STRONG DECAYS OF THE D_{SJ} (2317) MESONS USING QCD SUM RULES

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
PHYSICS

JULY 2007

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ABSTRACT

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July 2007, 28 pages

Unexpected properties of recently discovered mesons $D_{sJ}(2317)$ and $D_{sJ}(2460)$ have caused an excitement in the high energy community. These mesons are under experimental study in BaBar, Belle and CLEO. The experimental data on these mesons is quite limited at the moment, but it is expected to be improved in the following years. The unexpected properties of these mesons, such as the low mass, and small width, have caused speculations about their structure. Various models have been proposed which go beyond the simple quark-antiquark picture of the mesons, such as a meson molecule, or a four-quark state. Therefore, understanding the underlying structure of these mesons can reveal a deeper understanding of QCD. In this thesis, the strong decay of the $D_{sJ}(2317)$ meson, $D_{sJ}(2317) \to D_s\pi^0$, is studied using three-point QCD Sum Rules method in the conventional $c\bar{s}$ framework. $D_{sJ}(2317) \to D_s\pi^0$ decay violates isospin symmetry. Therefore, this decay is studied as a two stage process; an isospin conserving $D_{sJ}(2317) \to D_s\eta$ decay followed by the conversion of η into a π^0 due to isospin violation.

Keywords: QCD Sum Rules, D_{sJ} Mesons, Strong Decays of D_{sJ} Mesons.

ÖZ

D_{SJ} (2317) MEZONUNUN KUVVETLİ BOZUNMASININ TOPLAM KURALLARI KULLANARAK İNCELENMESİ

Aydemir, Ufuk

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Temmuz 2007, 28 sayfa

Yakın zamanda keşfedilen $D_{sJ}(2317)$ ve $D_{sJ}(2460)$ mezonlarının beklenmedik özellikleri yüksek enerji dünyasında büyük heyecan yarattı. Bu mezonlar BaBar, Belle, ve CLEO da deneysel olarak incelenmektedir. Şu anki deneysel veri sınırlı olmakla beraber ileriki yıllarda bu durumun düzelmesi beklenmektedir. Bu mezonların düşük kütle ve küçük genişliğe sahip olmaları gibi beklenmedik özellikleri bu mezonların yapıları hakkında değişik spekülasyonlara yol açmıştır. Bu mezonları, standart kuvark - anti-kuvark modelinin dışında, mezon molekülü ya da dört kuvarklı bir sistem olarak açıklamaya çalışan modeller öne sürülmüştür. Bu mezonların yapılarının anlaşılması, şu anda baz alınan modeller hakkında da bilgi verebilmesi açısından da önemlidir. Bu tezde $D_{sJ}(2317)$ mezonunun kuvvetli bozunması ($D_{sJ}(2317) \rightarrow D_s\pi^0$) üç nokta QCD toplam kuralları kullanılarak incelenmiştir. Bu bozunma izospin simetrisini ihlal etmektedir. Bu nedenle bu bozunma iki aşamalı çalışılacaktır; izospini koruyan $D_{sJ}(2317) \rightarrow D_s\eta$ bozunması ve bunu takiben izospin nedeniyle η'nın π^0'na dönüşmesi.

Anahtar Kelimeler: QCD Toplama Kuralları, D_{sJ} Mezonları, D_{sJ} Mezonlarının Kuvvetli Bozunmaları.

To my family

ACKNOWLEDGMENTS

I would like to express my deepest gratitude to my advisor Assoc. Prof. Dr. Altuğ Özpineci for his help, advice and guidance during my thesis. Without his supervision, encouragement and support I would not have been able to complete this thesis.

I am also deeply indebted to Prof. Dr. Namık Kemal Pak whose invaluable help and support not only for this thesis but also for my other studies in my undergraduate and graduate life helped me to gain a solid background in physics. I would like to especially thank him once again because of his friendly attitude, patience and supervision.

There are no words to describe the appreciation and gratitude I feel for my family. I am proud to have such a family which gave me valuable advice and support when I needed it most. And special thanks to Aslı Tandoğan for her endless love, faith in me, understanding and continued patience.

I am particularly indebted to Hayriye Sundu, for her very beneficial help and support.

I am also thankful to TÜBİTAK for the partial support provided under the project numbered 106T333 and the scholarship provided for my Master Education.

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CHAPTER 1

INTRODUCTION

The recently observed narrow resonance $D_{sJ}(2317)$ was firstly reported by BaBar Collaboration in the $D_s^+\pi^0$ invariant mass distribution in $e^+ - e^-$ annihilation process in April 2003 [1]. Afterwards, CLEO Collaboration confirmed the observation of the BaBar Collaboration and additionally reported a new second narrow peak at a mass of 2460 MeV, named $D_{sJ}(2460)$ [2], which was also confirmed by BaBar Collaboration [3]. Later on, Belle Collaboration confirmed both observations [4], and FOCUS Collaboration reported a narrow resonance at 2323 MeV which is slightly different than the previous observations [5]. In addition, these mesons were also observed in B decays by BaBar Collaboration [6].

When $D_{sJ}(2317)$ was observed, it was thought that this new narrow resonance ($\Gamma \leq 10$ MeV) is a candidate for the missing scalar p wave state in the $c\bar{s}$ spectrum (the candidate for the missing axial vector p-wave state is $D_{sJ}(2460)$), which was studied long before the observations of the D_{sJ} mesons [7]. The reason of this idea is that the possible quantum number of $D_{sJ}(2317)$ is $J^P = 0^+$ which is the quantum number of the missing scalar meson in this spectrum. Hence, it is tempting to identify $D_{sJ}(2317)$ as the missing scalar meson, but this identification is also problematic since expected mass of the missing bound state, according to quark model prediction [8, 9, 10], is in the range 2450-2500 MeV which is nearly 150 MeV higher than the mass of the $D_{sJ}(2317)$. If its mass was in this range, than it would strongly decay to DK because its mass would be above the DK decay channel threshold $M_{DK} = m_D + m_K = 2.359$ GeV.

However, the observed mass of the $D_{sJ}(2317)$ is below this threshold forbidding this channel. Hence $D_{sJ}(2317)$ has to decay by isospin-violating $D_s\pi^0$ mode, or it can make a radiative decay. Among the radiative decays, $D_s\gamma$ has not been observed yet. This is an accepted result (actually, it is also an expected result) due to the fact that if the quantum number of $D_{sJ}(2317)$ is $J^P = 0^+$, this decay is not possible because of the angular momentum and parity conservations. The possible radiative decay for the $D_{sJ}(2317)$ is the $D_s^*\gamma$ decay ending up with the $D_s\gamma\gamma$ final state resulting from the decay chain $D_{sJ}(2317) \to D_s^*\gamma \to D_s\gamma\gamma$. Observing $D_s\gamma\gamma$ final state becomes an evidence for $D_s^*\gamma$ transition of $D_{sJ}(2317)$. However, $D_s\gamma\gamma$ final state has not been observed yet.

In addition to the peculiarity of the mass of $D_{sJ}(2317)$, one more peculiarity comes from an observation of Belle Collaboration reporting a new non-strange state D (2308) with quantum number 0^+ [11]. Identifying this state as the scalar meson in $c\bar{s}$ mass spectrum causes another anomaly since the potential model [10] predicts the mass splitting of these two states to be nearly 110 MeV.

