# Aspects of scalar dynamics and vacuum energy in the string swampland program


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**Ginevra Buratti**

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Tesis Doctoral dirigida por **Dr. Ángel M. Uranga**,
Profesor de Investigación del Instituto de Física Teórica

Departamento de Física Teórica
Universidad Autónoma de Madrid

Instituto de Física Teórica UAM/CSIC


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The following papers, some of them unrelated to the content of this document, were published by the candidate while this thesis was being developed:

1. *Dynamical cobordism and swampland distance conjectures*
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2. *Dynamical tadpoles, stringy cobordism, and the SM from spontaneous compactification*
   **G. Buratti**, M. Delgado and A. Uranga
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3. *Discrete symmetries, weak coupling conjecture and scale separation in AdS vacua*
   **G. Buratti**, J. Calderón, A. Mininno and A. Uranga
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4. *Self-interacting chiral p-forms in higher dimensions*
   **G. Buratti**, K. Lechner and L. Melotti
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5. *Duality invariant self-interactions of abelian p-forms in arbitrary dimensions*
   **G. Buratti**, K. Lechner and L. Melotti
   JHEP 09 (2019) 022

6. *Transplanckian axion monodromy!?*
   **G. Buratti**, J. Calderón and A. Uranga
   JHEP 05 (2019) 176

7. *Supersymmetry breaking warped throats and the weak gravity conjecture*
   **G. Buratti**, E. García-Valdecasas and A. Uranga
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# Abstract

This thesis, written as a compendium of articles, investigates the properties of scalar dynamics and vacuum energy in the context of string theory compactifications, and their connection with the swampland program.

In the first article presented in the thesis, we consider warped throats with locally AdS geometry, and study their stability properties. Motivated by considering the near horizon limit of systems of fractional D-branes at singularities, we propose that such backgrounds cannot be stable in the absence of supersymmetry, and thus generalize the swampland criterion that forbids stable non supersymmetric AdS vacua. This allows us to rule out large classes of warped throats with supersymmetry breaking ingredients, shedding new light on already known instabilities, including the runaway in the $dP_1$ theory, and unveiling novel decay mechanisms such as the one associated to $\mathcal{N} = 2$ fractional branes.

In the second article we focus on the asymptotic Klebanov–Tseytlin solution, regarded as a compactification to five dimensions in which an axion runs in the radial direction of the locally AdS spacetime. The model can be reinterpreted as a fully backreacted solution of transplanckian axion monodromy, with the axion traversing arbitrarily large distances in field space, and provides an existence proof of transplanckian field excursions in string theory. In particular, we discuss how the ten-dimensional solution fully encodes the backreaction of the axion dynamics including the impact on the axion kinetic term, and the backreaction on other sectors, such as the compactification moduli and the vacuum energy.

In the third article we propose a refinement of certain swampland conjectures in the presence of discrete gauge symmetries. We consider theories with both discrete and continuous gauge symmetries, and relate the gauge coupling of the continuous symmetry with the order of the discrete symmetry. We also study discrete symmetries associated to domain walls, and we use them to justify the presence of separation of scales in an infinite family of $AdS_4$ flux vacua of type IIA string theory.

In the last two articles of the thesis, we study running solutions sourced by tadpoles for dynamical fields, and analyse their properties in large classes of string theory models. These solutions can only extend up to a finite distance in spacetime, scaling inversely with the strength of the tadpole, and are capped off by cobordism walls of nothing in a dynamical realization of the cobordism conjecture. We also discuss domain walls interpolating between different (but cobordant) theories. The key criterion to distinguish between the two kinds of walls is related to the distance in field space, and suggests a connection with the distance conjecture.

# Resumen


En esta tesis, escrita como un compendio de artículos, se exploran la dinámica de campos escalares y la energía de vacío en el contexto de compactificaciones de teoría de cuerdas, y sus conexión con el programa de swampland.

En el primer artículo de la tesis, consideramos gargantas curvadas con geometría localmente AdS, y analizamos sus propriedades de estabilidad. Motivados por considerar el límite cerca del horizonte de sistemas de D-branas fraccionarias en singularidades, conjeturamos que estas geometrías no pueden ser estables si supersimetría está rota, generalizando el criterio de swampland que excluye vacíos non-supersimétricos estables de tipo AdS. Aplicando esta idea a largas clases de gargantas curvadas non supersimétricas, aclaramos los distintos mecanismos que obstaculan la estabilidad en ejemplos conocidos de la literatura, como la teoría basada en $\mathrm{dP}_1$, y encontramos mecanismos innovadores de decaimiento, como la inestabilidad asociada a branas fraccionarias de tipo $\mathcal{N} = 2$.

En el segundo artículo nos centramos en la solución asintótica de Klebanov–Tseytiln, considerada como una compactificación cuya teoría efectiva en cinco dimensiones incluye un axion dependiente en la coordenada radial del espaciotiempo localmente AdS. Esta solución se puede reinterpretar como un modelo de monodromía axionica en que la distancia efectiva que recorre el campo sea arbitrariamente larga, aportando así una prueba de existencia de excursiones transplanckianas en teoría de cuerdas. Específicamente, mostramos como la solución en diez dimensiones incluye plenamente la retroacción sobre la dinámica del axion, los otros módulos y la energía de vacío.

En el tercer articulo proponemos un refinamiento de algunas conjeturas de swampland en presencia de simetrías discretas de gauge. Específicamente, consideramos teorías con simetrías de gauge discretas y continuas y relacionamos la constante de acoplamiento de la simetría continua con el orden de la simetría discreta. Además examinamos simetrías discretas asociadas a paredes de dominio, y explicamos como utilizarlas para justificar la presencia de separación de escalas en una familia infinita de vacíos $\mathrm{AdS}_4$ de teoría de cuerdas tipo IIA.

En los últimos dos artículos de la tesis, examinamos soluciones asociadas a tadpoles para campos dinámicos, y analizamos sus propriedades en varios modelos de teoría de cuerdas. Estas soluciones no se pueden extender mas de una distancia finita en el espaciotiempo, que depende inversamente de la fuerza del tadpole, y culminan en paredes de dominio en una realización dinámica de la conjetura de cobordismo. Además discutimos paredes de dominio que separan teorías distintas (pero cobordantes). El criterio llave para diferenciar los dos tipos de paredes está relacionado con la distancia en el espacio de campos, lo que sugiere una conexión con la conjetura de la distancia.

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# Contents

# 1
# General introduction

Progress in theoretical physics has often taken the form of unification. In the 1860s, James Clerk Maxwell realized the similarities between electricity and magnetism, and elaborated his theory of a single electromagnetic force. In the 1920s, when Albert Einstein began to work on a unified theory of gravitation and electromagnetism, these were the only known fundamental forces, and no subatomic particles were known apart from electrons and protons. Einstein strongly believed that all of nature must be described by a single theory, and devoted the last thirty years of his life to this idea, but his efforts were clearly premature.

It was only in the 1970s that theorists began to disclose the connection between electromagnetism, with its visible effects in everyday life, and the weak force, which is usually concealed within the atomic nucleus. These two forces appear to be very different at low energies, but begin to act on equal terms at the higher energies explored in particle collisions. Experiments also show that the strong force becomes weaker as energy increases, and this is a good hint that at extremely high energies, a thousand million times greater than those accessible to particle accelerators, the strengths of the electromagnetic, weak and strong interactions might be the same. Taking this idea even further, theorists envisage the possibility of including gravity at still higher energies, thereby unifying all of the fundamental forces into one.

To provide a deep understanding of the interactions and the structure of matter at a fundamental level is one of the key objectives of particle physics. A remarkable insight into these questions is provided by the Standard Model, the most accurate physical theory ever produced, which has successfully explained almost all experimental results, and precisely predicted a wide variety of phenomena. Formulated as a quantum field theory, it describes the elementary particles leading to both matter and the electromagnetic, weak and strong interactions into a single mathematical framework, which incorporates the two paradigms of modern physics, namely quantum mechanics and special relativity. The first implies a shift from a deterministic to a probabilistic perspective in our understanding of the world, while the second intertwines space and time in a very specific manner.

Despite its undeniable success, however, there are evidences that the Standard Model is still incomplete. In particular, the fact that an interacting theory of a massless spin-2 boson is non-renormalizable implies that it is not possible to include gravity in this framework. This is a difficult challenge, and the formulation of a description of gravity in a way consistent at the quantum level stands as the main task ahead of this century theoretical physics. In this respect, string theory represents the leading candidate for a consistent quantum theory of gravity, and at the same time has a structure which is just rich enough to contain the essential ingredients of the Standard Model.

This unified description requires the introduction of extra spatial dimensions parametrizing an internal space of very small size, inaccessible to our present observations. Despite this fact, the properties of the compactification space are very relevant, since they determine the properties of the physics in four dimensions, including the gauge group for the interactions, the particle content charged under them, and the specific values of the coupling constants of the theory, namely the strength of the interactions and the masses and couplings of particles. There seems to be an enormously large number of possible compactifications in string theory, each leading to a different vacuum with (possibly widely) different properties for the resulting physics in four dimensions. This has led to the concept of a *string landscape*, as the set of four-dimensional theories arising from the different string compactifications. The largeness of this set might suggest the idea that any four-dimensional effective field theory ultimately admits some embedding into string theory, for a suitable choice of the compactification data.

However, there is an increasing evidence that this is not the case, and that there exist certain constraints that any low energy effective field theory must satisfy in order to admit a consistent UV embedding in quantum gravity. Otherwise, the theory cannot be part of the landscape, and is said to belong to the *swampland*. Unfortunately, the complete set of such constraints is not known. These are usually formulated in the form of conjectures for which a complete proof is still missing, but the increasing web of connections between them suggest that they might be pointing towards some more fundamental quantum gravity principles yet to be uncovered. To identify such constraints and gather evidence to prove (or disprove) them, as well as to understand their phenomenological implications for low energy physics, is the aim of the swampland program. Although the notion of the swampland is in principle not restricted to string theory, swampland conjectures are often motivated by or checked in stringy setups, which are a perfect arena to test them in a quantitative and rigorous way.

Hence, it is fair to say that the existence of a swampland is good news, and paves the way towards an era of *quantum gravitational* string phenomenology, with interesting applications in both particle physics and cosmology. Swampland conjectures can indeed provide new guiding principles to construct theories beyond the Standard Model, as well as induce UV/IR mixing that breaks the expectation of scale separation. They can also be used to address the structure of large field inflation in early-time cosmology, or the mechanism responsible for the observed late-time acceleration of the universe, to name but a few. In the rest of the introduction we will briefly introduce some of these conjectures, focusing on those that are more relevant for the articles presented in the thesis.

*Weak gravity conjecture*

The motivation for the weak gravity conjecture (WGC) [5] is twofold. First, it provides a mechanism that prevents from recovering a global symmetry when the gauge coupling of a U(1) gauge symmetry is sent to zero, $g \to 0$. The statement that there are no global symmetries in quantum gravity, which means that any symmetry is either broken or gauged, is indeed the oldest and most widely accepted of swampland conjectures [7], with much evidence for it coming from string theory. This is known to be true in perturbative string theory [6, 23], where global symmetries on the string worldsheet correspond to gauge symmetries in spacetime, and there is no way to have global symmetries in spacetime. The situation for strings in AdS background [1, 13] is also similar: global symmetries in the boundary theory are associated with gauge symmetries in the bulk, and there is no way to have global symmetries in the bulk. Second, the WGC corresponds to the kinematic requirement that extremal black holes are able to decay, thus avoiding troubles with remnants and entropy bounds. However, as many arguments based on black hole physics, this should be taken as a (heuristic) motivation rather than a proof.

The WGC comes with both an electric and a magnetic version. Given a theory with a U(1) gauge vector field coupled to Einstein gravity, the electric WGC requires the existence of an electrically charged particle with charge-to-mass ratio greater than that of an extremal black hole, which depends on the theory under consideration and is typically of order one. In four dimensions, for instance, the Reissner-Nordström black hole leads to the inequality

$$m \leq \sqrt{2} g q M_p \,, \tag{1.1}$$

where $m, q$ are the mass and the quantized charge of the WGC particle. An equivalent interpretation of the conjecture (in the absence of scalar fields) is that for such particle the gravitational force is weaker than the electromagnetic one,

$$F_{\text{grav}} \leq F_{\text{em}} \,, \tag{1.2}$$

whence the name.

To derive the magnetic version, one applies (1.1) to the magnetic dual field, and gets a constraint on the mass of a magnetic monopole. Since the magnetic charge is proportional to the magnetic gauge coupling, which is the inverse of the electric gauge coupling, and the mass of the monopole is typically at least at the order of the cut-off of the theory over the gauge coupling squared, one finds that the cut-off $\Lambda$ is bounded from above by the gauge coupling, and is smaller than the Planck mass if the gauge coupling is small,

$$\Lambda \lesssim g M_p \,. \tag{1.3}$$

In particular, when the gauge coupling goes to zero, this results in new light particles according to (1.1), and the cut-off also goes to zero according to (1.3), invalidating the effective field theory description. This provides the aforementioned obstruction to restoring a global symmetry.

An obvious generalisation of the WGC applies to $p$-form gauge fields in $d$ dimensions, and implies the existence of a $(p-1)$-dimensional state with charge-to-tension ratio greater than that of an extremal $(p-1)$-black brane. For the Reissner-Nordström black brane, one gets the bound

$$\frac{p(d-p-2)}{d-2} T^2 \leq q^2 g^2 M_p^{d-2} \,, \tag{1.4}$$

which reproduces (1.1) for $p = 1$ and $d = 4$. The case for axions $p = 0$, albeit not covered by the above formula, can also be argued, and implies the existence of an instanton with quantized charge $q$ whose action satisfies

$$S \lesssim q \frac{M_p}{f} \,. \tag{1.5}$$

The equivalent of the mass for an instanton is indeed its action, and the equivalent of the gauge coupling is the inverse of its decay constant $f$. The main difficulty is to properly identify what one means by an extremal instanton [14]. Otherwise, the previous bound actually contains an undetermined order one factor, whence the twiddle. One of the most interesting applications of the WGC for axions, when interpreted as a bound on the size of the monotonic regions of the axion potential, is to constrain models of large-field inflation [10, 19, 24]. In natural inflation, an axion slowly rolls down a non-perturbative potential generated by instantons. While the overall periodicity is of the order of the decay constant, and can be made large with $f \gg 1$, the magnitude of monotomic regions, and therefore of the field range available for inflation, remains approximately constant when higher instanton corrections are included. A loophole in this argument is that the WGC instanton may not be the leading contribution in the potential: in this case stronger versions of the conjecture are needed.

Another implication of the WGC that has raised significant interest is related to AdS space. Assuming that the WGC bound can only be saturated by BPS states in a supersymmetric theory, and applying this criterion to D-branes with AdS near-horizon limits, one concludes that stable AdS vacua are necessarily supersymmetric [22]. In particular, any non-supersymmetric AdS vacuum must either be inconsistent or not stable. Since there are several large classes of string compactifications or supergravity backgrounds which seemingly lead to non-supersymmetric AdS vacua, this opens up the general question of what are the instability mechanisms for these constructions. Such mechanisms may be of non-perturbative nature, and therefore provide very interesting insights into string theory beyond perturbation theory. Similar conclusions hold for other extensions to close relatives of non-supersymmetric AdS vacua, such as the holographic gravitational duals of D-branes at singularities breaking supersymmetry in the infrared due to strong coupling dynamics. In these gauge theories, the D-brane construction leads to a runaway behaviour, which should turn into a direct decay of the proposed locally AdS vacua. This and other examples are studied in detail in the first article included in the thesis.

Finally, most works on the WGC focus on the properties of continuous gauge symmetries, whereas fewer results have been obtained for discrete symmetries. A partial explanation for the scarcity of swampland constraints on discrete symmetries, mostly focusing on the constraint that global discrete symmetries, just as in the continuous case, are forbidden in quantum gravity, is the lack of long-range fields or tunable parameters like coupling constants, that makes quantitative statements more difficult. In the third article presented in the thesis, we overcome this difficulty by considering theories with both discrete and continuous gauge symmetries, namely $\mathbb{Z}_k$ and U(1) symmetries, and uncover interesting quantitative relations between them. In particular, we propose that the gauge coupling scales as

$$g \sim k^{-\alpha} \tag{1.6}$$

when the order $k$ of the discrete symmetry is large, with $\alpha$ a positive order one coefficient. We also relate to diverse versions of swampland distance conjectures, which are discussed next.

*Distance conjectures*

There is no free dimensionless parameter in string theory. Upon compactification to lower dimensions, the values of couplings and masses are controlled by the vevs of scalar fields called moduli, which are massless before adding fluxes or other ingredients, and from a ten-dimensional perspective correspond to the size and shape of the extra dimensions. Moving in this moduli space corresponds to exploring different effective field theories. However, something dramatic is expected to happen when moving towards a point where a global symmetry is restored, for example by sending a gauge coupling to zero. An obvious solution that prevents a global symmetry from being recovered in this way is that such point is located at infinite distance in moduli space, and is actually unreachable. Albeit from a purely QFT perspective there seems to be nothing wrong with being as close as possible to this point, global symmetries are strictly forbidden in quantum gravity, and one may expect that the effective field theory description breaks down continuously as the approximate global symmetry looks more and more exact. One may also wonder what happens when approaching any infinite distance limit in moduli space. The answer to these and related questions is provided by the swampland distance conjecture (SDC) [21].

Given an effective field theory coupled to Einstein gravity and with moduli space $\mathcal{M}$ parametrized by massless scalar fields, the SDC predicts the existence of an infinite tower of states that becomes exponentially light for any point $P \in \mathcal{M}$ and any infinite field distance limit, that is

$$M(Q) \sim M(P)e^{-\lambda\Delta\phi} \quad \text{when } \Delta\phi \to \infty\,, \tag{1.7}$$

in terms of the geodesic field distance $\Delta\phi \equiv d(P,Q)$. The exponential rate $\lambda$, apart from being positive, is not further specified. It is believed to be of order one, as one would expect that the exponential behaviour is reached at a distance of order the Planck mass [15], but how small it can be remains an important open question, although some concrete lower bounds have been proposed in the literature [2, 9, 12, 16]. Associated to the tower there is a quantum gravity cut-off decreasing exponentially in the distance, which implies that the range of validity of an effective field theory cannot be extended an arbitrarily large distance away from the original point: the higher the cut-off, the smaller is the maximum field distance that the theory can describe. This is to be contrasted with the situation in which gravity is absent, and no obstruction to the extension of an effective field theory to an arbitrary point in moduli space appears.

In its original formulation, the SDC is a statement about the moduli space of effective field theories with zero potential. However, it is phenomenologically relevant to consider the case in which a potential is added and the moduli space is lifted, applying the SDC to any scalar field, and not just moduli [15]. This is closely related to the axionic WGC, suggesting that periodic axion potentials cannot host transplanckian field ranges. There are also partial studies concerning axion monodromy models, trying to rule out their transplanckian excursion by invoking the backreaction on the scalar kinetic terms, which reduces the effectively traversed distance [8]. These results would seem to suggest that transplanckian field ranges are not physically attainable in quantum gravity. If correct, this statement would have profound implications for phenomenological applications, notably for the construction of models of inflation. In the second paper of the thesis, however, we prove that this statement is in fact incorrect, and that transplanckian field excursions are physically realized in string theory.

It has been proposed that the SDC can be extended to more general field configurations beyond the moduli space [17]. In particular, one can define a notion of distance between different metric configurations of AdS spaces, such that the flat space limit, corresponding to the AdS scale going to zero $\Lambda \to 0$, is at infinite distance. This leads to the AdS distance conjecture (ADC), stating that any AdS vacuum has an infinite tower of states that becomes light in the flat space limit as

$$m \sim |\Lambda|^{\alpha}\,. \tag{1.8}$$

A strong version of the conjecture states that $\alpha = \frac{1}{2}$ in the supersymmetric case, implying that there is no scale separation between the AdS scale and the KK scale. A counterexample to the strong version (but not to the conjecture itself) is provided by an infinite family of four-dimensional type IIA orientifold vacua that is claimed to achieve scale separation [11]. In the third paper of the thesis, we propose a way to reconcile these vacua with (a refinement of) the strong ADC, based on discrete symmetries.

*Cobordism conjecture*

Cobordism is an equivalence relation on the space of compact manifolds of the same dimension, whereby two manifolds are identified if their disjoint union is the boundary of a compact manifold one dimension higher. This has an abelian group structure, with the trivial element given by a manifold that is a boundary by itself. In the presence of gravity, topology changing transitions are expected to occur, and since these can be interpreted as cobordisms, any topological global charge that is associated to a topological symmetry should also be a cobordism invariant, so that it doesn't change after a topology changing transition. This means that a natural global charge to look at is the cobordism group itself.

Motivated by the statement that global symmetries, including topological symmetries, are forbidden in quantum gravity, it has been proposed that in order not to define a global symmetry, the cobordism group actually has to vanish, and all the cobordism classes must be trivial [18]. This means that any compactification manifold must be cobordant to the empty manifold, and thus can be shrunk to a point. An equivalent perspective is to define the cobordism group of a theory of quantum gravity by identifying configurations that can be connected by a finite energy domain wall. This makes sense because, if the theory arises upon compactification of higher dimensions, then any non-trivial cobordism between two compactification spaces would correspond to a domain wall from the perspective of the lower dimensional theory. In this language, the vanishing of the cobordism group corresponds to the existence of a boundary the theory can end on, namely a *wall of nothing*.

The most interesting implication of the cobordism conjecture is that it can be used to predict the existence of defects. When the cobordism group of a theory is non-trivial, it is indeed possible to include additional ingredients to generate new cobordisms and kill the otherwise non-trivial cobordism classes. Obviously there are many cobordism groups, and which one to look at depends on the theory under consideration. If the theory contains a $p$-form U(1) gauge field, the total magnetic flux over $(p+1)$-dimensional manifold is a cobordism invariant, labelling non-trivial cobordism classes. However, by adding magnetic monopoles that act as a source, the total flux is not an invariant any more, and a trivial corbordim group is recovered. This is nothing but a rewriting in the language of cobordism of the well-known statement that completeness of the spectrum for abelian gauge symmetries follows from the absence of global symmetries, and can be used to predict the existence of D-branes from the presence of RR-gauge fields in type II string theories.

Suppose instead to turn off all the gauge fields, and use only the fact that string theory contains spinors. The spin cobordism for 0-dimensional manifolds is $\mathbb{Z}$, and is generated by the positively oriented point. To kill this class in M-theory, one needs a boundary, which is given by the Horava–Witten wall. Similarly, in type IIA one has an orientifold 8-plane as a boundary, corresponding to the Horava–Witten wall wrapped on the M-theory circle. Another example is the spin cobordism for 1-dimensional manifolds, which is $\mathbb{Z}_2$, and is generated by the circle with periodic spin structure. The way to kill it in a circle compactification of type IIB is to include two O7-planes. Now all the objects in these examples are very explicit and familiar, but this is not always the case, and sometimes one has to rely on the conjecture to claim that the required defect actually exists [18, 20].

The cobordism conjecture is topological in nature, and a better understanding of situations where it is treated in a more physical, real-life sense is clearly desirable. An important step forward in endowing cobordism walls with dynamics is taken in the last two articles presented in the thesis, where we study the properties of theories with tadpoles for dynamical fields, and discuss their interplay with cobordism and distance conjectures.

**Plan of the thesis** After this general introduction, the second part of the thesis contains the collection of five articles in the published versions. Three of them can be read independently and are presented in separated chapters. The other two are one the continuation of the other and are incorporated in a single chapter. We leave the conclusions for the last part of the thesis, where the main results of these works are summarized.

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# 2

# Supersymmetry breaking warped throats and the weak gravity conjecture

This chapter contains the article

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# Supersymmetry breaking warped throats and the weak gravity conjecture

**Ginevra Buratti,[a] Eduardo García-Valdecasas[a,b] and Angel M. Uranga[a]**

[a] *Instituto de Física Teórica IFT-UAM/CSIC, C/ Nicolás Cabrera 13-15, Campus de Cantoblanco, 28049 Madrid, Spain*

[b] *Departamento de Física Teórica, Facultad de Ciencias Universidad Autónoma de Madrid, 28049 Madrid, Spain*

*E-mail:* ginevra.buratti@studenti.unipd.it, eduardo.garcia.valdecasas@gmail.com, angel.uranga@csic.es

Abstract: We generalize the swampland criterion forbidding stable non-supersymmetric AdS vacua and propose a new swampland conjecture forbidding stable non-supersymmetric "locally AdS" warped throats. The conjecture is motivated by the properties of systems of fractional D3-branes at singularities, and can be used to rule out large classes of warped throats with supersymmetry breaking ingredients, and their possible application to de Sitter uplift. In particular, this allows to reinterpret the runaway instabilities of the gravity dual of fractional branes in the $dP_1$ theory, and to rule out warped throats with Dynamical Supersymmetry Breaking D-brane sectors at their bottom. We also discuss the instabilities of warped throats with supersymmetry broken by the introduction of anti-orientifold planes. These examples lead to novel decay mechanisms in explicit non-supersymmetric examples of locally AdS warped throats, and also of pure AdS backgrounds.

Keywords: Brane Dynamics in Gauge Theories, Gauge-gravity correspondence, Superstring Vacua, Supersymmetry Breaking

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## Contents





## 1 Introduction: Quantum Gravitational String Phenomenology

The recent flurry of activity, largely triggered by [1–3], in constraining phenomenological string model building using Quantum Gravity swampland criteria [4–11] (see [12] for a recent review) is giving birth to an emerging field, which can deservedly claim the designation of *Quantum Gravitational String Phenomenology*.

The application of constraints convincingly argued to hold in any theory of Quantum Gravity is leading to new breakthroughs. In particular, the Weak Gravity Conjecture (WGC) [6] (see [1–3, 13–26] for different formulations and applications) has motivated the remarkable statement that stable non-supersymmetric Anti de Sitter (AdS) vacua are not possible in Quantum Gravity [7, 8]. This AdS-WGC constraint is largely motivated by the application of the refined WGC to systems of branes in the near horizon limit, and has received direct support from the study of decays of non-supersymmetric AdS vacua in string theory via bubbles of nothing [27]. The AdS-WGC has been argued to have far-reaching implications for particle physics and its scales [24–26].

There are also recent proposals of swampland criteria attempting to rule out de Sitter vacua as well [9, 28, 29], possibly in certain regimes under parametric control. This claim clashes with familiar roadmaps for the construction of de Sitter vacua in string theory [30, 31], see [32, 33] for recent discussion. A key ingredient in the parametric control of these scenarios is the presence of warped throats [34, 35] at whose bottom the supersymmetry breaking sectors are localized, so that they undergo a redshift crucial for the tunability of the 4d vacuum energy. Starting from the original proposal of supersymmetry breaking by anti-D3-branes [30], there is a rich variety of proposals, see e.g. [36–38]. Hence, it is interesting to explore the interplay of non-supersymmetric warped throats with constraints from Quantum Gravity.

In this paper we consider non-compact warped throats and constrain these 5d backgrounds by proposing a new swampland conjecture, the *local AdS-WGC*, which generalizes the AdS-WGC to locally AdS warped throats. The conjecture is motivated by considering the near horizon limit of systems of fractional D-branes at singularities, but should hold more in general. Although it does not constrain metastable non-supersymmetric throats, hence has no direct implication for e.g. anti-D3-brane models, it can be used to rule out large classes of warped throats with supersymmetry breaking sectors at their bottom. We study this phenomenon in several explicit examples, shedding new light on already known instabilities in supersymmetry breaking D-brane models, such as the $dP_1$ theory, and unveiling novel decay channels in AdS or locally AdS backgrounds. For instance, we explicitly discuss warped throats with supersymmetry broken by the introduction of anti-orientifold planes.

A remarkable feature of these examples is that the non-supersymmetric backgrounds are stable at the classical level, and that the pathologies arise at the quantum level, often by nucleation of bubbles hosting interiors of more stable vacua. This is consistent with the interpretation of these constraints as arising from consistency in Quantum Gravity.

The paper is organized as follows: in section 2 we review systems of D-branes at singularities and fractional branes using the powerful toolkit of dimer diagrams. In section 3 we propose the local AdS-WGC criterion; we derive it in section 3.1, and use it in section 3.2 to reinterpret the properties of supersymmetric and non-supersymmetric warped throats dual to fractional D3-branes in toric singularities. In section 3.3 we discuss the situation for throats with meta-stable supersymmetry breaking. In section 4 we consider an illustrative example of a system of D3-branes with Dynamical Supersymmetry Breaking due to strong dynamics and consider its embedding into warped throats. The D-brane gauge theory is discussed in section 4.1, and in section 4.2 we describe the instabilities that arise when embedded into AdS or locally AdS warped throats, in agreement with the (local) AdS-WGC implications for non-supersymmetric throats; in section 4.3 we describe the local AdS-WGC statement in an explicit example illustrating how it applies to non-supersymmetric throats from $\mathcal{N} = 2$ fractional branes. Section 5 treats warped throats with supersymmetry broken by the presence of anti-orientifold-planes. In section 5.1 we discuss generalities about such throats. In section 5.2 we focus on anti-O3-planes, describe their different kinds and their interaction with systems of D3-branes. In section 5.3 we discuss the corresponding gravitational backgrounds and describe their instabilities, in agreement with the (local) AdS-WGC statement. Finally, in section 6 we give our conclusions.

## 2 Review of dimers and fractional branes

Here we briefly review some ingredients of the dimer diagram description of D3-branes at singularities. The initiated reader is welcome to skip it and jump into the next sections.



