve). The visibilities of the two curves are 0.779 ± 0.020 (α = π/2) and 0.795 ± 0.020 (α = 3π/2), respectively. The horizontal axis represents the settings of Bob (β), which vary from 0 to 4π. The error bars are derived from Poissonian distributions. Each data point is measured for 60 s.

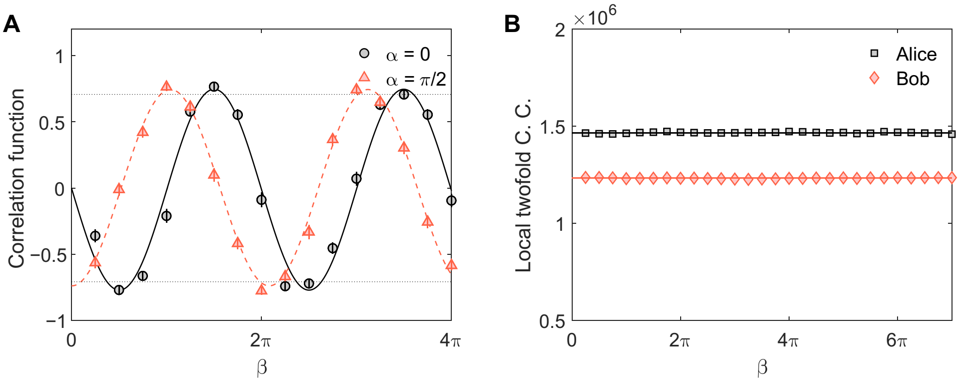

Fig. 3. Verification of quantum correlation. (**A**) Correlation functions $E(\alpha, \beta)$. The *x* axis represents phase β. The maximum values of the correlation functions are 0.766 ± 0.029 (α = 0) and 0.784 ± 0.028 (α = π/2), respectively. (**B**) Local results of Alice and Bob. The nonlocal interference in four-photon FI shows no interference with only local detection.

Figure 3B shows the local twofold C. C. of Alice [C. C. (a_1 and a_2)] and Bob [C. C. (b_1 and b_2)]. The counts show no interference. This means that the interference seen in Fig. 2 is not due to the local variation of Alice or Bob but arises from the correlation between the two parties. Alice cannot derive the phase β from her detection result. This is similar to measuring the local single counts of entangled states in a Bell test, which are independent of both Alice's and Bob's settings.

DISCUSSION

In the interferometer made by nonlinear crystals, we suppress or enhance the generation rate of the four-photon C. C. Our results show that using the measurement and the outcomes in the Bell-experiment formalism, we violate the CHSH inequality in the four-photon interferometer consisting of four nonlinear crystals using path identity. In this work, indistinguishability of the creation processes—rather than entanglement—is responsible for our observation. Conceptually, our experiment differs from a conventional Bell-violation experiment in one crucial way: Rather than merely measuring an entangled quantum state, we actively manipulate the state during its creation. Our experimental setup enables this manipulation to show quantum correlations via the violation of the Bell inequality.

One may have doubts about where the down-converted photons generated in sources I and II go when interference is destructive. The viewpoint that photons have been generated in sources I and II is incorrect. According to Bohr, "physics concerns what we can say about nature" (*32*). If we were to measure and can conclusively say that four photons have been generated by crystals I and II or III and IV, then we obtain which-source information and therefore destroy the final interference. The right interpretation is that we merely create a possibility of generating four down-converted photons rather than actually generating those four photons. Hypothetically, one could argue that we created a multiphoton superposition between the two different four-photon sources (sources I and II and sources III and IV after the postselection of fourfold C. C.), which leads to a multiphoton superimposed state $|\psi\rangle = 1/\sqrt{2}\left[e^{i(\alpha+\beta)}|1111\rangle_{\mathrm{I\&II}} + |1111\rangle_{\mathrm{III\&IV}}\right]$, where the subscripts represent the respective four-photon sources shown in Fig. 1A. However, this view is incorrect. As long as the photon paths are superimposed, the which-path information never exists (*11*, *33*) and hence need not be erased, which is the fundamental difference between this work compared to all existing quantum-eraser type works (*34*–*37*). On the other hand, when all probability amplitudes corresponding to down-conversion emission processes leading to four-photon coincidences interfere destructively, there is no consistent set of combinations of "physical" down-conversion emission processes that could lead to the detected four-photon coincidence. Therefore, it is incorrect to argue that the state $|\psi\rangle$ is responsible for the Bell violation because this process actually cannot happen when the interference is destructive. This is the actual "counterintuitive" aspect of the work.

