BEAM-BEAM INTERACTION AND LUMINOSITY LIMIT IN STORAGE RINGS

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1. Introduction

The first experiments concerning the observation and investigation of the beam-beam effects have been performed in Novosibirsk and Stanford using the colliding electron-electron beams facilities[1,2,3]. It has become clear that nonlinear interactions between colliding beams will determine the major limitations of the luminosity of such facilities. In particular, the following effects were observed:

a) formation of a set of islands at betatron frequencies near the nonlinear resonances;

b) a diffusive (non-resonant) increase of the sizes of one or two beams at betatron frequencies far from the resonances;

c) shortening of the life time.

The further experiments have been performed using the colliding electron-positron beam facilities. Despite the fact that for electron-positron beams the opposite beam from a defocusing nonlinear lens has turned into a focusing one, the character of the beam-beam effects has remained unchanged.

As far as the theoretical aspects are concerned, there are two basic directions of research.

Hamiltonian formalism, originally developed to study the nonlinear effects in accelerators[4,5], has been applied successfully to analyse the beam-beam effects[6]. This approach has made possible to obtain a qualitative notion about the mechanism of excitation of nonlinear resonances, to derive analytical expressions for the width of the bucket of betatron nonlinear resonance (BB) and the frequency of oscillations around the centre of it, for the case of one-demensional systems. The criteria of resonance overlap and the appearance of the stochastic regime have been estimated in[7] for the first time. With a great number of the factors involved (two-dimensionality, modulation, interaction between the resonances, etc.), the quantitative description has turned out to be a complicated problem.

Computer simulation has begun evolving later[8-17] and its possibilities expand as the method is developed and with the progress in computers. At present, computer simulation allows a great deal of factors to be taken into account and interesting, for practice, results to be derived. As a matter of fact, it offers the possibility of finding the sizes of the beam and its luminosity.

However, it is unworthy contrasting these methods, these rather complement one another and continue to develop. At the first stage the studies have been mainly connected with an interaction between a particular particle and the electromagnetic field of a bunch, i. e. the so-called model of the weak-strong beam. In recent years, computer simulation has allowed to advance the sdudy of the strong-strong beam model[18-19]. The analytical approach is being advanced as well[20-22].

Due to the building of big facilities (LEP, HERA, ASC, SSC and the others), an interest to the beam-beam effects has recently increased, which is evidenced by interesting reviews of the theoretical and experimental aspects reported at series of Schools and Seminars[23-27].

In the present report the authors would like to present breafly the current status of the beam-beam effects problem and to discuss some important questions. The report deals with the electron-positron storage rings in the work on which the authors are being involved. The authors have used also the information and the data of the special questionaries obtained from the corresponding accelerator centres.

2. Basic reasons for an increase of the beam sizes and the luminosity limits

In this Section we will consider the major features of the dynamics of colliding beams in an ideal magnetic structure, which implies the same interaction points and the same phase shifts of betatron oscillations between them, symmetrically to the force of the colliding beam as well as the absence of the nonlinearities in the magnetic structure, etc. The influence of the machine imperfections and errors will be considered in the next Section.

2.1. Major linear effects.

Considering the opposite beam as an equivalent short lens for the amplitudes of small oscillations of another beam, we obtain the known expression for a linear shift of the frequencies of betatron oscillations per interaction points:

$$\xi_{x,z} = \frac{\beta_{x,z} N r_0}{2\pi \gamma \sigma_{x,z} (\sigma_x + \sigma_z)} .$$

It is known that this tune shift differs from the true[29] $\Delta\nu$ one equal to

$$\Delta\nu \approx (\sqrt{1+2\xi \operatorname{ctg} \mu_0} - 1) \operatorname{tg} \mu_0 ,$$

where μ_0 is the betatron phase advance between the interaction points.

In addition, this effective lens causes:

a) a change of the β-function and the η-function at the interaction points:

$$\frac{\Delta\beta}{\beta} \cong 2\pi\xi \operatorname{ctg} \mu_0 , \qquad \frac{\Delta\eta}{\eta} \cong 2\pi\xi \operatorname{ctg} \frac{\mu_0}{2} ;$$

b) the perturbation of these functions along the ring;

c) a change in chromatism.

It is easy to see that at realistic values of ξ and μ_0 these perturbations are significant (see Table 1). They are strongly dependent on the operating point. With μ_0 taken a little large than $(\eta\pi)$, on the one hand, $\Delta\nu$ decrease at fixed ξ, and, on the other hand, the perturbations in the structure grow.

The structure pertubations can give rise to an increase in vertical size even in the linear model. This is due to that the vertical size in storage rings is mainly formed by the combined effect of specially-established skew-quadrupoles and spuriously rotated lattice quadrupoles. The change in the structure and in the phase advance between the rotated quadrupoles varies the force of linear coupling resonances and the magnitude of the vertical η-function. This violates the coupling compensation condition. It cannot be ruled out that a fine turning of a good luminosity regime is connected with the method of suppressing the coupling between the vertical and radial oscillations.

2.2. Qualitative analysis of the nonlinear effects.

A regular passage of a particle through the nonlinear lens, i. e. through the opposite beam, creates the conditions for the excitation of betatron oscillations near the nonlinear resonances, which satisfy the relation: $m\nu_x+n\nu_z=k$. The parameters $\xi_{x,z}$ and, correspondingly, $\Delta\nu_{x,z}$ characterize the spectrum of betatron frequencies which arises during the interaction with the nonlinear field of the colliding beam. In fact, we have the dependence of the frequencies on the oscillation amplitudes. On the plane of betatron frequencies the region occupied by the beam particles depends on a relation between the beam sizes (σ_z, σ_x) and ($\Delta\nu_z$, $\Delta\nu_x$) (Fig. 1) The size of the «tie» increases with increasing