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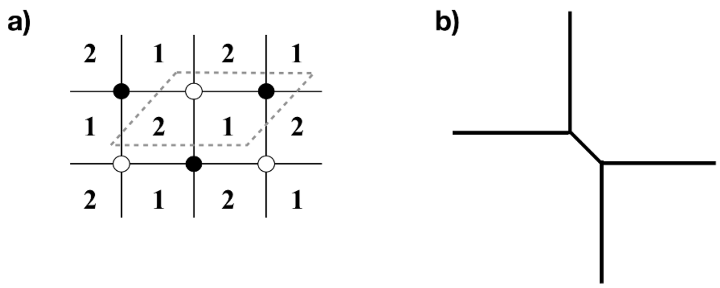


**Figure 1**. (a) Dimer diagram for the theory of D-branes at a conifold. The dashed line is the unit cell in the periodic array. (b) Web diagram of the conifold. We have displayed it with a finite size $\mathbf{S}^2$ (middle segment) for clarity; the actual singularity arises when this $\mathbf{S}^2$ is blown-down.



The gauge theories on D3-branes at toric CY threefold singularities are nicely encoded in a combinatorial graph known as dimer diagram [39, 40] (see also [41, 42] and references therein). They are (bipartite) graph tilings of $\mathbf{T}^2$, or equivalently infinite periodic graphs in $\mathbf{R}^2$. Their faces correspond to gauge factors, edges represent bi-fundamental chiral multiplets (oriented e.g. clockwise around black nodes, and counterclockwise around white nodes), and nodes represent superpotential couplings (with sign determined by the node color). As an illustration, the diagram for the conifold is shown in figure 1(a). The corresponding gauge theory [43] has gauge group $\mathrm{U}(n_1) \times \mathrm{U}(n_2)$, bi-fundamental chiral multiplets in two copies of the representation $(\,\square_1, \overline{\square}_2) + (\,\overline{\square}_1, \square_2)$, denoted by $A_i$, $B_i$, $i = 1, 2$, and a superpotential $W = \epsilon_{ik}\epsilon_{jl} A_i B_j A_k B_l$.

The geometric information about the CY singularity is encoded in simple combinatorial objects in the dimer, whose discussion we skip, directing the interested reader to the references. We just mention that the geometries are encoded in web diagrams, which specify the fibration structure of the corresponding toric geometry. The web diagram can be obtained by constructing the zig-zag paths in the dimer (these are paths constructed out of sequences of edges which turn maximally left at black nodes and maximally right at white nodes) and translating the non-trivial $(p, q)$ windings of the path on the two non-trivial 1-cycles in $\mathbf{T}^2$ into the $(p, q)$ labels of external legs in the web diagram. The web diagram for the conifold is shown in figure 1(b).

The choice of ranks $n_i$ in the gauge groups of the dimer theories is arbitrary, but constrained by cancellation of RR tadpoles. These are equivalent to cancellation of non-abelian gauge anomalies (understood as formally imposed for all gauge factors, even those of possible empty faces). These conditions also guarantee the cancellation of mixed U(1) anomalies thanks to Green-Schwarz couplings. There are in fact topological $BF$ couplings with RR 2-forms making all U(1) factors massive (even the non-anomalous ones, see [44]). Supersymmetry of the configuration implies that blow-up modes couple as (field dependent) FI terms to the D3-branes. Although these U(1)'s are massive, it still makes sense to discuss them if the corresponding couplings to localized closed string modes are taken into account.

The choice of all ranks equal $n_i \equiv N$ for all $i$ is always allowed, and corresponds to D3-branes which can move off the singularity, as signaled by corresponding flat directions

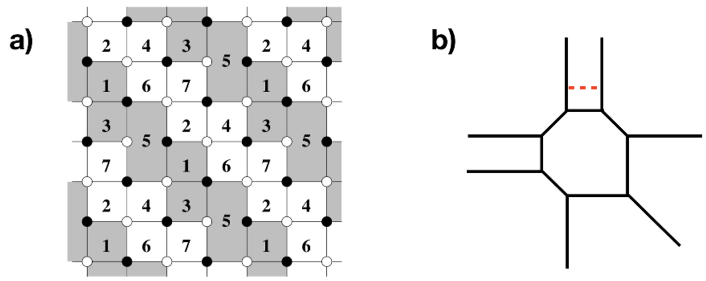


**Figure 2**. (a) Dimer diagram showing an $\mathcal{N}=2$ fractional brane in the $\mathrm{PdP}_4$ theory; (b) Web diagram, displaying the corresponding mobile $\mathbf{P_1}$ as a red discontinuous segment.



in the D3-brane gauge theory. This in fact underlies the way in which the dimer encodes the CY threefold geometry, as the moduli space of a single such D-brane. These D3-branes are referred to as dynamical, or regular (since, for orbifold singularities, they are associated to the regular representation of the orbifold group [45]).

Other rank assignments consistent with the RR tadpole constraints are known as fractional branes. They can be regarded as D5-branes wrapped on 2-cycles (collapsed at the singularity) such that their dual 4-cycle is non-compact. This allows the RR charge carried by the D-branes to escape to infinity. These can always be written as combinations of certain basis of fractional branes, which fall into different classes, as described in [46], as follows:

- The so-called $\mathcal{N}=2$ fractional branes correspond to an overall increase of ranks in a subset of faces bounded by zig-zag paths associated to the same $(p,q)$ 1-cycle in the dimer $\mathbf{T}^2$. They are associated to parallel external legs in the web diagram, or equivalently to curves of $\mathbf{C}^2/\mathbf{Z}_k$ singularities sticking out of the singularity at the origin. The gauge theory on these fractional D3-branes has a flat direction, parametrized by the meson obtained by concatenation of bifundamentals joining the faces bounded by the zig-zag paths in the dimer. The flat direction describes the possibility of moving the fractional D-brane off the origin along the curve of singularities, to become a fractional brane of $\mathbf{C}^2/\mathbf{Z}_k$, namely a D5-brane wrapped on one of the collapsed 2-cycles of the orbifold singularity. The gauge theory on this branch is the $\mathcal{N}=2$ $A_{k-1}$ quiver gauge theory [45], hence the name. An example of $\mathcal{N}=2$ fractional brane is shown in figure 2.

- The so-called deformation branes are associated to complex deformations of the CY threefold singularity. They are associated to splittings of the web diagram into sub-webs in equilibrium. The rank assignment corresponds to an overall increase of ranks in the subset of faces bounded by the splitting. Namely, the homological sum of the zig-zag paths associated to the sub-web removed (in a given complex deformation, the two sub-webs give the same result, due to the condition that the total sum of $(p,q)$ charges for external legs is zero). They correspond to checkerboard pictures on the dimer. The complex deformation of the geometry has a field theory counterpart, in

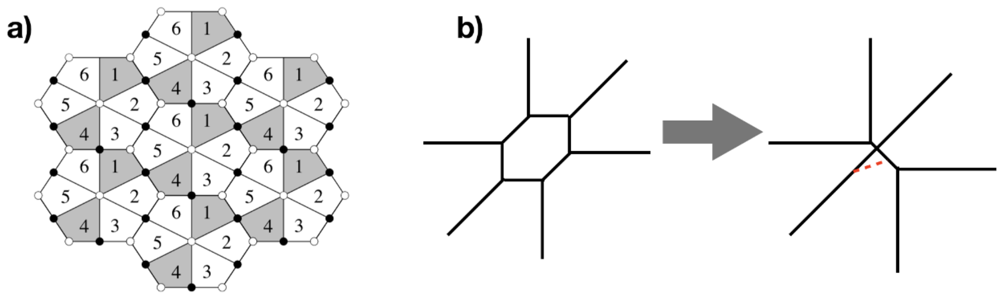


**Figure 3**. (a) Dimer diagram showing a deformation fractional brane in the $dP_3$ theory; (b) Web diagram, and its splitting into subwebs in equilibrium, with the finite size $\mathbf{S}^3$ displayed as a red discontinuous segment suspended between the subwebs.

which the gauge theory on the fractional branes confines and has a complex deformed moduli space. The resulting gauge theories are associated to the two sub-webs [47, 48]. An example of a deformation fractional brane is shown in figure 3.

The gauge theory arising from a set of $N$ regular D3-branes and $M$ (deformation) fractional branes, leads to RG flows with a sequence of Seiberg duality cascades, along which the overall number of D3-branes $N$ is reduced in multiples of $M$, and the number $M$ of D5-branes remains fixed. The gravity dual corresponds to a warped throat supported by RR 3-form fluxes on the 3-cycles associated to the complex deformed singularity, and NSNS flux in the dual (non-compact) 3-cycle. Their combination $G_3 = F_3 - \tau H_3$ is an ISD 3-form of type $(2,1)$, thus preserving supersymmetry [49–51]. The throat is locally similar to $AdS_5 \times \mathbf{X}_5$, but with logarithmic changes in the cosmological constant and the RR 5-form flux along the radial direction.

The simplest example is the conifold, studied exhaustively in [34] both from the viewpoints of field theory and of its gravity dual warped throat. The generalization of duality cascades in gauge theories associated with fractional branes in more general singularities has been studied in [47, 52]. We will consider the gravity dual of deformation branes in general singularities in section 3.1.

- The last class corresponds to the remaining kind of fractional branes. Their corresponding rank assignments on faces have no correspondence with a set of zig-zag paths defining a sub-web in equilibrium. Therefore, there is no geometric complex deformation of the singularity associated to them. Indeed, contrary to deformation fractional branes, their infrared dynamics involves non-abelian gauge dynamics (even for the minimal such fractional brane) and results in the absence of a supersymmetric vacuum (hence they were dubbed DSB branes in [46], see also [53, 54]). On the other hand, similarly to deformation fractional branes, they can trigger duality cascades in the presence of $N$ regular D3-branes, which define some warped throats (albeit with naked singularities in the infrared region) [52]. The discussion of the infrared dynamics, supersymmetry breaking, and its implications for the gravity dual and the deformed AdS-WGC are discussed in section 3.1

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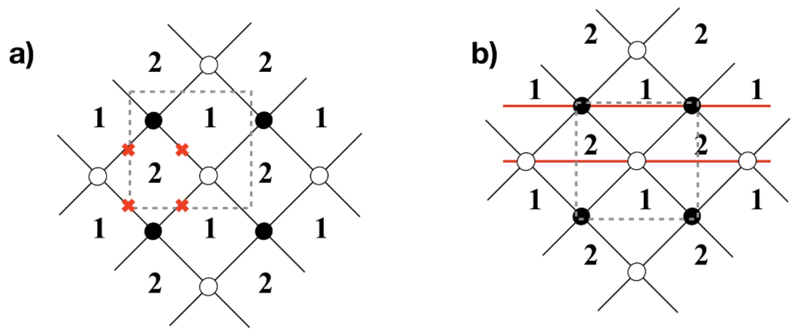


**Figure 4**. Dimer diagrams for orientifolds of the conifold with fixed points (a) or fixed lines (b).



In this paper we will also exploit systems of D-branes at orientifolds of toric singularities. They can be usefully encoded in suitable modifications of dimer diagrams. The general description was provided in [55], and corresponds to modding out the dimer diagram by a $\mathbf{Z}_2$ involution. There are two kinds of orientifold quotients, classified by their fixed sets being lines or points. Two such orientifolds of the conifold theory are shown in figure 4. It is easy to construct other examples, see later and [55].

In the following we will mainly focus on models with orientifold fixed points in the dimer. For this class, the rules are as follows (see [55] for detailed derivations). Each orientifold point carries a $\pm$ sign, with the constraint that the number of orientifold planes with the same sign is even (resp. odd) for dimers with number of nodes given by $4k$ (resp. $4k{+}2$). Orientifold points with charge $+$ (resp $-$) in the middle of a dimer face project down the corresponding gauge factor to $\mathrm{SO}(n_a)$ (resp. $\mathrm{USp}(n_a)$ ). Orientifold points with charge $+$ (resp. $-$) in the middle of a dimer edge project down the corresponding bifundamental onto the two-index symmetic (resp. antisymmetric) representation. Finally, faces and edges not mapped to themselves by the orientifold, combine with their images and descend to $\mathrm{U}(n_a)$ gauge factors and bi-fundamental matter multiplets in the orientifold theory.

## 3 The local AdS-WGC swampland criterion

### 3.1 Derivation

The WGC [6], in its minimal formulation establishes that in any theory including quantum gravity, any U(1) gauge factor should have a super-extremal charged particle, namely $q \geq m$, in natural units. This has been generalized to other $p$-form gauge fields, requiring the existence of the corresponding branes with tensions bounded by their charges, $Q \geq T$, an extension natural in string theory models via T-duality.

The proposal in [7] of a *refined* WGC establishes that the inequality is saturated only for BPS states in supersymmetric theories. This further motivates the *AdS-WGC* statement that theories of quantum gravity do not have stable non-supersymmetric AdS vacua, which are thus in the swampland, rather than the string landscape. The AdS-WGC is largely motivated by a particular (but large) class of AdS backgrounds in string theory, which correspond to flux compactifications arising as near horizon limits of systems of D-branes. A prototypical example is the type IIB $AdS_5 \times \mathbf{S}^5$ solution with $N$ units of RR 5-form flux on the $\mathbf{S}^5$, which arises as the near horizon limit of a system of $N$ D3-branes in flat 10d spacetime [56]. In short, the $T = Q$ condition is crucial in the structure of these vacua, in which the tension creating the spacetime curvature is balanced against the flux sourced by the brane charge in the underlying picture. This proposal is further supported by the study of instabilities of non-supersymmetric AdS vacua due to bubbles of nothing [27]. The AdS-WGC is a powerful statement, which e.g. has subsequently been applied to derive novel constraints on particle physics [24–26, 57].

In this paper we propose a generalization of the conjecture, which we dub the *local AdS-WGC*. It states that certain warped throats backgrounds, which are AdS locally in the radial direction but have a slow variation of the local 5d value of the cosmological constant, are not consistent in quantum gravity, except for supersymmetric cases. The precise formulation will be manifest from the derivation below.

The derivation follows the strategy of [7] for AdS fluxed backgrounds, by taking a near horizon limit of D-brane systems. In our case, we apply the near horizon description to systems of regular and fractional D3-branes at singularities, in particular the toric CY singularities of section 2. We note that the discussion below also applies to throats from $\mathcal{N} = 2$ fractional branes, despite the presence of singularities in the near horizon geometry, if one accounts for the additional fields from the twisted sectors, see 4.3 for extra details.

The backgrounds correspond to the holographic duals of (the UV regime of) gauge theories with cascading RG flows, like the familiar conifold example. The statements below have well-established translations to the holographic dual gauge theory on the D-branes, but we prefer to emphasize the properties of the gravity side.

Consider a system of $N$ regular and $M$ fractional D3-branes at a toric CY singularity with metric,

$$ds^2_{\mathbf{Y}_6} \,=\, dr^2 \,+\, r^2 ds^2_{\mathbf{X}_5} \tag{3.1}$$

The near horizon geometry is a solution of the kind considered in [58] for the conifold and generalized in [51, 52], as a particular class of the supersymmetric warped compactification ansatz in [35, 50],

$$ds^2 \,=\, Z(r)^{-1/2}\, \eta_{\mu\nu}\, dx^\mu\, dx^\nu \,+\, Z(r)^{1/2}\, [\, dr^2 \,+\, r^2 ds^2_{\mathbf{X}_5}\, ]. \tag{3.2}$$

One obtains a warped version of the singular manifold, which can be regarded as the 5d horizon $\mathbf{X}_5$ fibered over the 5d space given by 4d Minkowski space and the radial direction $r$.

There are $M$ units of RR 3-form flux along a non-trivial 3-cycle $\Sigma_3$ (topologically an $\mathbf{S}^3$ or a Lens space) in $\mathbf{X}_5$, and a corresponding NSNS 3-form flux, such that the combination (setting the 10d RR axion to zero for simplicity) $G_3 = F_3 - \frac{i}{g_s} H_3$ is a harmonic (2,1)-form, so that the flux is supersymmetric. This $H_3$ flux can be described as a variation in $r$ of the 5d scalar arising from the axion $\phi$ given by the component of the NSNS 2-form $B_2$ along the harmonic 2-form $\omega_2$ Poincaré dual to $\Sigma_3$ (equivalently, the period of $B_2$ over the 2-cycle $\Sigma_2$ dual to $\Sigma_3$ in $\mathbf{X}_5$), specifically.

$$H_3 \,=\, g_s\, M\, \frac{dr}{r} \,\wedge \omega_2(\Sigma_3) \tag{3.3}$$

The combination of fluxes is a source of the RR 5-form $dF_5 = F_3 \wedge H_3$, such that its flux $N$ over $\mathbf{X}_5$ varies logarithmically as

$$N \sim g_s\, M^2 \ln(r/r_0) \tag{3.4}$$

where $r_0$ is a cutoff distance. The fluxes also backreact on the geometry, via the warp factor, which obeys

$$(\nabla^2{}_{\mathbf{Y}} Z)\,\mathrm{vol}(\mathbf{Y}_6) \;=\; g_s\, F_3 \wedge H_3 \tag{3.5}$$

leading to

$$Z(r) \;=\; \frac{4\pi {g_s}^2}{r^4}\, M^2 \left( \ln\left(\frac{r}{r_0}\right) + 1 \right) \tag{3.6}$$

The whole of $\mathbf{X}_5$ shrinks at $r = 0$, but the $F_5$ flux has disappeared by then, so there is no topological obstruction to the shrinking from this side. However, the 3-cycle $\Sigma_3$ in $\mathbf{X}_5$ also collapses, and it supports the $F_3$ flux, which is constant. This leads to a naked singularity at the tip of the throat.

The 5d part of the above solution describes what we refer to as a *local AdS solution*. It corresponds to a background which locally in $r$ is an $\mathrm{AdS}_5$ background, but whose AdS curvature changes in $r$, as in (3.6). This variation is controlled by that of a 5d scalar, which in the earlier flux throat is $\phi = \int_{\Sigma_2} B_2$, changing from (3.3). In purely 5d terms, the defining property for this scalar is that (from the 10d topological coupling $F_3 \wedge B_2 \wedge F_5$) it has a 5d topological coupling

$$S_{\mathrm{CS}} \;=\; M\, \phi\, F_5 \tag{3.7}$$

This is the 5d version of the topological couplings [59, 60], arising in flux compactifications as described in [61, 62]. Upon integrating out the non-dynamical $F_5$, the resulting potential for $\phi$ controls the local (in $r$) value of the vacuum energy. The background value for this 5d field, following from (3.3) is

$$d\phi \;=\; g_s\, M\, \frac{dr}{r} \tag{3.8}$$

Alternatively, its boundary condition is fixed by the asymptotic behavior

$$\phi \sim M\, \ln(r/r_0) \tag{3.9}$$

The local AdS solution can thus be described as a (in this case, 5d) AdS solution modified by the backreaction of a (5d) scalar $\phi$ with topological coupling to a non-dynamical field strength top-form and obeying (3.8). The coupling to the top-form can be replaced by equivalent dual formulations, e.g. the explicit $r$-dependence of the 5d vacuum energy.

The local AdS backgrounds we have described contain a naked singularity at the origin, which in fact is known to admit a smooth deformation (preserving supersymmetry) in certain singularities, starting from the celebrated conifold example [34] and generalized in [47]. Thus, the local AdS solution should be regarded as defining the asymptotics of certain very general class of warped throats, in principle with or without supersymmetry, and imposing swampland constraints on the possible existence of such throats in quantum gravity. This brings us to the precise formulation of a new swampland conjecture.

> Local AdS-WGC swampland criterion: *in consistent theory of quantum gravity, there are no stable non-supersymmetric solutions with asymptotics given by local AdS backgrounds, as defined above.*

### 3.2 Evidence from deformation and DSB fractional brane systems

Besides the direct derivation in the spirit of the AdS-WGC, we now present additional support for the local AdS-WGC. Although the following results are known in the literature, their re-interpretation in terms of a swampland constraint is new and provides an interesting insight into the structure of the underlying warped throats and supersymmetry breaking, which we further exploit in later sections.

As mentioned in section 2, there is a large class of local AdS backgrounds arising as holographic duals of (the UV regime of) systems of regular and fractional D3-branes at singularities, specifically, fractional branes of the deformation or DSB kinds ($\mathcal{N} = 2$ fractional branes are discussed in section 4.3). We discuss their interplay with the local AdS-WGC in turn.

Toric CY singularities admitting a complex deformation can support deformation branes. The gauge theory on their worldvolumes has an UV RG flow whose holographic dual is given by a supersymmetric local AdS background supported by $M$ units of RR flux on the 3-cycle $\Sigma_3$ associated to the complex deformation. Thus the naked singularity at the origin in the local AdS background can be smoothed out by giving this 3-cycle a finite size. The resulting configuration is a smooth supergravity solution described by a warped version of the deformed CY threefold, preserving supersymmetry, and with asymptotics given by a local AdS background; this is thus in agreement with the local AdS-WGC statement. The field theory counterpart of this deformation process was described in [34, 47].

Toric CY singularities can also support DSB fractional branes which are not associated to complex deformations. Still, the gauge theory on their worldvolume has a UV RG flow whose holographic dual is a supersymmetric local AdS background supported by $M$ units of RR flux on a 3-cycle $\Sigma_3$. The latter, however, cannot be given a finite size while preserving supersymmetry. Naively, one may think that the infrared region is smoothed out to an alternative configuration breaking supersymmetry, either in the form of a supergravity background beyond the warped CY ansatz (in the spirit of e.g. [63] in the supersymmetric case), or perhaps involving stringy ingredients, such us explicit sources from branes or other singular objects. However, if such re-stabilization would indeed be possible, it would contradict our local AdS-WGC statemetnt.

The actual answer is that the warped throats created by DSB fractional branes actually do not admit any such stable non-supersymmetric smooth version, in agreement with the local AdS-WGC conjecture. This has actually been already studied in the lileraturе, from the gauge theory side. The complex cone over $\mathrm{dP}_1$ is the prototypical case of a duality cascade triggered by a DSB brane, and the lack of a supersymmetric vacuum in this $\mathrm{dP}_1$

theory was discussed in [46, 53, 54]. This however does not imply the existence of a non-supersymmetric stable vacuum, rather [46] already established that the theory shows a runaway behaviour, as follows. By keeping the U(1) factors in the description of the gauge theory, the system has a supersymmetry breaking minimum only if the Fayet-Iliopoulos terms are kept fixed, due to the constraints from the D-term potential. However, the FI terms are actually field dependent, and are controlled by the vevs of closed string twisted sectors. When they are taken as dynamical, the D-term potential can relax in new directions leading to the runaway. The same physics was reinterpreted in [64] as a baryonic runaway direction in the gauge theory with the (massive) U(1)'s integrated out. In either of these descriptions, the runaway direction corresponds to a dynamical blow-up of the singularity, since FI terms, or baryonic vevs, are related to blow-up modes. The fractional brane remains as a D5-brane wrapped on a 2-cycle in the $dP_1$ exceptional divisor.

The gravity dual of this runaway has not been determined in the literature, but its structure should correspond to a time-dependent solution, in which the geometry is resolved by growing a finite size $dP_1$ itself, with $M$ explicit D5-branes wrapped on one of its 2-cycles. The latter plays the role of sourcing the $M$ units of RR 3-form, peeling it off the 3-cycle and allowing it to shrink to zero size at the bottom of the (disappearing) throat.

It is interesting to point out that this system provides an interesting link between two seemingly unconnected swampland criteria. On one hand, the statement that in theories of quantum gravity all FI terms should be field-dependent, and thus dynamical [65]; on the other hand, our newly proposed local AdS-WGC. We expect other connections of the local AdS-WGC constraint with other swampland criteria.

We thus see that the class of throats obtained from the different kinds of fractional branes provide illustrative examples of the local AdS-WGC constraint. In later sections we illustrate the power of this conjecture to exclude candidates to non-supersymmetric throats proposed in the literature.

### 3.3 Meta-stable throats

It is important to emphasize that the present form of the local AdS-WGC still allows for certain forms of non-supersymmetric warped throats. For instance,

- The conjecture poses no conflict so far with the existence of supersymmetry breaking meta-stable throats with local AdS asymptotics. For instance the systems of anti-D3-branes at the bottom of conifold-like warped throats (i.e. created by deformation fractional branes), extensively used since [30], are in principle allowed.[1] See also [67], where non-supersymmetric orbifolds are considered and shown to be unstable through nucleation of bubbles of nothing. In contrast with the AdS-WGC, in local AdS throats there is no isometry in the radial direction introducing an infinite volume factor multiplying the decay probability, rather instabilities tend to nucleate near the tip of the throat. Hence, a finite and potentially small decay amplitude is in principle feasible, although this point deserves further study.[2]

[1] For discussions on asymptotics and stability of these throats, there is a long-standing debate, see e.g. [66] for a recent work, and references therein.

[2] We thank M. Montero for raising this point.

- Similarly for the nilpotent Goldstino scenario realized in terms of a single anti-D3-brane on top of an O3-plane [37], for which the stability remarks of [68] specially apply.

- Finally, global compactifications including warped throats may contain ingredients in the CY bulk which modify non-trivially the boundary conditions in the UV region of the throat, thus changing its asymptotics, and allowing it to evade the local AdS-WGC constraint. For instance, this may well be the case if one introduces euclidean D3-brane instantons on 4-cycles intersecting the underlying DSB D3-brane system (thus, stretching in the radial direction of the throat) to stop their runaway, as proposed in [69] (see also [70, 71] for related tools). Also, if one includes D7-branes introducing new flavours in DSB D-brane systems, to allow for metastable supersymmetry breaking vacua [72, 73] in the ISS spirit [74]. For a recent discussion of orientifolded throats, see [75].

In the following discussions, we consider several large classes of non-supersymmetric warped throats, and reconcile them with the local AdS-WGC by looking for decay channels. Whether these decay channels render the configurations unstable or just meta-stable is not constrained by the conjecture in its present form, hence we loosely refer to them as instabilities of the configuration, even in cases where they could host meta-stable backgrounds.

## 4 Warped throats with dynamical supersymmetry breaking

In the previous discussion, the system of D3-branes breaking supersymmetry had a fairly manifest runaway behaviour. There are however other systems of D3-branes at singularities which trigger genuine dynamical supersymmetry breaking, rather than runaway. In this section we explore the proposal of embedding such systems in warped throats [38], and how they face the local AdS-WGC.

Again, there are systematic tools for the construction of such theories in terms of D3-branes at toric singularities (possibly in the presence of orientifold quotients), producing $\mathcal{N} = 1$ supersymmetric gauge theories with supersymmetry broken only by non-perturbative dynamics. As explained in [38], dimer diagram tools moreover allow to realize them as the theories arising in the infrared of duality cascades of systems of further (deformation) fractional D3-branes at singularities. The gravity dual description of these configurations would correspond to a locally AdS supersymmetric warped throat supported by 3-form fluxes on a 3-cycle associated to a complex deformation, and at whose tip we have the supersymmetry breaking D-brane sector.

If stable, such configurations would lead to a supersymmetry breaking warped throat violating the local AdS-WGC. In this section, we provide a detailed analysis of an illustrative example and show that the configurations are actually unstable. Concretely, although the DSB D3-brane system is consistent in isolation, its embedding into a warped throat contains an instability against bubble nucleation of certain D-brane domain walls. The latter are however more involved than just D3-brane domain walls peeling off the 5-form flux, and provide a novel kind of decay for warped throats. The system also relates to warped throats from (orientifolds of) $\mathcal{N} = 2$ fractional branes, which we discuss as well.

### 4.1 The DSB D-brane system

To make the discussion concrete, we consider an illustrative explicit example given by the DSB theory introduced in [55]. We start with the $\mathbf{C}^3/\mathbf{Z}'_6$ geometry, where the $\mathbf{Z}'_6$ generator $\theta$ acts as

$$\theta : \quad z_i \to e^{2\pi i v_i} z_i \tag{4.1}$$

with $v = (1, 2, -3)/6$. We consider the quotient by an orientifold group $(1 + \theta + \ldots + \theta^5)(1 + \Omega\alpha(-1)^{F_L})$, where $\alpha$ acts as

$$(z_1, z_2, z_3) \to (e^{2i\pi/12}, e^{4i\pi/12}, e^{-6i\pi/12}). \tag{4.2}$$

Equivalently, we may introduce invariant coordinates

$$x = z_1^6,\ y = z_2^3,\ z = z_3^2. \tag{4.3}$$

in terms of which the orientifold corresponds to the geometric action

$$x \to -x,\ y \to -y,\ z \to -z. \tag{4.4}$$

We consider sets of D3-branes at this orientifold singularities. The resulting gauge theory can be determined from its dimer diagram, shown in figure 5. As discussed in the introduction, there are different choices of orientifold signs, which lead to different results of $SO$ or $Sp$ gauge factors and of $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ / $\square\!\square$ matter fields. For our choice of interest, corresponding to orientifold signs $(a, b, c, d) = (+ + --)$, the resulting gauge theory is

$$\begin{aligned}&\mathrm{SO}(n_0) \times \mathrm{U}(n_1) \times \mathrm{U}(n_2) \times \mathrm{USp}(n_3)\\ &(\,\square_0,\ \overline{\square}_1) + (\,\square_1,\ \overline{\square}_2) + (\,\square_2,\ \overline{\square}_3)\\ &\qquad +(\,\square_0,\ \overline{\square}_2) + (\,\square_1,\ \square_3) + \begin{smallmatrix}\square\\\square\end{smallmatrix}_2 + \overline{\square\square}_1\\ &\qquad +[\,(\,\square_0,\ \square_3) + (\,\square_1,\ \square_2) + (\,\overline{\square}_1,\ \overline{\square}_2)\,].\end{aligned} \tag{4.5}$$

As is familiar [44], cancellation of non-abelian gauge anomalies is equivalent to the requirement of cancellation of compact RR tadpoles, which leads to

$$-n_0 + n_2 + n_3 - n_1 - 4 = 0. \tag{4.6}$$

We consider the solution $n_1 = n_3 = 0$, $n_0 = k$, $n_2 = k + 4$, which yields the gauge group $\mathrm{SO}(k) \times \mathrm{U}(k + 4)$ with matter $(\,\square\,,\ \overline{\square}\,) + (1,\ \begin{smallmatrix}\square\\\square\end{smallmatrix}\,)$. The U(1) gauge factor is anomalous, with anomaly canceled by Green-Schwarz couplings, which make it massive and remove it from the massless spectrum. Focusing on $k = 1$, we have an SU(5) theory with chiral multiplets in the $10 + \overline{5}$ and no superpotential. This theory has been argued to show dynamical supersymmetry breaking [76, 77]. Since there is no moduli space, there is an

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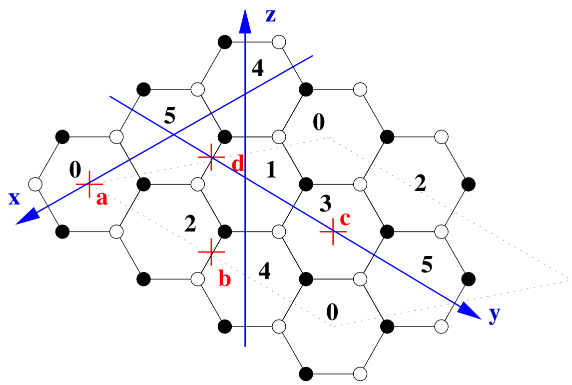


**Figure 5**. Dimer diagram for an orientifold of the $\mathbf{C}^3/\mathbf{Z}_6'$ theory, from [55].

isolated non-supersymmetric vacuum, which however lies at strong coupling and is non-calculable. Nevertheless, the vacuum energy should scale with the strong dynamics scale $\Lambda$ as

$$V \sim |\Lambda|^4 \tag{4.7}$$

This provides a consistent configuration displaying supersymmetry breaking localized at the tip of the corresponding singularity.