In this work, we postselect four-photon C. C. to violate the inequality, which could potentially be mitigated with highly efficient photon generation processes (*38*). We anticipate to address this limitation in future implementations. The setting events of α and β could be in principle space like separated in four-photon FI (see the scheme in Fig. 1). In the present work, the settings α and β are close to each other, and hence, the locality loophole remains open (alongside the sampling loophole). Classical local realism theories have improved and found new loopholes over the past decades in the context of Bell-inequality violation with entangled photon pairs (*29*, *39*–*45*). We not only expect that tailored loopholes and local hidden variable to the work reported here can be identified but also expect that they will be consistently excluded by hardware improvements of high-quality quantum photonic devices and experiments, as we witnessed in the 90-year endeavor in the violations of local realism with entangled particles (*6*–*9*, *46*–*50*). Moreover, our work could very well lead to other interesting experiments, such as in the development of the Bell experiment. In analogy to the Bell experiment with two particles, we expect that quantum mechanics will lastly prevail.

MATERIALS AND METHODS

Definition of probabilities

We observe that the FI exhibits a 2π-period cosine dependence on the phase (see fig. S3 in the Supplementary Materials). In particular, the coincidence count under a π phase shift, $N(+1,+1|\alpha+\pi,\beta)$, is complementary to the count in the original setting, $N(+1,+1\,|\,\alpha,\beta)$, and therefore can be identified as $N(-1,+1\,|\,\alpha,\beta)$, which is also complementary to $N(+1,+1\,|\,\alpha,\beta)$. Accordingly, we adopt the following relations between outcome counts and interferometer settings

$$N(+1,+1\,|\,\alpha,\beta)=N(+1,-1\,|\,\alpha,\beta+\pi)=N(-1,+1\,|\,\alpha+\pi,\beta)=N(-1,-1\,|\,\alpha+\pi,\beta+\pi) \tag{9}$$

$$N(+1,-1\,|\,\alpha,\beta)=N(+1,+1\,|\,\alpha,\beta+\pi)=N(-1,-1\,|\,\alpha+\pi,\beta)=N(-1,+1\,|\,\alpha+\pi,\beta+\pi) \tag{10}$$

$$N(-1,+1\,|\,\alpha,\beta)=N(-1,-1\,|\,\alpha,\beta+\pi)=N(+1,+1\,|\,\alpha+\pi,\beta)=N(+1,-1\,|\,\alpha+\pi,\beta+\pi) \tag{11}$$

$$N(-1,-1\,|\,\alpha,\beta)=N(-1,+1\,|\,\alpha,\beta+\pi)=N(+1,-1\,|\,\alpha+\pi,\beta)=N(+1,+1\,|\,\alpha+\pi,\beta+\pi) \tag{12}$$

In general, the joint conditional probabilities should be defined in the same setting

$$p(a,b\,|\,a,\beta)=\frac{N(a,b\,|\,a,\beta)}{N(+1,+1\,|\,a,\beta)+N(+1,-1\,|\,a,\beta)+N(-1,+1\,|\,a,\beta)+N(-1,-1\,|\,a,\beta)} \tag{13}$$

where $a=\pm1$, $b=\pm1$, and we use $N(+1,+1|\alpha,\beta)$, $N(+1,-1|\alpha,\beta)$, $N(-1,+1|\alpha,\beta)$, and $N(-1,-1|\alpha,\beta)$ to construct the probability $p(a,b|\alpha,\beta)$.