While conventional $c\bar{s}$ interpretation of $D_{sJ}(2317)$ keeps its popularity [13, 14, 15, 16, 17, 18, 19, 20, 21], the inconsistencies between mass values predicted by the models and observed by the experiments led to the new speculations to explain the structure of these mesons such as; four quark states [22, 23, 25, 26, 27, 24, 28], $D\pi$ atom [29] and DK molecules [30].

Understanding the D_{sJ} mesons is important since it might yield new information about the workings of QCD. The strong decays of D_{sJ} mesons will open more insights in understanding their structure.

The theory of the strong interactions within the framework of the Standard Model (SM) is Quantum Chromodynamics (QCD). Although the exact mechanism of the confinement is not known yet, we know that the quarks are confined inside the hadrons and produce bound states since all attempts to observe free quarks have been in vain. Confinement is the hypothesis which claims that there is no way to observe free quarks.

In QCD, the Lagrangian is invariant under SU(3) color symmetry, and the charges corresponding to SU(3) group are called color charges which are denoted symbolically as red, green and blue. According to the confinement, all observed particles have to be colorless. Therefore, the quarks are confined inside the hadrons such that the total charge of the hadrons (baryons (qqq) and mesons ($\bar{q}q$)) become zero (or white).

In QCD, unlike Quantum Electrodynamics (QED), the running coupling constant $\alpha_s(Q^2)$(where Q^2 is transferred momentum), which determines the strength of the interaction, is small at high energies (short distances) and large at low energies (long distances). Consequently, as the quarks get closer, the interaction strength decreases and at the asymptote they become nearly free particles. This phenomenon is called asymptotic freedom [31, 32, 33].

Since α_s is small at high energies (Q^2 is large), the calculations can be treated perturbatively at that region. In the series expansion in α_s, one can neglect the higher order graphs since as the power of α_s increases the contribution of corresponding term to total amplitude decreases. However, as far as long distance behavior is concerned, the perturbation method is not valid due to the fact that α_s approaches unity at long distances. Since a complete theory must include both long distance and short distance behaviors, non-perturbative QCD is a problem of strong interactions which should be dealt with separately.

One of the most important methods for non-perturbative QCD is the QCD SUM RULES (QCDSR) method, which is introduced by Shifman, Vainshtein and Zakharov (SVZ) [34]. Due to its advantages and its applicability to a large variety of problems, it has become a very important tool in hadron physics. In this method, a correlation function of the interpolating quark currents representing the hadrons is treated in two different ways. First, operator product expansion (OPE) [35] is used to represent the correlation function in terms of Wilson coefficients, which include information about short distances, and vacuum expectation values of local gauge invariant operators, which are solely related to long distance behavior. The Wilson coefficients can be calculated perturbatively since they include short distance behavior, and universal vacuum condensates are accounted for phenomenologically. The other way of treating the correlation function is to represent it phenomenologically in terms of hadronic parameters. Once these two representations are matched, a sum rule is found from which information about the observable characteristics of hadronic ground states can be extracted in terms of QCD parameters such as quark masses, vacuum condensates, etc.

The main idea of QCDSR method is to deal with the bound state problem of QCD by taking advantage of the simplicity of the theory in the ultraviolet region due to the asymptotic freedom. The method is to start from short distances where the perturbation theory can be used and the OPE is meaningful, and enlarge the domain to larger distances where asymptotic freedom starts to break down and perturbation theory cannot be used solely. Since the OPE is valid in the framework of perturbation theory [36, 37, 38], it is broken down at a critical distance.

In the standard perturbation theory, the vacuum expectation values of quark and gluon condensate operators such as $< 0 \mid \bar{q}q \mid 0 >$, $< 0 \mid G^a_{\mu\nu}G^{\mu\nu a} \mid 0 >$ vanish by definition. On the other hand, these matrix elements do not vanish as the asymptotic freedom is broken down, and they introduce power corrections, which are non-perturbative effects in the QCD vacuum [34]. In addition, the non-vanishing vacuum expectation value of $\bar{q}q$ signals the spontaneous breakdown of the chiral symmetry since these vacuum condensates mix quarks with left chirality with quarks with right chirality.

In this thesis, we have investigated the strong decay of $D_{sJ}(2317)$ meson, $D_{sJ}(2317) \to D_s\pi^0$, by three-point QCDSR. This decay is an isospin violating decay. Hence, it is treated as a two stage process ($D_{sJ}(2317) \to D_s\eta \to D_s\pi^0$). There is no isospin violation in $D_{sJ}(2317) \to D_s\eta$, and η and π^0 mix due to the isospin violation in $D_s\eta \to D_s\pi^0$. Therefore, $D_{sJ}(2317) \to D_s\pi^0$ can be treated in these two stages.

The strong decay of the $D_{sJ}(2317)$ was discussed previously in the framework of different approaches [14, 15, 18, 49]. It is also discussed in the framework of QCDSR in [39] in which light-cone QCDSR (LCQCDSR) is used. In LCQCDSR method, the wavefunction of the η meson is needed. The wavefunctions of η are known for on-shell η mesons. In this transition, η meson is off-shell, and hence, using its on-shell wavefunction in the calculations is not strictly correct but only an approximation. To compare the calculations of this decay of $D_{sJ}(2317)$, we studied this transition using three-point QCDSR method.

The thesis is organized as follows. In chapter 2, QCDSR will be reviewed in the framework of two-point correlation function case. First, OPE side of QCDSR will be reviewed in detail and then the phenomenological part will be discussed.

Then, the two representations will be matched using the spectral representation of the correlation function. Afterwards, three-point QCDSR formalism, which is used in this thesis to calculate $g_{D_{sJ}D_s\eta}$, will be discussed by generalizing the two-point case. In chapter 3, the sum rules for the $g_{D_{sJ}D_s\eta}$ will be derived. The final chapter is devoted to conclusions.

CHAPTER 2

QCD SUM RULES

2.1 OPE Side

The QCDSR method [34] is a method which relates the resonance parameters to the correlation function which defined as;

$$\Pi(q^2) = i \int d^4x \; e^{iqx} < 0 \mid T\{J(x), J(0)\} \mid 0 > \tag{2.1}$$

where T is the time ordering operator, $J(x)$'s are quark currents, $\mid 0 >$ is the non-perturbative (QCD) vacuum and q is the external momentum. The correlation function can be represented by Operator Product Expansion (OPE) if external momentum q or internal mass m_h is large [34], since in this region, the main contribution to Eq. (2.1) is from small distances where OPE is valid:

$$T\{J(x), J(0)\} = \sum_n C_n(x^2) O_n \tag{2.2}$$

and hence

$$i \int d^4x \; e^{iqx} < 0 \mid T\{J(x), J(0)\} \mid 0 >= \sum_n C_n(q^2) < 0 \mid O_n \mid 0 > \tag{2.3}$$

where C_n's are the Wilson Coefficients and O_n's are the local gauge invariant operators with zero spin. Actually, O_n's do not all have to be spin-0 operators, but only those with spin-0 contribute to the vacuum expectation values. When non-perturbative effects are taken into account, the OPE is broken down at a critical dimension which is generally assumed to be 6. The operators with zero spin and dimension less than 6 are;