It is natural to consider its embedding into warped throats, as a possible source of tunable uplifting energy to be used in attempts to build de Sitter string vacua. In the following we argue this not to be possible.

### 4.2 The DSB AdS throat

As a warm-up towards such throats, we may consider the simple addition of a large number of dynamical D3-branes to the earlier system, and take the near horizon limit. This corresponds to increasing the rank of all gauge factors in 4.5 by the same amount, namely

$$n_0 = N+1 \quad , \quad n_1 = n_3 = N \quad , \quad n_2 = N+5 \tag{4.8}$$

For consistency with the USp factor, $N$ should be taken even, but is otherwise unconstrained.

Since the DSB D-brane system (including the orientifold and the $k=1$ SU(5) D-brane set) is subleading in $1/N$, standard arguments show that in the large $N$ limit we obtain a gravity dual given by $\mathrm{AdS}_5 \times \mathbf{X}$, where $\mathbf{X}$ corresponds to an orientifold of the $\mathbf{Z}_6'$ orbifold of $\mathbf{S}^5$. Note that since the $\mathbf{Z}_6'$ orbifold contains fixed complex planes in $\mathbf{C}^3$, there are fixed circles in the action on $\mathbf{S}^5$. This leads to circles of $\mathbf{C}^2/\mathbf{Z}_2$ and $\mathbf{C}^2/\mathbf{Z}_3$ singularities, which are however well understood [78, 79]. The orientifold action (4.4) has instead the origin as only fixed point, hence it is freely acting on $\mathbf{S}^5$.

At leading order in $1/N$, which corresponds to the classical gravity level, we have a supersymmetric AdS configuration, associated to the near horizon limit of a D-brane system saturating the WGC bound, hence satisfying the AdS-WGC. In the exact configuration, however, the DSB D-brane sector breaks supersymmetry, and implies that at the quantum level the gravitational background becomes non-supersymmetric, hence according to the AdS-WGC, the system should exhibit an instability.

Naively, it would seem that the instability corresponds, as suggested in [7], to the emission of shells of D3-branes peeling off the 5-form flux background from the AdS solution. This would correspond, in the underlying picture of D-branes at singularities, to the DSB D-brane system repelling dynamical D3-brane off the origin towards generic points in the transverse space. This actually turns out to be incorrect, as can be shown using the field theory description, using standard supersymmetric field theory arguments. Expelling the dynamical D3-branes corresponds to the Higgsing down the gauge theory with the rank assignment (4.8) to the original $N = 0$ SU(5) theory, by giving vevs to suitable mesonic operators. To make the point, it suffices to turn on a vev for the gauge invariant operator involving fields in the first line in (4.5)

$$\langle\, (\,\square_0,\, \overline{\square}_1) \cdot (\,\square_1,\, \overline{\square}_2) \cdot (\,\square_2,\, \overline{\square}_3)\,\rangle \equiv \Phi^3 \tag{4.9}$$

Here $\Phi$ is the dimension 1 order parameter for this vev. The superpotential involves only triples of fields from the three different lines in (4.5), hence it is an F-flat direction. As follows from the D-brane picture, there are more general choices, allowing for three independent vevs — for similar mesonic operators built from fields in the three different lines in (4.5) — for each of the dynamical D3-branes. But for our present purposes it suffices to consider only this overall position vev $\Phi$.

From the viewpoint of the infrared SU(5) theory this corresponds to a Higgsing of the UV SU($N+5$) theory by the $N$ flavours acquiring vevs involved in $\Phi$. Denoting $\Lambda$, and $\Lambda_{\rm UV}$ the dynamical scales of the SU(5) and SU($N+5$) theories, the potential for $\Phi$ would follow from (4.7) from the implicit dependence of the IR scale $\Lambda$ on $\Phi$. However, taking the SU(5) theory, with a $10+\overline{5}$ matter content, and the UV SU($N+5$) theory, with matter content $(3N+1)\,\overline{\square} + 2N\,\square + \begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}$, the matching relation is just $\Lambda = \Lambda_{\rm UV}$, with no dependence of $\Phi$. This implies that the DSB D-brane systems does not exert forces on dynamical D3-branes, which are thus not repelled from the origin. The non-supersymmetric AdS configuration is not unstable towards the emission of such D3-brane shells peeling off the 5-form flux.

Actually, the contradiction with the AdS-WGC statement is avoided by a novel mechanism, related to a different kind of instability, which we explain as follows. Let us return to the picture of D3-branes at the orientifold of the $\mathbf{C}^3/\mathbf{Z}_6'$ singularity, i.e. the rank assignment (4.8). The $\mathbf{Z}_6'$ quotient does not actually define an isolated singularity; indeed, the generator (4.1) has the origin as only fixed point, but $\theta^3$ leaves invariant the complex plane parametrized by $z_2$, and $\theta^2$ leaves $z_3$ invariant. This implies that there is a complex plane (along $z_2$) of $\mathbf{C}^2/\mathbf{Z}_2$ singularities, and a complex plane (along $z_3$) of $\mathbf{C}^2/\mathbf{Z}_3$ singularities. In the field theory, there are flat directions corresponding to splitting some of the dynamical D3-branes into fractional D3-branes (of the $\mathcal{N} = 2$ kind, i.e. D5-branes wrapped on the collapsed cycles of the $\mathbf{C}^2/\mathbf{Z}_n$) which can slide off the origin along the corresponding complex plane. Once the non-perturbative supersymmetry breaking kicks in, these flat directions can turn into runaway, providing an instability, bringing back agreement with the AdS-WGC.

The existence of this instability can again be analyzed in terms of the field theory, by Higgsing and scale matching. Consider for concreteness the splitting of dynamical D3-branes into fractional D3-branes of the $\mathbf{C}^2/\mathbf{Z}_2$ singularity associated to $\theta^3$, and moving the latter along $z_2$. A similar analysis could be performed using the fractional branes of the $\mathbf{C}^2/\mathbf{Z}_3$ curve of singularities. Motion in $z_2$ corresponds to mesonic vevs for fields in the second line in (4.5). Denoting the fields $(\,\square_0,\ \overline{\square}_2)$ and $\begin{smallmatrix}\square\\\square\end{smallmatrix}_2$ by ${Q_A}^i$ and $A_{ij}$, respectively, and $(\,\square_1,\ \square_3)$, $\overline{\square\square}_1$ by $Q'_{j',B'}$, $S^{i'j'}$, respectively, the vevs for the two kinds of fractional branes have the structure

$$v \,=\, \langle\, \epsilon_{AB} {Q_A}^i {Q_B}^j A_{ij}\,\rangle \quad ; \quad v' \,=\, \langle\, S^{i'j'} Q_{i'A} Q_{j'B} \delta^{AB}\,\rangle \tag{4.10}$$

For simplicity we have assumed all fractional D3-branes of the same kind to be located at the same position. The fact that the two different fractional branes are related to vevs of fields Higgsing the combinations of gauge factors (0,2) and (1,3), respectively, is manifest in the dimer diagram in figure 5, where the above combinations correspond to two sets of faces forming two different strips in the $z_2$ mesonic direction.

Let us compute the scale matching. Considering for instance $v \gg v'$ (eventually shown to be the realistic regime), the Higgsing pattern is

$$\begin{aligned} &\mathrm{SO}(N+1)\times\mathrm{SU}(N)\times\mathrm{SU}(N+5)\times\mathrm{USp}(N) \xrightarrow{\ v\ } \\ &\xrightarrow{\ v\ } \mathrm{SO}(1)\times\mathrm{SU}(N)\times\mathrm{SU}(5)\times\mathrm{USp}(N) \xrightarrow{\ v'\ } \mathrm{SO}(1)\times\mathrm{SU}(5) \end{aligned} \tag{4.11}$$

where the SO(1) factor is kept for bookkeeping purposes. In the first step, the $\mathrm{SU}(N+5)$ is Higgsed down to SU(5). In the second step, the SU(5) theory maintains the number of colors, but $2N$ flavours become massive. The scale matching between the IR and UV scales $\Lambda$, $\Lambda_{\rm UV}$ is

$$\Lambda^{13} \,=\, \Lambda_{\rm UV}^{13}\, v'^{2N}\, v^{-2N} \tag{4.12}$$

Replacing in (4.7), the vev $v$ runs away to infinity, while the vev $v'$ is attracted to zero.

Note that, although the two kinds of fractional branes have similar features in isolated $\mathbf{C}^2/\mathbf{Z}_2$, they have a very different behavior in the presence of the orientifold action. This is in fact manifest already in the orientifold projection on the gauge group and matter content.

The resulting configuration is given by a set of D-branes describing the $\mathrm{SO}(1)\times\mathrm{SU}(N)\times\mathrm{SU}(5)\times\mathrm{USp}(N)$ gauge theory. The SU(5) gauge factor still has the antisymmetric matter, but it has extra vector-like flavours, and the theory has supersymmetric vacua [77]. This fits nicely with the vacuum energy from (4.7), (4.12) going to zero as $v' \to 0$. Note that the final configuration can be described as a quotient (a $\mathbf{Z}_3$ orbifold of an orientifold of) a set of $N$ $\mathcal{N}=2$ fractional branes at $\mathbf{C}^2/\mathbf{Z}_2$. This configuration has a supersymmetric gravity dual given by a locally AdS throat of the kind studied in [80, 81]. These can be regarded as $\mathcal{N}=2$ versions of the $\mathcal{N}=1$ Klebanov-Strassler throats, with the singularity at the origin resolved by a stringy phenomenon, the so-called enhançon configuration [82]. The fact that the final end point is a supersymmetric local AdS background avoids conflicts with the local AdS-WGC.

In the gravity picture of the initial configuration, the instability of the non-susy AdS corresponds to the nucleation of bubbles defined by suitable fractional D3-branes, namely D5-branes wrapped on a collapsed $\mathbf{P_1}$ on the $\mathbf{S}^1$ of $\mathbf{C}^2/\mathbf{Z}_2$ singularities, and with spatial topology $\mathbf{S}^3$ in the non-compact dimensions, expanding outwards with time. In the interior, we are left with a supersymmetric locally AdS throat induced by the $N$ fractional branes stabilized at the origin, and with the singularity at its tip smoothed out presumably by an enhançon configuration. In contrast with other examples in the literature, this is neither a bubble of nothing nor a bubble removing the 5-form flux completely. It thus corresponds to a novel decay channel for non-supersymmetric warped throats.

The $\mathbf{C}^3/\mathbf{Z}_6'$ orientifold singularity can be embedded in a locally AdS warped throat associated to a complex deformation, as discussed in section 4. In this setup, supersymmetry breaking on the infrared gauge theory would lead to contradiction with our proposed local AdS-WGC. However, our above analysis of the AdS case shows that the locally AdS throat is already unstable due to D5-brane bubble nucleation (on top of other possible decay channels related to the deformation fractional branes). Hence, the conflict with the local AdS-WGC is solved by the decay channel already solving the potential conflict with the AdS-WGC.

### 4.3 Non-supersymmetric warped throats for $\mathcal{N} = 2$ fractional branes

In this section we exploit the previous configuration to obtain a non-trivial example of non-supersymmetric warped throat induced by $\mathcal{N} = 2$ fractional branes. The discussion is straightforward and the arguments should be familiar by now.

Consider the previous orientifold singularity, with D-branes corresponding to the rank assignment

$$n_0 = M + 1 \quad , \quad n_1 = n_3 = 0 \quad , \quad n_2 = M + 5 \tag{4.13}$$

with $M$ even, for consistency of the (hidden) $USp$ factor. This leads to a gauge theory with group $\mathrm{SO}(M+1) \times \mathrm{SU}(M+5)$, with matter $(\,\square\,,\,\overline{\square}\,) + (1,\,\begin{smallmatrix}\square\\\square\end{smallmatrix}\,)$. In the limit of large $M$, at leading order we have a gravity dual given by a quotient of the supersymmetric $\mathcal{N} = 2$ warped throats in [80, 81]. The configuration is of the local AdS kind, hence the local AdS-WGC constraints should apply.

On the other hand, the gauge theory does not have a supersymmetric vacuum. The $\mathrm{SU}(N)$ theory with odd $N$, antisymmetric matter, and no extra flavours, breaks supersymmetry, as shown in [76, 77, 83]. Actually, this reference argued for an isolated supersymmetry breaking vacuum for the theory with Yukawa couplings, which remove the classical flat direction. In our present example, such superpotential couplings are absent, and the classical flat direction can turn into runaway ones. This is precisely the conclusion from matching of scales, as in the previous section, which we skip.

This means that, on the gravity side, the classical background has a decay channel given by nucleation of bubbles of fractional branes, exactly as in the previous section. In this case, however, since there are no fractional branes of the supersymmetry preserving kind, the bubbles completely peel off the 5-form flux background of the configuration leading to a complete decay of the local AdS throat.

This example thus provides an explicit example of the application of the local AdS-WGC constraint to non-supersymmetric warped throats induced by $\mathcal{N} = 2$ fractional branes.

## 5 Supersymmetry breaking orientifolds in warped throats

In the previous sections, we have focused on warped throats whose underlying D-brane configuration is supersymmetric in perturbation theory, with supersymmetry breaking arising from non-perturbative strong dynamics effects. It is interesting to check the behavior of warped throats with more dramatic supersymmetry breaking patterns. In this section, we explore a class of warped throats, where supersymmetry breaking is induced by orientifold planes not preserving the supersymmetry preserved by the CY geometry and the 3-form fluxes. In fact, they correspond to the CPT conjugates of the familiar supersymmetric orientifold planes, so we refer to them as anti-orientifold planes. Systems of anti-orientifold planes in the presence of D-branes are identical to systems of anti-D-branes in the presence of orientifold planes, which have been considered in many non-supersymmetric string constructions, pioneered in [84–89].

### 5.1 Non-supersymmetric throats from anti-O3-planes

We focus on anti-O3-planes in the presence of a large number $N$ of D3-branes, possibly at singularities and with extra $M$ fractional branes. In the underlying D-brane construction, they lead to an explictly non-supersymmetric spectrum, which can be easily determined using open string techniques and (non-supersymmetric projections of) dimer diagrams. For $M = 0$, the systems of anti-O3-planes with $N$ D3-branes behave as "supersymmetric" and conformal in the leading large $N$ approximation, in the sense that the effects of orientifold planes (noticed via crosscaps) are subleading in the large $N$ limit. This implies that the gravity dual description corresponds to AdS backgrounds which behave as supersymmetric in the classical supergravity approximation, but have supersymmetry breaking effects at 1-loop. Similarly, in systems in the presence of $M$ additional deformation branes, we obtain locally AdS warped throats which are supersymmetric in the leading approximation, but break supersymmetry at the 1-loop order. These AdS and locally AdS configurations thus correspond to classically stable backgrounds, which, if stable in the full theory, would violate the AdS-WGC or the local AdS-WGC, respectively. Our purpose is thus to test the stability of these configurations, providing a check of these conjectures at the quantum level.

Concrete examples are easy to build. For instance, [37] provided tools to embed a single (anti-)O3-plane at the bottom of a warped throat with 3-form fluxes, for instance based on the $xy = z^3w^3$ singularity, a $\mathbf{Z}_3$ orbifold of the conifold. The deformed conifold itself $xy - zw = t^2$ also admits an involution $(x, y, z, w) \to (y, x, -z, -w)$ leading to O3-planes (in fact, two, located at $z = w = 0, x = y = \pm i\,t$) [90]. Considering any of these geometries, we may just replace the O3-planes by anti-O3-planes and obtain explicit locally AdS warped throats with supersymmetry broken by anti-orientifold planes.

### 5.2 Dynamics of D3-branes and anti-O3-planes

It is useful to start considering anti-O3-planes in flat space, in the presence of $N$ D3-branes. In the large $N$ limit, the near horizon limit leads to gravity duals of the form $\mathrm{AdS}_5 \times \mathbf{RP}_5$, which behave as supersymmetric at leading order and feel the absence of supersymmetry at order $1/N$. The configuration is the CPT symmetric of O3-planes in the presence of anti-D3-branes (denoted by $\overline{\mathrm{D3}}$'s), which was studied in [87] following the analysis in [91] for the supersymmetric O3-D3 system. We now revisit the main points, in anti-O3-plane language.

An anti-O3-plane is a fixed plane of the $\mathbf{Z}_2$ orientifold action on $\mathbf{R}^6$, preserving the 16 supersymmetries broken by D3-branes. There are four kinds of anti-O3-planes, classified according to the (discretized) values 0, $\frac{1}{2}$ for the NSNS and RR 2-form backgrounds on the $\mathbf{RP}_2$ (twisted) 2-cycles on the $\mathbf{RP}_5 = \mathbf{S}^5/\mathbf{Z}_2$ surrounding the origin in $\mathbf{R}^6$. In short, comparing with [91], the tension of an anti-O3-plane equals that of the corresponding O3-plane, while they have opposite RR charge. The tensions and charges, measured in D3-brane units, for the anti-O3-planes are in the following table.

| D-brane description | $(\theta_{NS}, \theta_R)$ | Tension | RR charge |
|---|---|---|---|
| anti-($\mathrm{O3}^-$) | $(0,0)$ | -1/2 | +1/2 |
| anti-($\mathrm{O3}^-$) + 1 $\overline{\mathrm{D3}}$ | $(0,1/2)$ | +1/2 | $-1/2$ |
| anti-$\mathrm{O3}^+$ | $(1/2,0)$ | +1/2 | $-1/2$ |
| anti-$\widetilde{\mathrm{O3}}^+$ | $(1/2,1/2)$ | + 1/2 | $-1/2$ |

Just like for O3-planes, the $\mathrm{O3}^-$ is a singlet under the type IIB $\mathrm{SL}(2,\mathbf{Z})$ and the three remaining ones transform into each other under it.

The stability of the throats built out using these anti-O3-plane can be heuristically understood by considering the dynamics of D3-branes in the presence of these anti-O3-planes. Namely, we can consider the previous anti-O3-planes with a $N$ D3-branes on top (as counted in the double cover), and study the stability properties of the system.

The corresponding analysis can in fact be borrowed from [87] (in its CPT conjugate version). It is straightforward to obtain the spectrum of the non-supersymmetric gauge theories on D3-branes in the presence of the different anti-O3-planes. The stability properties of the system can be assessed from the open string perspective, by the computation of the Coleman-Weinberg potential. We instead focus on the dynamics in the dual closed string channel, by comparing the interaction between D3-branes and anti-O3-planes due to exchange in the NSNS and RR channels. We consider the different cases in turn:

- Consider $N = 2p$ D3-branes in the presence of the anti-($\mathrm{O3}^-$). They have opposite sign tensions and equal sign RR charges, hence the gravitational and Coulomb interactions are both repulsive. Thus, D3-branes are expelled away from the anti-($\mathrm{O3}^-$) and the configuration is unstable.
- Take $N = 2p$ D3-branes in the presence of the anti-($\mathrm{O3}^-$) + 1 $\overline{\mathrm{D3}}$. The D3-branes are attracted to the origin, but when they reach below sub-stringy distances, a tachyon

arises from open strings between the stuck $\overline{\text{D3}}$- and the dynamical D3-branes. The result is a configuration of the anti-(O3$^-$) with one stuck D3-brane at the origin, and $(2p-2)$ dynamical D3-branes. The system at the origin has tension $+1/2$ and charge $+3/2$, so the Coulomb repulsion overcomes the gravitational attraction and D3-branes are repelled. The result is a (CPT conjugate) of the nilpotent Goldstino configuration [37].

- Consider $N = 2p$ D3-branes in the presence of the anti-(O3$^+$). The gravitational and Coulomb interactions are both attractive, so the D3-branes are driven to the origin. Contrary to the previous case, however, there is no obvious annihilation between the anti-(O3$^+$) and the D3-branes. This would suggest that the non-supersymmetric $\text{AdS}_5 \times \mathbf{RP}_5$ gravity dual is stable, in conflict with the AdS-WGC. Happily, as we will discuss later on, a non-perturbative instability will come to the rescue.

- For $N = 2p$ D3-branes in the presence of the anti-($\widetilde{\text{O3}}^+$) we have a similar situation. The D3-branes are driven to the origin, and no obvious decay channel seems to be available. This perturbatively stable configuration is however again rendered unstable by a non-perturbative process described later on, thus solving the potential conflict with the AdS-WGC and the local AdS-WGC constraints.

### 5.3 Instabilities in throats with anti-O3-planes

The large $N$ limit of the above configurations of D3-branes on top of anti-O3-planes leads to near horizon geometries classically given by $\text{AdS}_5 \times \mathbf{RP}_5$, with $N$ units of RR 5-form flux (as counted in the covering space) and the corresponding discrete NSNS and RR 2-form backgrounds on $\mathbf{RP}_2 \subset \mathbf{RP}_5$. Absence of supersymmetry is only detectable at the 1-loop (i.e. $1/N$ order), namely via string diagrams involving crosscaps and thus noticing the underlying non-supersymmetric orientifold. Thus, the AdS-WGC condition implies such AdS backgrounds should have instabilities.

The same statement applies in more general local AdS warped throats with anti-O3-planes. For any local AdS warped throat admitting a supersymmetric orientifold involution introducing O3-planes, it is possible to consider the non-supersymmetric version obtained by the introduction of any of the different anti-O3-planes. The resulting gravitational background remains the same at the level of classical supergravity, but subleading corrections encode the breaking of supersymmetry. Thus, the local AdS-WGC conditions imply such local AdS backgrounds should be unstable.

We now analyze the instabilities in these AdS backgrounds, and the same conclusions clearly apply to local AdS configurations. The analysis follows the discussion in the previous section.

- In the case of the anti-(O3$^-$) orientifold projection, the repulsion exerted by the anti-O3-plane on D3-branes translates into a decay channel of the corresponding non-supersymmetric $\text{AdS}_5 \times \mathbf{RP}_5$ background, by nucleation of D3-brane bubbles, which discharge the $N$ units of RR 5-form flux, much along the lines suggested in [7].

- In the case of the anti-($O3^-$) with an extra anti-D3-brane, the decay channel of the corresponding non-supersymmetric $\mathrm{AdS}_5 \times \mathbf{RP}_5$ background is identical to the previous one, since the two configuration simply differ in the value mod 2 of the RR 5-form flux $N$. Notice that the decay does not change the values of the NSNS and RR 2-form backgrounds, since the anti-($O3^-$) with either the initial stuck anti-D3-brane or the final stuck D3-brane, both have vanishing NSNS background and non-trivial RR 2-form background.

- In the case of the anti-($O3^+$) projection, the flat space configuration seems stable. However, the S-dual of the anti-($O3^+$) is given by the configuration of an anti-($O3^-$) + 1 $\overline{\mathrm{D3}}$ of the previous paragraph. This suggests that the anti-($O3^+$) can turn into an anti-($O3^-$) via strong coupling processes. Indeed, notice that if one considers an NS5-brane (whose core is inherently non-perturbative) stretching along three of the anti-O3 directions and three directions transverse to it, the NS5-brane splits the anti-O3 in two halves, which actually have opposite signs for the orientifold plane charge, with one extra half anti-D3-brane on top of the anti-($O3^-$) half to provide a continuous O3-plane charge across the NS5-brane (see [92] for a review including such brane constructions). This allows to nucleate holes in the anti-($O3^+$), in whose interior the stuck $\overline{\mathrm{D3}}$ on the anti-($O3^-$) can annihilate against one of the D3-branes around it, leading to repulsion of the remaining D3-branes, and thus, to instability. This suggests that, in the $\mathrm{AdS}_5 \times \mathbf{RP}_5$ gravity dual language, there is a decay channel via the nucleation of bubbles bounded by a domain wall given by an NS5-brane wrapped on a maximal $\mathbf{RP}_2$. From the analysis of topological constraints on wrapped branes in [91] (derived in the supersymmetric setup, but valid in general), this is indeed allowed. The NS5-brane may moreover carry arbitrarily large D3-brane charge, thus discharging dynamically the RR 5-form flux and rendering the AdS unstable.

- Similar conclusions hold in the case of the anti-($\widetilde{O3}^+$) projection, where now the required domain wall involves a bound state of one NS5- and one D5-brane (aka a (1,1)-fivebrane) wrapped on $\mathbf{RP}_2 \subset \mathbf{RP}_5$, thus changing both the NSNS and RR 2-form backgrounds. The fivebrane can carry D3-brane charge, so it can peel off the RR 5-form flux of the AdS compactification triggering its instability.

The instabilities of the above non-supersymmetric orientifolds of AdS backgrounds generalize straightforwardly to non-supersymmetric orientifolds of local AdS warped throats. Hence, in this class of examples, the local AdS-WGC is closely related to the ordinary AdS-WGC constraint.

## 6 Discussion

In this paper we have proposed a new swampland conjecture forbidding stable non-supersymmetric locally AdS warped throats. This *local AdS-WGC* statement generalizes the analogous statement for stable non-supersymmetric AdS vacua. We have illustrated its application, which allows to reinterpret several known results about warped throats from fractional branes, and to derive new results on the (in)stability of large classes of non-supersymmetric throats, with supersymmetry breaking triggered by strong dynamics in infrared D-brane sectors, or by the presence of stringy sources like anti-O3-planes.

Although the local AdS-WGC forbids stable non-supersymmetric throats, it has no direct bearing on meta-stable non-supersymmetric throats. In contrast with the AdS-WGC, there is no isometry in the radial direction introducing an infinite volume factor multiplying the decay probability, so a finite and potentially small decay amplitude is in principle feasible. The question of whether swampland criteria can impose further restrictions on the meta-stable throats used in dS uplifts is a very interesting one, to which we plan to return in the future.

Several of the instabilities of the non-supersymmetric throats we have discussed are of the runaway kind. In actual 4d compactifications, this corresponds to shortening the throat, thus moderating the hierarchies between the bulk and the throat. Hence, even if the dynamics of the global compactification eventually stabilizes the runaway and renders such configurations more stable, there may remain a question on the tunability of scale hierarchies in the final states. The possibility that swampland criteria directly constrain such hierarchies is a tantalizing direction we hope to explore in the future.

We have made some interesting progress, and provided yet another hint that the body of knowledge on swampland criteria on effective theories is paving the way towards an era of Quantum Gravitational String Phenomenology.

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# 3
# Transplanckian axion monodromy!?

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# Transplanckian axion monodromy!?

**Ginevra Buratti, José Calderón and Angel M. Uranga**

*Instituto de Física Teórica IFT-UAM/CSIC,*
*C/ Nicolás Cabrera 13–15, Campus de Cantoblanco, 28049 Madrid, Spain*

*E-mail:* ginevra.buratti@studenti.unipd.it, jose.calderoni@estudiante.uam.es, angel.uranga@csic.es

Abstract: We show that warped throats of the Klebanov-Strassler kind, regarded as 5d flux compactifications on Sasaki-Einstein manifolds $\mathbf{X}_5$, describe fully backreacted solutions of transplanckian axion monodromy. We show that the asymptotic Klebanov-Tseytlin solution features a 5d axion physically rolling through its dependence on an spatial coordinate, and traversing arbitrarily large distances in field space. The solution includes the backreaction on the breathing mode of the compactification space and on the vacuum energy, which yields a novel form of flattening. We establish the description of the system in terms of an effective 5d theory for the axion, and verify its validity in transplanckian regimes. In this context, rolling axion monodromy configurations with limited field space range would correspond, in the holographic dual field theory, to duality walls, which admit no embedding in string theory so far. We present an identical realization of transplanckian axion monodromy in 4d in fluxed version of $\mathrm{AdS}_4 \times \mathbf{X}_7$. We speculate that similar models in which the axion rolls in the time direction naturally correspond to embedding the same mechanism in de Sitter vacua, thus providing a natural arena for large field inflation, and potentially linking the swampland de Sitter and distance conjectures.

Keywords: Flux compactifications, Gauge-gravity correspondence

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## Contents





## 1 Introduction and conclusions

The application of Quantum Gravitational constraints to string theory vacua has motivated several conjectures limiting the possibilities to attain field ranges larger than the Planck scale [1–7] (see [8] for a recent review). A prototypical case is the application of the Weak Gravity Conjecture [3] to axions, which implies that periodic axion potentials, such as those in natural inflation, cannot host transplanckian field ranges [9]. Another general result is the Swampland Distance Conjecture, which states that as some modulus approaches a point at infinite distance in moduli space, there is an infinite tower of states becoming massless, exponentially with the distance [2]. There are also partial studies concerning axion monodromy models [10], see also [11–18], trying to rule out their transplanckian excursion by invoking the backreaction on the scalar kinetic terms reducing the effectively traversed distance [19].[1] There are also discussions ruling out particular models using 10d lifts [21, 22] or other mechanisms [23].