However, in our experiment, there is no −1 output. Therefore, on the basis of Eqs. 9 to 12, we use the following instead

$$p(a,b\,|\,\alpha,\beta)=\frac{N\left(+1,+1\,|\,\alpha+\frac{1-a}{2}\pi,\beta+\frac{1-b}{2}\pi\right)}{N(+1,+1\,|\,\alpha,\beta)+N(+1,+1\,|\,\alpha,\beta+\pi)+N(+1,+1\,|\,\alpha+\pi,\beta)+N(+1,+1\,|\,\alpha+\pi,\beta+\pi)} \tag{14}$$

Here, we use the terms that yield a (+1, +1) outcome and are accessible in our experiment.

This approach is analogous to Bell experiments using polarizers (*29*, *30*), which exhibit a polarization rotation periodicity of π. Specifically, when the polarizer is set to 0°, only the +1 (H) outcome is detected, while the orthogonal −1 (V) outcome is blocked. These experiments typically rely on the assumption based on Malus's law, the cosine dependence of the intensity of a polarized beam after an ideal polarizer (*51*). This assumption permits one to use the +1 outcome measured with the polarizer oriented at 90° to infer the otherwise inaccessible −1 outcome at 0°. In other words, by performing selective projective measurements along these mutually orthogonal directions, the combined outcomes form a complete projective measurement. We adopt the assumptions (Eqs. 9 to 12) when constructing the conditional probabilities, correlation functions, and the inequality.

The output state of the four-photon FI

The unnormalized final state of the photons in modes a_1, a_2, b_1, and b_2, truncated to second order in the SPDC amplitude, is

$$|\psi_f\rangle_{a_1a_2b_1b_2}=|0000\rangle+g\left[e^{i\alpha}|1010\rangle+e^{i\beta}|0101\rangle+|1100\rangle+|0011\rangle\right]+g^2\left[e^{2i\alpha}|2020\rangle+e^{2i\beta}|0202\rangle+|2200\rangle+|0022\rangle\right]+g^2\left[e^{i(\alpha+\beta)}|1111\rangle+|1111\rangle\right]+g^2\sqrt{2}\left[e^{i\alpha}|2110\rangle+e^{i\alpha}|1021\rangle+e^{i\beta}|1201\rangle+e^{i\beta}|0112\rangle\right] \tag{15}$$

where each term denotes a Fock state, indicating the photon numbers in the respective modes a_1, a_2, b_1, and b_2. The parameter g characterizes the down-conversion gain (efficiency) of the SPDC process. We emphasize that Eq. 15 is an approximate expression. Notably, when evaluating the probabilities of local detection events observed by Alice, the calculated probability distribution $p_{a_1,a_2}(+1,+1)$ appears to depend on the phase β, which, if naively interpreted, might suggest superluminal signaling. This apparent paradox, however, arises from truncating higher-order terms in our theoretical framework. We confirm this through the following analytical derivations:

For each nonlinear crystal, we apply the two-mode squeezing operator

$$\hat{S}(g)=e^{-g\left(ab-a^\dagger b^\dagger\right)} \tag{16}$$

acting on input modes $a_{\rm in}$ and $b_{\rm in}$ (*52*)

$$\begin{pmatrix} a_{\rm out} \\ b^\dagger_{\rm out}\end{pmatrix}=\begin{pmatrix}\cosh g & \sinh g\\ \sinh g & \cosh g\end{pmatrix}\begin{pmatrix} a_{\rm in} \\ b^\dagger_{\rm in}\end{pmatrix} \tag{17}$$