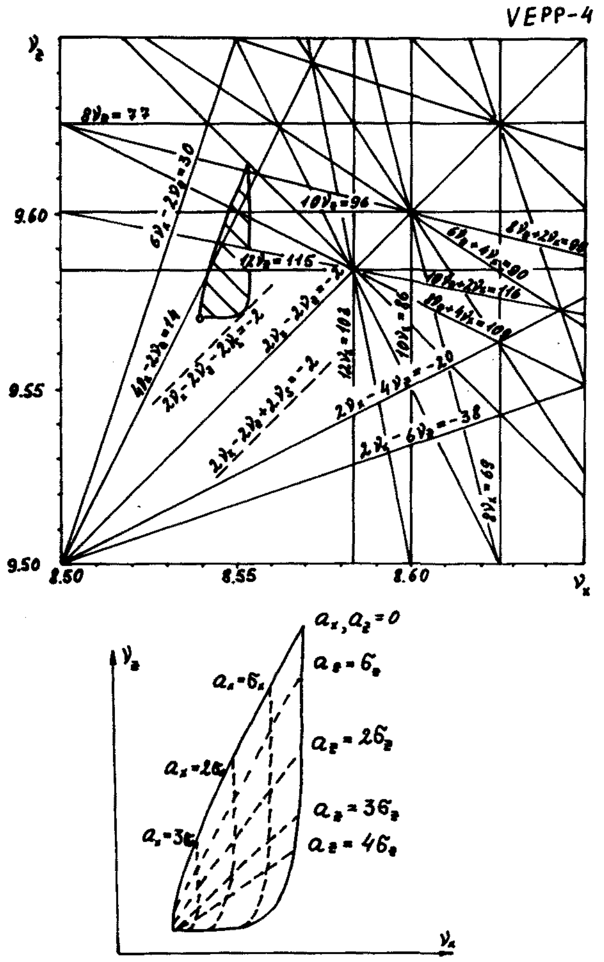


Fig. 1.

the number of particles N ($\Delta\nu_z$ and $\Delta\nu_x$, respectively), so that the number of resonances, crossing the «tie», also increases. The resonances, intersecting the right upper part of the «tie» where there are the small-amplitude particles, determine the size of the beam, while those intersecting the left lower part have influence on the dynamics of large-amplitude particles and, hence, on the lifetime.

For one-dimensional oscillations, the bucket of betatron nonlinear resonance (BB) may be presented in the phase plane (Figs 6,8). The width of the BB regions of a particular resonance is independent of the number of particles in the opposite beam, but is determined by the position of these buckets on the phase plane and decreases with an increase of the order of the resonance $(m+n)$.

As the number of particles grow, the BB regions become closer and with the buckets overlapped the stochastic regime can occur[13]. However, even before such an overlap the formation the BB changes the particle distribution function and can lead to an increase in the beam size.

As a rule, in electron-positron storage rings $\frac{\sigma_z^2}{\beta_z} \ll \frac{\sigma_x^2}{\beta_x}$ and $\sigma_z \ll \sigma_x$. This favours the excitation of the coupling resonances including the difference ones for which even an insignificant exchange of the transverse motion energy, between the horizontal and vertical degrees of freedom is important. A detailed analysis[21] of the two-demensional motion for the beam model with $\sigma_z \ll \sigma_x$ has been made. As it was shown, the degree of coupling on the resonance $m\nu_x+n\nu_z=k$ is given by a parameter $g=\frac{m^2}{n^2}\frac{\xi_x}{\xi_z}\varkappa$ (where $\varkappa$ is emittance ratio: $\varkappa=\varepsilon_z/\varepsilon_x$), so that for a conventional situation $g\ll 1$ the resonance affects only the vertical size of the beam, leaving horizontal one unperturbed. The important role of the mechanism of the vertical beam size excitation by coupling resonances is discussed in the Ref.[28].

2.3. Modulational resonances

The beam-beam effects are further complicated because of synchrotron oscillations. The general condition of the resonance is $m\nu_x+n\nu_z+l\nu_s=k$, for this case. The character of the particle motion in resonance depends on a relation between the frequency of synchrotron motion ν_s and that of oscillations in BB region. In modern storage rings $\nu_s/q(m+n)<\xi$, therefore the beam region can be intersected by the major and the modulational resonances simultaneously.

The particle motion is charaterized by two pairs of parameters: phase advance $\mu_{x,z}$ from one interaction point to another and by the interaction with the opposite beam $\xi_{x,z}$. In view of this, all types of modulation can be grouped into a) μ-modulation and b) ξ-modulation.

We would like to indicate some of them:

1) μ-modulation because of machine chromaticity:

$$\Delta\mu=2\pi\frac{\partial\nu}{\partial P/P}\,\Delta P\sin\frac{\nu_s t}{q};$$

2) μ-modulation of the phase advance between the interaction points because of the longitudinal motion:

$$\Delta\mu\approx\frac{2\pi}{q}\frac{\Delta S}{\beta_0}\nu_s\sin\frac{\nu_s t}{q},$$

where ΔS is the amplitude of longitudinal oscillations;

3) ξ-modulation because of the η-function at the interaction point:

$$\frac{\Delta\xi_z}{\xi_z}\approx-\frac{1}{2}\left(\frac{\eta_x}{\sigma_x}\frac{\Delta P}{P}\right)^2\sin\left(\frac{2\nu_s t}{q}\right);$$

4) ξ-modulation caused by a strong dependence of the β-function on the longitudinal coordinate at the interaction point ($\beta=\beta_0+\Delta S^2/\beta_0$)

$$\frac{\Delta\xi_z}{\xi_z}\approx\frac{\Delta S^2}{4\beta_0^2}\sin\frac{2\nu_s t}{q};$$

5) ξ-modulation because of the β-function chromaticity at the interaction point:

$$\frac{\Delta\xi}{\xi}\approx\frac{1}{\beta}\,\frac{\partial\beta}{\partial P/P}\,\Delta P\sin\frac{\nu_s t}{q}.$$

Some of the types of modulation are compared and their joint action is studied in Ref.[31]. In real storage rings the μ-modulation is the strongest because of the longitudinal motion (type 2). Since the power of the sideband resonances is $\sim J_l\left(\frac{n\cdot\Delta\mu_0}{2\pi\nu_s}\right)$, it is easy to see that at a large amplitude of synchrotron oscillations the sideband resonances become comparable with the major ones, while the latter can vanish. Such a situation is easily realized at typical parameters of storage rings. One can show that the total area, on the phase plane, occupied by the regions of nonlinear resonances, can substantially increase when the modulation is introduced.

Modulational (synchrobetatron) resonances have been observed at many storage rings and determined, to a considerable extent (for example, on the DORIS device), the maximum value of ξ. A large number of papers is devoted to the investigation of synchrobetatron resonances in storage rings[31–33].