$$
\begin{aligned}
&I(the\,unit\,operator) && (d=0), \\
&\overline{q}q && (d=3), \\
&G^a_{\mu\nu}G^{\mu\nu a} && (d=4), \\
&\overline{q}\sigma_{\mu\nu}\tfrac{\lambda^a}{2}G^{\mu\nu a}q && (d=5), \\
&\overline{q}\Gamma_m q\overline{q}\Gamma_n q && (d=6), \\
&f_{abc}G^a_{\mu\nu}G^{\nu b}_{\sigma}G^{\sigma\mu c} && (d=6) \qquad (2.4)
\end{aligned}
$$

where $G^a_{\mu\nu}$ is the gluon field strength tensor, λ^a are the Gell-Mann SU(3) matrices, $Tr(\lambda^a\lambda^b) = 2\delta^{ab}$, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ and Γ_m denotes the Dirac-tensor structure. The operators with $d > 6$ are assumed to give negligible contributions since they give rise to $1/q^d$ power corrections, for most of the applications [34, 40]. The lowest-dimension operator of the OPE is the unit operator (d=0), and it corresponds to perturbative contribution since $C_0(q^2) = \Pi^{pert}(q^2)$ and $< 0 \mid O_0 \mid 0 >\equiv 1$. In the other terms in the expansion, vacuum expectation values of the higher dimensional operators appear due to the non-perturbative effects.

One of the most impressive properties of the OPE is that in this representation, a scale μ has to be introduced which separates the long and short distance effects. In the region where $q^2 > \mu^2$ the interaction is included in Wilson Coefficients $C(q^2)$ and hence can be calculated perturbatively, where at $q^2 < \mu^2$ it is included in vacuum condensates parameterizing the non-perturbative contributions. There is no way to find an exact μ value, but μ should be large enough such that the Wilson coefficients can be calculated by Feynman Perturbation method. Since μ cannot be determined exactly, it is unavoidable that one does make double counting of soft part of perturbative diagrams and long distance effects which are included in vacuum condensates. However, this does not affect the calculations significantly since vacuum condensates contribution is much larger than the soft part of perturbative diagram [41].

2.2 Phenomenological Side and Analytic Continuation

In the previous section, OPE side of the correlation function is discussed. As mentioned before, the correlation function can also be represented in terms of phenomenological parameters. If a complete set of intermediate hadronic states is inserted in Eq. (2.1), the phenomenological representation of the correlation function is obtained in terms of hadronic states;

$$\Pi^{phen}(q^2) = \sum_h \frac{< 0|J|h(q) >< h(q)|\overline{J}|0 >}{q^2 - m_h^2} \tag{2.5}$$

where all the resonances are included. Since QCDSR deal with lowest lying resonance, it is more useful to separate the lowest lying part from the excited ones;

$$\Pi^{phen}(q^2) = \frac{< 0|J|h_1(p) >< h_1(p)|\overline{J}|0 >}{q^2 - m_{h_1}^2} + \int_{s_0^h}^{\infty} \frac{\rho^h(s)ds}{s - q^2} + polynomials \tag{2.6}$$

where $\rho^h(s)$ is the spectral density of higher states and continuum, and s_0^h is the threshold of the lowest continuum state. The polynomials in Eq. (2.6) are the subtraction terms which will be discussed later. Once $\Pi^{phen}(q^2)$ is found, physical information can be extracted by matching the two representations of the correlation function.

The operator product expansion part of $\Pi(q^2)$ is treated at large negative q^2 since, under this condition, $\Pi(q^2)$ receives contributions from short distances due to the large virtuality. Therefore, the calculations can be treated perturbatively at large negative q^2. The phenomenological part, however, is valid at positive q^2 domain since the masses of bound states are positive quantities. To match these two representations one must enlarge the domain of $\Pi(q^2)$ such that it includes the positive q^2 as well. For this purpose, the analytic continuation can be used to find a representation of $\Pi(q^2)$ by a function which is analytic everywhere except for the poles on the positive real axis representing the hadronic thresholds.

Using analytical properties of $\Pi(q^2)$, it can be represented as a contour integral around the point q^2 in the complex plane. If the contour is deformed and extended such that it does not cross the singular points which are hadronic threshold at $q^2 > 0$ (Fig. 2.1)[41];

$$\Pi(q^2) \quad = \frac{1}{2\pi i} \oint_C dz \frac{\Pi(z)}{z - q^2}$$

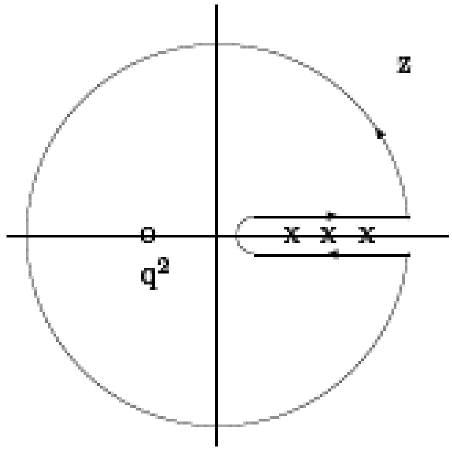

Figure 2.1: The contour in the plane of the complex variable $q^2 = z$. The open point indicates the reference point of the QCD calculation and crosses indicate the hadronic thresholds

$$= \frac{1}{2\pi i}\oint_{|z|=R} dz \frac{\Pi(z)}{z-q^2} + \frac{1}{2\pi i}\oint_0^R dz \frac{\Pi(z+i\varepsilon)-\Pi(z-i\varepsilon)}{z-q^2} \tag{2.7}$$

One can put R$\rightarrow\infty$ so that the first term on RHS in Eq. (2.7) is zero if $\lim_{|q^2|\rightarrow\infty}\Pi(q^2) \sim \frac{1}{|q^2|^\varepsilon}$ with $\varepsilon > 0$. The second integral can be dealt with by using Schwartz reflection principle which states that;

$$\Pi(q^2+i\varepsilon) - \Pi(q^2-i\varepsilon) = 2iIm(q^2) \tag{2.8}$$

at $q^2 > t_{min}$ if $\Pi(q^2)$ is real at $q^2 < t_{min}$ where t_{min} is hadronic threshold, and finally one obtains;

$$\Pi(q^2) = \frac{1}{\pi}\int_{t_{min}}^{\infty} ds \frac{Im\Pi(s)}{s-q^2-i\varepsilon} \tag{2.9}$$

2.3 Borel Transformation

The spectral density representation $\Pi(q^2)$ is ultraviolet divergent since $Im\Pi(s)$ does not vanish as $s\rightarrow\infty$. This problem can be eliminated by subtracting a few number of terms from $\Pi(q^2)$ [41];

$$\overline{\Pi}(q^2) = \Pi(q^2) - subtraction\, terms \tag{2.10}$$

and most generally;

$$\Pi(q^2) = \frac{(q^2)^n}{\pi}\int \frac{Im\Pi(s)}{s^n(s-q^2)}ds + \sum_{k=0}^{n-1} a_k(q^2)^k \tag{2.11}$$

where $\sum_{k=0}^{n-1} a_k(q^2)^k$ are the subtraction terms [40]. The polynomials in (2.6) exist due to the fact that, in genera, not all $a_k = 0$.