From this, we obtain the four output modes of our experiment

$$\begin{aligned}
a_{1\rm out} &= \cosh^2 g a_{1\rm in}+\sinh^2 g b_{2\rm in}+\sinh g\cosh g\left(b^\dagger_{1\rm in}+a^\dagger_{2\rm in}\right)\\
a_{2\rm out} &= \cosh^2 g a_{2\rm in}+\sinh^2 g b_{1\rm in}+\sinh g\cosh g\left(a^\dagger_{1\rm in}+b^\dagger_{2\rm in}\right)\\
b_{1\rm out} &= \cosh^2 g b_{1\rm in}+e^{i\beta}\sinh^2 g a_{2\rm in}+\sinh g\cosh g\left(a^\dagger_{1\rm in}+e^{i\beta}b^\dagger_{2\rm in}\right)\\
b_{2\rm out} &= e^{-i\beta}\cosh^2 g b_{2\rm in}+\sinh^2 g a_{1\rm in}+\sinh g\cosh g\left(b^\dagger_{1\rm in}+e^{-i\beta}a^\dagger_{2\rm in}\right)
\end{aligned} \tag{18}$$

where we have set $\alpha=0$. When Alice performs coincidence detections on paths a_1 and a_2, the second-order correlation is independent of β

$$\left\langle a^\dagger_{1\rm out}a_{1\rm out}a^\dagger_{2\rm out}a_{2\rm out}\right\rangle=6\sinh^4 g\cosh^4 g+\sinh^2 g\cosh^6 g+\sinh^6 g\cosh^2 g \tag{19}$$

Numerical simulations further validate this result (see section S1 in the Supplementary Materials).

Supplementary Materials

This PDF file includes:

Supplementary Text
Figs. S1 to S4
Table S1

REFERENCES AND NOTES

1. R. P. Feynman, R. B. Leighton, M. Sands, *Lectures on Physics* (Addison-Wesley, 1965).
2. L. Mandel, Coherence and indistinguishability. *Opt. Lett.* **16**, 1882 (1991).
3. L. Mandel, Quantum effects in one-photon and two-photon interference. *Rev. Mod. Phys.* **71**, S274–S282 (1999).
4. A. Zeilinger, Experiment and the foundations of quantum physics. *Rev. Mod. Phys.* **71**, S288–S297 (1999).
5. A. Zeilinger, G. Weihs, T. Jennewein, M. Aspelmeyer, Happy centenary, photon. *Nature* **433**, 230–238 (2005).
6. A. Einstein, B. Podolsky, N. Rosen, Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **47**, 777–780 (1935).
7. N. Bohr, Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **48**, 696–702 (1935).
8. E. Schrödinger, die gegenwärtige situation in der quantenmechanik. *Naturwissenschaften* **23**, 807–812; 823–828; 844–849 (1935).