Let us note that in a study of the possibilities of obtaining a monochromatic regime by the creation of a large vertical dispersion of an opposite sign for electron and positron beams[34], it has turned out that the modulational resonances increase the vertical betatron size and deteriorate the monochromatisity[35] already at $\xi\geqslant 0.015$.

The problem arises whether there is a correlation between the betatron-oscillation and synchrotron-oscillation amplitudes. An analysis of such a correlation can give useful information on the mechanism of arising of the modulational resonances and their influence. For this purpose, use has been made of the experimental results concerning the Υ-resonance at VEPP-4. It is known that the natural width of the Υ-resonance is tens times smaller than the summary energy spread. Correlations can vary an effective energy spread and, correspondingly, the number of Υ-mesons (N_Υ) per unit luminosity integral $\int L\,dt\sim N_{ee}$. If the particles with large energy deviation acquire large amplitudes because of the beam-beam effects, the ratio N_Υ/N_{ee} will grow as the size increases and vice versa.

Fig. 2 presents the results of these measurements. The yield of Υ-mesons at the top of the resonance, which is normalized over the luminosity integral, is laid off along the ordinate axis. It is seen that the ratio remains first unchanged as the size grows, but at 10 mA currents a small decrease in the yield of Υ-mesons is observed. Thus the tendency is observed for an increase of the betatron amplitudes of the particles with small (rather than large) amplitudes of synchrotron oscillations. Such an effect may be explained as follows. The VEPP-4 betatron frequency are chosen such (Fig. 1) that in the small-amplitude region (a_x, a_z) (these amplitudes mainly determine the density in the centre of the bunch and the luminosity) there is no strong modulational resonances. Since modulation weakens the force of the major resonances intersecting the beam core, the amplitudes of the central particles will most probably grow at small modulation, thereby resulting in reducing the ratio N_Υ/N_{ee}. A strong modulational resonance $2\nu_z-2\nu_x-2\nu_s=k$ has influence basically on the particles with large amplitudes. This tells first of all on the lifetime rather than on the density in the centre.

At high currents, the observed lowering of N_Υ/N_{ee} is not significant, but exceeds the statistical accuracy roughly by a factor of 3. It would be interest to analyse measurements results concerning narrow resonances at the other facilities.

2.4. Damping and quantum fluctuations. Similarity problems.

As it is well known, beam sizes in electron-positron storage rings are determined by quantum fluctuations of radiation and radiation damping. Diffusion of the amplitudes of synchrotron oscillations changes continuously the power of modulational resonances, thereby varying, correspondingly, the widths of the BB regions

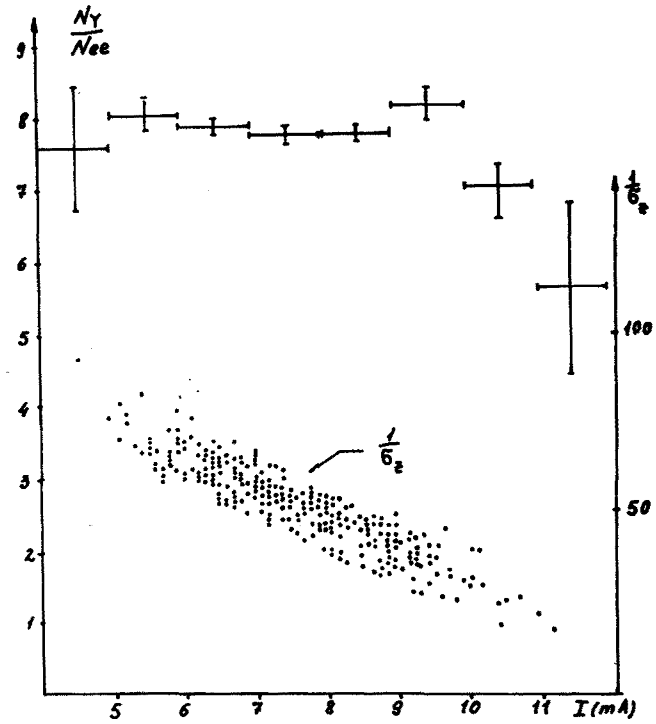


Fig. 2. The vertical size ($1/\sigma_z$) of the beam and relative yield of Υ-mesons (N_Υ/N_{ee}) vs. the beam current.

of the main and modulational resonances. The latter leads to the particle capture in the BB regions or to their damping. This results in amplifying the diffusion coefficient and in increasing the beam sizes. A similar situation occurs with the sum and difference resonances as well. Diffusion of the amplitudes of radial oscillations changes the position of the amplitudes in the BB regions of two-dimensional resonances and the beam sizes grow as a result.

As has been shown[6], damping makes smaller the BB width and limits the order of the resonances being operated. The storage ring experiments indicate that, due to damping, ξ increases with increasing the energy according to the empiric relation $\xi\sim\gamma^n$, where $n=0.5-1.5$ for different facilities[24]. This means that the luminosity grows faster than γ^4. At VEPP-4, within the 1.8—3.5 GeV range ξ increased from 0.03 to 0.55. However, no considerable ξ increase was observed in the range of 3.5—5 GeV energies (Fig. 3). The tendency to saturation of ξ with the energy was observed at the other facilities as well.

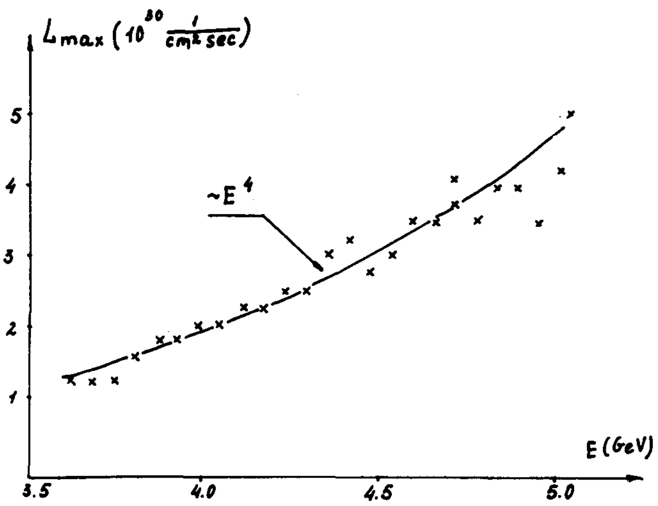


Fig. 3. Luminosity vs. energy (VEPP-4, 1984 scanning).