Since Eq. (2.10) contains 'subtraction terms' and since ρ^h can only be determined by approximations, this form of $\Pi(q^2)$ is not useful. However, there is a transformation, called Borel transformation, which removes the subtraction terms and suppresses the contribution of ρ^h exponentially. Borel transformation is based on the differentiation of $\Pi(q^2)$ with respect to q^2 via the following limit;

$$\Pi(M^2) \equiv B_{M^2}\Pi(q^2) = \lim_{\substack{-q^2, n\to\infty \\ \frac{-q^2}{n}=M^2}} \frac{(-q^2)^{n+1}}{n!}(\frac{d}{dq^2})^n\Pi(q^2) \tag{2.12}$$

where M^2 is the borel mass.

The Borel transformations of polynomials vanish, i.e,

$$B_{M^2}(q^2)^k = 0\,, \qquad k > 0 \tag{2.13}$$

and,

$$B_{M^2}\left(\frac{1}{(s-q^2)^k}\right) = \frac{1}{(k-1)!}\frac{e^{\frac{-s}{M^2}}}{(M^2)^{k-1}}\,, \qquad k > 0 \tag{2.14}$$

then, Borel transformation of the correlation functions $\Pi^{OPE}(q^2)$ and $\Pi^{phen}(q^2)$ becomes respectively;

$$\Pi^{OPE}(M^2) = \int_0^\infty \rho^{OPE}(s)e^{\frac{-s}{M^2}}ds$$
$$and$$
$$\Pi^{phen}(M^2) =< 0|J|h_1(p) >< h_1(p)|\overline{J}|0 > e^{\frac{-m_{h_1}^2}{M^2}} + \int_{s_0^h}^\infty \rho^h(s)e^{\frac{-s}{M^2}}ds \tag{2.15}$$

Hence, after Borel transformation the polynomials are eliminated, and the contributions of the continuum and higher states are suppressed by the factor $e^{\frac{-s}{M^2}}$ in the integrand, so the possible errors in the estimation of the contribution of the higher states and the continuum are reduced.

In order to extract the parameters of the lowest lying resonance, the contribution of the higher states and the continuum has to be modeled. In this work, quark-hadron duality approximation has been used. Quark-hadron duality states that above a critical energy, the spectral function of the continuum and higher states can be represented as the spectral function of the Operator Product Expansion. That is;

$$\rho^h \cong \rho^{OPE}(s - s_0)\theta(s - s_0) \tag{2.16}$$

where s_0 is the effective threshold which does not need to coincide with s_0^h.

Thanks to Borel transformation, the quark-hadron duality is more reliable. After Borel transformation and using the quark-hadron duality approximation, equating the two expressions in Eq. (2.15) we get a very useful sum rule which enables us to extract information about the unknown resonance parameters:

$$< 0|J|h_1(p) >< h_1(p)|\overline{J}|0 > e^{\frac{-m^2}{M^2}} = \int_0^{s_0} \rho^{OPE}(s) e^{\frac{-s}{M^2}} ds \tag{2.17}$$

2.4 Choice of the Borel Parameter

By matching the OPE and phenomenological representations of the correlation function, a sum rule is obtained from Eq. (2.17) in terms of a new auxiliary parameter M^2. Since the physical parameters should not depend on the Borel parameter, a range for the Borel parameter should be found such that the predictions for the physical parameters are independent of M^2.

To get contribution from the lowest lying single resonance, M^2 should be chosen as small as possible. Since the Borel transformation brings the exponential suppression factor to the integral in Eq. (2.17) , the integral receives the main contribution at $s \cong M^2$.Therefore, lower M^2 means that the integral receives the dominant contribution from lower s and therefore, decreases the contribution from the region at which $s \geq M^2$. Consequently, quark-hadron duality is reliable and the integral is dominated by the lowest single resonance. At large M^2, the case in which the asymptotic freedom exists (high energy case),the quark-hadron duality is not reliable since the main contribution to the integral does not come from the lowest lying resonance. On the other hand M^2 cannot be too small since when M^2 is small, the corrections to the perturbative part blow up and the convergence of the series cannot be guaranteed. Therefore, truncation of the sum rules cannot be reliable. Consequently, one can conclude that M^2 should not be large but not be too small. A balance can be possible between upper and lower values of M^2, which provides both convergence of the series and dominance of single resonance contribution. The interval of which M^2 can take values can be extracted from the dependence of the physical parameters to the M^2. The optimum interval of M^2 is the one at which the unknown parameter is nearly stable with respect to M^2 [34].

2.5 Three-Point QCD Sum Rules

Up to now QCD Sum Rules formalism was discussed in the two-point function case. Two-point correlation functions are suitable to extract masses, residues, leptonic decay constants of hadrons. If one is interested in studying transitions involving more than one kind of hadron, other correlation functions would be more useful. In this work, three-body coupling is necessary. For this calculation, a three body correlation function is more suitable.

If one is trying to find the masses of a particle, Two-Point QCDSR might be the choice since there is nothing with the Three Point case in that problem, but for example if one wishes to investigate three particle coupling, Two-Point QCDSR is not suitable but Light-Cone QCDSR and Three-Point QCDSR are suitable.

The area in which Three-Point QCDSR is used is very wide. Couplings of two hadrons to an on-shell γ (e.g. $J/\psi \to \eta_c \gamma$ [42, 43]), form factors (e.g. pion electromagnetic form factor [44, 45]) and D and B transitions [46, 47] are the examples in which Three-Point QCDSR has been applied.

This formalism is similar to the Two-Point correlation function formalism. In three-point case, the correlation function becomes,

$$\Pi(q^2, p^2, p'^2) = i^2 \int d^4x \, d^4y \; e^{iqx} e^{ipy} < 0 \mid T\{J_1(x), J_2(0), J_3^\dagger(y)\} \mid 0 > \tag{2.18}$$

and Wilson coefficients in operator product expansion of the correlation function depend on q^2, p^2 and p'^2;

$$\begin{aligned} i^2 \int d^4x \, d^4y \; e^{iqx} e^{ipy} &< 0 \mid T\{J_1(x), J_2(0), J_3^\dagger(y)\} \mid 0 > \\ &= \sum_n C_n(q^2, p^2, p'^2) < O_n > \end{aligned} \tag{2.19}$$

where $p' = q + p$.