9. J. S. Bell, On the Einstein Podolsky Rosen paradox. *Physics Physique Fizika* **1**, 195–200 (1964).
10. N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Bell nonlocality. *Rev. Mod. Phys.* **86**, 419–478 (2014).
11. X. Y. Zou, L. J. Wang, L. Mandel, Induced coherence and indistinguishability in optical interference. *Phys. Rev. Lett.* **67**, 318–321 (1991).
12. D. M. Greenberger, M. A. Horne, A. Zeilinger, Multiparticle interferometry and the superposition principle. *Phys. Today* **46**, 22–29 (1993).
13. T. J. Herzog, J. G. Rarity, H. Weinfurter, A. Zeilinger, Frustrated two-photon creation via interference. *Phys. Rev. Lett.* **72**, 629–632 (1994).
14. G. B. Lemos, V. Borish, G. D. Cole, S. Ramelow, R. Lapkiewicz, A. Zeilinger, Quantum imaging with undetected photons. *Nature* **512**, 409–412 (2014).
15. Y. Yang, H. Liang, X. Xu, L. Zhang, S. Zhu, X.-s. Ma, Interaction-free, single-pixel quantum imaging with undetected photons. *npj Quantum Inf.* **9**, 2 (2023).
16. D. A. Kalashnikov, A. V. Paterova, S. P. Kulik, L. A. Krivitsky, Infrared spectroscopy with visible light. *Nat. Photonics* **10**, 98–101 (2016).
17. A. V. Paterova, H. Yang, C. An, D. A. Kalashnikov, L. A. Krivitsky, Tunable optical coherence tomography in the infrared range using visible photons. *Quantum Sci. Technol.* **3**, 025008 (2018).
18. M. Krenn, A. Hochrainer, M. Lahiri, A. Zeilinger, Entanglement by path identity. *Phys. Rev. Lett.* **118**, 080401 (2017).
19. J. Su, L. Cui, J. Li, Y. Liu, X. Li, Z. Y. Ou, Versatile and precise quantum state engineering by using nonlinear interferometers. *Opt. Express* **27**, 20479–20492 (2019).
20. G. B. Lemos, R. Lapkiewicz, A. Hochrainer, M. Lahiri, A. Zeilinger, One-photon measurement of two-photon entanglement. *Phys. Rev. Lett.* **130**, 090202 (2023).
21. A. V. Paterova, S. M. Maniam, H. Yang, G. Grenci, L. A. Krivitsky, Hyperspectral infrared microscopy with visible light. *Sci. Adv.* **6**, eabd0460 (2020).
22. I. Kviatkovsky, H. M. Chrzanowski, E. G. Avery, H. Bartolomaeus, S. Ramelow, Microscopy with undetected photons in the mid-infrared. *Sci. Adv.* **6**, eabd0264 (2020).
23. S. Töpfer, M. Gilaberte Basset, J. Fuenzalida, F. Steinlechner, J. P. Torres, M. Gräfe, Quantum holography with undetected light. *Sci. Adv.* **8**, eabl4301 (2022).
24. K. J. Resch, J. S. Lundeen, A. M. Steinberg, Nonlinear optics with less than one photon. *Phys. Rev. Lett.* **87**, 123603 (2001).
25. T. Herzog, J. Rarity, H. Weinfurter, A. Zeilinger, Herzog *et al.* reply. *Phys. Rev. Lett.* **73**, 3041 (1994).
26. X. Gu, M. Erhard, A. Zeilinger, M. Krenn, Quantum experiments and graphs II: Quantum interference, computation, and state generation. *Proc. Natl. Acad. Sci. U.S.A.* **116**, 4147–4155 (2019).
27. L.-T. Feng, M. Zhang, D. Liu, Y.-J. Cheng, G.-P. Guo, D.-X. Dai, G.-C. Guo, M. Krenn, X.-F. Ren, On-chip quantum interference between the origins of a multi-photon state. *Optica* **10**, 105 (2023).
28. K. Qian, K. Wang, L. Chen, Z. Hou, M. Krenn, S. Zhu, X. S. Ma, Multiphoton non-local quantum interference controlled by an undetected photon. *Nat. Commun.* **14**, 1480 (2023).
29. S. J. Freedman, J. F. Clauser, Experimental test of local hidden-variable theories. *Phys. Rev. Lett.* **28**, 938–941 (1972).
30. A. Aspect, P. Grangier, G. Roger, Experimental tests of realistic local theories via Bell's theorem. *Phys. Rev. Lett.* **47**, 460–463 (1981).
31. J. F. Clauser, M. A. Horne, A. Shimony, R. A. Holt, Proposed experiment to test local hidden-variable theories. *Phys. Rev. Lett.* **23**, 880–884 (1969).
32. A. Petersen, The philosophy of Niels Bohr. *Bull. At. Sci.* **19**, 8–14 (1963).
33. A. Hochrainer, M. Lahiri, M. Erhard, M. Krenn, A. Zeilinger, Quantum indistinguishability by path identity and with undetected photons. *Rev. Mod. Phys.* **94**, 025007 (2022).
34. M. O. Scully, K. Drühl, Quantum eraser: A proposed photon correlation experiment concerning observation and "delayed choice" in quantum mechanics. *Phys. Rev. A* **25**, 2208–2213 (1982).
35. M. A. Horne, A. Shimony, A. Zeilinger, Two-particle interferometry. *Phys. Rev. Lett.* **62**, 2209–2212 (1989).