An attempt[24,36] has been made to compare the facilities by introducing the universal damping decrement δ on the section between two interaction points. One can easily see that if relative decreases in the amplitude, caused by the damping between two interaction points are the same, than beam-beam dynamics should be identical. The authors have constructed, for some facilities the dependence of the parameter ξ on the dimensionless damping decrement $\delta = 1/\tau f q$ (q is the number of the interaction points, f—the revolution frequency, τ—the time of damping) of various facilities. The results obtained were rather uncertain, with large spread. Such an analysis is likely to be not correct because of the fact that the maximum value of ξ is usually limited by the minimum admissible lifetime (1—2 hours). In fact, equal lifetimes correspond to diffirent number of collisions in the facilities with different orbit lengths. So the facilities VEPP-2M and LEP differ by 200 times with respect to the time between collisions and, hence, there is no sense to compare them, basing on the parameter δ. Nevertheless, one can say that with equal and admissible lifetime on larger-size facilities the larger ξ can be expected. An attempt to construct an universal dependence on $\Delta\nu/\gamma = f(1/2q\varrho)$ (ϱ is the radius of a machine)[24] has not however revealed a definite dependence. The distinction of the facilities in many other parameters (ν, σ_z/σ_x, β, η, etc.) brings about a considerable discrepance in the presented results.

3. Large amplitudes and lifetime

It is known that the nature of the motion at large amplitudes determines the lifetime and the background conditions. That has usually a resonance structure (Fig. 4).[41] We would like to emphasize two circumstances.

1. The appearance of non-gaussian tails in the density distribution. Due to scattered light, there is no usual possibility of observing the beam density at high amplitudes ($>4\sigma$) using synchrotron radiation. The technique with the probes in the storage ring aperture[24,37] has made it possible to trace the tails at larger amplitudes. It has turned out that at these amplitudes the particle density is much large than that of a gaussian distribution[47].

2. Decrease in the effective aperture, the appearance of the so-called «dynamic» aperture. This means that there is a definite amplitude of oscillations beginning from which the particle moves to the wall and gets lost. Both factors (dynamic apertures and X-tails) have

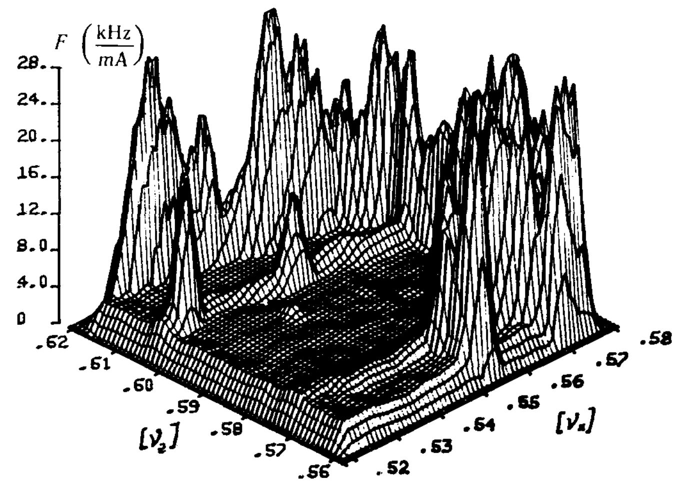


Fig. 4. Background vs. working point (VEPP-4).

influence on a maximum ξ and determines the dependence of this parameter on the aperture limitation.

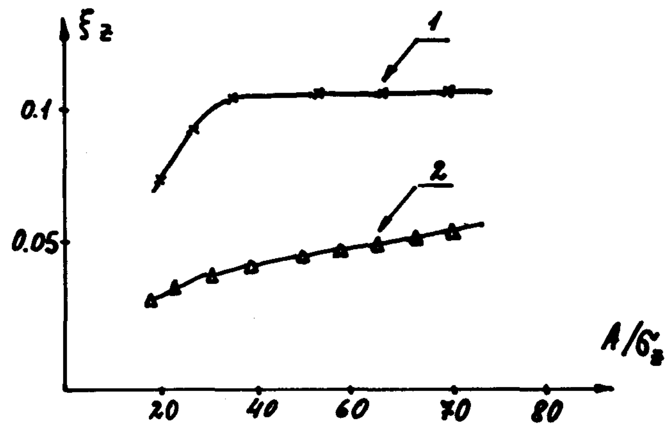


Fig. 5. The maximum ξ_z vs. the aperture for different interaction points (VEPP-2M, 0.5 GeV, weak-strong beams): 1. The basic interaction points ($\sigma_z \ll \sigma_x$, $\beta_z = 5$ cm, $\beta_x = 40$ cm); 2. The after interaction points ($\sigma_z \approx \sigma_x$, $\beta_z = 150$ cm, $\beta_x = 25$ cm).

Fig. 5 demonstrates ξ against the aperture in the «weak-strong beam» regime for the interaction points with different β-functions[46]. The parameter ξ, was taken to be such, as to lead to the constant value of the lifetime of the weak beam. It is seen on the diagram that for the main interaction point ξ begins decreasing at $A_z/\sigma_z = 30$, though $A_{z_{max}}/\sigma_z = 70$. In the given case (curve 1), the character of the dependence $\xi \approx f(A_z/\sigma_z)$ may be accounted for by the appearance of aperture limitation beginning at A_z/σ_z (dynamic aperture). For the other interaction point (curve 2) the character of the above dependence is completely different and the distribution tails extend to a larger amplitudes.

A study of the particle motion at large amplitudes for the case (1) has been made[38]. The indications have been obtained that initially the amplitudes of vertical oscillations increase and then, at rather high vertical-oscillation amplitudes, the radial ones build up fast and the particle is lost as a result, which is apparently due to the influence of the coupling resonance. The VEPP-2M experiments also evidence in favour of such a mechanism of appearance of the «dynamic» aperture.

At present it is planned to install the «snakes» at some facilities to increase the phase volume and, correspondingly, the luminosity. In view of this, the question concerning the value of ξ when increasing A/σ is important.