In Three-Point QCDSR case, since the correlation function (Eq. (2.19)) depends on p^2, p'^2 and q^2, the spectral integration is a double integration and spectral density function is a function of two variables in addition to q^2, say, and s_1 and s_2, corresponding to two momenta p and p';

$$\begin{aligned}
\Pi^{phen}(p^2, p'^2, q^2) &= \frac{<0|J_3|h_3><0|J_2|h_2><h_2 h_3|h_1><h_1|J_1^\dagger|0>}{(q^2 - m_{h_1}^2)(p^2 - m_{h_2}^2)(p'^2 - m_{h_3}^2)} \\
&+ \int_{s_0^h}^{\infty}\int_{s_0^{h'}}^{\infty} \frac{\rho^h(s_1, s_2, q^2) ds_1 ds_2}{(s_1 - p^2)(s_2 - p'^2)} + polynomials(p^2 \ or \ p'^2) \\
and & \\
\Pi^{OPE}(p^2, p'^2, q^2) &= \int_0^{\infty}\int_0^{\infty} \frac{\rho^{OPE}(s_1, s_2, q^2) ds_1 ds_2}{(s_1 - p^2)(s_2 - p'^2)} + polynomials(p^2 \ or \ p'^2)
\end{aligned} \tag{2.20}$$

To eliminate the subtraction terms, the Borel transformation should be taken with respect to p^2 and p'^2. After Borel transformation and invoking the quark-hadron duality, which is the same procedure with that of two-point case, one gets the final sum rule;

$$\begin{aligned}
&\frac{<0|J_3|h_3><0|J_2|h_2><h_2 h_3|h_1><h_1|J_1^\dagger|0> e^{\frac{-m_{h_2}^2}{M_1^2}} e^{\frac{-m_{h_3}^2}{M_2^2}}}{q^2 - m_{h_1}^2} \\
&= \int_0^{s_0'}\int_0^{s_0} \rho^{OPE}(s_1, s_2, q^2) e^{\frac{-s_1}{M_1^2}} e^{\frac{-s_2}{M_2^2}} ds_1 ds_2
\end{aligned} \tag{2.21}$$

CHAPTER 3

CALCULATING $g_{D_{sJ}D_s\eta}$

In this section $D_{sJ}(2317) \to D_s\eta \to D_s\pi^0$ will be studied in the framework of three-point QCDSR formalism. In order to analyze the first stage of the transition, the three particle coupling constant $g_{D_{sJ}D_s\eta}$ should be calculated.

In the first section the OPE side of the calculations will be treated, that is, ρ^{OPE} will be calculated. In the second section phenomenological part of the correlation function will be treated and finally, in the 'Numerical Analysis and Discussion' part, the $g_{D_{sJ}D_s\eta}$ and $\Gamma(D_{sJ}(2317) \to D_s\pi^0)$ will be found and the results will be discussed.

3.1 OPE Side

The correlation function chosen for the purpose of calculating $g_{D_{sJ}D_s\eta}$ is;

$$\Pi_\mu = i^2 \int\int d^4x d^4y e^{ipx+iqy} < 0|T\{J_{D_s,\mu}(x) J_\eta(y) J^\dagger_{D_{sJ}}(0)\}|0 > \qquad (3.1)$$

where p and q are the momenta of D_s and η respectively, $J_{D_s,\mu} = \bar{c}\gamma_\mu\gamma_5 s$, $J_\eta = \frac{1}{\sqrt{6}}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s)$ and $J_{D_{sJ}} = \bar{c}s$. The operator product expansion of Eq. (3.1) which is truncated after dimension 5 (since the contribution of the dimension 6 operators give negligible contribution in heavy-light quark systems) is;

$$\Pi_\mu = \Pi^0_\mu + \Pi^{(1)}_\mu < \bar{s}s > + \Pi^{(2)}_\mu < \frac{\alpha_s}{\pi} G_{\nu\lambda} G^{\nu\lambda} > + \Pi^{(3)}_\mu < \bar{s}\sigma_{\nu\xi} \frac{\lambda^a}{2} G^{\nu\xi,a} s > \qquad (3.2)$$

where α_s is the coupling constant of the strong interactions. Since the c-quark is too heavy to form a condensate, it is not written in Eq. (3.2). In Fig. 3.1 the Feynman diagrams contributing to the correlation function are shown. The

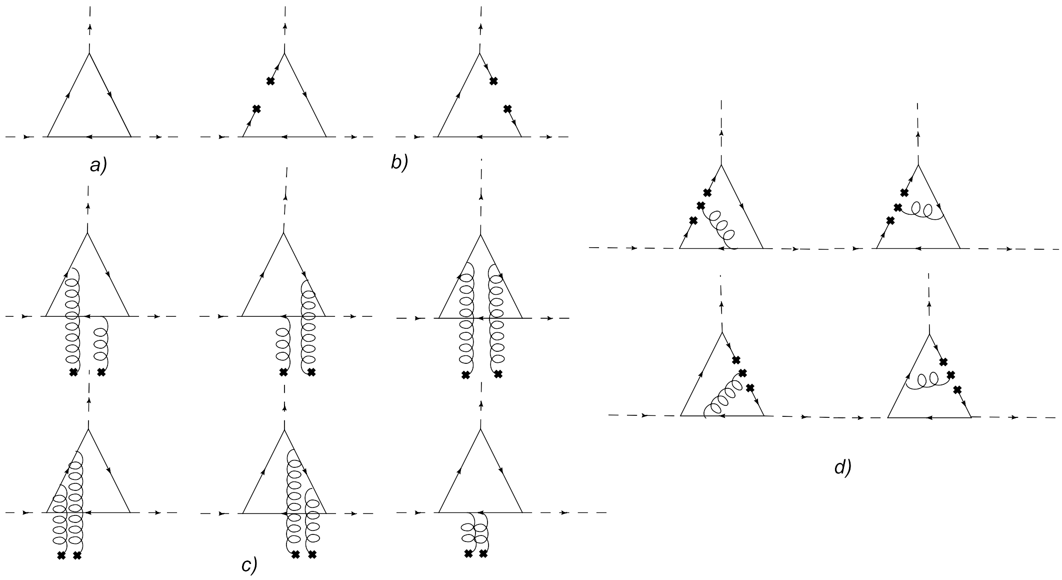

Figure 3.1: Contributions to the three point correlator: a) Perturbative contribution, b) Quark condensates, c) Gluon condensates d) Mixed quark-gluon condensates

first term of the Eq. (3.2) is perturbative part of the expansion corresponding to operator with dimension one (Fig. 3.1a). The second term is the one corresponding to quark condensates (Fig. 3.1b), the third one to gluon condensate (Fig. 3.1c) and finally the last term corresponds to mixed quark-gluon condensates (Fig. 3.1d). The quark condensates terms do not contribute to the correlation function after Borel transformation since they depend only on either p^2 or p'^2 and are eliminated by the Borel transformation.

Invoking the Wick's theorem, Eq. (3.1) becomes;

$$\Pi_\mu = -i \int\int d^4x d^4y e^{ipx+iqy} < 0 \mid Tr(S_c^{ki}(0,x)\gamma_\mu\gamma_5 S_s^{ij}(x,y)\gamma_5 S_s^{jk}(y,0)) \mid 0 > \tag{3.3}$$

where $S_q(x,y)$'s are quark propagators in gluon fields, c and s represent charm and strange quarks respectively, and (i,j,k) are color indices, and sum over repeated indices is implied. If a quark is propagating in a gluon field (Fig 3.1 b,c,d) then the propagator is;

$$S_q^{ki}(x,0) = S_q^{free,ki}(x,0) + g_s S_q^{G,ki}(x,0) + g_s^2 S_q^{G^2,ki}(x,0) \tag{3.4}$$

where $S_q^{free,ki}(x,0)$, $g_s S_q^{G,ki}(x,0)$ and $S_q^{G^2,ki}(x,0)$ are given in Appendix A. Finally, the total correlation function becomes;