These results would seem to motivate a Swampland Transplanckian conjecture, stating that transplanckian field ranges are not physically attainable in Quantum Gravity. If correct, this statement would have profound implications for certain phenomenological applications, like the construction of inflation models with sizable gravitational wave backgrounds (which for single-field inflation are directly related to the distance traversed by the inflaton). The purpose of the present article is to prove that this conjecture is in fact incorrect, and

[1] For other discussions of backreaction related to flattening of the potential, see [20].

that transplanckian field excursions are physically realized in string theory. We will do it by presenting a completely explicit example of axion monodromy model, with full backreaction taken into account, in terms of the complete 10d supergravity solution. The complete background turns out to be given by a simple and well-known warped throat, the Klebanov-Strassler throat [24, 25], when regarded as a flux compactification on a Sasaki-Einstein manifold $\mathbf{X}_5$, with a 5d axion rolling in the radial direction of a (locally) $AdS_5$ spacetime.

Let us make some relevant remarks:

- The discussion is intended as an existence proof of transplanckian axion monodromy in string theory. In particular, we focus on discussing how the 10d solution fully encodes the backreaction of the axion dynamics including the impact on axion kinetic terms, and the backreaction on other sectors, including the compactification moduli and the vacuum energy. This last point is extremely relevant and has not been taken into account in earlier attempts to describe 10d lifts of axion monodromy models.

- We consider configurations where the axion has an explicit dependence on the non-compact spacetimes coordinates (in fact, on a particular spatial coordinate). This is crucial for the configuration to allow the axion to climb its potential while maintaining the solution on-shell. Again, this is an ingredient not properly accounted for in earlier analysis of 10d backreaction of transplanckian axion monodromy, and ties directly to the question of including the vacuum energy variation in the analysis.

- On the other hand, it is *physically meaningful* to consider configurations where the axion is actually varying in spacetime. After all, the main motivation for scalars with transplanckian field excursions are large field inflation models, in which the eventual cosmological solution corresponds to a time-dependent configuration of the scalar rolling down its potential.

- We work in configurations with negative vacuum energy. This is not an obstruction from the fundamental viewpoint of establishing the existence of transplanckian field excursions in string theory. On the other hand, it does not yield realistic models for inflation. Related to this, our configurations have axions depending on spatial directions, rather than time-dependent ones. In fact, formally the sign flip required to switch from space to time dependent scalar profiles correlates with the sign flip for the vacuum energy. This suggests a tantalizing link between positive cosmological constant and time dependent background, which in the present context is reminiscent of the dS/CFT correspondence [26]. It would be interesting to explore the relation of our transplanckian axion monodromy scenario with recent discussions of bounds on slow-roll and the swampland de Sitter conjectures [27].

- We focus on 5d models because the kinds of Klebanov-Strassler throats we need (either for the conifold or for generalizations) have been most studied in this setup. On the other hand, there are less studied but completely analogous throats based on locally $AdS_4 \times \mathbf{X}_7$ configurations in M-theory, which we also discuss and lead to 4d transplanckian axion monodromy configurations in precisely the same fashion as the 5d models.

- The dynamics of the transplanckian axion can be described within an effective field theory, which we discuss explicitly based on a consistent truncation provided in [24]. This, together with the full 10d solution, allows for a discussion of the validity of effective actions for the transplanckian excursion. We show that the configuration is free from oftentimes feared problems: no pathology arises neither when the axion winds its period a large number of times, and no infinite tower of states becomes exponentially light when the axion reaches beyond transplanckian distances in field space.

- Freund-Rubin vacua such as $AdS_5 \times \mathbf{X}_5$ with 5-form flux on $\mathbf{X}_5$ are often described as not yielding good effective field theories, since the compactification radius is comparable to the AdS radius. However, we are not interested in describing an effective field theory which describes the stabilization of the compactification breathing mode, which cannot be decoupled (in the Wilsonian sense) from the KK tower of states. We are interested in the effective dynamics of a massless axion and its spacetime variations at much lower scales, and in its backreaction effects, which are also controlled by those scales. Our effective theory is suitable for that purpose, and can be regarded as describing the low energy dynamics of a scalar in a gravitational background which is fixed at higher scales, save for backreaction effects which are duly included in the effective field theory description.

The paper is organized as follows. In section 2 we describe the KS solutions from the perspective of producing 5d axion monodromy models, focusing on the conifold example. In section 2.1 we describe the 5d compactification on $\mathbf{X}_5$ with no 3-form fluxes, leading to the $AdS_5$ vacuum. In section 2.2 we describe the KS solution [25] (actually, its KT asymptotic form [24]) and in section 2.3 we establish that it describes an axion monodromy solution in which the field range traversed is arbitrarily large, in particular transplanckian. In section 2.4 we relate hypothetical backgrounds with finite axion field ranges with duality walls in the UV of the holographically dual field theories, which have so far not been shown to admit a gravitational description. In section 3 we turn to the effective field theory description. In section 3.1 we review the effective field theory in [24] for the axion and compactification moduli. In section 3.2 we obtain an effective action at energies hierarchically below the KK scale, which actually encodes the axion dynamics and its backreaction effects. In section 4 we discuss 4d configurations from M-theory compactifications, with exactly the same axion monodromy physics as the previous 5d examples. Appendix A discusses a dual Hanany-Witten configuration of D4- and NS5-branes useful to illustrate the absence of pathologies as the axion winds around its period.

## 2 Warped throats and transplanckian axion monodromy

In the following we review the Klebanov-Strassler (KS) throat [25]. We intentionally emphasize its structure as a 5d compactification in which the introduction of the RR 3-form flux yields a 5d axion monodromy model, for which the KS throat is an explicit fully backreacted solution. We then show that the axion roll in this configuration is transplanckian. Actually, for this purpose it suffices to focus on the region far from the tip of the throat, so we use the simpler expressions of the Klebanov-Tseylin (KT) throat [24], supplemented with the boundary conditions derived from the KS smoothing of its naked singularity. For the latter reason, we still refer to the configuration as KS throat.

### 2.1 The 5d theory

Consider as starting point the type IIB Freund-Rubin $\mathrm{AdS}_5 \times T^{1,1}$ background

$$ds^2 = R^2 \frac{dr^2}{r^2} + \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + R^2 \, ds^2_{T^{1,1}} \tag{2.1}$$

with

$$R^4 = 4\pi(\alpha')^2 g_s N \tag{2.2}$$

and with $N$ units of RR 5-form flux through $T^{1,1}$. The type IIB complex coupling is constant, and we will keep it set at $\tau = i/g_s$ (introduction of non-trivial constant $C_0$ is straightforward via minor changes in the fluxes below).

This is the near horizon limit of a set of $N$ D3-branes at a conifold singularity [28]. The line element $ds^2_{T^{1,1}}$ corresponds to a (unit volume) 5d horizon $T^{1,1}$, which is an $\mathbf{S}^1$ bundle over $\mathbf{P_1} \times \mathbf{P_1}$ with first Chern classes $(1,1)$, hence the name. Topologically, it is an $\mathbf{S}^2 \times \mathbf{S}^3$. Denoting by $\sigma_2$ and $\sigma_2'$ the volume forms of the two $\mathbf{P_1}$'s, we have a harmonic 2-form $\omega_2 = \sigma_2 - \sigma_2'$ and its (dual in $T^{1,1}$) harmonic 3-form $\omega_3$. They are Poincaré duals of the 3- and 2-spheres, and $\omega_2 \wedge \omega_3$ is the volume form on $T^{1,1}$.

On top of the complex dilaton, the resulting effective 5d theory has a massless axion, given by the period of the NSNS 2-form over $\mathbf{S}^2 \subset T^{1,1}$

$$\int_{\mathbf{S}^2} B_2 = \phi \quad \text{namely} \quad B_2 = \phi \, \omega_2. \tag{2.3}$$

The periodicity $\phi \sim \phi + 1$ is set by the exponential of the action of a fundamental string wrapped on the $\mathbf{S}^2$. Above the scale of massless fields, there is the scale $1/R$. This is the scale of KK modes, but also the scale of stabilization of the breathing mode of $T^{1,1}$. It is possible to write an effective action for this dynamical mode;[2] in this action, the potential is minimized at the value (2.2), and with a negative potential energy cosmological constant, such that the maximally symmetric solution is the $\mathrm{AdS}_5$ space in (2.1). For a simplified discussion in the completely analogous case of $\mathrm{AdS}_5 \times \mathbf{S}^5$, see [29]; we will discuss such effective actions in a more general context later on.

The above background is a particular case of the general class of $\mathrm{AdS}_5 \times \mathbf{X}_5$ vacua, where $\mathbf{X}_5$ is a Sasaki-Einstein variety. These are gravitational duals to systems of D3-branes at singularities, and have been intensely explored in the literature. Large classes of these models admit also the introdution of 3-form fluxes to be described below, and thus lead to axion monodromy models. To emphasize this direct generalization, we will oftentimes write $\mathbf{X}_5$ instead of $T^{1,1}$.

[2] Since this scale is not hierarchically lower than the KK masses, this effective action should be interpreted as arising from a consistent truncation, rather than a Wilsonian one.

### 2.2 The KS solution

Once we have described the compactification to 5d, we would like to describe the introduction of a RR flux on $\mathbf{S}^3 \subset T^{1,1}$

$$\frac{1}{(2\pi)^2\alpha'}\int_{\mathbf{S}^3} F_3 = M. \tag{2.4}$$

Our key observation is that the resulting 5d theory is an axion monodromy model for $\phi$. This simply follows because the self-dual 5-form field strength

$$\tilde{F}_5 = dC_4 - \frac{1}{2}\,C_2 \wedge H_3 + \frac{1}{2}\,B_2 \wedge F_3 \tag{2.5}$$

satisfies the modified Bianchi identity

$$d * \tilde{F}_5 = d\tilde{F}_5 = H_3 \wedge F_3. \tag{2.6}$$

From the KK perspective the flux (2.4) induces a 5d topological coupling

$$\int_{10d} F_3 \wedge B_2 \wedge F_5 \quad \longrightarrow M \int_{5d} \phi\, F_5. \tag{2.7}$$

As already noted in [16, 30],[3] this is a 5d version of the Dvali-Kaloper-Sorbo term [32, 33] associated to a monodromy for the axion. Clearly, as $\phi$ winds around its basic period, there is a corresponding increase for the flux of $\tilde{F}_5$ through $T^{1,1}$ (and, by self-duality, through the non-compact 5d space), as follows,

$$N = \int_{T^{1,1}} \tilde{F}_5 = N_0 + M\phi. \tag{2.8}$$

In the following we take the reference value $N_0$ to be reabsorbed into a redefinition of $\phi$.

The presence of a scalar potential of the axion monodromy kind, arising from the reduction of the 10d $|\tilde{F}_5|^2$ terms, will be manifest in the 5d effective action discussed in section 3. We are interested in the behaviour of this theory as the value of $\phi$ changes over a large range. Clearly, the presence of this potential term implies that moving the scalar vev adiabatically away from the minimum leads to off-shell configurations, for which the computation of the backreaction is not clearly defined. A natural solution is to instead consider configurations in which the scalar $\phi$ is allowed to roll, so that the spacetime dependent background allows to remain on-shell.[4] The KS solution is precisely an explicit 10d solution of this rolling configuration in which the axion $\phi$ is allowed to roll along one of the *spatial* directions. (As discussed in the introduction, the realization of time dependent roll suggests an interesting interplay with the question of realizing de Sitter vacua). We now review the 10d KS solution (actually, its KT limit with KS boundary conditions) from this perspective.

[3] While finishing this paper, we noticed the recent [31], which involves a similar structure of flux and axion, albeit in a different approach to axion monodromy.

[4] This is in fact a natural viewpoint in inflationary axion monodromy models, in which the interesting solutions correspond to physical time-dependent rolls of the scalar down its potential.

The KS throat describes a configuration in which the axion has a dependence on the radial direction. Concretely, $\phi$ is a harmonic form in the radial direction in the underlying $\mathrm{AdS}_5$, hence

$$\Delta\phi = 0 \quad \rightarrow \quad \phi \sim M \log r. \tag{2.9}$$

This corresponds to the fact that the combination $G_3 = F_3 - \frac{i}{g_s} H_3$ is imaginary self-dual, and in fact (2,1) i.e. supersymmetry preserving, when regarded as a flux in the conifold CY threefold $\mathbf{X}_6$, i.e. when combining the radial coordinate $r$ with the angular manifold $T^{1,1}$. The metric then simply corresponds to a warped version of $M_4 \times \mathbf{X}_6$ of the general class in [34, 35]

$$ds_{10}^2 = h^{-1/2}(r) dx_n dx_n + h^{1/2}(r)(dr^2 + r^2 ds^2_{T^{1,1}}) \tag{2.10}$$

with

$$h(r) = \frac{1}{4r^4} M^2 \log \frac{r}{r_*} \tag{2.11}$$

with $r_*$ some reference value. In short, the metric is of the form (2.1) with the radius (2.2) including a radial dependence

$$N \sim M^2 \log r, \tag{2.12}$$

which follows from (2.8). As explained, this is the KT solution, which has a naked singularity at $r \to 0$. The KS solution provides a smoothing of this based on the deformed conifold.[5] In fact we will be interested in the region of large $r$, and how it extends to infinity, so the KT solution suffices.

The above solution describes precisely all the effects of the backreaction for arbitrarily large values of the axion and number of windings along its period. As one moves towards large $r$, the axion is climbing up its potential and inducing larger flux $N$ due to the monodromy. The flux and stored energy backreact on the stabilization of the breathing mode of the compactification space, whose minimum tracks the value of $\phi$ from (2.2), (2.8) and (2.12)

$$R^4 \sim g_s M \phi \sim g_s M^2 \log r. \tag{2.13}$$

The non-compact geometry is locally $\mathrm{AdS}_5$ with varying radius $R$. Hence, there is also a backreaction in the vacuum energy, with runs towards less negative values as

$$V_0 \sim (\log r)^{-1}. \tag{2.14}$$

[5] When regarded from the 5d perspective, this implies that the direction $r$ "ends" at a finite distance. Of course this is not relevant for the discussion below, which only deals with the large $r$ regime. Moreover, even if one would be interested in having a radial dimension with no end, it is straightforward to modify (2.11) or even its full KS version, e.g. by introducing a large number $P$ of additional explicit D3-branes, producing an $\mathrm{AdS}_5$ at the bottom of the KS throat, effectively removing the endpoint for $r$. This corresponds to the mesonic branches of the cascade [36].



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The slow growth of the vacuum energy can be regarded as a flattening of the potential, albeit different from the polynomial ones in [20].

From the holographic perspective, each winding of $\phi$ on its period corresponds to a cycle in the cascade of Seiberg dualities, in which, as one moves to the UV (larger $r$), the effective number of colors increases by (actually twice) a factor $M$

$$\begin{aligned} \mathrm{SU}(N_0) \times \mathrm{SU}(N_0+M) &\to \mathrm{SU}(N_0+2M) \times \mathrm{SU}(N_0+M) \to \\ &\to \mathrm{SU}(N_0+2M) \times \mathrm{SU}(N_0+3M). \end{aligned} \tag{2.15}$$

Although we will not exploit this holographic picture (as the supergravity solution speaks for itself), we will use it in appendix A to explain why no disaster arises when the axion rolls around its period.[6] In particular there are no states becoming massless or light as one crosses the “zero” value, an effect often feared to play a lethal role for the discussion of monodromy dynamics in effective field theory. The fact that this effect is absent in our model supports the expectation that it is not a generic problem of axion monodromy models (but rather, either of particular models realizing the idea, or of partial analysis of those models without full inclusion of backreaction).

### 2.3 Transplanckian axion field range

Let us use the above solution to quickly show that the 5d field $\phi$ traverses a transplanckian distance in field space. A more systematic discussion is presented in section 3.

The distance traversed by $\phi$ from a reference point $r_0$ to infinity is given by

$$\Delta = \int_{r_0}^{\infty} \left( G_{\phi\phi} \frac{d\phi}{dr} \frac{d\phi}{dr} \right)^{\frac{1}{2}} dr = \int_{r_0}^{\infty} \left( G_{\phi\phi} \right)^{\frac{1}{2}} \frac{d\phi}{dr}\, dr, \tag{2.16}$$

where $G_{\phi\phi}$ is the metric in field space, which is determined by the 5d kinetic term for $\phi$, in the 5d Einstein frame

$$S_5 = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g_5} \left( \mathcal{R}_5 - G_{\phi\phi}\, \partial_m \phi\, \partial_n \phi\, g^{mn} \right). \tag{2.17}$$

Since the compactification volume varies, certain care is required. We must define a fixed reference radius $R$ determining the 5d Planck scale, and introduce a 5d dynamical breathing mode $\tilde{R}$ encoding any variation (see [29] for a similar parametrization). Hence, focusing just on the parametric dependence, we write

$$V_{\mathbf{X}_5} = R^5\, \tilde{R}^5, \tag{2.18}$$

$$ds^2 = g^{(5)}_{mn}\, dx^m\, dx^n + (R\tilde{R})^2\, (g_{\mathbf{X}_5})_{ij}\, dy^i\, dy^j. \tag{2.19}$$

We now focus on the reduction on $\mathbf{X}_5$ of the 10d action for the metric and kinetic term of $B_2$. In the 10d Einstein frame we have

$$S_{10d} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g_{10}} \left( \mathcal{R}_{10} - \frac{1}{12\, g_s}\, H_{MNP}\, H^{MNP} \right). \tag{2.20}$$

[6]See [12] for some discussion of periodic effects in axion monodromy.

As explained, the reference value $R$ fixes the 5d Planck scale

$$\frac{R^5}{2\kappa_{10}^2} = \frac{1}{2\kappa_5^2} \tag{2.21}$$

and the factor $\tilde{R}^5$ is reabsorbed by rescaling the 5d metric to the 5d Einstein frame

$$(g_5)_{mn} \rightarrow \tilde{R}^{-\frac{10}{3}} (g_5)_{mn}. \tag{2.22}$$

We follow the effect of this rescaling in the kinetic term of the component of $B_2$ given by (2.3). The dependence on $\tilde{R}$ is as follows:

$$\int d^{10}x \sqrt{-g_{10}}\, g^{mn} g^{ik} g^{jl} \partial_m B_{ij} \partial_n B_{kl} \overset{\text{compact.}}{\longrightarrow} \tag{2.23}$$

$$\overset{\text{compact.}}{\longrightarrow} \int d^5x \sqrt{-g_5}\, (R\tilde{R})\, (g_5)^{mn}\, \partial_m\phi\, \partial_n\phi \overset{\text{Einstein}}{\longrightarrow} \int d^5x \sqrt{-g_5}\, (R\tilde{R}^{-4})\, (g_5)^{mn}\, \partial_m\phi\, \partial_n\phi\,.$$

Hence, we have $\tilde{R}^4 \sim M^2 \log r$ and thus

$$G_{\phi\phi} \sim (\, M^2 \, \log r\, )^{-1}. \tag{2.24}$$

We have $\phi \sim M \log r$, hence the distance (2.16) is

$$\Delta = \int G_{\phi\phi}^{\frac{1}{2}} \frac{d\phi}{dr}\, dr \sim \int dr\, (M^2 \, \log r)^{-\frac{1}{2}}\, M\, \frac{dr}{r} = \int \frac{ds}{s^{\frac{1}{2}}} \tag{2.25}$$

for $s = \log r$. This becomes arbitrarily large for large $r$, showing that the 5d scalar $\phi$ rolls through a transplanckian distance in field space.

The 10d backreacted solution for this transplanckian axion monodromy configuration allows to address many of the objections to transplanckian field excursions in string theory or quantum gravity, and study how the present models avoid those potential pitfalls. As many of these are related to the regimes of validity of effective field theories for the axion dynamics, we postpone their discussion until section 3.

The above $\text{AdS}_5$ vacua admit generalizations associated to D3-branes at more general CY threefold singularities, which have been extensively studied in the toric case. The dual backgrounds correspond to type IIB Freund-Rubin $\text{AdS}_5 \times \mathbf{X}_5$, where $\mathbf{X}_5$ is the 5d horizon of the 6d CY cone. The construction of KT backgrounds by introducing (possibly a richer set of) 3-form fluxes is a straightforward extension of our above discussion (see for instance [37] for complex cones over del Pezzo surfaces), so there is a large class of constructions leading to transplanckian axion monodromy. Being more careful, we should make clear that only CY singularities admitting complex deformations can complete their KT throats into smooth supersymmetric KS-like throats [38]; other choices admit no supersymmetric KS completion [37, 39, 40], and actually lead to runaway instabilities [37, 41], a fact which has recently motivated the "local AdS-Weak Gravity Conjecture" [30], generalizing the "AdS-WGC" in [4]. However, even with the restriction to CY singularities admitting complex deformations, there is an enormous class of such explicit constructions (built with standard toolkits, see e.g. [42]), and thus leading to transplanckian axion monodromy.

### 2.4 Duality walls

The fact that the axion traverses an arbitrarily large distance in field space as one moves to larger distances in $r$ is intimately related to the RG flow structure in the holographic field theory. As mentioned in section 2.2, the axion winding around its period corresponds to completing a cycle in the Seiberg duality cascade of the $\mathrm{SU}(N) \times \mathrm{SU}(N+M)$ field theory. The steps in the energy scale in each duality cycle relate to the radial distance required for the scalar to wind around its period. The infinite range in energy as one moves up to the UV in the field theory provides an infinite range in radial distance on the gravity side, which allows for an arbitrarily large axion field range with finite gradient energy density. Hence, the nice properties of the holographic field theory RG flow relates to the fact that the gravity side is described by a supergravity background.

In contrast with this picture, it is interesting to point out that a different kind of RG flow behaviour of duality cascades has been contemplated, purely from the field theory perspective. These are known as duality walls, and correspond to duality cascade RG flows in which, as one moves to the UV, the energy steps in each duality cycle decrease; more concretely, the number of duality cycles in a given energy slice increases as one moves up to the UV, in such a way that there is a limiting energy, at which the number of cycles per energy interval diverges. Such RG flows have been introduced in [43], and proposed to relate to quiver gauge theories of D-branes at singularities in e.g. [44–46]. However, there is no concrete string theory D-brane realization of such RG flows. In particular, systematic searches for gravity backgrounds dual to gauge theories with duality walls have produced no such results [37].

The absence of such backgrounds, at least in the context of supergravity, has an interesting implication for our perspective on field ranges in axion monodromy models. Gravitational solutions dual to duality walls would require an axion winding around its period an infinite number of times in a finite range in the radial distance. This is compatible with finite gradient energy densities only if the kinetic term of the axion varies so as to render finite the traversed distance in field space. This kind of behaviour would produce axion monodromy models where superplanckian field ranges cannot be attained. Hence, the absence of supergravity backgrounds of this kind is a signal that superplanckian axion monodromy models are actually generic in the present setup, whereas those with limiting field ranges are exotic, if at all existent.

## 3 Effective field theory analysis

In the previous section we have shown a fully backreacted explicit 10d solution for axion monodromy models with arbitrarily large field ranges. In this section we bring the discussion to the context of the 5d effective field theory, where much of the discussion of swampland conjectures is carried out.

### 3.1 Effective field theory for axion and breathing mode

From the 10d solution it is clear that the relevant dynamics in 5d involves the axion $\phi$ and the breathing mode of $\mathbf{X}_5 = T^{1,1}$, coupled to 5d gravity. It is interesting to device an effective field theory describing the dynamics for these degrees of freedom in the KS solution.[7] This provides a concrete context in which to test the regime of validity of the effective field theory to describe transplanckian axion monodromy, or to test other swampland conjectures.

The 5d effective field theory can be obtained starting from the 10d type IIB effective action, and using a suitable ansatz for the compactification, which allows for general dynamics for the relevant 5d fields. This strategy was in fact put forward in [24] to produce the 5d action we are interested in. We review the key ingredients relevant for our purposes, and adapted to our present notation.

We consider the metric ansatz

$$ds_{10}^2 = L^2\big(\, e^{-5q}\, ds_5^2 + \, e^{3q} ds_{T^{1,1}}^2\big)\,. \tag{3.1}$$

Here $q$ is a 5d field encoding the breathing mode of $T^{1,1}$. Also, $ds_5^2$ is the line element in the 5d non-compact spacetime, defined in the 5d Einstein frame thanks to the prefactor $e^{-5q}$. The explicit $L$ scales out the line elements to geometries of unit radius.

There are $M$ units of $F_3$ flux over the $\mathbf{S}^3 \in T^{1,1}$ and there is a 5d axion defined by (2.3). The modified Bianchi identity (2.6) implies that the flux of $\tilde{F}_5$ over $T^{1,1}$ is given by (2.8).

The 5d effective action for the 5d scalars $\phi$ and $q$, collectively denoted by $\varphi^a$, is given by

$$S_5 = -\frac{2}{\kappa_5^2}\int d^5x\,\sqrt{-g_5}\left[\frac{1}{4}R_5 - \frac{1}{2}G_{ab}(\varphi)\partial\varphi^a\partial\varphi^b - V(\varphi)\right], \tag{3.2}$$

with the kinetic terms and potential given by

$$G_{ab}(\varphi)\partial\varphi^a\partial\varphi^b = 15(\partial q)^2 + \frac{1}{4}g_s^{-1}e^{-6q}(\partial\phi)^2\,, \tag{3.3}$$

$$V(\varphi) = -5e^{-8q} + \frac{1}{8}M^2 g_s\, e^{-14q} + \frac{1}{8}(N_0 + M\phi)^2 e^{-20q}. \tag{3.4}$$

The different terms in the potential have a clear interpretation. The first negative contribution corresponds to the curvature of the compactification space $T^{1,1}$, the second is the contribution from the $M$ units of $F_3$ flux on the $\mathbf{S}^3$, and the third corresponds to the contribution from the 5-form flux over $T^{1,1}$, and has the typical axion monodromy structure. We note that, despite the bare quadratic dependence, the backreaction of $\phi$ on the geometry will produce a different functional dependence of the potential energy at the minimum, as shown below. Also, as already explained, the above action should be regarded as a consistent truncation in supergravity, so we will take special care to discuss the role of other physical degrees of freedom, like KK modes.

Since the above effective theory is general, it should reproduce the basic $AdS_5$ background for $M = 0$. The potential becomes

$$V(\varphi) = -5e^{-8q} + \frac{1}{8}N_0^2 e^{-20q}. \tag{3.5}$$

[7]Inclusion of the dilaton is discussed in section 3.3.

The potential has a minimum at

$$e^{6q} = \frac{N_0}{4} \tag{3.6}$$

with negative potential energy at the minimum

$$V_0 = -3\, e^{-8q}. \tag{3.7}$$

Comparing (3.1) with the standard expression for $AdS_5 \times T^{1,1}$ metric (2.1), we recover the scaling of the $T^{1,1}$ radius $R$ with $N_0$

$$R^2 \sim e^{3q} \;\rightarrow\; R^4 \sim N_0 \tag{3.8}$$

with other factors reabsorbed in $L$ in (3.1). Taking the value for $V_0$ (3.7) and removing a factor of $e^{-5q}$ to change to the 10d frame, we recover the same scaling for the radius of the $AdS_5$ vacuum.

The KS throat (actually its asymptotic KT form) is a solution of the above effective action. Following [24], we take the following ansatz for the metric

$$ds^2_{10} = s^{-1/2}(r)\, \eta_{\mu\nu}\, dx^\mu\, dx^\nu + h^{1/2}(r)\, (dr^2 + r^2 ds^2_{T^{1,1}}). \tag{3.9}$$

In terms of (3.1), this corresponds to

$$e^{3q} = r^2\, h^{1/2}(r) \quad , \quad ds^2_5 = e^{5q}\, [\, s^{-1/2}(r)\, \eta_{\mu\nu}\, dx^\mu\, dx^\nu + h^{1/2}(r)\, dr^2\, ]. \tag{3.10}$$

The effective theory admits a solution where

$$\phi = M\, \log r \quad , \quad s(r) = h(r) = \frac{1}{4r^4} M^2 \log \frac{r}{r_*}, \tag{3.11}$$

with $r_*$ some reference value. This is just the throat solution discussed in section 2.2.

The effective action can be exploited to recover the result of the transplanckian field range covered by the axion. Since the 5d effective action is already in the 5d Einstein frame, we can read out and evaluate the kinetic term for $\phi$ in (3.3)

$$G_{\phi\phi} \sim e^{-6q} = [\, r^4\, h(r)\, ]^{-1} \sim (\, M^2\, \log r\, )^{-1}. \tag{3.12}$$

We thus recover, in a more precise setting, the result (2.24), and thus the corresponding unbounded (and hence transplanckian) field range.

### 3.2 The axion effective field theory

As explained, the above action should be regarded as a consistent truncation in supergravity, but not as a Wilsonian effective action. In other words, at the scale $1/R$ at which the stabilization of the breathing mode occurs, there are many other modes, corresponding to KK excitations of the 10d fields in $\mathbf{X}_5$ which are not included in the action. Note that this scale goes as $1/R \sim (\log r)^{-1/4}$. On the other hand, the effective dynamics for the axion occurs at far lower scales, set by $\partial\phi = 1/r$. Similarly, the scale of the backreaction on the



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compactification radius or the vacuum energy is measured by their derivatives with respect to $r$, which are similarly suppresed by $1/r$ (or even with additional inverse powers of $\log r$). It is therefore interesting to construct an effective field theory including just the axion and intended to describe its dynamics at those scales (hence, including the backreaction on the volume and vacuum energy).

For this, we minimize the scalar potential for $q$ keeping $\phi$ fixed. This gives the condition

$$\frac{5}{2}\,(N_0+M\phi)^2\,x^2+\frac{7}{4}\,g_sM^2\,x-40=0 \quad , \quad \text{with } x=e^{-6q}. \tag{3.13}$$

Rather than solving the above exactly, since we are focusing on the large $r$ regime, where $\phi$ is large and $x$ is comparably small, we drop the subleading second term, and obtain

$$e^{6q}=\frac{1}{4}(N_0+M\phi). \tag{3.14}$$

This reproduces the result of the KS solution that $e^{6q}\sim M^2\log r$ for $\phi\sim M\log r$, so we are capturing the relevant physics.