4. The influence of errors

There exist some reasons giving rise to the distinction of the real motion of particles in storage rings from the ideal model. The influence of imperfections on a decrease of the maximum values of ξ has recently been discussed in a number of papers. Taking into account the errors in the structure, computer simulation has been performed for the facilities PETRA, PEP and some others. In particular, the simulation results for PEP[18] have shown that possible errors reduce the luminosity by a factor of 2.

A fine tuning of the maximum luminosity, mainly determined by the skill of an operator, basically consists in eliminating the errors. An important role here is played by the orbit distortions in sextupoles influencing both the structure and the coupling. For example, on the facility CESR there is the so-called «golden orbit» at which the maximum luminosity is achieved[39]. This orbit does not correspond to the minimum distortions measured using pick-up electrodes. Similar results have been obtained at some other facilities too.

In what follows the influence of some certain types of imperfections will be considered qualitatively, with the experimental results for VEPP-2M and for VEPP-4 involved.

4.1. Different positions of the interaction points and the errors in phase advance.

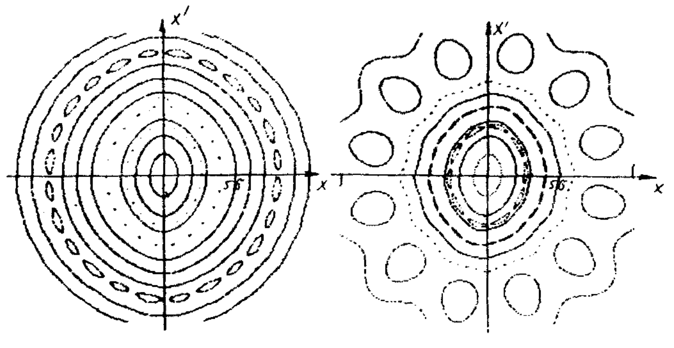
It is known that the ideal model the set of betatron tunes at which nonlinear resonances appear is determined by the condition:

$$2m\nu = k \cdot q \ ,$$

where k and m are the integers and q is the number of the interaction points. The coefficient 2 in this condition results from the symmetry of the space charge «force» of the opposite beam. The quantity q on the right hand reflects the symmetry of the azimuthal motion. The resonance power is determined by the order of $P=2m$. If the β-functions are different at the interaction points or there are errors in the betatron phase advance between them, the symmetry of the azimuthal motion violates and a set of additional resonances appear:

$$2m\nu = kq + l \ ,$$

where l is an integer. In modern facilities it is easy to measure and correct the β-functions. In view of this, a real danger, relative to the beam-beam effects, is the inequality of the phase advances between the interaction points. The effect of additional resonances is illustrated in Fig. 6. Here the particle trajectories in phase space are given for the model with two interaction points and a round opposite beam. In the first case (Fig. 6a), the phase advances between the collisions are equal. It is seen here the BB regions of nonlinear high-order resonance: $\nu=2/24$. In the second case (Fig. 6b), the phase advances are distinguished by $\Delta\mu=0.03\cdot 2\pi$. Here the lower-order resonance appear 1/12, with large BB regions. In both cases the total phase advance was the same and corresponded roughly to the position of the VEPP-2M operating point at which the dependence of



a) $\Delta\mu=0$, b) $\Delta\mu=0.03\cdot 2\pi$

Fig. 6. The appearance of an additional resonances because of errors in the betatron phase advance ($\Delta\mu$) between the interaction points (weak-strong beams, $\xi=0.03$, two interaction points, $\nu=3.08$).

the intensity of the beam-beam effects on the misalignment of the magnetic structure was observed experimentally. With the wiggler installed, the necessity arises to compensate for the error in the phase advance. The diagram in Fig. 7 represents the experimental data obtained. The relative increase in the size of the weak

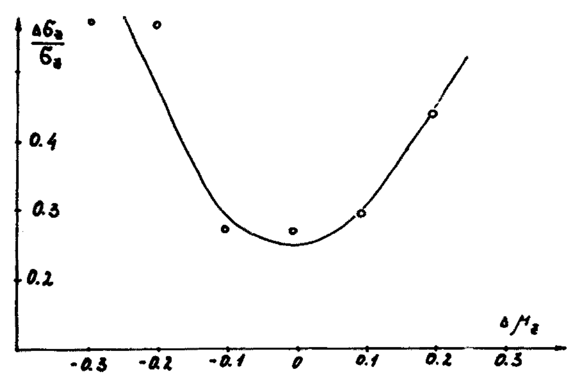


Fig. 7. The increase in the size of a weak beam vs. phase advance at the VEPP-2M.

beam is plotted on the vertical and the difference in the betatron phase advances between the interaction points is laid off on the horizontal. The total phase advance has remained unchanged. The minimum seen on this diagram corresponds to the equality of the phase advances between the interaction points.

4.2. Orbit separation at the interaction points.

As has been mentioned above, the set of resonance frequencies in the ideal model depends basically on the symmetry of space charge forces. The orbit separation at the interaction points violates this symmetry and leads to the appearance of additional resonances.

In the case of a facility with a single interaction point (VEPP-4), at ideal orbits alignment, the set of resonant frequencies will be the following:

$$2m\nu = k \; .$$

When separating the orbits, additional resonances must appear because of symmetry breakdown:

$$(2m+1)\nu = k \; .$$

This effect is illustrated in Fig. 8. Here one can see the particle trajectories in phase space for the model with a round beam and a single interaction point. In the first

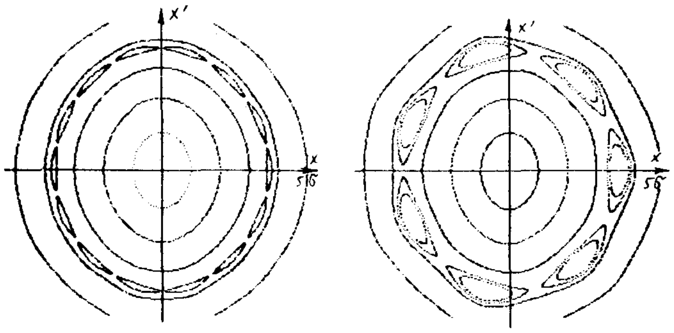


Fig. 8. The influence of the small orbit separation ($\xi = 0.03$, $\nu = 0.565$): a) $\Delta x = 0$; b) $\Delta x = 0.1\sigma$.

case (Fig. 8a), the orbits at the interaction point are exactly aligned and the BB regions of the nonlinear resonance $\nu = 2/14$ are seen on the phase plane. In the second case (Fig. 8b), the orbits are separated by 0.1σ. On the phase plane, on the place of the 2/14 resonance, there are the considerably large BB regions of the 1/7 resonance.