$$\begin{aligned}
\Pi_\mu = \quad & -i\int\int d^4x d^4y e^{ipx+iqy} \\
& [Tr(S_c^{free,ki}(0,x)\gamma_\mu\gamma_5 S_s^{free,ij}(x,y)\gamma_5 S_s^{free,jk}(y,0)) \\
& +g_s^2(<0 \mid Tr(S_c^{free,ki}(0,x)\gamma_\mu\gamma_5 S_s^{G,ij}(x,y)\gamma_5 S_s^{G,jk}(y,0)) \mid 0> \\
& + <0 \mid Tr(S_c^{G,ki}(0,x)\gamma_\mu\gamma_5 S_s^{free,ij}(x,y)\gamma_5 S_s^{G,jk}(y,0)) \mid 0> \\
& + <0 \mid Tr(S_c^{G,ki}(0,x)\gamma_\mu\gamma_5 S_s^{G,ij}(x,y)\gamma_5 S_s^{free,jk}(y,0)) \mid 0>) \\
& +g_s^2(<0 \mid Tr(S_c^{free,ki}(0,x)\gamma_\mu\gamma_5 S_s^{free,ij}(x,y)\gamma_5 S_s^{G^2,jk}(y,0)) \mid 0> \\
& + <0 \mid Tr(S_c^{G^2,ki}(0,x)\gamma_\mu\gamma_5 S_s^{free,ij}(x,y)\gamma_5 S_s^{free,jk}(y,0)) \mid 0> \\
& + <0 \mid Tr(S_c^{free,ki}(0,x)\gamma_\mu\gamma_5 S_s^{G^2,ij}(x,y)\gamma_5 S_s^{free,jk}(y,0)) \mid 0>)]
\end{aligned} \tag{3.5}$$

where we have neglected contributions which will vanish after Borel transformation.

After putting the propagator in Eq. (3.5) and taking traces, there appears two types of integrals;

$$\begin{aligned}
I_\mu^{nml} &= \int d^4k \frac{k_\mu}{((k-q)^2-m_s^2)^n(k^2-m_s^2)^m((k+p)^2-m_c^2)^l} \\
\textit{and} \quad & \\
I^{nml} &= \int d^4k \frac{1}{((k-q)^2-m_s^2)^n(k^2-m_s^2)^m((k+p)^2-m_c^2)^l}
\end{aligned} \tag{3.6}$$

Using Wick's rotation and Feynman Parametrization [48] these integrals can be computed. The results of these integrals and their Borel transforms with respect to p^2 and p'^2 are given in Appendix B.

In this thesis, only the perturbative part will be calculated. The calculation of the gluon condensate contribution is beyond the scope of this thesis. It has been shown in [52] that in the similar processes $D \longrightarrow \pi(\rho)e\nu$ and $B \longrightarrow \pi(\rho)e\nu$ the contribution of the gluon condensate is ~10%. This uncertainty will be included in the uncertainty of our prediction. For this purpose, we need only the integrals I_μ^{111} and I^{111}. Once these integrals are computed, then $\rho_{pert}^{OPE}(s_1,s_2)$ can be found after two successive double Borel transformations first with respect to (p^2, p'^2) and then with respect to $(\frac{1}{M_1^2},\frac{1}{M_2^2})$. In these calculations q^2 will be neglected since q is the momentum of the π^0 and the mass of the π^0 is negligible in our calculations. The spectral density function of perturbative part of OPE side is;

$$\rho_{per}^{OPE}(s_1, s_2) = \frac{3}{4\sqrt{6}\pi^2}\delta(s_1 - s_2)[(m_s + m_c)^2 - s_1] \int_0^1 dx\, \theta(s_1 - s(x))(\frac{1}{\overline{x}} + \frac{x}{\overline{x}}) \tag{3.7}$$

where;

$$\overline{x} \equiv 1 - x$$
$$and$$
$$s(x) \equiv \frac{m_s^2}{\overline{x}} + \frac{m_c^2}{x} \tag{3.8}$$

3.2 Phenomenological Side

In the phenomenological part, inserting a complete set of hadronic states into Eq. (3.1) the correlation function becomes:

$$\frac{< 0|J_{D_{sJ}}|D_{sJ} >< D_s|J^{\mu}_{D_s}|0 >< \eta|J_\eta|0 > g^{pert}_{D_{sJ}D_s\eta} e^{\frac{-m^2_{D_{sJ}}}{M_1^2}} e^{\frac{-m^2_{D_s}}{M_2^2}}}{q^2 - m_\eta^2} = (\int_0^{s_0'} \int_0^{s_0} \rho^{OPE}(s_1, s_2) e^{\frac{-s_1}{M_1^2}} e^{\frac{-s_2}{M_2^2}} ds_1 ds_2) q^\mu \tag{3.9}$$

where

$$< 0|J_{D_{sJ}}|D_{sJ} >= f_{D_{sJ}} m_{D_{sJ}}$$
$$< D_s|J^{\mu}_{D_s}|0 >= f_{D_s} p^\mu$$
$$< \eta|J_\eta|0 >= \frac{m^2_\eta}{2m_s} f_\eta \tag{3.10}$$

and quark-hadron duality has been used to subtract the contribution of higher states and continuum.

Finally, after setting $q^2 = m^2_{\pi^0} = 0$, $g^{pert}_{D_{sJ}D_s\eta}$ can be written in terms of phenomenological parameters and borel masses as:

$$g^{pert}_{D_{sJ}D_s\eta} = \frac{-2m_s}{f_{D_{sJ}} m_{D_{sJ}} f_\eta f_{D_s}} e^{\frac{m^2_{D_{sJ}}}{M_1^2} + \frac{m^2_{D_s}}{M_2^2}} (\int_0^{s_0'} \int_0^{s_0} \rho^{OPE}(s_1, s_2) e^{\frac{-s_1}{M_1^2}} e^{\frac{-s_2}{M_2^2}} ds_1 ds_2) \tag{3.11}$$

All the masses and leptonic decay constants in Eq. (3.11) are known, and will be discussed in the next section. However, as mentioned in Section 2.3, we should determine an interval for the borel mass. To find the $g^{pert}_{D_{sJ}D_s\eta}$, the $g^{pert}_{D_{sJ}D_s\eta}$ versus borel masses should be plotted, and $g^{pert}_{D_{sJ}D_s\eta}$ is found in the borel mass range at which the $g^{pert}_{D_{sJ}D_s\eta}$ is most stable.

3.3 Numerical Analysis and Discussion

The numeric values of the quark masses and leptonic decay constants are $m_{D_{sJ}} = 2.317$ GeV, $m_{D_s} = 1.968$ GeV, $m_c = 1.25$ GeV and $m_s = 0.125$, $f_{D_{sJ}} = 0.225$ GeV, $f_{D_s} = 0.266$ GeV, and $f_\eta = 0.137$ GeV. The continuum threshold values are chosen as $s_0 = s'_0 = 6.50 \pm 0.25$ [51].