We should replace that value in the potential. Again restricting to large $r$, we drop the second term in (3.4) and obtain

$$V=-e^{-8q}\left[5-\frac{1}{8}\,(N_0+M\phi)^2\,e^{-12q}\right]. \tag{3.15}$$

This has the same structure as (3.5) with the replacement $N_0\to N_0+M\phi$. The potential should be regarded as a function of $\phi$ only, by simply replacing (3.14) in this expression. It is therefore clear that considering a profile $\phi=M\log r$ leads to the appropriate change in the vacuum energy, so that the backreaction of the axion monodromy is duly included.

The complete axion action should include its kinetic term, obtained from that in (3.3) by using (3.14). We recover a kinetic term

$$\sim(N_0+M\phi)^{-1}(\partial\phi)^2, \tag{3.16}$$

which again reproduces the familiar result about the transplanckian distance traveled in the rolling solution considered.

This effective action suffices to describe the dynamics of the transplanckian axion monodromy, so it is a well-defined setup to test/propose swampland conjectures on effective actions. For instance, one natural idea is to consider if there is an analog of the swampland distance conjecture, and there is a tower of states becoming exponentially light as the axion travels at arbitrarily large distances. This is not the case, as follows. The invariant distance in axion field space goes (for large $\phi$) as $d\sim\phi^{1/2}$; on the other hand, the masses of KK modes (which are the primary suspects for fields becoming light at large $\phi$, since $R$ increases), scale as $m_{\rm KK}\sim e^{-4q}\sim(\phi)^{-2/3}$, hence $m_{\rm KK}\sim d^{-4/3}$ and there is no tower of exponentially light states. This is compatible with the swampland distance conjecture, if interpreted as applying to field ranges approaching points at infinite distance in moduli space [2, 47]. It is also compatible with the oftentimes used version for transplanckian geodesic distances, since in the next section we will show that our axion travel does not follow a geodesic. However the model provides a beautiful way in which a fully backreacted monodromic axion can travel arbitrarily large distance in field space without triggering the appearance of exponentially light states.

There are other interesting questions that can be addressed in the present setup, such as the application of swampland constraints on the scalar potential, or the realization of the weak gravity conjecture in the present setup, etc. Since the underlying model is a string theory compactification on a smooth geometry with fluxes, we expect no new surprises or novel mechanisms related to these other swampland conjectures.

### 3.3 Inclusion of the dilaton

As announced, in this section we show that the underlying reason for the compatibility of the transplanckian axion monodromy model with the swampland distance conjectures is that the axion does not follow a geodesic in the moduli of light fields. The crucial ingredients to understand this are the spacetime dependence of the axion, and the inclusion of the dilaton in the moduli space.

The original KT 5d effective action [24] includes further fields beyond those included in the earlier discussion. Indeed, it contains fields $\varphi^a = q, f, \Phi, \phi$, where $f$ describes a possible asymmetric volume for the $\mathbf{S}^2$ and $\mathbf{S}^3$ of $T^{1,1}$, and $\Phi$ is the dilaton. The 5d action for these fields has the structure (3.2) with

$$\begin{aligned} G_{ab}(\varphi) &= \mathrm{diag}\left(15, 10, \frac{1}{4}, \frac{1}{4}e^{-\Phi-4f-6q}\right), \\ V(\varphi) &= e^{-8q}\left(e^{-12f} - 6e^{-2f}\right) + \frac{1}{8}M^2 e^{\Phi+4f-14q} + \frac{1}{8}(N_0 + M\phi)^2 e^{-20q}. \end{aligned} \tag{3.17}$$

The pure AdS$\times T^{1,1}$ solution for $M = 0$ shows that in this action the breathing mode $q$ and asymmetric mode $f$ are heavy modes, while the axion $\phi$ and dilaton $\Phi$ remain as light fields. Morally, we should thus consider the later as parametrizing a moduli space at scales hierarchycally below the KK scale, with a potential induced by the introduction of non-zero $M$. This is manifest because the terms including $M$ in the potential are subdominant with respect to the first, $M$-independent, one.

This allows to integrate out $q$ and $f$. We may minimize the leading potential for $f$, and set $f = 0$ (as implicit in the previous section). For the minimization of $q$, we proceed as in the previous section and recover (3.14).

Note that, in the resulting theory for the axion and the dilaton, there is a non-trivial potential for the dilaton. This is however compatible with its constant value in the axion monodromy solution in an interesting way: the spacetime dependence of the axion has a non-trivial backreaction in the dilaton, through the dilaton dependence of the axion kinetic term, which induces an effective potential for the dilaton balancing the original one and allowing for a constant dilaton solution. Quantitatively, the equation of motion for a general field in the presence of a spacetime-dependent axion background reads

$$\frac{1}{\sqrt{g}}\partial_\nu\left(\sqrt{g}g^{\mu\nu}G_{ac}\partial_\mu\varphi^a\right) = \frac{1}{2}\frac{\partial G_{\phi\phi}}{\partial\varphi^c}(\partial\phi)^2 + \frac{\partial V}{\partial\varphi^c}. \tag{3.18}$$

For the dilaton, the condition to allow for a constant dilaton $e^\Phi = g_s$ is the vanishing of the right-hand side, which is proportional to

$$-e^{-6q-\Phi}(\partial\phi)^2 + e^{-14q+\Phi}M^2. \tag{3.19}$$

This indeed vanishes in the KT solution, allowing for a constant dilaton. As anticipated, the spacetime dependence of the axion exerts a force on the dilaton keeping it constant on the slope of its bare potential.

The scale of this effect is set by the gradient of the axion $\partial\phi$, which is hierarchycally below the KK scale. This implies that the corresponding backreaction effect for the other fields $q$ and $f$ is negligible, and can be ignored when they are integrated out, as implicit in our above discussion. It also implies that it is not appropriate, in a Wilsonian sense, to integrate out the dilaton dynamics, as it occurs at the scale relevant for axion dynamics.

This last observation raises an important point. In checking the interplay of our axion monodromy model with the swampland distance conjectures, the moduli space on which distances should be discussed is that spanned by the axion and the dilaton, as their potential on this moduli space is hierarchically below the KK scale cutoff. As we have shown, in this moduli space the KT solulion describes an axion monodromy model traversing transplanckian (and actually arbitrarily long) distances without encountering infinite towers of light states. However, as we now argue, this does not contradicts swampland distance conjectures, since the trajectory does not correspond to a geodesic in the axion-dilaton moduli space.

After replacement of $q$ and $f$ by their values at the minimum of their potentials, the kinetic term for $\phi$, $\Phi$ reads

$$\mathcal{L}_{\rm kin} = \frac{1}{8}(\partial\Phi)^2 + \left(\frac{e^{-\Phi}}{2\,(N_0+M\phi)} + \frac{5M^2}{24\,(N_0+M\phi)^2}\right)(\partial\phi)^2. \tag{3.20}$$

At large $\phi$ we can neglect the subleading second term in the kinetic term of $\phi$ and get

$$\mathcal{L}_{\rm kin} = \frac{1}{8}(\partial\Phi)^2 + \frac{e^{-\Phi}}{2\,(N_0+M\phi)}(\partial\phi)^2. \tag{3.21}$$

To look at the geodesics of this theory it is convenient to change variables

$$\begin{aligned} x &= \frac{4}{M}\sqrt{N_0+M\phi}, \\ y &= 2e^{\Phi/2} = 2\sqrt{g_s}. \end{aligned} \tag{3.22}$$

This leads to

$$\mathcal{L}_{\rm kin} = \frac{1}{2y^2}\left[(\partial x)^2 + (\partial y)^2\right], \tag{3.23}$$

which is the metric of the hyperbolic plane. Geodesics of this space, considering $y$ the vertical axis, are vertical lines or half-circles centered in the horizontal axis. On the other hand, the KT solution corresponds to horizontal lines at different constant values of the dilaton.

## 4 The 4d case

The above discussion has been carried out in the 5d context because, being holographically dual to 4d gauge theories, these are the best studied warped throats. However, there are well studied supergravity solutions of the form $\mathrm{AdS}_4 \times \mathbf{X}_7$, and supergravity solutions of the KT kind when the horizon variety $\mathbf{X}_7$ admits the introduction of fluxes [48]. In the following we review these backgrounds and show that they realize in 4d the same kind of transplanckian axion monodromy as the 5d configurations described above.

The starting point is the $\mathrm{AdS}_4 \times \mathbf{X}_7$ background, which can be regarded as arising from the near-horizon limit of a stack of $N$ coincident M2-branes [49]

$$ds^2 = h(r)^{\frac{2}{3}}\, \eta_{\mu\nu}\, dx^\mu\, dx^\nu + h(r)^{\frac{1}{3}}\, (\, dr^2 + r^2\, ds^2_{\mathbf{X}_7}\, ), \tag{4.1}$$

where now Greek indices label non-compact coordinates spanning, together with $r$, the 4d spacetime. The harmonic function is

$$h(r) = \frac{2^5\, \pi^2\, N\, \ell_p^6}{r^6}. \tag{4.2}$$

Namely, we have

$$ds^2 = \frac{R^4}{r^4}\, \eta_{\mu\nu}\, dx^\mu\, dx^\nu + R^2\, \frac{dr^2}{r^2} + R^2\, ds^2_{\mathbf{X}_7}\, , \tag{4.3}$$

where

$$R^6 = 2^5\, \pi^2\, N\, \ell_p^6. \tag{4.4}$$

There are $N$ units of flux of the 7-form field strength $F_7$ (dual to the 4-form field strength $F_4$) through $\mathbf{X}_7$.

Consider an $\mathbf{X}_7$ with a non-trivial 4-cycle,[8] on which we turn on $M$ units of 4-form field strength flux $F_4$. Taking the dual 3-cycle $\Pi_3$ in $\mathbf{X}_7$, there is a 4d axion

$$\phi = \int_{\Pi_3} C_3. \tag{4.5}$$

This axion is monodromic, as follows from the reduction of the 11d Chern-Simons coupling

$$\int_{11d} F_4 \wedge F_4 \wedge C_3 \;\to\; \int_{4d} M\, \phi\, F_4. \tag{4.6}$$

The monodromy implies that the value of $N$ varies with $\phi$ as

$$N = N_0 + M\, \phi, \tag{4.7}$$

with $N_0$ a reference value, which we take zero in what follows.

[8] Such horizons can be obtained for instance by taking the near horizon limit of M2-branes at toric $\mathrm{CY}_3 \times \mathbf{C}$ (leading to 3d $\mathcal{N} = 1$ theories), where the $\mathrm{CY}_3$ admits a complex deformation corresponding to the size of a 3-cycle. The horizon $\mathbf{X}_7$ then contains (an $\mathbf{S}^1$ worth of) such 3-cycle, and hence its dual 4-cycle.



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This leads to a 4d analog of the KT throat found in [48] and given by a flux background

$$F_4 = d^3x \wedge dh^{-1} + M \; *_7 \, \omega_3 - M \, \frac{dr}{r} \wedge \omega_3. \tag{4.8}$$

Here $\omega_3$ is the Poincare dual to the 4-cycle in $\mathbf{X}_7$, so the second term corresponds to the $F_4$ flux through the 4-cycle. The third term corresponds to a rolling scalar profile $d\phi = dr/r$, hence

$$\phi \,\sim\, M \log r. \tag{4.9}$$

Hence we have the axion rolling logarithmically up its monodromic potential, exactly as in the 5d KS solutions discussed above. The first term correspond to the dual of the flux of $F_7$ through $\mathbf{X}_7$, which varies with the radial coordinate due to the axion monodromy.

The harmonic function $h(r)$ is

$$h(r) = M^2 \left( \frac{\log r}{6r^6} + \frac{1}{36r^6} \right) \tag{4.10}$$

(up to some $\rho/r^6$ factor, which defines a reference value which we take to be zero). It also determines the metric by replacement in (4.1).

The solution, just like in the 5d KT example, has a naked singularity at $r = 0$, which is presumably smoothed out at least for certain geometries $\mathbf{X}_7$, although no analog of the full KS solution has been found. It would be interesting to develop the dictionary of fractional M2-brane theories and their gravity duals further to gain insight into such smoothings. This however lies beyond the scope of the present paper.

It is straightforward to compute the 4d kinetic term of the axion $\phi$ as in the simplified 5d calculation in section 2.3. Specifically, the Einstein-Hilbert and 3-form kinetic term in the 11d action read

$$S_{11} = \frac{1}{2\kappa_{11}^2} \int d^{11}x \, \sqrt{-g_{11}} \left( \mathcal{R}_{11} + \frac{1}{2} \, |F_4|^2 \right). \tag{4.11}$$

Define the volume of $\mathbf{X}_7 = (R\tilde{R})^7$, where $R$ defines the backgound value and $\tilde{R}$ its breathing mode. The KK reduction to 4d contains the terms

$$S_4 = \frac{1}{2\kappa_4^2} \int d^{11}x \, \sqrt{-g_4} \, \big( \tilde{R}^7 \, \mathcal{R}_4 + c \, \tilde{R} \, g^{mn} \, \partial_m \partial_n \phi \big). \tag{4.12}$$

Here we have introduced

$$\kappa_4^2 = \frac{\kappa_{11}^2}{R^7}. \tag{4.13}$$

Also, the factor $\tilde{R}$ in the axion kinetic term arises from an $\tilde{R}^7$ from the compactification volume and a factor $\tilde{R}^{-6}$ from three inverse metrics of $\mathbf{X}_7$ required for the contractions of $|\omega_3|^3$. Finally $c$ is a constant that depends on geometrical properties of the cycles in $\mathbf{X}_7$.

Going to the 4d Einstein frame we have

$$S_4 = \frac{1}{2\kappa_4^2} \int d^{11}x \, \sqrt{-g_4} \, \big( \mathcal{R}_4 + c \, \tilde{R}^{-6} \, g^{mn} \, \partial_m \partial_n \phi \big). \tag{4.14}$$

So the kinetic term for the axion gives

$$G_{\phi\phi} \sim \tilde{R}^{-6} \sim (M^2 \log r)^{-1}. \tag{4.15}$$

This is exactly as in the 5d example, and again leads to arbitrarily large, in particular transplanckian, field ranges traversed by the axion roll.

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## A Periodic crossing and the dual Hanany-Witten picture

In this section we discuss a T-dual realization of the KS duality cascade, in terms of the NS5- and D4-brane configurations [50] realizing 4d gauge theories à la Hanany-Witten [51]. The picture is similar to that mentioned in [10], albeit with additional relevant refinements.

The configuration is flat 10d space with one dimension, labelled 6, compactified on an $\mathbf{S}^1$. There is one NS5-brane along the directions 012345 (and at the origin in 89), and one NS5-brane (denoted NS5') along the directions 012389 (and at the origin in 45), with D4-branes along 0123 and suspended among them in 6 (and at the origin in 4589), in a compact version of [52]. The positions of all branes in the directions 7 are taken equal. The numbers of D4-branes at each side of the interval are $N$ and $N + M$ respectively. The scalar $\phi$ corresponds to the distance (in units of $2\pi$ the radius of $\mathbf{S}^1$) between the NS and the NS'-branes, so it has periodicity $\phi \sim \phi + 1$.

In a naive description, as the scalar winds around its period, the crossings of the NS and NS'-branes produce Seiberg dualities that complete a full cycle in the duality cascade. This naive picture would seem to suggest that each crossing leads to additional light degrees of freedom, which could spoil the axion monodromy, or at least its description in terms of an effective action not including these new modes.

However, the actual picture is somewhat more intricate and is free of these problems. The answer lies in the phenomenon of brane bending in [50], which implies that the $M$ additional D4-branes on one of the intervals forces the NS- and NS'-branes to bend. This bending has a logarithmic dependence, and is a long distance result of the description of the whole system as a single M5-brane in a holomorphic curve in the M-theory lift of the configuration [50, 53]. In $\mathcal{N} = 2$ 4d theories, this corresponds in a precise manner to the field theory running of gauge couplings on the Coulomb branch. In the present $\mathcal{N} = 1$ setup, the RG direction (to become the radius in the gravitational dual side) can be thought of as the radial distance away from the point $x^4 = x^5 = x^8 = x^9 = 0$ at which all branes are located. Then, there is a logarithmic bending of the positions of the NS- and



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NS'-branes in the directions 6, which matches the above naive description. However, the other positions of the NS- and NS'-branes in the other directions do not coincide, hence no actual crossing of branes occurs. The discussion of Seiberg dualities carries over but in this more precise sense. The phenomenon is similar to the discussion in [54].



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# 4

# Discrete symmetries, weak coupling conjecture and scale separation in AdS vacua

This chapter contains the article

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# Discrete symmetries, weak coupling conjecture and scale separation in AdS vacua

**Ginevra Buratti, José Calderón, Alessandro Mininno and Angel M. Uranga**

*Instituto de Física Teórica IFT-UAM/CSIC,*
*C/ Nicolás Cabrera 13-15, Campus de Cantoblanco, 28049 Madrid, Spain*

*E-mail:* ginevra.buratti@uam.es, j.calderon.infante@csic.es, alessandro.mininno@uam.es, angel.uranga@csic.es

Abstract: We argue that in theories of quantum gravity with discrete gauge symmetries, e.g. $\mathbf{Z}_k$, the gauge couplings of U(1) gauge symmetries become weak in the limit of large $k$, as $g \to k^{-\alpha}$ with $\alpha$ a positive order 1 coefficient. The conjecture is based on black hole arguments combined with the Weak Gravity Conjecture (or the BPS bound in the supersymmetric setup), and the species bound. We provide explicit examples based on type IIB on $\mathrm{AdS}_5 \times \mathbf{S}^5/\mathbf{Z}_k$ orbifolds, and M-theory on $\mathrm{AdS}_4 \times \mathbf{S}^7/\mathbf{Z}_k$ ABJM orbifolds (and their type IIA reductions). We study $\mathrm{AdS}_4$ vacua of type IIA on CY orientifold compactifications, and show that the parametric scale separation in certain infinite families is controlled by a discrete $\mathbf{Z}_k$ symmetry for domain walls. We accordingly propose a refined version of the strong AdS Distance Conjecture, including a parametric dependence on the order of the discrete symmetry for 3-forms.

Keywords: Discrete Symmetries, Flux compactifications, Superstring Vacua, Supersymmetric Effective Theories

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## Contents





## 1 Introduction and conclusions

By now there is a substantial amount of swampland conjectures constraining effective field theories to be compatible with Quantum Gravity [1–6] (see [7, 8] for reviews). They have led to interesting insights into phenomenological applications of string theory models.

Interestingly, many of these works focus on the properties of continuous gauge symmetries, whereas far fewer results have been obtained to constrain discrete symmetries (for some results, see [9–11], and also [12]), and mostly focus on the constrain that global discrete symmetries, just like global continuous symmetries, are forbidden in quantum gravity (see [13–19] for early literature). Discrete gauge symmetries are an interesting area with exciting applications in BSM phenomenology and string model building [20–26]. The scarcity of swampland constraints on them is partially explained by the fact that discrete symmetries lack long-range fields or tunable parameters like coupling constants, so there are less handles to quantitatively constrain their properties or their impact on other quantities of the theory.

In this work, we overcome this difficulty by considering theories with both discrete and continuous gauge symmetries, and uncover interesting quantitative links among them. For simplicity we focus on abelian $\mathbf{Z}_k$ and U(1) symmetries. In theories with a U(1) gauge symmetry, considerations about evaporation of charged black holes lead to the Weak Gravity Conjecture [3], by demanding that the black hole should remain (sub)extremal throughout the process. To put it simply, considering an extremal black hole with $M = gQ$ (in Planck units), the theory must contain particles with mass $m$ and charge $q$, with $m \leq gq$, such that the black hole can decay without becoming super-extremal. This is the Weak Gravity Conjecture (WGC). The marginal case in which the Weak Gravity Conjecture particles saturate the inequality $m = q$ has been further proposed to correspond to supersymmetric situations, in which it often corresponds to a BPS bound.

If the theory enjoys a further $\mathbf{Z}_k$ discrete gauge symmetry, one can consider any such classical black hole solution and endow it with discrete $\mathbf{Z}_k$ charge, with no change in the classical solution, as this charge does not source long-range fields (see e.g. [27], and also [28] for a recent perspective), and study their decay as in the WGC. In particular, we may consider extremal black holes carrying $\mathbf{Z}_k$ charge and derive a striking result, the $\mathbf{Z}_k$ Weak Coupling Conjecture (WCC) which schematically is the statement that in a theory with a discrete $\mathbf{Z}_k$ gauge symmetry and a U(1) gauge symmetry with coupling $g$, the gauge coupling scales as $g \sim k^{-\alpha}$ for large $k$, with $\alpha$ a positive order 1 coefficient.

The derivation and some qualifications on this statement are discussed in section 2. In particular, we also relate this statement with diverse versions of swampland distance conjectures.

As we will see, the derivation is most precise in the supersymmetric case, in which the WGC bound saturates, but we believe it holds far more generally, as we will illustrate in concrete string theory examples. In particular, in section 3 we study $\text{AdS}_5 \times \mathbf{S}^5/\mathbf{Z}_k$ vacua (and generalizations to general toric[1] theories $\text{AdS}_5 \times \mathbf{X}_5/\mathbf{Z}_k$), in which there is a discrete Heinsenberg group $\mathbf{H}_k$, associated to torsion classes in $\mathbf{S}^5/\mathbf{Z}_k$ [29–31]. This is generated by elements $A$, $B$, each generating a $\mathbf{Z}_k$ symmetry, with commutation relations $AB = CBA$, with $C$ a central element. In the effective 5d theory (namely at scales below the KK scale, and thus at long distance compared with the AdS radius as well) there is at least one U(1) gauge symmetry, corresponding to the R-symmetry of the holographic dual SCFT, whose coupling, as we show, obeys the WCC. In addition, for $\mathbf{S}^5/\mathbf{Z}_k$, and in fact for any toric

[1]By toric, in this context we mean that the CY3 obtained as the real cone over the Sasaki-Einstein 5d variety, is toric.



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theory $\mathbf{X}_5/\mathbf{Z}_k$, there are two additional U(1)'s (the mesonic global symmetries in the dual SCFT), which also satisfy the WCC.

In section 4 we discuss an analogous exercise in 4d by considering in section 4.1 the case of M-theory on $\mathrm{AdS}_4 \times \mathbf{S}^7/\mathbf{Z}_k$, which provides the gravity dual to the ABJM theories [32]. The U(1) symmetry corresponds to an isometry of the internal space, and the discrete symmetry is also related to torsion classes in $\mathbf{S}^7/\mathbf{Z}_k$, although it has an intricate structure not reducible to just $\mathbf{Z}_k$. This is further clarified using the type IIA perspective in section 4.2, in which the discrete gauge symmetry is shown to have order $k^2 + N^2$, and the U(1) symmetry is a linear combination of different RR p-form gauge symmetries, with a second linear combination that is massive due to a Stückelberg coupling. We discuss these systems and show how the corresponding WCC is duly satisfied.

In section 5 we turn to exploiting these considerations in theories in which the $\mathbf{Z}_k$ charged objects are not particles (or their dual objects, e.g. strings in 4d), but rather 4d domain walls. In particular, we consider the type IIA $\mathrm{AdS}_4$ vacua obtained in CY orientifold compactification with NSNS and RR fluxes. In section 5.1 we review a class of compactifications with fluxes scaling with a parameter $k$, shown in [33] to have parametric scale separation controlled by $k$. These vacua would violate the strong AdS Distance Conjecture proposed in [34], an issue on which our analysis sheds important insights. In section 5.2 we show that these systems are higher p-form analogues to the type IIA vacua of section 4.2, with a continuous 3-form symmetry arising from a massless linear combination, and the discrete symmetry arising from a second linear combination made massive by a 3-form Stückelberg mechanism (see [35, 36], also [24]), also called Dvali-Kaloper-Sorbo (DKS) mechanism. In section 5.3 we discuss the role of the discrete $\mathbf{Z}_k$ symmetry in fixing the scaling of the moduli with $k$. In section 5.4 we use tensions of BPS domain walls to recover the vacuum energy scalings, and show that AdS vacua with trivial 3-form discrete symmetry have no scale separation, while the above scaling family of AdS vacua with a non-trivial 3-form discrete symmetry displays scale separation controlled by $k$, as follows. The scale separation relation between the KK scale $m_{KK}$ and the 4d cosmological constant $\Lambda$ is given by the species bound

$$\Lambda = \frac{m_{\mathrm{KK}}^2}{k} \, . \tag{1.1}$$

We accordingly formulate the following $\mathbf{Z}_k$ Refined Strong $\mathrm{AdS}_4$ Distance Conjecture: in supersymmetric $\mathrm{AdS}_4$ vacua with a discrete symmetry associated to $\mathbf{Z}_k$-charged domain walls, the ratio between the KK scale and $\Lambda$ is $m_{\mathrm{KK}} \sim (k\Lambda)^{1/2}$.

This provides an underlying rationale for the seeming violation of the strong ADC by the family of scaling AdS solutions in type IIA vacua with field strength fluxes. It would be interesting to test it in other setups, and even exploit it in applications to holography.

Our work is an important step in understanding the nature of discrete gauge symmetries in quantum gravity, and their non-trivial interplay with continuous gauge symmetries. As in other swampland constraints, although the arguments for the $\mathbf{Z}_k$-WCC are admittedly heuristic, there is a substantial amount of evidence from concrete, very rigorous, string vacua supporting it. We have argued that discrete symmetries for 3-forms play an important role in the problem of scale separation, and provided a rationale to embed it in a refined AdS Distance Conjecture. We thus expect they may be relevant in other swampland criteria, like the de Sitter constraint. We hope to report on these topics in the near future.

**Note:** as we were finishing writing this paper, ref. [37] appeared, which studies scale separation in type IIA AdS vacua, albeit from a different perspective (note also [38], appeared shortly after our work). It would be interesting to explore the relation between the two approaches.

## 2 The $\mathbf{Z}_k$ weak coupling conjecture

In this section we consider theories of quantum gravity with discrete and continuous gauge symmetries. For simplicity we focus on a $\mathbf{Z}_k$ discrete symmetry and a U(1) gauge symmetry. Generalizations to multiple U(1)'s and discrete groups could be worked out similarly. Notice that throughout the paper we are interested in the properties of the theory at large $k$, hence many of our expressions should be regarded as the leading approximation in a $1/k$ expansion.

### 2.1 A black hole argument

For concreteness we focus on 4d theories, although the results extend to other dimensions (as we will see e.g. in the examples of section 3). The strategy is to use black hole evaporation as a guiding principle to derive new swampland constraints, as we now review in two familiar situations.

#### 2.1.1 Review of some mass bound derivations

Let us briefly recall one such derivation for the Weak Gravity Conjecture (WGC) [3]. The idea is to consider extremal black holes, with mass $M$ and charge $Q$, satisfying $M = gQM_p$, where $g$ is the U(1) gauge coupling (in units in which the minimal charge is 1). Requiring the decay of such extremal black holes, while preventing them from becoming super-extremal, leads to the familiar statement of the Weak Gravity Conjecture, namely that there must exist some particle in the theory with mass $m$ and charge $q$ such that

$$m \leq g\, q\, M_p. \tag{2.1}$$

There are different versions of the WGC (see [8] for a review with references), including the lattice [39] and sublattice [40] versions, but we stick to the basic one above.

Let us consider a black hole (possibly charged under the U(1) or not), carrying a discrete $\mathbf{Z}_k$ charge. The analysis now follows [41]. Even though this is a gauge symmetry, it does not have long-range fields, so it does not affect the classical black hole solution, neither its evaporation in the semiclassical approximation, which thus does not allow to eliminate the $\mathbf{Z}_k$ charge. Since we are interested in the large $k$ behavior, this would lead to a too large number of remnants. Hence, when the black hole radius reaches some cutoff value $\Lambda^{-1}$ it starts peeling off its $\mathbf{Z}_k$ charge. If we denote by $m$ the mass of the $\mathbf{Z}_k$ charged particles, the mass of the black hole at the cutoff scale should suffice to emit $\mathcal{O}(k)$ of such particles, that is

$$M_p^2\,\Lambda^{-1}\,\gtrsim\,k\,m\,. \tag{2.2}$$

The cutoff radius is intuitively of the order of the inverse mass of the emitted particle, hence we consider $\Lambda\sim\beta m$, with $\beta$ some unknown coefficient encoding model dependent information about the black hole and its evaporation process. Consequently, we obtain

$$m^2\,\lesssim\,\frac{M_p^2}{k}\,. \tag{2.3}$$

This is often known as the species bound [41], although in the present context $k$ does not correspond to the number of species, rather it relates to the order of the discrete symmetry.[2]

Keeping in mind the unknown factors in the discussion, we take the above relation as controlling the scaling of suitable $\mathbf{Z}_k$ charged particles in the limit of large $k$. Namely, there must exist some $\mathbf{Z}_k$ charged particle whose mass must scale as $m\lesssim k^{-1/2}M_p$.

In the following, we will apply this constraint to black holes charged under continuous U(1) symmetries. One may worry that the derivation in [41] did not include such charges, i.e. it implicitly assumed Schwarzschild black holes. However, there are analogous arguments for charged (in fact extremal) black holes in theories with U(1) gauge groups, leading to identical results, as we discuss in appendix A. Hence for practical purposes we may continue with the above simple picture.

#### 2.1.2 The $\mathbf{Z}_k$ weak gravity conjecture

In the above discussion, the mass of the $\mathbf{Z}_k$ particle we are constraining is thought of as the lightest one. However, in the following we argue that we can use a similar argument to constrain not only the lightest $\mathbf{Z}_k$ charge particle, but also the one with smallest ratio $q/m$ between its U(1) charge and its mass. Namely, the Weak Gravity Conjecture particles.

Consider an extremal black hole with mass $M$ and charge $Q$, and endow it with a large $\mathbf{Z}_k$ charge. The black hole can try to peel off its $\mathbf{Z}_k$ charge by emitting $\mathbf{Z}_k$ charged particles, but this would decrease its mass while keeping its charge fixed, thus becoming super-extremal. The simplest way to prevent this is that there exist some $\mathbf{Z}_k$ charged particle which is also charged under the U(1) with charge $q$, and such that it satisfies the WGC bound $m\leq gqM_p$. In other words, the simplest resolution is that the WGC particles carry $\mathbf{Z}_k$ charge. We may dub this result as the $\mathbf{Z}_k$ *Weak Gravity Conjecture*.