The appearance of additional resonances on account of an insignificant radial orbit separation at the interaction point was experimentally observed at VEPP-4.

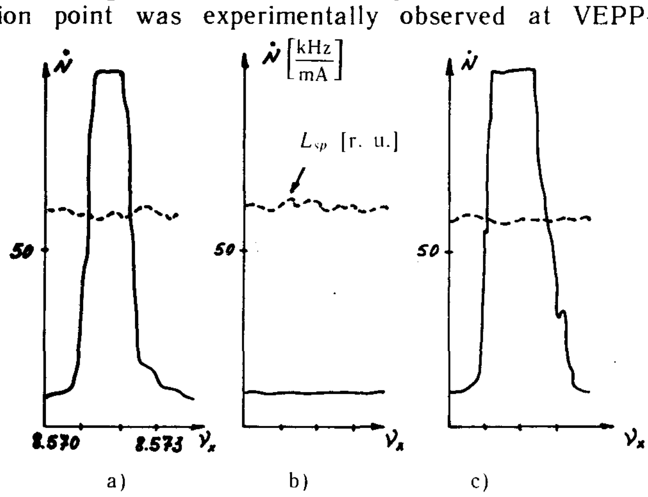


Fig. 9. The specific luminosity (L_{sp}) and the particle loss rate ($\dot{N}$) near resonance $\nu_x = 70/6$ for the different orbit separation. a) $U_{sep} = 0$ kV ($\Delta x \approx 0.1\sigma$); b) $U_{sep} = 20$ kV ($\Delta x \approx 0$); c) $U_{sep} = 40$ kV ($\Delta x \approx -0.1\sigma$).

Fig. 9 presents the diagrams of the particle losses from a weaker positron beam and those showing the dependence of specific luminosity on the tune when the operating point intersects the resonance 60/7, for various voltages on the plates of radial orbit separators. The experimental procedure and conditions are similar to those described in[40]. It is seen from these measurements that the resonance 60/7 manifests only in particle losses and has no influence on the specific luminosity. In practice, the latter is not dependent on the voltage at the plates, i. e. it is impossible to use this parameter to control the orbit separation. At a 20 kV voltage on the plates the resonance $\nu_x = 60/7$ disappears. One can conclude that this voltage corresponds to a correct radial alignment of the orbits at the interaction point.

A reduction of radial misalignments at the interaction point at VEPP-4 has resulted in increasing the lifetime of particles in the beam-beam limited regime.

4.3 Influence of the nonlinear components of a magnetic field.

The nonlinear components of the guiding magnetic field of a storage ring have an influence on the beam-beam effects, first, in reducing the dynamic aperture because of the action of the resonance harmonics of the «machine» nonlinearities[41] and, second, in changing the dependence of the frequency of betatron oscillations on the amplitudes[40,42].

In the case of one-dimensional motion, an addition to the variation in tune with respect to the amplitudes because of the «machine» nonlinearity can be described using the following expression:

$$\Delta\nu = R \cdot a^2 \; ,$$

where a is the amplitude of betatron oscillation and R is the nonlinearity. The main contribution to R usually comes from sextupole fields used to correct the chromaticity and the octupole fields which can appear because of the imperfection of the magnetic elements. It is known that the nonlinearity of betatron oscillations introduced by the space charge «force» of the opposite beam is negative, i. e. as the amplitude grows the frequency of betatron oscillations lowers. If the effect of «machine» nonlinearities is such that the frequency increases with increasing the amplitude, then there must exist the region where a negative beam nonlinearity is compensated by a positive «machine» nonlinearity. In this region the resonant motion of a particle is not stabilized by quadratic nonlinearity, therefore the resonant oscillations of the betatron amplitude increase strongly, thereby changing the «dynamic» aperture. The phase trajectories on Fig. 10 illustrate this effect.

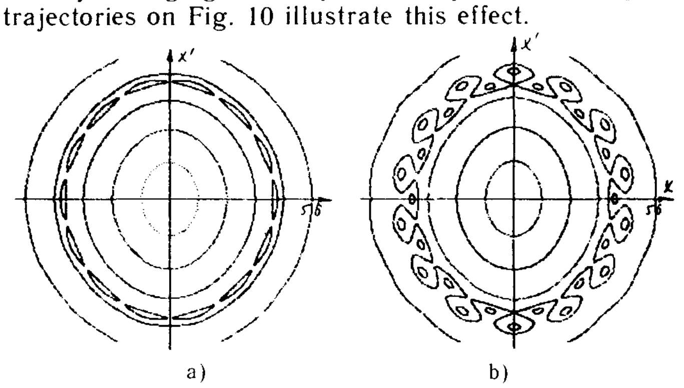


Fig. 10. The increase in resonant oscillation amplitude at a positive «machine» nonlinearity. The model: round beam, $\xi = 0.03$; a single interaction point, a) $\nu = 0.565$, the «machine» nonlinearity is zero: b) the «machine» nonlinearity $\beta \frac{\partial \nu}{\partial a^2} = 20$ cm^{-1}, $\nu = 0.5575$ the phase volume of the beam $\varepsilon = 2.2 \times 10^{-5}$ rad·cm.

In the motion at large amplitudes, where the frequency-amplitude dependence is determined by the expression (1), the condition of overlap of two beam-beam driven nonlinear resonances will be:

$$\frac{V_1^{1/2}+V_2^{1/2}}{|\nu_1-\nu_2|}|2R|^{1/2}\geqslant 1,$$

where $\nu_{1,2}$ and $V_{1,2}$ are the resonant frequencies and harmonics. It is seen from this expression that at large absolute magnitudes of nonlinearity the conditions of resonance overlap and instability appearance become less restrictive, in the high-amplitude region. The experimental data dealing with an observation of the influence of the cubic nonlinearity of the guiding magnetic field of a storage ring on the beam-beam effects can be found in Ref.[40].

5. Strong-weak and strong-strong beams

In the foregoing section we have discussed the behaviour of a single particle for the interaction with an intense strong bunch. The distinction between the strong-weak and strong-strong systems have been analysed in a number of papers.