36. Z. Y. Ou, L. J. Wang, X. Y. Zou, L. Mandel, Evidence for phase memory in two-photon down conversion through entanglement with the vacuum. *Phys. Rev. A* **41**, 566–568 (1990).
37. J. G. Rarity, P. R. Tapster, Experimental violation of Bell's inequality based on phase and momentum. *Phys. Rev. Lett.* **64**, 2495–2498 (1990).
38. H. Hübel, D. R. Hamel, A. Fedrizzi, S. Ramelow, K. J. Resch, T. Jennewein, Direct generation of photon triplets using cascaded photon-pair sources. *Nature* **466**, 601–603 (2010).
39. A. Aspect, J. Dalibard, G. Roger, Experimental test of Bell's inequalities using time-varying analyzers. *Phys. Rev. Lett.* **49**, 1804–1807 (1982).
40. G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, A. Zeilinger, Violation of Bell's inequality under strict Einstein locality conditions. *Phys. Rev. Lett.* **81**, 5039–5043 (1998).
41. M. A. Rowe, D. Kielpinski, V. Meyer, C. A. Sackett, W. M. Itano, C. Monroe, D. J. Wineland, Experimental violation of a Bell's inequality with efficient detection. *Nature* **409**, 791–794 (2001).
42. T. Scheidl, R. Ursin, J. Kofler, S. Ramelow, X.-S. Ma, T. Herbst, L. Ratschbacher, A. Fedrizzi, N. K. Langford, T. Jennewein, A. Zeilinger, Violation of local realism with freedom of choice. *Proc. Natl. Acad. Sci. U.S.A.* **107**, 19708–19713 (2010).
43. M. Giustina, A. Mech, S. Ramelow, B. Wittmann, J. Kofler, J. Beyer, A. Lita, B. Calkins, T. Gerrits, S. W. Nam, R. Ursin, A. Zeilinger, Bell violation using entangled photons without the fair-sampling assumption. *Nature* **497**, 227–230 (2013).
44. B. G. Christensen, K. T. McCusker, J. B. Altepeter, B. Calkins, T. Gerrits, A. E. Lita, A. Miller, L. K. Shalm, Y. Zhang, S. W. Nam, N. Brunner, C. C. W. Lim, N. Gisin, P. G. Kwiat, Detection-loophole-free test of quantum nonlocality, and applications. *Phys. Rev. Lett.* **111**, 130406 (2013).
45. J. Gallicchio, A. S. Friedman, D. I. Kaiser, Testing Bell's inequality with cosmic photons: Closing the setting-independence loophole. *Phys. Rev. Lett.* **112**, 110405 (2014).
46. B. Hensen, H. Bernien, A. E. Dréau, A. Reiserer, N. Kalb, M. S. Blok, J. Ruitenberg, R. F. Vermeulen, R. N. Schouten, C. Abellán, W. Amaya, V. Pruneri, M. W. Mitchell, M. Markham, D. J. Twitchen, D. Elkouss, S. Wehner, T. H. Taminiau, R. Hanson, Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres. *Nature* **526**, 682–686 (2015).
47. M. Giustina, M. A. M. Versteegh, S. Wengerowsky, J. Handsteiner, A. Hochrainer, K. Phelan, F. Steinlechner, J. Kofler, J.-Å. Larsson, C. Abellán, W. Amaya, V. Pruneri, M. W. Mitchell, J. Beyer, T. Gerrits, A. E. Lita, L. K. Shalm, S. W. Nam, T. Scheidl, R. Ursin, B. Wittmann, A. Zeilinger, Significant-loophole-free test of Bell's theorem with entangled photons. *Phys. Rev. Lett.* **115**, 250401 (2015).
48. L. K. Shalm, E. Meyer-Scott, B. G. Christensen, P. Bierhorst, M. A. Wayne, M. J. Stevens, T. Gerrits, S. Glancy, D. R. Hamel, M. S. Allman, K. J. Coakley, S. D. Dyer, C. Hodge, A. E. Lita, V. B. Verma, C. Lambrocco, E. Tortorici, A. L. Migdall, Y. Zhang, D. R. Kumor, W. H. Farr, F. Marsili, M. D. Shaw, J. A. Stern, C. Abellán, W. Amaya, V. Pruneri, T. Jennewein, M. W. Mitchell, P. G. Kwiat, J. C. Bienfang, R. P. Mirin, E. Knill, S. W. Nam, Strong loophole-free test of local realism. *Phys. Rev. Lett.* **115**, 250402 (2015).
49. W. Rosenfeld, D. Burchardt, R. Garthoff, K. Redeker, N. Ortegel, M. Rau, H. Weinfurter, Event-ready Bell test using entangled atoms simultaneously closing detection and locality loopholes. *Phys. Rev. Lett.* **119**, 010402 (2017).
50. M.-H. Li, C. Wu, Y. Zhang, W.-Z. Liu, B. Bai, Y. Liu, W. Zhang, Q. Zhao, H. Li, Z. Wang, L. You, W. J. Munro, J. Yin, J. Zhang, C.-Z. Peng, X. Ma, Q. Zhang, J. Fan, J.-W. Pan, Test of local realism into the past without detection and locality loopholes. *Phys. Rev. Lett.* **121**, 080404 (2018).
51. S. Gröblacher, T. Paterek, R. Kaltenbaek, Č. Brukner, M. Żukowski, M. Aspelmeyer, A. Zeilinger, An experimental test of non-local realism. *Nature* **446**, 871–875 (2007).
52. C. Weedbrook, S. Pirandola, R. García-Patrón, N. J. Cerf, T. C. Ralph, J. H. Shapiro, S. Lloyd, Gaussian quantum information. *Rev. Mod. Phys.* **84**, 621–669 (2012).