This is a remarkable result, but is actually a little bit of an overstatement. It may well happen that the WGC particles are neutral and do not saturate the WGC bound, and the evaporation of the black hole by emission of WGC particles makes it sufficiently sub-extremal so as to be able to subsequently emit enough $\mathbf{Z}_k$ charged particles (not obeying the WGC bound) to peel off its discrete charge without ever getting super-extremal.

[2] Actually, to account for the fact that the particle needs not be minimally charged under $\mathbf{Z}_k$, we should point out that the role of $k$ above should actually be played by the number of emitted particles. Hence the factor appearing in relations like (2.3) may differ from the order of the discrete group by a factor of the particle charge, see some examples in sections 4, 5.

Interestingly, notice that this is only possible if the WGC particles satisfy the strict WGC bound, not the equality, and hence, according to the extended WGC version in [4], it is possible only in non-supersymmetric theories. Thus our derivation above is strictly valid in the supersymmetric setup, and in our examples we will indeed focus on supersymmetric examples. We however still consider the argument as interestingly compelling also in non-supersymmetric models, and hence keep an open mind about its general validity, and that of its implications, to which we turn.

#### 2.1.3 The $\mathbf{Z}_k$ weak coupling conjecture

The fact that the WGC particles, whose defining feature has to do with the U(1) gauge symmetry, know about the $\mathbf{Z}_k$ symmetry implies that there are cross constraints among the U(1) and the $\mathbf{Z}_k$ symmetry. Indeed, let us consider a relaxed version of the $\mathbf{Z}_k$ bound (2.3), by stating that the $\mathbf{Z}_k$ charged particles involved in the black hole decay should have mass scaling as

$$m \sim k^{-\alpha} M_p \,, \tag{2.4}$$

with $\alpha$ an order 1 coefficient, obeying some bound $\alpha \geq 1/2$ to satisfy (2.3). On the other hand, the particles that extremal black holes use to peel off their $\mathbf{Z}_k$ charge are WGC particles, hence obey

$$m \sim g\, q\, M_p \,. \tag{2.5}$$

We thus obtain that the gauge coupling of the U(1) must depend on $k$ and should become weak fast enough in the large $k$ limit, as

$$g \sim k^{-\alpha} \,. \tag{2.6}$$

We thus propose this to be a general swampland constraint, as follows:

**$\mathbf{Z}_k$ weak coupling conjecture.** *In a quantum gravity theory with a discrete $\mathbf{Z}_k$ gauge symmetry and a* U(1) *gauge symmetry with coupling $g$, the gauge coupling scales as $g \sim k^{-\alpha}$ for large $k$, with $\alpha$ a positive order 1 coefficient.*

We note that, in the case of multiple U(1) gauge symmetries, a similar BH argument leads to a $\mathbf{Z}_k$ Weak Coupling Conjecture for any rational direction in charge space, much in the spirit of the WGC for multiple U(1)'s [42]. Since the gauge coupling of any linear combination follows from those in some basis in the charge lattice, in this case it suffices that the couplings of these independent U(1) obey the $\mathbf{Z}_k$ Weak Coupling Conjecture. We also note that in the case of multiple discrete symmetries, the conjecture applies to each discrete symmetry independently.

The above intertwining between the properties of discrete and continuous symmetries is completely unexpected from the viewpoint of the low energy effective field theory, where these parameters are uncorrelated and would seem to be completely free choices. As with other swampland constraints, it is amusing that quantum gravity manages to impose its own plans.

A simple illustration of how this interplay works in intersecting brane modes is discussed at the heuristic level in appendix B. More concrete examples will follow in the upcoming sections.

### 2.2 Distance conjectures

Before moving to concrete examples, it is interesting to explore the relation between the $\mathbf{Z}_k$ WCC and the Swampland Distance Conjectures (SDC). The WCC states that gauge couplings scale to zero for large $k$, thus approaching a global symmetry and hence presumably leading to the appearance of a tower of states becoming light.

An intuitive picture of this implication is as follows. Consider a 4d version of the $\mathbf{Z}_k$ WCC with $g \sim k^{-\alpha}$. For simplicity, and following many examples in string theory we consider $g$ to belong to a complex modulus

$$S = \frac{1}{g^2} + i\theta \tag{2.7}$$

and assume a Kähler potential

$$K(S, \bar{S}) = -\log(S + \bar{S}) \ . \tag{2.8}$$

In this moduli space, the distance as a function of $s = \text{Re}\, S$ as one approaches infinity reads

$$d \sim \int \frac{ds}{s} \sim \log s \ . \tag{2.9}$$

The SDC states that there is a tower of states becoming light as $s \to \infty$ with masses

$$m_{\text{tw}} \sim M_p \, e^{-\gamma d} \ , \tag{2.10}$$

with $\gamma$ an order 1 coefficient, for $d$ measured in Planck units. In our case we have

$$m_{\text{tw}} \sim \ M_p \, k^{-\frac{1}{2}\alpha\gamma} \ . \tag{2.11}$$

Hence there is a $\mathbf{Z}_k$ Distance Conjecture stating that there is a tower of states with masses becoming light as a negative power of $k$. This is just a re-derivation of the 'species' bound cutoff [41].

In fact, the above argument where $g$ is dealt with as a modulus going to infinite distance in moduli space does not correspond to the general $\mathbf{Z}_k$ WCC, since at least some of the gauge couplings may not correspond to fundamental moduli. For instance, consider the intersecting brane toy model in appendix B. There, the moduli remain at fixed location in moduli space, and we instead change the discrete wrapping numbers for some D-branes. Hence, the origin of the tower should be a different one, as is easily argued. In a configuration in which one stack of branes has wrappings scaling with $k$, the angles between that stack of branes and others will scale as $\theta \sim k^{-1}$ (to see that, consider e.g. the cycles $(1, 0)$ and $(k, 1)$ in a rectangular $\mathbf{T}^2$ with radii $(R_1, R_2)$. They have intersection angle $\theta$ with $\tan\theta = k^{-1}R_2/R_1$, hence $\theta \sim k^{-1}$). As discussed in [43, 44] there is a tower of string states with masses given by

$$m_{\text{tw}}^2 \sim M_s \, \theta \sim k^{-1} \ . \tag{2.12}$$

This again nicely reproduces the 'species' bound cutoff.

## 3 $\mathbf{AdS_5 \times S^5}$ orbifolds

In this section we consider type IIB string theory on $\mathrm{AdS}_5 \times \mathbf{S}^5/\mathbf{Z}_k$. The discussion can be easily extended to general toric orbifold theories $\mathrm{AdS}_5 \times \mathbf{X}_5/\mathbf{Z}_k$, but the 5-sphere case will suffice to illustrate the main points. We study general $\mathbf{Z}_k$ actions compatible with supersymmetry, namely acting as SU(3) in the underlying $\mathbf{C}^3$. We also note that, although these vacua do not display scale separation, we may discuss the 5d physics essentially in the same sense as in the AdS/CFT correspondence, whose dictionary and results we use freely in this section. Moreover, our final statement involves gauge couplings for U(1) symmetries, which can be observed at arbitrarily long distances, in particular at energies well below the KK scale.

As pioneered in [29] (see also [30, 45–47] for other examples) and generalized in [31], there is a discrete gauge symmetry in the $\mathrm{AdS}_5$ theory, corresponding to the discrete Heisenberg group $\mathbf{H}_k$. This is defined by two non-commuting $\mathbf{Z}_k$ symmetries generated by $A$, $B$ (hence $A^k = 1$, $B^k = 1$) satisfying

$$AB = CBA \, , \tag{3.1}$$

with $C$ a central element (also generating a further $\mathbf{Z}_k$, and possibly mixing with other anomaly free baryonic U(1)'s, if present).

Generalizing [29], the particles charged under the discrete symmetry are D3-branes wrapped on torsion 3-cycles carrying non-trivial flat gauge bundles (discrete Wilson lines and 't Hooft loops). The minimally charged particle is obtained by wrapping the D3-brane on a maximal $\mathbf{S}^3/\mathbf{Z}_k$. We are interested in the mass of this particle, and in particular in its scaling with $k$. It is a simple exercise, as this is just analogous to a giant graviton in the parent $\mathrm{AdS}_5 \times \mathbf{S}^5$ theory [48].

**The D3-brane particle mass computation.** In the KK reduction from 10d to 5d, the 5d Planck mass $M_{p,5}$ in terms of the string scale is

$$M_{p,5}^3 = \frac{M_s^8 R^5}{g_s^2 \, k} \, . \tag{3.2}$$

We are ignoring numerical factors e.g. in the volume of $\mathbf{S}^5$. Above, $R$ is the curvature radius of $\mathbf{S}^5$, which is also the $\mathrm{AdS}_5$ radius. Note that in order to get a theory with $N$ units of RR 5-form flux over $\mathbf{S}^5/\mathbf{Z}_k$, the parent theory is the $\mathrm{AdS}_5 \times \mathbf{S}^5$ solution corresponding to $Nk$ D3-branes, and the usual relation between the radius $R$ and $N$ is modified to

$$R^4 = 4\pi(\alpha')^2 \, g_s \, N \, k \, . \tag{3.3}$$

Hence

$$R \sim M_s^{-1} g_s^{\frac{1}{4}} \, N^{\frac{1}{4}} \, k^{\frac{1}{4}} \, , \tag{3.4}$$

where we have dropped numerical factors.



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The mass $m$ of the D3-brane particle[3] in 5d is

$$m = \frac{M_s^4 R^3}{g_s\, k} \ . \tag{3.5}$$

We wish to express the mass in terms of the 5d Planck scale. From (3.2) and (3.4) we get

$$M_s \sim M_{p,5}\, g_s^{\frac{1}{4}}\, N^{-\frac{5}{12}}\, k^{-\frac{1}{12}} \quad , \quad R \sim M_{p,5}^{-1}\, N^{\frac{2}{3}}\, k^{\frac{1}{3}} \ . \tag{3.6}$$

Hence

$$m \sim M_{p,5}\, N^{\frac{1}{3}}\, k^{-\frac{1}{3}} \ . \tag{3.7}$$

Note that the $k$-dependence reproduces the 5d version of the relation (2.3) [41]

$$m^3 \sim \frac{M_{p,5}^3}{k} \ . \tag{3.8}$$

This result fits nicely with the expectation for the mass of a particle charged under $\mathbf{Z}_k$.

Notice that, as mentioned in section 2.1, the coefficient in (3.8) is not necessarily the order of the discrete symmetry (which we recall is the Heisenberg group $\mathbf{H}_k$) but the number of particles emitted to peel off the black hole charge. We also note that the factor of $N$ in (3.7) is presumably related to the precise nature of the cutoff $\Lambda$ in the black hole argument in section 2.1.1. It would be interesting to explore this dependence in more detail, but we leave this for future work.

**Comparison with the BPS formula and WCC.** The above states are not the lightest carrying charges under the $\mathbf{Z}_k$ subgroups of the Heisenberg group. In fact, there are charged particle states arising from fundamental strings and D1-branes wrapped on torsion 1-cycles on the internal geometry. What is special about the above D3-brane particle states is that they are BPS. Just like giant gravitons in $\mathrm{AdS}_5 \times \mathbf{S}^5$, they carry $N$ units of momentum along a maximal $\mathbf{S}^1$, determined by the $\mathbf{Z}_k$ action. In the 5d theory, there is a KK $\mathrm{U}(1)_R$, which is precisely the gravity dual of the R-symmetry of the holographic SCFT. In the SCFT, the D3-brane particle states are dibaryons of the form $\det \Phi_{ij}$, with $\Phi$ denoting a generic bifundamental chiral multiplet in the quiver gauge theory. It has R-charge $N$, and conformal dimension $\Delta = N$. Using the AdS/CFT dictionary, we then expect the masses of these particles to be given by

$$m = \frac{N}{R} \ . \tag{3.9}$$

The fact that these states are BPS means that they should saturate the WGC conjecture bound, in other words, the BPS mass formula

$$m \ = \ (g\, M_{p,5}^{\frac{1}{2}})\, N M_{p,5} \ . \tag{3.10}$$

[3]Notice that for our purposes it does not matter if we are in the string or Einstein frame, since this introduces factors that depend on dynamical fields, but does not change the scaling with $k$, which goes into the constant part (reference value).

This is the standard $m = gQ$ in Planck units, with charge $q = N$ and $g$ being the gauge coupling of the U(1).

In these relations, there is no manifest dependence on $k$, which could be puzzling from the viewpoint of the black hole arguments. As we however know, the resolution is that, on these general grounds, the gauge coupling $g$ *must* scale with $k$, at large $k$, in particular

$$g \sim k^{-\frac{1}{3}} \ . \tag{3.11}$$

This is easily checked by computing the gauge coupling. In the KK reduction from 10d to 5d, the prefactor of the gauge kinetic term is

$$\frac{1}{g^2} = \frac{M_s^8 \, R^5}{g_s^2 \, k} \, R^2 \ . \tag{3.12}$$

The first factor is just the 10d prefactor times the volume of $\mathbf{S}^5/\mathbf{Z}_k$, and the $R^2$ comes from the rescaling of mixed components of the metric into dimensionful gauge field, such that charges are quantized in integers.

Using our above expressions, we get

$$g \sim R^{-1} \, M_{p,5}^{-\frac{3}{2}} \ , \tag{3.13}$$

which means

$$g \, M_{p,5}^{\frac{1}{2}} = N^{-\frac{2}{3}} \, k^{-\frac{1}{3}} \ . \tag{3.14}$$

So, in terms of this gauge coupling, the mass (3.7) turns into (3.10). Hence we recover a very explicit confirmation of our heuristic argument in section 2.

Let us conclude with some general remarks.

- In addition to $\mathrm{U}(1)_R$ there are in general (in fact, for general toric theories) two extra mesonic U(1) symmetries, arising from isometries of the internal 5d manifold. The direct computation of their 5d gauge couplings proceeds as above, thus leading to a scaling compatible with the WCC.

- In addition to D3-brane charged particles, there are 5d membranes of real codimension 2, which implement monodromies associated to the discrete group elements. As in the abelian case, these objects are charged under a dual discrete gauge symmetry (this can be made more manifest by introducing non-harmonic forms to represent the torsion classes [22, 49]). However, since these objects are not charged under any continuous symmetry, we lack a good handle to constrain their properties, and we will not discuss them further.

**The $\mathbf{Z}_k$ distance conjectures.** It is interesting to explore the relation between the $\mathbf{Z}_k$ WCC and the AdS Distance Conjecture in the present setup where, using (3.6), going to large $k$ implies going to large $R$. This is a decompactification limit (note that the orbifold only reduces lengths in $\mathbf{S}^5$ in some directions, so the KK scale remains $R^{-1}$), in which also the AdS cosmological constant goes to zero, approaching flat space. Hence we can apply



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the AdS Distance Conjecture, which e.g. in its strong version (as we have supersymmetry) establishes that there should be a tower of states with masses scaling as

$$m_{\rm tw} \sim \frac{1}{R} \sim M_{p,5}\, N^{-\frac{2}{3}}\, k^{-\frac{1}{3}}\ , \tag{3.15}$$

where we have also kept the dependence on $N$. From the $1/R$ dependence, it is clear the tower corresponds to KK modes. These are the familiar particles dual to single trace chiral primary mesonic operators of the dual SCFT, extensively studied in the literature [50], see [51]. Note that, even though the scaling with $k$ is the same as for wrapped D3-branes, KK modes are lighter due to the relative factor of $N$.

**A further subtlety.** The above discussion has overlooked an important subtlety. The discrete symmetry $\mathbf{Z}_k$ (in fact the full discrete Heisenberg group) is intertwined with the U(1) in the following sense. Since the D3-branes are charged under the U(1) with charge $N$, a set of $k$ D3-branes carries no discrete $\mathbf{Z}_k$ charge, but carries $kN$ units of momentum and cannot decay to the vacuum. In fact, the instanton processes removing the discrete $\mathbf{Z}_k$ charge (which correspond to a D3-brane wrapped on the 4-chain whose boundary is $k$ times the torsion 3-cycle) produce simultaneously $N$ particles each carrying momentum $k$ on the circle (whose radius is $R/k$ due to the orbifold).

The situation is very analogous to the one we will encounter in M-theory and type IIA compactifications in section 4, so we postpone the discussion. Suffice it to say that in this kind of situation, the actual discrete symmetry has order $k^2 + N^2$, heuristically corresponding to the fact that the discrete charge may be eliminated via emission of $k$ D3-branes (each with charge $k$ under the discrete group) and $N$ KK modes (each with charge $N$ under the discrete group). In the regime where the gravity description of $\mathbf{S}^5/\mathbf{Z}_k$ is valid, we need large $R^4 \sim Nk$ and large $R/k \sim N^{1/4}k^{-3/4}$, hence $N \gg k^3$, and the order of the gauge group is effectively dominated by the $N^2$ term, corresponding to emission of $N$ KK modes. Hence, the actual discrete symmetry in this regime is an effective $\mathbf{Z}_N$.

It is straightforward to repeat the above computations for the KK mode particles. The mass is given by $k/R$, as corresponds to mesonic operators of dimension $k$ (or multiples of it) due to the orbifold action. We obtain the relations and scalings

$$m \,\sim\, M_{p,5}\, N^{-\frac{2}{3}}\, k^{\frac{2}{3}} \quad , \quad m \sim g\, k\, M_{p,5}^{3/2} \quad , \quad g\, M_{p,5}^{\frac{1}{2}} \,=\, N^{-\frac{2}{3}}\, k^{-\frac{1}{3}}\ . \tag{3.16}$$

Here $g$ is obviously the same as in (3.14), but we repeat it for convenience. Happily, it is clear that $g$ obeys a $\mathbf{Z}_N$ WCC. Notice also that the discretely charged KK modes fit more nicely with the black hole argument in section 2.1. It seems more manageable to emit KK particles than D3-brane particles, as the later extend to a very large size in the internal dimension.

As anticipated, we will re-encounter a very similar situation in M-theory compactifications in the next section, with the additional handle of a type IIA reduction which makes these aspects far more intuitive. We refer the reader to those sections for details.

## 4 M-theory orbifolds and ABJM

In this section we study the WCC in M-theory on $\mathrm{AdS}_4 \times \mathbf{S}^7/\mathbf{Z}_k$ and its type IIA reduction, which provide the gravity dual of the ABJM gauge theories [32]. These theories display interesting new subtleties as compared with earlier cases. Some have been partially discussed in the ABJM literature, so we can again profit from the holographic dictionary.

### 4.1 M-theory on $\mathrm{AdS}_4 \times \mathbf{S}^7/\mathbf{Z}_k$

Let us now consider M-theory on $\mathrm{AdS}_4 \times \mathbf{S}^7/\mathbf{Z}_k$, where $\mathbf{Z}_k$ is generated by $z_i \to e^{2\pi i/k} z_i$ in the underlying $\mathbf{C}^4$. This theory is the dual to the ABJM theories, which correspond to $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ Chern-Simons matter theories,[4] with $\pm k$ denoting the CS level.

The curvature radius of the covering $\mathbf{S}^7$ and the $\mathrm{AdS}_4$ are given by

$$R^6 \,=\, 2^5 \pi^2 M_{p,\,11}^{-6} N k \ , \tag{4.1}$$

where the factor of $Nk$ is analogous to that in section 3.

We are interested in studying gauge symmetries in the 4d theory. The 4d Planck scale is given by

$$M_{p,\,4}^{\,2} \,=\, \frac{M_{p,\,11}^{\,9} R^7}{k} \ . \tag{4.2}$$

Hence we have

$$R \,\sim\, M_{p,\,11}^{-1}\, N^{\frac{1}{6}}\, k^{\frac{1}{6}} \ , \tag{4.3}$$

and then

$$M_{p,\,11} \,\sim\, M_{p,\,4}\, N^{-\frac{7}{12}}\, k^{-\frac{1}{12}} \quad , \quad R \sim M_{p,\,4}^{-1}\, N^{\frac{3}{4}} k^{\frac{1}{4}} \ . \tag{4.4}$$

There are two relevant symmetries. There is a U(1) isometry, surviving from the underlying isometry of $\mathbf{S}^7$ which decomposes as $\mathrm{SO}(8) \to \mathrm{SU}(4) \times \mathrm{U}(1)$ under the orbifold action $z_i \to e^{2\pi i/k} z_i$. It is a continuous gauge symmetry in $\mathrm{AdS}_4$. In addition, the internal space has a non-trivial torsion group $H_5\left(\mathbf{S}^7/\mathbf{Z}_k\right) = \mathbf{Z}_k$ which allows to obtain 4d particles by wrapping M5-branes on the torsion 5-cycle. In the covering space the minimal charge particle is essentially an M5-brane giant graviton, similar to those in the $\mathrm{AdS}_4 \times \mathbf{S}^7$ theory. In particular, it carries $N$ units of momentum on the $\mathbf{S}^1$ associated to the U(1) symmetry.

This seems a perfect candidate for a WGC particle charged under the discrete symmetry, so we consider its properties, in analogy with the D3-brane particles in section 3. Its mass is given by

$$m_{\mathrm{M5}} \sim \frac{M_{p,\,11}^{\,6}\, R^5}{k} \,=\, M_{p,\,4}\, N^{\frac{1}{4}}\, k^{-\frac{1}{4}} \ , \tag{4.5}$$

[4] Actually, as mentioned below and pointed out in [32] the global structure is different such that there are gauge invariant dibaryons for arbitrary $N$, $k$.

where, in the last equation, we have used (4.4). Note that we recover the AdS/CFT dictionary relation

$$m_{\rm M5} \,=\, \frac{N}{R}\ , \tag{4.6}$$

indicating that the M5-brane particle is dual to an operator of conformal dimension $N$, as befits a dibaryon.

We can compare this mass with the WGC bound (BPS bound), by computing the gauge coupling. This is just given by the KK reduction of the 11d Einstein terms and gives

$$g^{-2} \,\sim\, M_{p,\,11}^{9}\,(\,R^7\,k^{-1}\,)\,R^2\ . \tag{4.7}$$

Note that we have taken the normalization factor $R^2$, which holds when $\gcd(N,k)=1$. This is because in that normalization, the charges under the U(1) are KK modes of momentum multiple of $k$ (since the radius is $R/k$ due to the orbifold action), and M5-branes, whose charges are multiples of $N$. Then by Bezout's lemma, the minimal charge quantum is 1. For the general case $\gcd(N,k)=r$, we would have a factor $(R/r)^2$. We proceed with the coprime case in what follows. As pointed out in [32], the existence of gauge invariant dibaryon operators for general $N$ (not a multiple of $k$) implies a specific choice of the global structure of the gauge group of the holographically dual ABJM field theory, see footnote 4.

Using (4.4) we have

$$g^{-2} \,\sim\, N^{\frac{3}{2}}\,k^{\frac{1}{2}} \quad\rightarrow\quad g\,\sim\, N^{-\frac{3}{4}}\,k^{-\frac{1}{4}}\ . \tag{4.8}$$

So we get the WGC/BPS relation

$$m_{\rm M5} \,=\, M_{p,\,4}\,g\,N\ . \tag{4.9}$$

It is interesting that in the large $k$ limit we recover a weak coupling scaling result $g\sim k^{-1/4}$, but that this decrease is slower than the critical $g\sim k^{-1/2}$ required by the black hole evaporation argument. The resolution of this point reveals two interesting related subtleties: the actual discrete gauge symmetry of the theory is not just $\mathbf{Z}_k$, and the wrapped M5-branes are not the only states charged under the discrete symmetry. Indeed, as mentioned in [32], a set of $k$ wrapped M5-brane particles can unwrap, but they do not decay to the vacuum, but rather turn into $N$ KK states with momentum along the U(1) circle (which, due to the $\mathbf{Z}_k$ orbifold, is quantized in multiples of $k$). In other words, there are instantons (given by M5-branes wrapped on the $\mathbf{CP}^3$ base of the Hopf fibration of $\mathbf{S}^7/\mathbf{Z}_k$), which emit $k$ M5-branes and $N$ minimal momentum KK modes. As will be more intuitively explained in section 4.2, there is a discrete symmetry of order $N^2+k^2$, under which a wrapped M5-brane has charge $k$ and a minimal momentum KK mode has charge $N$. Thus KK modes provide a possible alternative to allow for black hole decay, which in fact is dominated by processes of emission of $N$ such KK modes. Hence, the gauge coupling needs to obey a WCC with respect to $N$. Let us thus check this point.

The KK particle mass is given by

$$m_{\rm KK} \,=\, \frac{k}{R}\ . \tag{4.10}$$

This in fact constitutes the holographic dictionary relation for an operator of conformal dimension $k$. These are constructed with $k$ copies of a bifundamental field, as required by gauge invariance under the level-$k$ U(1)'s of the holographic dual field theory [32].

Using (4.4) we have

$$m_{\rm KK} \;=\; M_{p,4}\, N^{-\frac{3}{4}}\, k^{\frac{3}{4}} \;, \tag{4.11}$$

and with (4.8) we obtain

$$m_{\rm KK} \;=\; M_{p,4}\, g\, k \;. \tag{4.12}$$

Hence these are WGC particles charged under the discrete symmetry, and the gauge coupling (4.8) obeys a WCC bound with respect to $N$.

### 4.2 Type IIA description of ABJM vacua

We may now describe the type IIA version of the previous section, which makes some of the above points more intuitive, and also provides a good warm-up for coming sections.

The type IIA limit arises as follows. The $\mathbf{S}^7$ is a $\mathbf{S}^1$ Hopf fibration over $\mathbf{CP}^3$, where the $\mathbf{Z}_k$ quotient acts on the $\mathbf{S}^1$. The radius of the $\mathbf{CP}^3$ factor is large whenever $Nk \gg 1$. From (4.1) we conclude that the M-theory description is valid whenever $k^5 \ll N$. When $k$ increases, we end up in a weakly coupled regime and we can reduce to type IIA string theory [32].

The type IIA background corresponds to a compactification on $\mathrm{AdS}_4 \times \mathbf{CP}^3$ with internal and AdS radii $R_s$ (see below), with $N$ units of $F_6$ RR flux over $\mathbf{CP}^3$ (i.e. of $F_4$ flux over $\mathrm{AdS}_4$) and $k$ units of $F_2$ RR flux over $\mathbf{CP}^1 \subset \mathbf{CP}^3$ (due to the Hopf fibration of the M-theory $\mathbf{S}^1$).

The matching of string theory quantities to the 11d Planck scale is as follows. The 10d string coupling $g_s$ is related to the M-theory radius $\mathcal{R} = R/k$ as

$$g_s = M_{p,11}^{3/2}\, \mathcal{R}^{3/2} \;, \tag{4.13}$$

that scales as

$$g_s \sim N^{\frac{1}{4}}\, k^{-\frac{5}{4}} \;. \tag{4.14}$$

The string scale $M_s$ is related to the 11d Planck scale as

$$M_{p,11}^3 \;=\; \frac{M_s^3}{g_s} \;. \tag{4.15}$$

So in terms of $M_s$, the radius (4.1) becomes

$$R \sim N^{\frac{1}{6}}\, k^{\frac{1}{6}}\, g_s^{\frac{1}{3}}\, M_s^{-1} \;. \tag{4.16}$$

Finally we need the radius $R_s$ of $\mathbf{CP}^3$ from the string viewpoint. The type IIA metric is given by

$$ds^2_{IIA} = R_s^2 \left( \frac{1}{4} ds^2_{\mathrm{AdS}_4} + ds^2_{\mathbf{CP}^3} \right) \;, \tag{4.17}$$

where

$$R_s^2 \,\sim\, N^{1/2}\, k^{-1/2}\, M_s^{-2} \ . \tag{4.18}$$

We can now compute the 4d Planck mass:

$$M_{p,\,4}^4 \sim M_s^8\, g_s^{-2}\, R_s^6 \ , \tag{4.19}$$

and combine with (4.18), (4.19) and (4.14) to obtain

$$R_s \,\sim\, N\, k^{-1}\, M_{p,\,4}^{-1}\, g_s^{-1} \sim M_{p,\,4}^{-1}\, N^{\frac{3}{4}}\, k^{\frac{1}{4}} \ , \quad M_s \,\sim\, N^{-3/4}\, k^{3/4}\, M_{p,\,4}\, g_s \,\sim\, M_{p,\,4}\, N^{-\frac{1}{2}}\, k^{-\frac{1}{2}} \ . \tag{4.20}$$

Let us now consider the gauge symmetries in the 4d theory in this type IIA string compactification. The SU(4) symmetry arises as the isometry of the internal $\mathbf{CP}^3$. On the other hand, there are additional U(1) gauge fields arising from the 10d RR fields, concretely the 10d RR 1-form potential and the 10d RR 3-form potential integrated over $\mathbf{CP}^1 \subset \mathbf{CP}^3$. We should however notice that there are Stückelberg couplings arising from the 10d Chern-Simons coupling $B_2 F_2 F_6$, of the form[5]

$$N\, B_2\, F_2 \,+\, k\, B_2 F_2' \ , \tag{4.21}$$

where $F_2' = \int_{\mathbf{CP}^2} F_6$. This implies that the massless U(1) linear combination is

$$J = kQ_0 - NQ_4 \ . \tag{4.22}$$

Here the generators $Q_0$, $Q_4$ are labeled by the objects charged under the corresponding U(1)'s, namely D0-branes and D4-branes wrapped on $\mathbf{CP}^2$. Note that our sign convention differs from [32].