In the experiments, two regimes were observed in the behaviour of two strong beams:

a) the size of one of the beams remained unchanged while that of the other increased;

b) the sizes of both beams increased simultaneously. So, at the VEPP-2 weakly-focusing ring the case (a) was realized. Note that if the oscillations are preliminarily excited in one beam with a help of a kicker, the size of this beam is larger, than that of the opposite one, even if its intensity is higher. This phenomenon called the flip-flop effect was observed on a number of facilities[43]. The flip-flop occurs readily if the operating point is a little below the strong resonance because the regime of simultaneous formation of BB regions for both beams is unstable.

In modern strongly-focusing storage rings, one succeeds in avoiding strong resonances, with the frequencies properly chosen. This makes it possible to find such a regime (b) when the sizes of both beams grow simultaneously. For example, at VEPP-2M, VEPP-4 and some other facilities the vertical sizes double in the standard regime of data collection (see Table).

However, even in this case the peak currents can exist at which the flip-flop occurs.

The phenomenological model demonstrating how flip-flop effect emerges is described in Ref.[44]. Using the prerequisitics[45], the author has assumed that the vertical size σ_1 of one of the beams depends on the current J_2 and size σ_2 of the second, according to the expression

$$\sigma_1^2=\sigma_1^2+\left(\frac{aJ_2}{\sigma_2}\right)^p,$$

where a and $p=\text{const}$, determining the character of size variation are constant.

Such a model allows to find the critical values of the current J^* and the size σ_0^* at which the flip-flop effect can occur:

$$J^*=\frac{\sigma^{1+2/p}}{a}\left(\frac{p}{p-2}\right)^{1/2}\left(\frac{2}{p-2}\right)^{1/p}; \quad \sigma_0^*=\sigma_0\left(\frac{p}{p-2}\right)^{1/2}.$$

It follows from the above relations that the flip-flop effect can occur only for the parameter $p>2$. The estimates[44] are in accord with the experimental results. Unfortunately, such a description provides no guide to particle dynamics in the strong-strong beam system.

The study of the beam-beam effects at VEPP-2M and VEPP-4 has shown that in the strong-weak beams regime the limiting value of ξ is somewhat higher (about 1.5 times) than in the case of two strong beams. The equality of the lifetime has usually served as a criterion for comparison. The difference in the values of ξ indicates some peculiarities of an interaction between two strong beams.

Coherent instabilities. The possibility of coherent instability for the interaction of two colliding beams has been noted in[22,25]. In[22] the conditions have been derived for excitation of the 2nd, 3rd and 4th modes of two strong beams. The data obtained show that the restrictions on the parameter ξ are of the resonant nature, and the quantity ξ approaches the real values at the facilities.

However, no experimental results concerning the observation of such instabilities are known to the authors for the time being. A study of two strong beams is a complicated experimental problem and there is still little experimental information. In addition, the single-beam effects also complicate the picture (head-tail, fast damping, etc.).

6. Comparison of the facilities

Table lists the parameters of some facilities, their luminosity and the maximum values of ξ, $\Delta\nu$. Of interest is to compare them, to discuss the effect of certain parameters on ξ.

Betatron frequencies (Fig. 11). The phase advance is closest to $n\pi$ at VEPP-2M, VEPP-4 and PEP. The maximum ξ_z have been achieved exactly at these facilities.

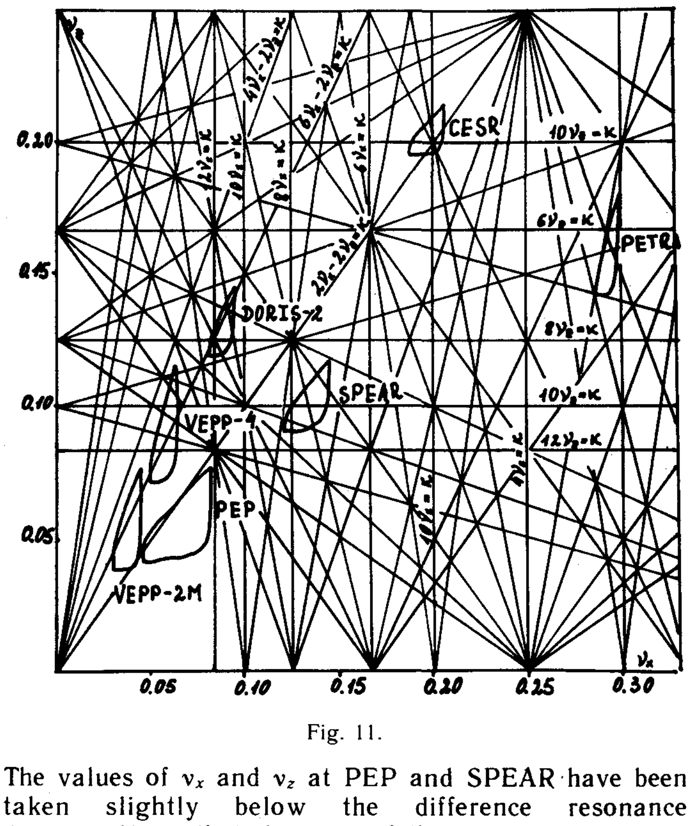


Fig. 11.

The values of ν_x and ν_z at PEP and SPEAR have been taken slightly below the difference resonance ($\nu_x-\nu_z=k$) so that the «tie» of the tune spread conveniently lies within the cell free from the resonances. The operating point of VEPP-2M, VEPP-4 and DORIS lies slighly over the difference resonance, therefore the «tie» is elongated in the vertical direction ($\xi_z\gg\xi_x$) since as ξ_x grows the coupling resonance or its satellits starts operating. The betatron tunes of PETRA have been chosen far from the difference resonance and this makes it possible to have readily a small phase volume at high energies and, correspondingly, the tie of tunes with $\xi_z \gg \xi_x$. The CESR «tie» of tunes intersects the linear coupling resonance and, hence, the coupling is considerable and $\xi_z \approx \xi_x$, as a consequence.

It is likely that the choice of betatron tunes for different facilities has been made proceeding from different arguments.

to a small ν_s/q (≈ 0.005) which determines the power of the most dangerous modulational resonances associated with longitudinal oscillations. At VEPP-4 and CESR, $\beta_z/\sigma_z \approx 2$ and $\nu_s/q \approx 0.02$ and that is why the further decrease in β_z did not result in increasing the luminosity[48].