Acknowledgments: We thank Č. Brukner for discussions. **Funding:** This work was funded by the National Key Research and Development Program of China (grants nos. 2022YFE0137000 and 2019YFA0308704) to X.-S.M., the Natural Science Foundation of Jiangsu Province (grants nos. BK20240006 and BK20233001) to X.-S.M., the Leading-Edge Technology Program of Jiangsu Natural Science Foundation (grant no. BK20192001) to X.-S.M., the Fundamental Research Funds for the Central Universities (grant no. 2024300324) to X.-S.M., the Innovation Program for Quantum Science and Technology (grant nos. 2021ZD0300700 and 2021ZD0301500) to X.-S.M., the Jiangsu Funding Program for Excellent Postdoctoral Talent (grant no. 20220ZB60) to K.W., the National Natural Science Foundation of China (grant no. 12304397) to K.W., and the Nanjing University - China Mobile Communications Group Co. Ltd. Joint Institute to X.-S.M. **Author contributions:** Motivation and critical insights: A.Z. and M.A. Methodology: K.W., M.K., and X.-S.M. Investigation: K.W., Z.H., K.Q., and L.C. Visualization: K.W., Z.H., and X.-S.M. Project administration: S.Z. and X.-S.M. Supervision: S.Z. and X.-S.M. Writing—original draft: K.W., M.K., A.Z., and X.-S.M. Writing—review and editing: K.W., Z.H., M.K., M.A., A.Z., and X.-S.M. **Competing interests:** The authors declare that they have no competing interests. **Data and materials availability:** The data that support the plots within this paper and other findings of this study are available on Dryad at https://doi.org/10.5061/dryad.7m0cfxq59 and on Github at https://github.com/NJU-Malab/Violate-CHSH-in-FI.

Submitted 18 June 2024
Accepted 1 July 2025
Published 1 August 2025
10.1126/sciadv.adr1794