The orthogonal linear combination,

$$Q_{\rm broken} \,=\, NQ_0 + kQ_4 \ , \tag{4.23}$$

corresponds to a massive U(1), which is broken by instanton effects, and only a discrete subgroup remains. The instanton corresponds to an NS5-brane wrapped on $\mathbf{CP}^3$, since it couples magnetically to $B_2$. It suffers from Freed-Witten anomalies due to the $F_6$ and $F_2$ fluxes, so it emits $N$ D0-branes and $k$ wrapped D4-branes. Hence, the total violation of $Q_{\rm broken}$ is $N^2 + k^2$. This is the order of the gauge group. However, notice that at the level of the black hole (and of the WCC), what is actually relevant is the number of particles required to be emitted, namely $N$ D0-branes (contributing charge $N$ each) and $k$ D4-branes (contributing charge $k$ each). The type IIA internal space is large compared with the string scale if $N \gg k$, so the limit of large order of the discrete gauge group scales as $N^2$ and the black hole decay is dominated by the emission of $N$ D0-branes. In the arguments below, this is one particular instance in which the relevant coefficient in scaling relations is not the order of the discrete symmetry, but the number of emitted particles.

Notice also that we are recovering in possibly more intuitive terms the discussion of the earlier M-theory setup, with wrapped D4-branes corresponding to wrapped M5-branes and D0-branes corresponding to KK modes of the M-theory circle.

[5]For further discussion of Chern-Simons couplings and swampland constraints see [52].



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Let us discuss the masses of the D4- and D0-brane particles and the U(1) gauge couplings. They scale as

$$
\begin{aligned}
m_{\rm D0} &= g_s^{-1}\, M_s \sim M_{p,4}\, N^{-\frac{3}{4}}\, k^{\frac{3}{4}}, \\
m_{\rm D4} &= g_s^{-1}\, M_s^5\, R_s^4 \sim M_{p,4}\, N^{\frac{1}{4}}\, k^{-\frac{1}{4}} \ .
\end{aligned} \tag{4.24}
$$

We already notice that the D0-brane mass decreases with $N$ faster than the 'species' bound reviewed in section 2.1.1, ensuring that black holes can get rid of their discrete charge by emitting D0-branes. Let us turn to check the implication for gauge couplings and verify the $\mathbf{Z}_N$ WCC.

The 4d gauge couplings for the U(1)'s generated by $Q_0$ and $Q_4$ are given by

$$
\begin{aligned}
\frac{1}{g_0^2} &\sim M_s^8\, R_s^6\, M_s^{-2} \\
\frac{1}{g_4^2} &\sim M_s^8\, R_s^6\, (\, M_s^{-5}\, R_s^{-4}\,)^{-2} \ .
\end{aligned} \tag{4.25}
$$

The first common factor arises from the reduction of the 10d kinetic term for RR fields on the $\mathbf{CP}^3$, while the last factors arise from the normalization of the gauge fields by the coefficient of the D-brane Chern-Simons term, so that charges are integer numbers. Using the familiar relations above, we obtain the scalings

$$
g_0^{-2} \sim N^{3/2} k^{-3/2} \quad , \quad g_4^{-2} \sim N^{-1/2} k^{1/2} \ . \tag{4.26}
$$

The coupling constant associated to the massless combination (4.22) is

$$
g^{-2} = \frac{k^2}{g_0^2} + \frac{N^2}{g_4^2} \sim N^{\frac{3}{2}} k^{\frac{1}{2}} \ , \tag{4.27}
$$

and, as explained, its scaling satisfies the WCC with respect to $N$

$$
g \sim N^{-\frac{3}{4}} k^{-\frac{1}{4}} \ . \tag{4.28}
$$

As expected, the D0- and D4-brane particles satisfy the BPS/WGC bound, in agreement with the result for wrapped M5-branes and KK modes in (4.9), (4.12)

$$
m_{\rm D4} = M_{p,4} g\, N \quad , \quad m_{\rm D0} = M_{p,4}\, g\, k \ . \tag{4.29}
$$

Notice also that $g \sim 1/R$ in Planck units, so the above masses imply conformal dimensions $N$ and $k$ for the holographically dual operators, as is by now familiar.

## 5 Discrete 3-form symmetries and scale separation in AdS solutions

In [34] it is proposed that in AdS vacua with cosmological constant $\Lambda$, the limit $\Lambda \to 0$ is accompanied by a tower of states becoming light as

$$
m \sim |\Lambda|^{\alpha} \ . \tag{5.1}
$$

The strong version of this conjecture is that $\alpha = 1/2$, which is the case in many/most string solutions (see below for examples). We focus on this version and phrase the conjecture as a ratio of scales[6]

$$\frac{m^2}{\Lambda} \sim \mathcal{O}(1)\ . \tag{5.2}$$

The states in the tower are typically KK states, and we use this term in the following. The conjecture implies that one cannot achieve a (parametric) separation of the KK scale and the scale of the cosmological constant. In fact, a problem that has been pervasive in holography literature is the search of gravity duals of QCD or 4d SCFT with conformal anomaly coefficients $a \neq c$. Scale separation is also an important intermediate step in constructions attempting to realize de Sitter vacua in string theory [53, 54]. Hence it is an important question which merits attention.

There are systematic constructions of $AdS_4$ vacua in string theory in type IIA compactifications on CY orientifolds with NSNS and RR fluxes [33, 55] (see [56] for a recent generalization to general CYs). As already noticed in the literature, there is a family of vacua in [33] (see also [55]) claimed to achieve scale separation, thus violating the strong form of the conjecture. In this section we show that this family enjoys a $\mathbf{Z}_k$ discrete symmetry arising from 3-form gauge symmetries broken by a topological coupling to an axion, of the kind considered in [35, 36], together with a continuous 3-form symmetry. Hence it provides a setup in which a $\mathbf{Z}_k$ WCC for 3-form gauge fields is at work. The tension of the corresponding BPS domain walls can be related to the vacuum energy, and introduces additional factors of $k$ in (5.2), thus explaining the parametric scale separation, that is controlled by the parameter $k$. This symmetry is consistently absent in other AdS vacua with no scale separation, hence provides a rationale for the existence of scale separation in this family, and suggests the proper generalization of (5.2) in the presence of domain wall $\mathbf{Z}_k$ symmetries.

### 5.1 Review of scaling $AdS_4$ vacua with scale separation

In this section we review some key elements of the family of models with scale separation, following [33] (see also [55] for related classes of type IIA AdS vacua).

Consider type IIA on a CY threefold modded out by an orientifold action introducing O6-planes. The O6-planes introduce a tadpole for the RR 7-form, which is canceled by (possibly present) D6-branes, and a combination of the $F_0 \equiv m$ Romans mass flux parameter and $H_3$ NSNS field strength flux on 3-cycles. Although it is possible to introduce it, we consider the RR $F_2$ field strength fluxes to be zero.[7] On the other hand, we introduce RR $F_4$ field strength fluxes on a basis of 4-cycles $\tilde{\Sigma}_i$

$$\int_{\tilde{\Sigma}_i} F_4 \,=\, e_{\tilde{i}} \in \mathbf{Z}\ . \tag{5.3}$$

[6] Note that $\Lambda$ has dimension mass$^2$.

[7] Actually, by monodromies in suitable axions [24] the $F_2$ flux can be generated due to the presence of $F_0$ flux. This follows from a Dvali-Kaloper-Sorbo coupling, and intertwines non-trivially with similar DKS coupling to appear in section 5.2. We keep our simplified discussion for $F_2 = 0$, and refer the reader to [57, 58] for further information on the more general framework.



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We do not introduce RR $F_6$ flux over the CY, and only consider it when generated by monodromies, see section 5.3. Some details on the 4d effective action of this theory are provided in appendix C, and here we streamline the key facts. Whereas the fluxes $F_0 = m$ and $H_3$ are constrained to be $\mathcal{O}(1)$ due to the tadpole conditions, the fluxes for $F_4$ are unconstrained and can be taken large. The scaling solutions are achieved in the large $k$ limit of

$$e_{\tilde{i}} \sim \bar{e}_{\tilde{i}}\, k \ , \tag{5.4}$$

where the $\bar{e}_{\tilde{i}}$ are $\mathcal{O}(1)$ quantities. Note that we have renamed the scaling parameter of [33] as $k$ to make better contact with earlier sections, and to emphasize its forthcoming role as related to a discrete gauge symmetry.

Although we keep much of the upcoming discussion general, it is useful to consider explicit examples. A simple class is obtained by taking toroidal orbifolds $\mathbf{T}^6/\mathbf{Z}_3$, whose untwisted sector is given by 3 Kähler moduli associated to the 3 underlying $\mathbf{T}^2$'s. Their volumes, measured in string units, are denoted by $v_i$, $i = 1, 2, 3$, with the overall volume being $\bar{\mathcal{V}} \sim v_1 v_2 v_3$. They are complexified by the axions from the NSNS 2-form over the 2-tori $b_i$. We ignore twisted sectors, and refer the reader to [33] for details. Since $h_{2,1} = 0$, there is only one axion $\xi$ from the period of the RR 3-form over the 3-cycle; it combines with the 4d dilaton $e^D$ to form a complex modulus.

In the scaling limit, [33] found a supersymmetric AdS$_4$ minimum (which we refer to as the DGKT solution) with the following values for the 4d moduli

$$v_i, b_i \sim k^{\frac{1}{2}} \quad , \quad \bar{\mathcal{V}} \sim k^{\frac{3}{2}} \quad , \quad e^{-D}, \xi \sim k^{\frac{3}{2}} \ . \tag{5.5}$$

This implies that

$$M_s^2 \sim e^{2D} M_{p,4}^2 \sim k^{-3} M_{p,4}^2 \ , \tag{5.6}$$

and that the following relevant quantities of the 4d effective action, evaluated at the minimum, and measured in 4d Planck units, scale as

$$W \sim k^{\frac{3}{2}} \quad , \quad e^{\mathcal{K}} \sim k^{-\frac{15}{2}} \quad \Lambda \sim k^{-\frac{9}{2}} \ . \tag{5.7}$$

One may evaluate the KK scale as

$$m_{\text{KK}} \sim \bar{\mathcal{V}}^{-\frac{1}{6}} M_s \sim k^{-\frac{7}{4}} M_{p,4} \tag{5.8}$$

(incidentally, it coincides with the mass scale for other massive moduli, so it provides a general cutoff of the 4d theory).

This leads to a relation of the type (5.1)

$$m_{\text{KK}}^2 \sim \Lambda^{\frac{7}{9}} \ , \tag{5.9}$$

and hence to a seeming parametric violation of the strong version of the conjecture. In [59] the problem was considered in a family of IIA compactifications with geometric fluxes. The back-reaction of the latter [60] implied a modification of $m_{\text{KK}}$ which restored the scaling predicted by the strong AdS Distance Conjecture. This mechanism however is not obviously available in the present context, where geometric moduli are absent. In the following sections we propose the scale separation is physical in these cases, and find a rationale in terms of underlying symmetries.

### 5.2 The discrete 3-form symmetry

In this section we address the backbone of the solution to the above conundrum. First, notice that we had rewritten the strong conjecture as in the form (5.2) with hindsight. Indeed, taking this ratio we find that in the DGKT family

$$\frac{m_{\rm KK}^2}{\Lambda} \sim k \ . \tag{5.10}$$

Alternatively, we may express the vacuum energy $\Lambda$ in terms of the UV cutoff scale $m_{\rm KK}$ as

$$\Lambda \sim \frac{m_{\rm KK}^2}{k} \ . \tag{5.11}$$

Recalling that $\Lambda$ has dimension 2, this is extremely reminiscent of the type of relation one finds in theories with a $\mathbf{Z}_k$ discrete gauge symmetry, see (2.3). Moreover, since the left hand side quantity is the vacuum energy, the relevant charged objects should be related to the structure of the vacuum.

We now show that there is indeed an effective $\mathbf{Z}_k$ symmetry acting on domain walls changing the fluxes in the vacuum. The structure is controlled by topological couplings of the 10d theory. In fact, we will study them without assuming the vacuum solution described in the previous section, and show that the scaling relations found there are a consequence of these topological couplings, or equivalently of the discrete symmetry structure.

So we start with the general CY (orientifold) compactification, and consider the basis of 4-cycles $\tilde{\Sigma}_i$ and their dual 2-cycles $\Sigma_i$. We recall the $F_4$ flux structure and introduce 4d axions from $B_2$ as

$$\int_{\tilde{\Sigma}_i} F_4 \,=\, k\, \bar{e}_{\tilde{i}} \quad , \quad \int_{\Sigma_i} B_2 \,=\, \phi_i \tag{5.12}$$

(these axions were denoted by $b_i$ in the toroidal setup above). In addition, we introduce a symplectic basis of orientifold-odd 3-cycles $\alpha_a$ and orientifold-even 3-cycles $\beta_a$, and introduce the NSNS $H_3$ fluxes and RR axions

$$\int_{\alpha_a} H_3 \,=\, p_a \quad , \quad \int_{\beta_a} C_3 = \xi_a \ . \tag{5.13}$$

In addition, there is a Romans mass flux parameter $F_0 = m$.

Let us initially focus on the dynamics of Kähler moduli, hence ignore $\xi_a$, which will be reintroduced later on. Most of the discussion is general, although we eventually apply it to the toroidal orbifold for illustration.

The dimensional reduction of the 10d Chern-Simons coupling $F_4F_4B_2$ leads to the 4d topological coupling

$$k\left(\sum_i \bar{e}_{\tilde{i}}\phi_i\right)F_4 \ . \tag{5.14}$$

This makes the 3-form massive, by eating up the 2-form dual to a linear combination of axions. The overall factor $k$ implies that there is a discrete $\mathbf{Z}_k$ symmetry under which domain walls are charged [21]. This confirms we are on the right track. In fact, although certain modifications are about to come in, in the large $k$ limit this $\mathbf{Z}_k$ discrete symmetry determines the properties of the system.

The situation is actually slightly more subtle, because of the following. The scalars $\phi_i$ also appear in couplings with other 4-forms, arising from the 8-form as

$$F_{4,\tilde{i}} = \int_{\tilde{\Sigma}_i} F_8 \ . \tag{5.15}$$

Hence, including the reduction of the 10d coupling $F_0 B_2 F_8$, the complete set of topological couplings is

$$m \sum_i \phi_i F_{4,\tilde{i}} + k \Big( \sum_i \bar{e}_{\tilde{i}} \phi_i \Big) F_4 \ . \tag{5.16}$$

This means that the combination $\phi' \equiv \sum_i \bar{e}_{\tilde{i}} \phi_i$ also couples to other 4-forms. To isolate that dependence, introduce the generators $Q'$ and $Q_i$ of 3-form U(1) symmetries for $C_3$ and $C_{3,i}$, and consider the linear combination

$$Q' = \sum_i \bar{e}_{\tilde{i}} Q_i \ . \tag{5.17}$$

The topological coupling for the corresponding 4-form $F_4'$ is

$$m \left( \sum_i \bar{e}_{\tilde{i}}\, \phi_i \right) F_4' = m\, \phi'\, F_4' \ . \tag{5.18}$$

Hence, we can isolate the axion $\phi'$ with its couplings to the 4-forms $F_4$, $F_4'$ as

$$\phi' \, \left( m\, F_4' + k\, F_4 \right) \ . \tag{5.19}$$

It is interesting that we have this universal sector, decoupled (at the topological level) from other axions and 4-forms, and hence independent of the details of the underlying CY compactification space.

Since there is only one axion and two 4-forms, there is clearly a massless 3-form corresponding to the combination

$$Q_{\mathrm{U}(1)} = k\, Q' - m\, Q = \sum_i e_{\tilde{i}}\, Q_{\tilde{i}} - m\, Q \ . \tag{5.20}$$

In the second equality we have recast the combination in terms of the original 4-forms. It is straightforward to check, using (5.16), that $Q_{\mathrm{U}(1)}$ is indeed free from topological couplings to scalars, hence remains an unbroken 3-form gauge symmetry.

The combination appearing in (5.19), namely

$$Q_\perp = mQ' + kQ = \sum_i m\, \bar{e}_{\tilde{i}}\, Q_{\tilde{i}} + k\, Q \ , \tag{5.21}$$

is broken to a discrete subgroup. To better understand its structure, consider the string emitting a number of domain walls, and let us compute the violation of conservation of $Q_\perp$. The relevant string couples to the dual to $\phi$, namely it is given by an NS5-brane wrapped on the linear combination of 4-cycles $\sum_i \bar{e}_{\tilde{i}} \tilde{\Sigma}_i$. Due to the presence of $m$, it emits $m\bar{e}_{\tilde{i}}$ D6-branes wrapped on $\tilde{\Sigma}_i$; due to the presence of $e_{\tilde{i}}$ units of 4-form flux over $\tilde{\Sigma}_i$, it emits $\sum_i \bar{e}_{\tilde{i}} e_{\tilde{i}}$ D2-branes. Since each D6-brane on $\tilde{\Sigma}_i$ violates $Q_{\tilde{i}}$ in 1 unit, and each D2-brane violates $Q$ in 1 unit, we have a total violation of $Q_{\text{broken}}$ by

$$\Delta Q_\perp = \sum_i \left(\bar{e}_{\tilde{i}}\right)^2 \left(k^2 + m^2\right) . \tag{5.22}$$

Although it would seem that at large $k$ the symmetry is of order $k^2$, notice that it suffices to have $k$ D2-branes (plus a number of D6's sub-leading in the $1/k$ approximation) to annihilate into a string. It's only that one D2-brane implies a violation of $k$ units of $Q_{\text{broken}}$, from the way we built the linear combination. So it is an effective $\mathbf{Z}_k$ for D2-branes.

Notice that this system realizes a 3-form version of the theories with discrete and continuous U(1) symmetries (for 1-forms) we described in earlier sections. In particular, the structure of two underlying U(1)'s with one linear combination broken by a topological coupling is completely analogous to the discussion of the type IIA gravity dual of ABJM theories in section 4.2.[8]

### 5.3 Scaling relations for moduli from discrete symmetries

In analogy with the ABJM system, the D2- and D6-brane domain walls are BPS, and their tensions must relate to their charges under the unbroken $Q_{\text{U}(1)}$,

$$T_{\text{DW}} = g\, Q_{\text{U}(1)}\, M_{p,4}^4 . \tag{5.23}$$

The gauge coupling $g$ for $Q_{\text{U}(1)}$ is derived from those of the 3-form symmetries associated to $Q$ and $Q_{\tilde{i}}$, see (5.20). We denote them $g_2$, $g_{6,\tilde{i}}$ respectively, to indicate that the charged objects are D2-branes and D6-branes on $\Sigma_{\tilde{i}}$. We have

$$\frac{1}{g^2} = k^2 \sum_i \left(\bar{e}_{\tilde{i}}\right)^2 \frac{1}{g_{6,\tilde{i}}^2} + m^2 \frac{1}{g_2^2} . \tag{5.24}$$

The fact that both D2- and D6-branes can satisfy the BPS condition (5.23), implies that, in the large $k$ limit, their gauge couplings must relate as

$$g_{6,\tilde{i}} \sim k\, g_2 . \tag{5.25}$$

It is easy to express the ratio of these gauge couplings in terms of microscopic compactification parameters and *derive* that the scaling for $v$ reproduces (5.5). We offer a simplified discussion here, referring the reader to appendix C for a supergravity-friendly derivation. For concreteness, we also focus on the toroidal case. The inverse gauge couplings squared are

$$\begin{aligned} \frac{1}{g_2^2} &= M_s^2 \bar{\mathcal{V}} \left(M_s^{-3}\right)^2 = M_s^{-4} \bar{\mathcal{V}} , \\ \frac{1}{g_{6,\tilde{i}}^2} &= M_s^2 \bar{\mathcal{V}} \left(M_s^{-3} \frac{v_i}{\bar{\mathcal{V}}}\right)^2 = M_s^{-4} \frac{(v_i)^2}{\bar{\mathcal{V}}} , \end{aligned} \tag{5.26}$$

[8] With the notational difference that the roles of $N$, $k$ are now played by $k$, $m$, respectively.

where the first factor arises from the 10d coupling and the terms in parenthesis arise from normalization of charges to integers, and we recall that $\bar{\mathcal{V}} = v_1 v_2 v_3$. We have that

$$\frac{g_{6,\tilde{i}}}{g_2} = \frac{\bar{\mathcal{V}}}{v_i} \tag{5.27}$$

and comparing with (5.25) for different $i$'s gives

$$v_i \sim k^{\frac{1}{2}} \quad , \quad \bar{\mathcal{V}} \sim k^{\frac{3}{2}} \; . \tag{5.28}$$

A more direct, and possibly more general, route to the scaling relations for moduli is to use the monodromy relations. The fact that e.g. $F_4$ has topological couplings to axions implies that the flux $N$ of $F_6$ over the CY changes as the axions wind across their periods. Indeed, the above discussion is slightly oversimplified, since the fluxes experience a more intricate set of axion monodromies. These have been studied systematically in [57], and appeared implicitly in [33]. They just follow from the nested structure of 10d Chern-Simons terms, or equivalently of the 10d modified Bianchi identity for $F_6$, which implies

$$F_6 \,=\, dC_5 \,+\, F_4 B_2 \,+\, F_2\, B_2\, B_2 \,+\, F_0\, B_2\, B_2\, B_2 \,+ H_3 C_3 \; . \tag{5.29}$$

Hence, restricting to our setup with only $F_0$, $F_4$ and $H_3$, the effective 4d theory can depend only on the combination

$$N \,+\, k\, \bar{e}_{\tilde{i}}\, \phi_i \,+\, m\, \kappa_{ijk}\, \phi_i \phi_j \phi_k \,+\, p_a\, \xi_a \tag{5.30}$$

(where sums over repeated indices are implicit). Here $\kappa_{ijk}$ is the triple intersection number. For instance, $\kappa_{123} = 1$ for the torus. This implies that it is possible to generate $F_6$ flux from $m$ by performing a monodromy in $b_1$ to generate $F_2$ on the first $\mathbf{T}^2$, followed by a monodromy in $b_2$ to generate $F_4$ on the $\mathbf{T}^4$ transverse to the third coordinate, and one in $b_3$ to generate $F_6$ on the CY.

This is a more complete version of the topological couplings to 4-forms we have been considering, and which underlies the discrete symmetry of the system. We are interested in its behavior in the large $k$ limit. Consistent scaling of the monodromy relations for large $k$ requires that

$$\phi_i \,\sim\, k^{\frac{1}{2}} \; . \tag{5.31}$$

This is the generalization of the scaling for $b_i$ in (5.5), and provides the complexified counterpart of our scalings for $v_i$ in (5.28) (which recovered those in (5.5)). We point out that the fact that the two components of complex moduli have identical scalings with large flux quanta fits nicely with results on asymptotic flux compactification [61]. It is extremely interesting that this result follow from just the discrete symmetry in the present context.

Motivated by this, we can use a similar argument to extract the scaling of the dilaton multiplet in the large $k$ limit. From (5.30) we get

$$\xi_a \,\sim\, k^{\frac{3}{2}} \; . \tag{5.32}$$

This is the complexification of a similar dependence of the dilaton, which thus reproduces (5.5).

Interestingly, with this information, which in particular implies the scaling (5.6), i.e. $M_s \sim k^{-3/2} M_{p,4}$, we obtain the scaling of gauge couplings (5.26), (5.24)

$$g_2 \sim k^{-\frac{15}{4}} \quad , \quad g_{6,\tilde{i}} \sim k^{-\frac{11}{4}} \quad , \quad g \sim k^{-\frac{15}{4}} \, , \tag{5.33}$$

providing a nice version of the WCC for domain walls.

Note however that when including the $H_3$ fluxes, the above discussion is equivalent to the inclusion of additional topological couplings $p_a \xi_a F_4$. In other words, D2-brane domain walls, in the presence of $H_3$ flux, can annihilate in sets of $p_a$ by nucleating a string given by a D4-brane wrapped on the 3-cycle $\alpha_a$, due to the Freed-Witten inconsistency of the latter. The presence of these couplings spoils the structure of continuous and discrete 3-form gauge symmetries found in the Kähler moduli sector. In other words, the coupling of $F_4$ to a different linear combination of axions implies that the former continuous symmetry is actually also broken by the new additional axion, given by the linear combination of $\xi_a$. We skip the detailed discussion of the resulting complete discrete symmetry group. Note however that for large $k$ the effects of both $m$ and $p$ are sub-leading in a $1/k$ expansion, so the $\mathbf{Z}_k$ symmetry we have been using prevails.

Since we have recovered the scalings of the Kähler and complex structure moduli, it is a simple exercise to use the expressions of 4d supergravity to derive others like (5.7), and eventually recover the scale separation (5.11). On the other hand, the 4d approach has been criticized as potentially hiding subtleties of the 10d solution. Therefore in the following we use an alternative approach, and exploit properties of BPS domain walls to recover the vacuum energy.

### 5.4 Discrete symmetries and scale separation

In this section we exploit the interplay between the tensions of domain walls and the vacuum energy, and study the interplay of discrete symmetries and scale separation. We argue through explicit examples that AdS vacua with trivial discrete symmetry for domain walls do not have scale separation; this is true even if there are non-trivial discrete symmetries for particles or strings, and in general for real codimension higher than 1 objects. On the other hand, we show that the above type IIA modes with non-trivial discrete symmetry for domain walls, with the corresponding scaling for moduli, do have vacuum energy with scale separation. We extend this general relation and put forward the following refined version of the swampland constraint (5.2), as follows:

**$\mathbf{Z}_k$ refined strong ads distance conjecture.** *Consider quantum gravity on an AdS vacuum with a $\mathbf{Z}_k$ discrete symmetry for domain walls (with $k$ large). In the flat-space limit $\Lambda \to 0$ (with $\Lambda k \to 0$ as well) there exists an infinite tower of states at a scale $M_{\rm cutoff}$, with the relation*

$$\Lambda \sim \frac{M_{\rm cutoff}^2}{k} \, . \tag{5.34}$$

We now proceed to check this conjecture in the examples of supersymmetric AdS vacua of this paper, by deriving their vacuum energies from the properties of domain walls.

#### 5.4.1 Vacuum energy from domain walls

Let us describe our main tool to evaluate the vacuum energies without invoking an underlying scalar potential. There is in fact a general relation between domain wall tensions and vacuum energies, which essentially follows from junction conditions in general relativity. We refer the reader to appendix D for a discussion well adapted to our application in AdS. The key point is that the domain wall tension $T$ is the variation of certain quantities $\lambda$, see (D.9), whose square essentially gives the vacuum energy $\Lambda$, see (D.8). In the supersymmetric setup, and for BPS domain walls, these statements become the familiar

$$\lambda = e^{\mathcal{K}/2}\, W \quad , \quad T = \Delta(e^{\mathcal{K}/2}\, W) = \Delta\lambda \quad , \quad \Lambda = -3e^{\mathcal{K}}\, |W|^2 \sim -|\lambda|^2 \, . \tag{5.35}$$

We consider BPS domain walls whose quantized charge describes the change in some field strength flux $n$ as one crosses the domain wall. In the limit of large flux $n$, the tension $T$ provides the derivative of $d\lambda/dn$. We can then solve to obtain the scaling with $n$ of $\lambda$, and thus of its square, $\Lambda$.

#### 5.4.2 Warm-up examples: no scale separation

We now turn to discuss the AdS examples of sections 3, 4, *deriving* their AdS radius from the above strategy, and showing there is no scale separation. This is in agreement with our Refined Strong AdS Distance Conjecture (RSADC), as these examples have discrete symmetries for particles (and for their dual real codimension 2 objects) but not for domain walls.

**Type IIB on $\mathbf{S^5/Z_k}$.** Consider type IIB on $\mathbf{S}^5/\mathbf{Z}_k$ with $N$ units of RR 5-form flux and

$$R^4 \sim M_s^{-4}\, g_s\, N\, k \, . \tag{5.36}$$

This is of course the class of theories considered in section 3, but we are now not imposing the solution for the 5d vacuum, rather we are deriving its vacuum energy from the domain wall properties. In passing, we also discuss the gauge coupling of the 3-forms and draw conclusions regarding the WCC.

We consider a BPS domain wall given by a D3-brane in 5d. Its tension is

$$T_{\rm D3} \, \sim \, M_s^4\, g_s^{-1} \, \sim \, M_{p,5}^4 N^{-\frac{5}{3}} k^{-\frac{1}{3}} \, . \tag{5.37}$$

The same result is obtained from the BPS condition

$$T_{\rm D3} = gQ_{\rm D3} \tag{5.38}$$

upon computation of the gauge coupling of the 5d RR 4-form under which the D3-brane is charged. Since the tension essentially agrees with the gauge coupling, we observe an interesting WCC scaling for $g$ (in that respect, recall that the relevant large order discrete symmetry is $\mathbf{Z}_N$). This is interesting, since the discrete symmetry acts on particles/membranes, whereas $g$ is a 3-form gauge coupling. It would be interesting to explore the interplay between discrete and continuous symmetries of different degrees; we hope to come back to this in future work.



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Since this domain wall interpolates among vacua with $N$ and $N+1$, one can now obtain

$$\frac{d\lambda}{dN} \sim N^{-\frac{5}{3}} k^{-\frac{1}{3}} \quad \Rightarrow \quad \lambda \sim N^{-\frac{2}{3}} k^{-\frac{1}{3}} \quad \Rightarrow \quad \Lambda \sim M_{p,5}^2 N^{-\frac{4}{3}} k^{-\frac{2}{3}} \,. \tag{5.39}$$

Using (3.6) we have

$$\Lambda \sim R^{-2} \,. \tag{5.40}$$

Hence the AdS radius is the same as that of the internal space, and there is no decoupling of scales. This is the strong ADC statement in [34].

Note that, even though there are discrete gauge symmetries in the system, their orders do not enter the ratio of scales. This is in agreement with our RSADC, since these discrete symmetries involve particles and membranes, not domain walls.