To simplify the calculations, it is reasonable to set $M_1^2 = M_2^2 = 2M^2$ since $m_{D_{sJ}} \simeq m_{D_s}$. In order to obtain a prediction for the coupling constant $g^{pert}_{D_{sJ}D_s\eta}$, a range of M^2 values should be found such that within this range $g^{pert}_{D_{sJ}D_s\eta}$ is almost independent of M^2. From Fig. 3.2, it is seen that for 3 $GeV^2 < M^2 < 5\ GeV^2$, $g^{pert}_{D_{sJ}D_s\eta}$ is stable with respect to variations of both s_0 and M^2. Hence, $g^{pert}_{D_{sJ}D_s\eta}$ is:

$$5.9\ GeV < g^{pert}_{D_{sJ}D_s\eta} < 7.0\ GeV \tag{3.12}$$

The errors in this prediction include only the variations with respect to s_0 and M^2. Considering a contribution of ∼10 % from the double gluon condensates [52], the error bars need to be increased:

$$5.3\ GeV < g_{D_{sJ}D_s\eta} < 7.7\ GeV \tag{3.13}$$

Once $g_{D_{sJ}D_s\eta}$ is calculated, the decay rate $\Gamma(D_{sJ}(2317) \rightarrow D_s\pi^0)$ can be calculated through $\pi^0\eta$ mixing. This brings a suppression factor $(\frac{m_d - m_u}{m_s - \frac{m_d+m_u}{2}})$ as in the $D_s^* \rightarrow D_s\pi^0$ decay [50]. With this suppression factor the decay width becomes;

$$\Gamma(D_{sJ}(2317) \rightarrow D_s\pi^0) = \frac{g^2}{8\pi}\frac{1}{m^2_{D_{sJ}}}(\frac{1}{m^2_\eta - q^2})^2(\frac{f_\eta}{2m_s}m^2_\eta)^2(\frac{m_d - m_u}{m_s - \frac{m_d+m_u}{2}})^2|\overrightarrow{p_{\pi^0}}| \tag{3.14}$$

and numeric value of the decay width becomes;

$$\Gamma(D_{sJ}(2317) \rightarrow D_s\pi^0) = 6.5 - 14.0\ \ KeV \tag{3.15}$$

Table 3.1: Decay Width (KeV) of the Strong Decay $D_{sJ}(2317) \to D_s\pi^0$ from various theoretical approaches.

	[14]	[18]	[15]	[39]	This Thesis
$D_{sJ}(2317) \to D_s\pi^0$	21.5	7 ± 1	$\simeq 10$	34-44	6.5-14.0

Table 3.2: Comparison between the ratio of radiative decay width (which is calculated in [51] in the framework of LCQCDSR) and strong decay width (calculated in this thesis) of $D_{sJ}(2317)$.

	Belle [12]	CLEO[2]	LCQCDSR [39]	This Thesis
$\frac{\Gamma(D_{sJ}(2317)\to D_s^*\gamma)}{\Gamma(D_{sJ}(2317)\to D_s\pi^0)}$	<0.18	< 0.059	0.13	0.29 - 0.92

Our prediction is compatible with the predictions of chiral theory [14], heavy-quark symmetry [18], and the phenomenological approach of [15] (Table 3.1). There is an inconsistency of a factor of about three between our prediction and the prediction of LCQCDSR [39] (Table 3.2). This inconsistency can be explained by an underdetermination of the errors in LCQCDSR approach and our prediction. If the uncertainties are increased to 30%, which is typical in QCDSR calculations, the inconsistency is removed.

To compare these predictions with experiment, the ratio $\frac{\Gamma(D_{sJ}(2317)\to D_s^*\gamma)}{\Gamma(D_{sJ}(2317)\to D_s\pi^0)}$ has to be calculated. Using the result of [51] for the radiative decay width, we obtain for this ratio:

$$0.29 < \frac{\Gamma(D_{sJ}(2317) \to D_s^*\gamma)}{\Gamma(D_{sJ}(2317) \to D_s\pi^0)} < 0.92 \tag{3.16}$$

which is above experimental limits. In order to definitely conclude that this result disproves the $c\bar{s}$ interpretation of $D_{sJ}(2317)$, both the radiative and strong decay width have to be calculated more precisely.

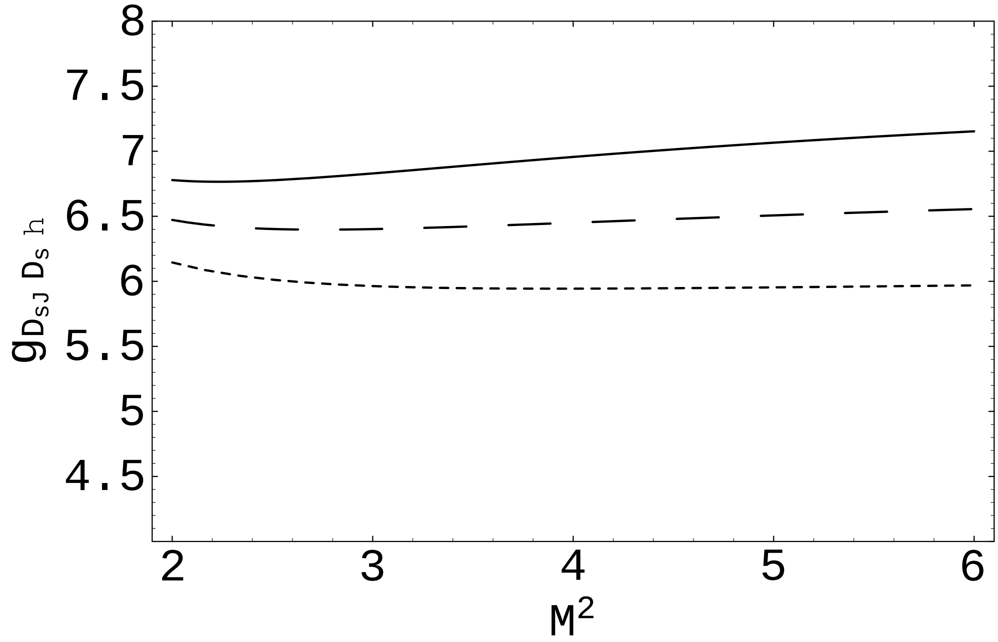

Figure 3.2: $g_{D_{sJ}D_s\eta}(GeV)$ versus $M^2(GeV^2)$. Dashed, long-dashed and continuous lines correspond to s_0=6.25,6.50,6.75 GeV^2, respectively)

CHAPTER 4

CONCLUSIONS

In this thesis, the strong decay of $D_{sJ}(2317) \rightarrow D_s\eta \rightarrow D_s\pi^0$ has been studied. The three particle coupling constant $g_{D_{sJ}D_s\eta}$ has been found in the framework of three-point QCD SUM RULES. This coupling is used to calculate the decay width of the decay $D_{sJ}(2317) \rightarrow D_s\pi^0$. This decay is studied previously by using Light-Cone QCDSR and the result is consistent with the limits obtained by Belle [12].

The calculations in this work are done assuming $c\bar{s}$ quark content for the $D_{sJ}(2317)$ meson. The predictions of these calculations are not consistent with experimental limits and the light-cone QCD Sum Rules predictions.

In this work, the contributions of the double gluon condensate have been neglected since their calculation is beyond the scope of this thesis, and later on, these are added in estimating the errors. This might be the reason of the inconsistency with the experimental results. These neglected contributions of the double gluon condensate have to be calculated for a more reliable estimation of the $D_{sJ}(2317) \rightarrow D_s\pi^0$ decay width and hence make a more conclusive remark on the $c\bar{s}$ interpretation of the $D_{sJ}(2317)$meson.