Parameters of the interaction point; the coupling. The relationships between σ_x and σ_z are considerably different for different facilities. The VEPP-4 beam is the flattest. At all the facilities, except for DORIS and CESR, the coupling is corrected by skew-guadrupoles.

	VEPP-2M	SPEAR	DORIS-2	VEPP-4	CESR	PEP	PETRA
E (GeV)	0.5	1.54	5	5	5.28	14.5	17.5
q	2	2	2	1	2	6	4
$\mu_z/2\pi$	1.54	2.59	2.62	9.57	4.695	3.037	5.642
$\mu_x/2\pi$	1.53	2.615	3.58	8.55	4.685	3.547	6.281
ν_s	0.01	0.03		0.02	0.049		
β_z (m)	0.03	0.09	0.055	0.12	0.03	0.09	0.08
β_x (m)	0,3	0.87	0.640	3.0	0.9	2.7	1.3
η (m)	0.4	0	−0.36	0	0.8	0	0
β_z/σ_l	1.0	3	3	2	1.5	4.5	6
$\partial\nu_x/\partial(\Delta P/P)$	0.5	1.1	2.5	0.5	2	2	3
$\partial\nu_z/\partial(\Delta P/P)$	0.5	0.6	2.5	0.5	2	2	3
$\sqrt{\varepsilon_z/\varepsilon_x}$	0.1	0.24	0.2	0.04	0.1	0.18	<0.1
A_x/σ_x	10	>10	10	18	15	15	13
A_z/σ_z	40	>80	60	45	50	65	90
$\delta = 1/q\tau f$	$0.2\cdot10^{-5}$	$4\cdot10^{-5}$	$24\cdot10^{-5}$	$29\cdot10^{-5}$	$5\cdot10^{-5}$	$14\cdot10^{-5}$	$30\cdot10^{-5}$
σ_x/σ_z	30	12	20	120	30		40
ξ_z	0.050	0.033	0.026	0.06	0.021	0.047	0.04
$\Delta\nu_z$	0.035	0.029	0.024	0.046	0.021	0.033	0.037
$\Delta\nu_z\cdot q$	0.070	0.058	0.048	0.046	0.042	0.198	0.148
ξ_x	0.015	0.026	0.014	0.017	0.021	0.047	0.016
$\Delta\nu_x$	0.012	0.024	0.013	0.015	0.020	0.035	0.015
$\Delta\nu_x\cdot q$	0.024	0.048	0.026	0.015	0.040	0.210	0.060
$\Delta\beta_z/\beta_z$ (%)	−83	−27	−15	−57	−4	−85	−16
$\Delta\beta_x/\beta_x$ (%)	−41	−16	−15	−28	−5	−69	−9
$\sigma_z(\xi_{max})/\sigma_{z0}$	1.5	1.1	1.9	2.0	5		
$T(\xi_{max})$ (min)	20	180	20	120	50	240	420
L_{max} ($cm^{-2}s^{-1}$) 10^{30}	3	0.6	30	5	15	30	17

Synchrotron frequency ν_s. Large values of synchrotron frequencies reduce the width of the operating cell because of the modulational resonances. As it is known, at SPEAR, after a new RF system had been introduced one had to increase betatron tunes[47]. The conventional practice is to avoide both machine and beam satellites. If for the machine satellites the quantity ν_s is important then for the beam ones ν_s/q is of value, both being strongly different at large q (PEP, PETRA). In view of this, the quantity ν_s and ν_s/q have effect on a choice of the operating point.

The ratio β_z/σ_l is the lowest (≈ 1) at VEPP-2M, while the quantity ξ_z here is rather high. It is likely to be due

Chromaticity. At all the facilities there is a small positive chromaticity and the modulation ξ is negligible because of this chromaticity.

A relative damping decrement δ. As has been mentioned above, it is difficult to trace the dependence on this parameter when the fasilities are compared.

The total shift of betatron tune ($\Delta\nu_x\cdot q$, $\Delta\nu_z\cdot q$).

At the facilities with high q (PEP, PETRA) the total shift of betatron frequencies and, correspondingly, the size of the «tie» can achieve large values. In this case, it is natural that the number of dangerous «machine» resonances grows. The question concerning admissible maximum values of the total shift of frequencies is not completely clarified yet.

Lifetime and ξ. As it is seen from the Table, when finding ξ different criteria for admissible lifetime have been taken at different facilities. This is likely to disturb the picture of comparison.

Luminosity. The luminosity is determined as follows:

$$L \approx \frac{\xi_z \cdot \xi_x \gamma^2 \cdot f_0}{\beta_z \cdot r_0^2} \varepsilon_x ,$$

where ε_x is the radial phase volume.

In the foregoing we have discussed the influence of the beam-beam effects on the parameters ξ_x and ξ_z. As it is seen from the expression for luminosity, the other parameters also play an important role. So, for example, the facilities VEPP-4, CESR and DORIS-II operating within the equal range of energies have different β_z and ξ_x. It is this circumstance which is due to the difference in the magnitude of luminosity.

7. Conclusion

We would like to discuss briefly the possibilities of a further increase of the parameters ξ_z and ξ_x and, hence, the luminosity.

1. The elimination of machine imperfections and the development of the methods of fine tuning of the magnetic structure.

2. A study of the dynamics at large amplitudes and the mechanisms of particle losses as well as the influence of the external nonlinear fields on the lifetime.

3. A choice of the optical relationship between ξ_z/ξ_x and β_z/β_x.

In the authors' opinion, a significant role in beam-beam effects is played by a considerable difference in the vertical and radial emittances and by the two-dimensionality of motion. Of interest is the suggestion[46] to create facilities with $k=1$, $\beta_z=\beta_x$, $\eta=0$, and $\xi_z=\xi_x$. In this case, the motion degenerates into one-dimensional and two-dimensional resonances disappear. As far as ξ is concerned, it will not be dependent on the bunch length. It is not difficult to realize such a variant for colliding proton-antiproton beams.

4. It is apparent that in electron-positron facilities the possibilities of increasing the luminosity are mainly due to the other parameters entering the expression for luminosity rather than an increase of ξ.

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