Once the neglected contributions are calculated in detail in the framework of three-point QCDSR, it will be possible to state whether the QCD Sum Rules method is consistent with $c\bar{s}$ interpretation or not.

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APPENDIX A

PROPAGATORS

The full quark propagator for a quark in an external gluon field can be written as:

$$S_q^{ki}(x) = S_q^{free,ki}(x) + g_s S_q^{G,ki}(x) + g_s^2 S_q^{G^2,ki}(x) \qquad (A.1)$$

where the free quark propagator is;

$$S_q^{free,ij}(x) = \delta^{ij} \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{m_q + \not{k}}{m_q^2 - k^2} \qquad (A.2)$$

and the corrections are;

$$\begin{aligned} S_q^{G,ij}(x) = \quad & -\delta^{ij} \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 dv [\frac{m_q + \not{k}}{(m_q^2 - k^2)^2} G_{\rho\nu}^{ij}(vx)\sigma^{\rho\nu} + \\ & \frac{1}{m_q^2 - k^2} vx^\rho G_{\rho\nu}^{ij} \gamma^\nu] \end{aligned} \qquad (A.3)$$

,

$$\begin{aligned} S_q^{G^2,ij}(x) = \quad & -\frac{< G^2 >}{12} m_q^2 \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{m_q + \not{k}}{(m_q^2 - k^2)^4} + \\ & < G^2 > \frac{m_q}{12} \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{1}{(m_q^2 - k^2)^3} \end{aligned} \qquad (A.4)$$

where q denotes the quark flavor, γ^μ's are the Dirac matrices, $< G^2 >$ are the gluon condensates, and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$.

APPENDIX B

FEYNMAN PARAMETRIZATION

In calculating loop integrals, one encounters products of factors in the denominator of the integrands. To evaluate the integrals, these factors can be united into a single factor using Feynman parametrization. During the calculations in this work, the necessary Feynman parameterizations are

$$\begin{aligned}\frac{1}{a^n b^m} &= \frac{\Gamma(n+m)}{\Gamma(n)\Gamma(m)}\int_0^1 dx \frac{x^{n-1}\overline{x}^{m-1}}{[ax+b\overline{x}]^{n+m}} \\ &\mathit{and} \\ \frac{1}{a^n b^m c^l} &= \frac{\Gamma(n+m+l)}{\Gamma(n)\Gamma(m)\Gamma(l)}\int_0^1 x^{n+m-1}\overline{x}^{l-1}dx\int_0^1 \frac{dy y^{n-1}\overline{y}^{m-1}}{[axy+bx\overline{y}+c\overline{x}]^{n+m+l}}\end{aligned} \tag{B.1}$$

where $\overline{x} = 1-x$ and $\overline{y} = 1-y$

APPENDIX C

SOME INTEGRAL FORMULAE

In the evaluation of ρ_{pert}^{OPE}, one needs I_μ^{111} and I^{111}. In this Appendix, the results of the general integrals I_μ^{nml} and I^{nml} are presented. I_μ^{111} can be written as:

$$\begin{aligned} I_\mu^{nml} &= \int d^4k \frac{k_\mu}{((k-q)^2 - m_s^2)^n (k^2 - m_s^2)^m ((k+p)^2 - m_c^2)^l} \\ &= A^{nml} p_\mu + B^{nml} q_\mu \end{aligned} \tag{C.1}$$

where;

$$\begin{aligned} A^{nml} &= \frac{i(-1)^{n+m+l+1}\pi^2}{\Gamma(n)\Gamma(m)\Gamma(l)} \int_0^1 dv \overline{v}^{m-1} v^{n-1} \int_0^1 du u^{n+m-1} \overline{u}^l \\ &\int_0^\infty dt t^{n+m+l-3} e^{-p^2 t(\overline{x} x \overline{y}) - p'^2 t(x\overline{x} y) - m_c^2 t - xt(m_s^2 - m_c^2)} \\ &, \\ B^{nml} &= \frac{i(-1)^{n+m+l}\pi^2}{\Gamma(n)\Gamma(m)\Gamma(l)} \int_0^1 dv \overline{v}^{m-1} v^n \int_0^1 du u^{n+m} \overline{u}^{l-1} \\ &\int_0^\infty dt t^{n+m+l-3} e^{-p^2 t(\overline{x} x \overline{y}) - p'^2 t(x\overline{x} y) - m_c^2 t - xt(m_s^2 - m_c^2)} \end{aligned} \tag{C.2}$$

This form of A^{nml} and B^{nml} are suitable for Borel transformations. Using the Borel transformation $B_{p^2}(M^2)e^{-\alpha p^2}=\delta(\frac{1}{\alpha} - M^2)$, the Borel transforms of A^{nml} and B^{nml} can be obtained:

$$\begin{aligned} B_{p^2} B_{p'^2} A^{nml} &= \frac{i(-1)^{n+m+l+1}\pi^2}{\Gamma(n)\Gamma(m)\Gamma(l)} \int_0^1 du u^{1-l} \overline{u}^{2-n-m} \frac{e^{-\frac{u m_s^2 + \overline{u} m_c^2}{M^2 u \overline{u}}} (M^2)^{2-l}}{(M_1^2)^{m-1} (M_2^2)^{n-1}} \\ &\textit{and} \\ B_{p^2} B_{p'^2} B^{nml} &= \frac{i(-1)^{n+m+l+}\pi^2}{\Gamma(n)\Gamma(m)\Gamma(l)} \int_0^1 du u^{2-l} \overline{u}^{1-n-m} \frac{e^{-\frac{u m_s^2 + \overline{u} m_c^2}{M^2 u \overline{u}}} (M^2)^{3-l}}{(M_1^2)^{m-1} (M_2^2)^{n}} \end{aligned} \tag{C.3}$$

The other integral is;

$$\begin{aligned} I^{nml} = & \int d^4k \frac{1}{((k-q)^2-m_s^2)^n(k^2-m_s^2)((k+p)^2-m_c^2)} \\ = & \frac{i(-1)^{n+m+l}\pi^2}{\Gamma(n)\Gamma(m)\Gamma(l)} \int_0^1 dv \overline{v}^{m-1} v^{n-1} \int_0^1 du u^{n+m-1} \overline{u}^{l-1} \\ & \int_0^\infty dt t^{n+m+l-3} e^{-p^2 t(\overline{x}x\overline{y}) - p'^2 t(x\overline{x}y) - m_c^2 t - xt(m_s^2-m_c^2)} \end{aligned} \quad \text{(C.4)}$$

and the Borel Transform of this integral with respect to p^2 and p'^2 is;

$$B_{p^2}B_{p'^2}I^{nml} = \frac{i(-1)^{n+m+l}\pi^2}{\Gamma(n)\Gamma(m)\Gamma(l)} \int_0^1 du u^{1-l}\overline{u}^{1-n-m} e^{-\frac{um_s^2+\overline{u}m_c^2}{M^2 u\overline{u}}} \frac{(M^2)^{2-l}}{(M_1^2)^{m-1}(M_2^2)^{n-1}} \quad \text{(C.5)}$$

where $\overline{u} = 1-u$, $\overline{v} = 1-v$ and $\frac{1}{M^2} = \frac{1}{M_1^2} + \frac{1}{M_2^2}$, and the Borel parameters of p^2 and p'^2 are M_1^2 and M_2^2 respectively.