**M-theory on $\mathbf{S^7/Z_k}$.** Let us consider M-theory on $\mathbf{S}^7/\mathbf{Z}_k$ with $N$ units of flux (or $Nk$ in the covering space) and

$$R^6 \sim M_{p,11}^{-6} N k \,. \tag{5.41}$$

This is of course the same system as in section 4, but again we wish to *derive* the 4d vacuum energy from the relevant BPS domain walls. We consider a BPS domain wall given by an M2-brane in 4d. Its tension is

$$T_{\rm M2} \sim M_{p,11}^3 \sim M_{p,4}^3 N^{-\frac{7}{4}} k^{-\frac{1}{4}} \,. \tag{5.42}$$

where we used (4.4). The same result is obtained from the BPS condition

$$T_{\rm M2} = g\, Q_{\rm M2} \tag{5.43}$$

upon computation of the gauge coupling $g$ for the 4d 3-form. Recalling the relevant large order discrete symmetry is $\mathbf{Z}_N$, we note again that we get an interesting WCC scaling for $g$.

Since the M2-brane domain wall interpolates between vacua with $N$ and $N+1$ units of flux, we have

$$\frac{d\lambda}{dN} \sim N^{-\frac{7}{4}} k^{-\frac{1}{4}} \quad \Rightarrow \quad \lambda \sim N^{-\frac{3}{4}} k^{-\frac{1}{4}} \quad \Rightarrow \quad \Lambda \sim M_{p,4}^2 N^{-\frac{3}{2}} k^{-\frac{1}{2}} \sim R^{-2} \,. \tag{5.44}$$

In the last relation, we have used (4.4). Again, we recover the result that the AdS radius is of the same order of magnitude as the KK scale of the internal space. Also, notice that there are discrete symmetries in the theory, but they involve particles and strings, rather than domain walls. Hence, they do not alter the relation between scales, in agreement with our RSADC.

**Type IIA on $\mathbf{CP^3}$.** We would like to repeat the previous computation in the type IIA picture. Let us consider type IIA theory on $\mathbf{CP}^3$ with $N$ units of $F_6$ RR flux over $\mathbf{CP}^3$ and $k$ units of $F_2$ RR flux over $\mathbf{CP}^1 \subset \mathbf{CP}^3$ and

$$R_s^2 \sim M_s^{-2} N^{1/2} k^{-1/2} \,. \tag{5.45}$$

This is the same system as in section 4.2. The relevant BPS domain wall is a D2-brane in 4d, whose tension is

$$T_{\mathrm{D2}} \sim M_s^3\, g_s^{-1} \sim M_{p,4}^3 N^{-7/4} k^{-1/4} \,. \tag{5.46}$$

This is the same scaling as the M2-brane in the previous section, and the D2-brane domain wall interpolates vacua with $N$ and $N+1$ units of flux, so we recover

$$\Lambda \sim M_{p,4}^2 N^{-3/2} k^{-1/2} \sim R_s^{-2} \,. \tag{5.47}$$

The AdS radius is the same as that of the internal space, with no scale separation, in agreement with our RSADC.

### 5.4.3 Revisiting the scale separation in type IIA CY flux compactifications

Consider now the configurations with the large $k$ discrete $\mathbf{Z}_k$ symmetry for domain walls in section 5.2. We wish to derive the scaling of the vacuum energy with $k$, just using the scaling of moduli vevs (5.5), (5.6) derived in section 5.3 from the $\mathbf{Z}_k$ symmetry.

We consider the BPS domains wall given by a D4-brane wrapped on the combination of 2-cycles $\sum_i \bar{e}_{\bar{i}} \Sigma_i$. This domain wall interpolates between vacua with $F_4$ flux given by $k$ and $k+1$. Notice that the $F_4$-flux is not monodromic, hence the D4-branes are stable against nucleation of strings, and can provide BPS objects (in contrast with e.g. D2- and D6-brane domain walls encountered in earlier sections).

The tension of these domain walls can be obtained from the BPS equation and the gauge couplings, computed in detail in appendix C. Here we carry out a simplified derivation, taking the toroidal case for concreteness. The gauge coupling of a D4$_i$-brane domain wall is

$$\frac{1}{g_{4,\,i}^2} = M_s^2\, \bar{\mathcal{V}}\, (\, M_s^{-3}\, v_i^{-1}\, )^2 = M_s^{-4}\, \bar{\mathcal{V}} v_i^{-2} \sim k^{\frac{13}{2}} \,. \tag{5.48}$$

As usual, in the first equality, the first term comes from the reduction of the 10d coupling, and the parenthesis from the charge normalization. Note that the scaling is common for all $i$, so by the BPS condition we get the tension

$$T_{\mathrm{DW}} \sim k^{-\frac{13}{4}} \,. \tag{5.49}$$

Notice that, if interpreted in terms of gauge couplings, this implies an interesting WCC, as in earlier examples. From the above tension we get

$$\frac{d\lambda}{dk} \sim k^{-\frac{13}{4}} \quad \Rightarrow \quad \lambda \sim k^{-\frac{9}{4}} \quad \Rightarrow \quad \Lambda \sim k^{-\frac{9}{2}} \,. \tag{5.50}$$

So we recover the scaling (5.7) for $\Lambda$ (the reader can check those of $\mathcal{K}$ and $W$ as well). Once $m_{\mathrm{KK}}$ is recovered as in (5.8), this reproduces the scale separation (5.11), in agreement with our RSADC conjecture.

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## A Species bound for extremal black holes

In the following we consider the evaporation of extremal black holes endowed with $\mathbf{Z}_k$ charge. For concreteness, the classical solutions we are taking are the extremal Reissner-Nordström black holes in 4d space-time dimensions. They have vanishing Hawking temperature, so the analysis in [41] is not directly applicable.

Extremal black holes can discharge through Schwinger radiation [62–64]. Whenever the electric field is much larger than the background curvature, this happens essentially in flat space [65]. In this case the production rate has an exponential suppression

$$\Gamma \sim e^{-\frac{m^2}{qE}} \sim e^{-\chi}\,, \tag{A.1}$$

where $m$ and $q$ are the mass and charge of the emitted particle and $E$ is the electric field, given by

$$E = \frac{g^2}{4\pi}\frac{Q}{r^2}\,. \tag{A.2}$$

As argued in section 2.1.2, the simplest way in which this kind of black hole is able to get rid of both continuous and discrete charge while remaining sub-extremal is in the presence of a $\mathbf{Z}_k$ WGC particle. Let us assume that this particle is actually BPS,

$$m = gqM_p\,. \tag{A.3}$$

As a consequence, the black hole will remain extremal throughout the whole evaporation process.

From (A.1) and (A.2), we notice that the maximum particle production will happen close to the horizon, so in this order of magnitude analysis we will approximate the whole radiation as the contribution of that region.

From the extremality condition we can relate the horizon radius and the charge of the BH with its mass through

$$r_h \sim \frac{M_{\rm BH}}{M_p^2}, \qquad gQ \sim \frac{M_{\rm BH}}{M_p}\,. \tag{A.4}$$

They lead to

$$E \sim g\frac{M_p^3}{M_{\rm BH}}\,. \tag{A.5}$$

Introducing (A.3) and (A.5) in (A.1) we can estimate the factor in the exponential suppression of the production rate of the $\mathbf{Z}_k$ WGC particle to be

$$\chi \sim \frac{mM_{\rm BH}}{M_p^2}\,. \tag{A.6}$$

The black hole will be able to efficiently evaporate discrete charge when

$$M_{\rm BH} \lesssim \frac{M_p^2}{m}\,. \tag{A.7}$$

With this condition being true, the black hole should still have enough mass to radiate $\mathcal{O}(k)$ particles (assuming the $\mathbf{Z}_k$ WGC particle to have unit discrete charge), which means

$$M_{\rm BH} \gtrsim km\,. \tag{A.8}$$

Finally, from the two conditions (A.7) and (A.8), we obtain the following bound for the mass of the $\mathbf{Z}_k$ WGC particle:

$$m^2 \lesssim \frac{M_p^2}{k}\,. \tag{A.9}$$

This is the species bound in [41]. We have shown that the bound also applies to extremal black holes emitting $\mathbf{Z}_k$ WGC particles via Schwinger effect.

## B Discrete symmetries in intersecting brane models

Discrete symmetries are ubiquitous in models of intersecting branes (see [66] for a review), as pioneered in [20]. In this appendix we use them to illustrate the interplay of $\mathbf{Z}_k$ and U(1) gauge symmetries, and the scalings implied by the $\mathbf{Z}_k$ WCC.

Let us start by recalling the basic setup. Consider a compactification of type IIA on a Calabi-Yau space $\mathbf{X}_6$ quotiented by the orientifold[9] action $\Omega\mathcal{R}(-1)^{F_L}$, where $\mathcal{R}$ is an antiholomorphic $\mathbf{Z}_2$ involution of $\mathbf{X}_6$, which introduces O6-planes. Let us denote $[\Pi_{\rm O6}]$ the total homology class of the 3-cycles wrapped by the O6-planes. Introducing a symplectic basis $[\alpha_i]$, $[\beta_i]$ of 3-cycles even and odd under $\mathcal{R}$, respectively, we may expand

$$[\Pi_{\rm O6}] \;=\; \sum_i r^i_{\rm O6}[\alpha_i] \;+\; s^i_{\rm O6}[\beta_i]\;, \tag{B.1}$$

with $r^i_{\rm O6}$, $s^i_{\rm O6}$ some coefficients of order 1-10.

The O6-planes are charged under the RR 7-form, so to cancel its tadpoles we introduce D6-branes. We consider stacks of $N_A$ overlapping $\mathrm{D6}_A-$branes wrapped on 3-cycles $\Pi_A$, and their orientifold image $\mathrm{D6}_{A'}-$branes on 3-cycles $\Pi_{A'}$. In terms of the basis, we have

$$[\Pi_A] \;=\; \sum_i r^i_A[\alpha_i] \;+\; s^i_A[\beta_i] \quad , \quad [\Pi_{A'}] \;=\; \sum_i r^i_A[\alpha_i] \;-\; s^i_A[\beta_i]\;. \tag{B.2}$$

The RR tadpole condition reads

$$\sum_A 2r^i_A + r^i_{\rm O6} \;=\; 0 \quad \forall i\;. \tag{B.3}$$

In addition there are K-theory RR tadpole conditions [67], which we skip in this sketchy discussion.

[9] Note that the orientifolds are not essential for the argument, but we choose to introduce them to better connect with the literature on intersecting brane models.

In these models, there are Stückelberg couplings for the $\mathrm{U}(1)_A$, of the form

$$\sum_A N_A s^i_A b_{2,i}\, F_A \ , \tag{B.4}$$

where wedge product is implicit. $F_A$ is the field strength of the U(1) gauge field on the $\mathrm{D6}_A-$branes, and the 4d 2-forms $b_{2,i}$ arise from the KK compactification of the RR 5-form $C_5$ as

$$b_{2,i} \ = \int_{\beta_i} C_5 \ . \tag{B.5}$$

This makes some of the U(1)'s massive. Let us consider linear combinations of the $\mathrm{U}(1)_A$ generators $Q_A$

$$Q \ = \sum_A c_A Q_A \ , \tag{B.6}$$

with $c_A$ being coprime integers, so as to preserve charge integrality. The Stückelberg coupling for the field strength $F$ of the U(1) generated by $A$ is

$$\left( \sum_A c_A N_A s^i_A \right) b_{2,i}\, F \ . \tag{B.7}$$

Hence, the condition for a U(1) to remain massless is

$$\sum_A c_A N_A s^i_A \ = 0 \quad \forall i \ . \tag{B.8}$$

If not, the U(1) is broken, remaining only as approximate global symmetry, broken by non-perturbative D2-brane instanton effects [68–70]. The condition that a discrete $\mathbf{Z}_k$ subgroup remains as exact discrete gauge symmetry is

$$\sum_A c_A N_A s^i_A \ = 0 \bmod k \quad \forall i \ . \tag{B.9}$$

Generically, to achieve this for large $k$ a possibility[10] is to have $s^i_A \sim k$, at least for some $A$, for all $i$. This implies that there is some brane which is wrapped on a very large (i.e. multiply wrapped) cycle. This implies that in general any unbroken U(1), given by a linear combination (B.6) satisfying (B.8), will also involve that particular $Q_A$ with a coefficient of order $k$. This implies that the gauge coupling of the unbroken U(1) scales as

$$\frac{1}{g^2} \ = k \quad \text{hence } g \sim k^{-\frac{1}{2}} \ , \tag{B.10}$$

in agreement with the $\mathbf{Z}_k$ WCC.

Although this is not quite a rigorous argument, it is a good illustration of how the interplay between U(1) gauge couplings and $\mathbf{Z}_k$ symmetries arises, as a consequence of the fact that, to achieve a large order $\mathbf{Z}_k$ discrete symmetry, one needs to use parametrically large cycles, thus parametrically scaling gauge couplings to zero. Hence, intersecting brane models provide an intuitive mechanism for the $\mathbf{Z}_k$ WCC. More detailed string theory examples are presented in the main text.

[10] This is not the only one, but we stick to it as an illustrative example.

## C Gauge couplings in type IIA CY compactifications

In this appendix we derive the gauge coupling constants for domain walls present in type IIA CY flux compactifications. We review the computation in [71] following the conventions in [33]. From [33], the 10d string frame action is given by[11]

$$S^{10d} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Big(e^{-2\phi}(R+4(\partial_\mu\phi)^2-\frac{1}{2}|H_3^{\text{total}}|^2) - (|\tilde{F}_2|^2+|\tilde{F}_4|^2+m_0^2)\Big)+S_{\text{CS}}\ , \tag{C.1}$$

where $2\kappa_{10}^2=(2\pi)^7\alpha'^4$ and the definitions of the field strengths are

$$\begin{aligned} H_3^{\text{total}} &= dB_s + H_3^{\text{bg}}\ ,\\ \tilde{F}_2 &= dC_1 + mB_2\ ,\\ \tilde{F}_4 &= dC_3 + F_4^{\text{bg}} - C_1\wedge H_3 - \frac{m}{2}B_2\wedge B_2\ . \end{aligned} \tag{C.2}$$

The Chern-Simons action contains also a prefactor $(2\kappa_{10})^{-1}$ in front. We define an adimensional internal volume by $\bar{\mathcal{V}} = M_s^6\mathcal{V}$ and perform the dimensional reduction in the string frame. For instance, the kinetic term for the 4d field strength associated to the 10d $F_4$ reads

$$S_{4d}^{\text{kin}} \supset -\frac{M_s^2}{2}\int d_4x\sqrt{-g_4}\bar{\mathcal{V}}|\mathcal{F}_4|^2\ , \tag{C.3}$$

where $\mathcal{F}_4 = dC_3$. To move back to the Einstein frame, we choose a reference scale $a$, and define the 4d dilaton $D(x)$ as

$$a = \frac{\langle\bar{\mathcal{V}}\rangle}{e^{2\langle\phi\rangle}}\quad ,\quad e^{2D} = \frac{e^{2\phi}}{\bar{\mathcal{V}}}\ . \tag{C.4}$$

So the Einstein frame kinetic terms take the form

$$S_E^{\text{kin}} \supset \frac{M_s^2}{2a}\int d^4x\sqrt{-g_E}R_E - \frac{a^2M_s^2}{2}\int d^4x\sqrt{-g_E}\bar{\mathcal{V}}e^{-4D}|\mathcal{F}_4|^2\ , \tag{C.5}$$

where the products are now done using $g_E$ as a metric.

To obtain 4d gauge 3-forms, we perform a KK reduction of 10d p-forms along suitable harmonic (p − 3)-forms in the internal space. In the notation of [57],

$$C_3 = c_3^0\ ,\ C_5 = c_3^a\wedge\omega_a\ ,\ C_7 = \tilde{d}_{3a}\wedge\tilde{\omega}^a \text{ and } C_9 = \tilde{d}_3\wedge\omega_6\ . \tag{C.6}$$

They corresponds to the relevant 4-forms $\mathcal{F}_4^0$, $\mathcal{F}_4^a$, $\tilde{\mathcal{F}}_{4,a}$ and $\tilde{\mathcal{F}}_4$ associated to D2-, D4-, D6- and D8-branes.

Notice that we need to normalize the gauge fields by the coefficient in front of the D-brane Chern-Simons term, in order for the charges to be properly quantized. For a Dp-brane this introduces factors of $\mu_p \propto \alpha'^{(p+1)/2} \sim M_s^{(p+1)}$ in the forthcoming gauge

[11] Our convention is that $|F_p|^2 = F_{\alpha_1\ldots\alpha_p}F^{\alpha_1\cdots\alpha_p}/p!$.

couplings. Namely, in order to be consistent, we need to keep the harmonic forms as adimensional, so the generic Chern-Simons action is

$$S_{\rm CS}^{(p)} \sim M_s^3 \int_{W_3\times\gamma_{p-2}} c_3 \wedge \omega_{p-2} \ , \tag{C.7}$$

where we have called $c_3 \wedge \omega_{p-2}$ collectively each decomposition in (C.6). The normalization consists in redefining the RR 3-form by a factor $M_s^3$, so that there is no prefactor in front of the Chern-Simons action. The effect of such redefinition on (C.5) is just a change in the prefactor in front of the kinetic terms of the gauge fields,

$$S_E^{\rm kin} \supset \frac{M_s^2}{2a}\int d^4x\sqrt{-g_E}R_E - \frac{a^2}{2M_s^2}\int d^4x\sqrt{-g_E}\bar{\mathcal{V}}e^{-4D}|\mathcal{F}_4|^2 \ . \tag{C.8}$$

We are almost done in the definition of the coupling constants, but first we need the following quantities:

$$M_s^2 \propto \frac{e^{2\langle\phi\rangle}}{\langle\bar{\mathcal{V}}\rangle}M_{p,4}^2 = a^{-1}M_{p,4}^2 \ , \quad K_K = -\ln(8\bar{\mathcal{V}}) \ , \quad K_Q = 4D \ , \quad K = K_K + K_Q \ .$$

Substituting in (C.8) and including the other 4-forms, we obtain [71]

$$S_E^{\rm kin} = \frac{\pi}{2M_p^4}\int \frac{e^{-K}}{8}\left[\mathcal{F}_4^0\wedge\star\mathcal{F}_4^0 + 4g_{ab}\mathcal{F}_4^a\wedge\star\mathcal{F}_4^b + \frac{1}{4\bar{\mathcal{V}}^2}g^{ab}\tilde{\mathcal{F}}_{4|a}\wedge\star\tilde{\mathcal{F}}_{4|b} + \frac{1}{\bar{\mathcal{V}}^2}\tilde{\mathcal{F}}_4\wedge\star\tilde{\mathcal{F}}_4\right] , \tag{C.9}$$

where

$$g_{ab} = \frac{\partial^2 K_K}{\partial t^a \partial \bar{t}^b} \tag{C.10}$$

is the metric in the Kähler moduli space with $t^a = v^a + ib^a$.

We need now to specialize to the toroidal orbifold in [33]. The Kähler potential is

$$K_K = -\ln(8v^1v^2v^3) = -\ln((t^1+\bar{t}^1)(t^2+\bar{t}^2)(t^3+\bar{t}^3)) \ , \tag{C.11}$$

so

$$g_{ab} = \frac{1}{4}\mathrm{diag}\left((v^1)^{-2},(v^2)^{-2},(v^3)^{-2}\right) \ . \tag{C.12}$$

We rewrite the action according to this metric obtaining

$$S_E^{\rm kin} = \frac{\pi}{2M_p^4}\int \frac{e^{-K}}{8}\left[\mathcal{F}_4^0\wedge\star\mathcal{F}_4^0 + \sum_{i=1}^3\left(\frac{1}{(v^i)^2}\mathcal{F}_4^i\wedge\star\mathcal{F}_4^i + \frac{(v^i)^2}{\bar{\mathcal{V}}^2}\tilde{\mathcal{F}}_{4|i}\wedge\star\tilde{\mathcal{F}}_{4|i}\right) + \frac{1}{\bar{\mathcal{V}}^2}\tilde{\mathcal{F}}_4\wedge\star\tilde{\mathcal{F}}_4\right] . \tag{C.13}$$

We are finally able to read the coupling constants of all kinds of domain walls:

$$\frac{1}{g_0^2} = \frac{\pi e^{-K}}{8M_p^4} \ , \qquad \frac{1}{g_i^2} = \frac{\pi e^{-K}}{8M_p^4(v^i)^2} \ ,$$
$$\frac{1}{g_{\tilde{i}}^2} = \frac{\pi e^{-K}(v^i)^2}{8M_p^4\bar{\mathcal{V}}^2} \ , \qquad \frac{1}{g_{\tilde{4}}^2} = \frac{\pi e^{-K}}{8M_p^4\bar{\mathcal{V}}^2} \ . \tag{C.14}$$

From the main text, the scalings with the flux $k$ are

$$e^K \sim k^{-15/2} \; , \; v_i \sim k^{1/2} \text{ and } \bar{\mathcal{V}} \sim k^{3/2} \; , \tag{C.15}$$

so the couplings scale as

$$\begin{aligned} \frac{1}{g_0^2} &= \frac{\pi e^{-K}}{8M_p^4} \sim k^{15/2} \quad , & \frac{1}{g_i^2} &= \frac{\pi e^{-K}}{8M_p^4 (v^i)^2} \sim k^{13/2} \; , \\ \frac{1}{g_{\tilde{i}}^2} &= \frac{\pi e^{-K}(v^i)^2}{8M_p^4 \bar{\mathcal{V}}^2} \sim k^{11/2} \quad , & \frac{1}{g_4^2} &= \frac{\pi e^{-K}}{8M_p^4 \bar{\mathcal{V}}^2} \sim k^{9/2} \; . \end{aligned} \tag{C.16}$$

## D Junction conditions for AdS vacua

Here we adapt to the 4d setup the discussion of [72], which studies a Randall-Sundrum construction [73, 74] with an arbitrary number of branes (domain walls). The discussion is also similar to systems of D8-branes in type I' theory [75].

Consider a 4d spacetime with $N$ parallel domain walls with tensions $T_i$, located at positions $y_i$ in a coordinate $y$. The region between the $i^{th}$ and $(i+1)^{th}$ brane has cosmological constant $\Lambda_i$. A solution of the 4d Einstein equations

$$\begin{aligned} \sqrt{-G}\left(R_{MN} - \frac{1}{2}G_{MN}R\right) = -\frac{1}{4M_{p,4}^2}\Bigg[&\sum_{i=1}^{N}\Lambda_i\left[\theta(y-y_i)-\theta(y-y_{i+1})\right]\sqrt{-G}\,G_{MN} \\ &+\sum_{i=1}^{N}T_i\sqrt{-g^{(i)}}g^{(i)}_{\mu\nu}\delta^\mu_M\delta^\nu_N\delta(y-y_i)\Bigg] \end{aligned} \tag{D.1}$$

is given by the ansatz

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 dy^2 \; . \tag{D.2}$$

The warp factor in the above expression is given by the following piecewise linear function

$$\begin{aligned} \sigma(y) =& (\lambda_1-\lambda_0)(y-y_1)\theta(y-y_1) + (\lambda_2-\lambda_1)(y-y_2)\theta(y-y_2) \\ &+ \ldots + (\lambda_N-\lambda_{N-1})(y-y_N)\theta(y-y_N) \; , \end{aligned} \tag{D.3}$$

where $\lambda_0$ and $\lambda_N$ provide the asymptotic behavior at $y \mp \infty$. In any region between two domain walls, we can perform a change of coordinates

$$\frac{x_0}{r_c} = e^{\sigma(y)} \; , \tag{D.4}$$

to bring the metric (D.2) to a more standard form, i.e.

$$ds^2 = \frac{r_c^2}{x_0^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dx_0^2\right). \tag{D.5}$$

from which it is clear that the solution describes slices of $\text{AdS}_4$ with different values of the cosmological constant, made explicit below.

From (D.1), we obtain the following constraints for $\sigma(y)$ [72]:

$$(\sigma'(y))^2 = -\frac{r_c^2}{12M_{p,4}^2}\sum_{i=1}^{N}\Lambda_i\left[\theta(y-y_i)-\theta(y-y_{i+1})\right] \ , \tag{D.6}$$

$$\sigma''(y) = \frac{r_c}{8M_{p,4}^2}\sum_{i=1}^{N}T_i\delta(y-y_i) \ . \tag{D.7}$$

Substituting (D.3) in (D.6) and (D.7), we obtain the relations

$$\lambda_i = \pm\sqrt{\frac{-\Lambda_i r_c^2}{12M_{p,4}^2}} \ , \tag{D.8}$$

$$\frac{T_i r_c}{8M_{p,4}^2} = \lambda_i - \lambda_{i-1} \ . \tag{D.9}$$

Hence these junction conditions relate the variation of the cosmological constant to the potential of the branes that give us the domain walls. This is a general interpretation of what we proposed in section 5.4.



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# 5

# From dynamical tadpoles to cobordism and distance conjectures

This chapter contains the articles

- G. Buratti, M. Delgado and A. Uranga

  *Dynamical tadpoles, stringy cobordism, and the SM from spontaneous compactification*

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# Dynamical tadpoles, stringy cobordism, and the SM from spontaneous compactification

**Ginevra Buratti, Matilda Delgado and Angel M. Uranga**

*Instituto de Física Teórica IFT-UAM/CSIC,*
*C/ Nicolás Cabrera 13–15, Campus de Cantoblanco, 28049 Madrid, Spain*

*E-mail:* ginevra.buratti@uam.es, matilda.delgado@uam.es, angel.uranga@csic.es

Abstract: We consider string theory vacua with tadpoles for dynamical fields and uncover universal features of the resulting spacetime-dependent solutions. We argue that the solutions can extend only a finite distance $\Delta$ away in the spacetime dimensions over which the fields vary, scaling as $\Delta^n \sim \mathcal{T}$ with the strength of the tadpole $\mathcal{T}$. We show that naive singularities arising at this distance scale are physically replaced by ends of spacetime, related to the cobordism defects of the swampland cobordism conjecture and involving stringy ingredients like orientifold planes and branes, or exotic variants thereof. We illustrate these phenomena in large classes of examples, including $\mathrm{AdS}_5 \times T^{1,1}$ with 3-form fluxes, 10d massive IIA, M-theory on K3, the 10d non-supersymmetric USp(32) strings, and type IIB compactifications with 3-form fluxes and/or magnetized D-branes. We also describe a 6d string model whose tadpole triggers spontaneous compactification to a semirealistic 3-family MSSM-like particle physics model.

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## Contents



## 1 Introduction and conclusions

Supersymmetry breaking string vacua (including 10d non-supersymmetric strings) are generically affected by tadpole sources for dynamical fields, unstabilizing the vacuum [1, 2]. We refer to them as *dynamical tadpoles* to distinguish them from *topological tadpoles*, such as RR tadpoles, which lead to topological consistency conditions on the configuration (note however that dynamical tadpoles were recently argued in [3] to relate to violation of swampland constraints of quantum gravity theories). Simple realizations of dynamical tadpoles arose in early models of supersymmetry breaking using antibranes in type II (orientifold) compactifications [4–7], or in 10d non-supersymmetric string theories [8].

Dynamical tadpoles indicate the fact that equations of motion are not obeyed in the proposed configuration, which should be modified to a spacetime-dependent solution (more precisely, solution in which some fields do not preserve the maximal symmetry in the corresponding spacetime dimension, but we stick to the former nomenclature), e.g. rolling down the slope of the potential. This approach has been pursued in the literature (see e.g. [9–13]), although the resulting configurations often contain metric singularities or strong coupling regimes, which make their physical interpretation difficult.

In this work we present large classes of spacetime[1] dependent field configurations sourced by dynamical tadpoles, which admit a simple and tractable smoothing out of such singularities. Remarkably, these examples reveal a set of notable physical principles and universal scaling behaviours. We argue that the presence of a dynamical tadpole implies the appearance of ends of spacetime (or walls of nothing) at a finite spacetime distance, which is (inversely) related to the strength of the tadpole. These ends of spacetime moreover correspond to cobordism defects (or end of the world branes) of the theory implied by the swampland cobordism conjecture [14, 15]. In most setups the cobordism defects end up closing off the space into a compact geometry (possibly decorated with branes, fluxes or other ingredients), thus triggering spontaneous compactification.

We can sum up the main features described above, and illustrated by our examples, in two lessons:

**Finite distance.** *In the presence of a dynamical tadpole controlled by an order parameter $\mathcal{T}$, the spacetime-dependent solution of the equations of motion cannot be extended to spacetime distances beyond a critical value $\Delta$ scaling inversely proportional to $\mathcal{T}$, with a scaling relation*

$$\Delta^{-n} \sim \mathcal{T} \,. \tag{1.1}$$

In our examples, $n = 1$ or $n = 2$ for setups with an underlying AdS-like or Minkowski vacuum, respectively.

**Dynamical cobordism.** *The physical mechanism cutting off spacetime dimensions at scales bounded by the $\Delta$ above, is a cobordism defect of the initial theory (including the dynamical tadpole source).*

To be precise, when there are multiple spacetime directions to be closed off, the actual defect is the cobordism defect corresponding to circle or toroidal compactifications of the initial theory, with suitable monodromies on non-trivial cycles. This is analogous to the mechanism by which F-theory on half a $\mathbf{P}_1$ provides the cobordism defect for type IIB on $\mathbf{S}^1$ with $\mathrm{SL}(2,\mathbf{Z})$ monodromy [14] (see also [16]).

As explained, we present large classes of models illustrating these ideas, including (susy and non-susy) 10d string theories and type II compactifications with D-branes, orientifold planes, fluxes, etc. For simplicity, we present models based on toroidal examples (and orbifolds and orientifolds thereof), although many of the key ideas easily extend to more

[1] Actually, we restrict to configurations of fields varying over spatial dimensions (rather than time); yet we abuse language and often refer to them as spacetime-dependent.

general setups. This strongly suggests that they can apply to general string theory vacua. Very remarkably, the tractability of the models allows to devise spontaneous compactification whose endpoint corresponds to some of the (supersymmetric extensions of the) SM-like D-brane constructions in the literature. As will be clear, our examples can often be regarded as novel reinterpretations of models in the literature.

Although our examples are often related to supersymmetric models, supersymmetry is not a crucial ingredient in our discussion. Dynamical tadpoles correspond to sitting on the slope of potentials, which, even in theories admitting supersymmetric vacua, correspond to non-supersymmetric points in field space. On the other hand, supersymmetry of the final spacetime-dependent configuration is a useful trick to guarantee that dynamical tadpoles have been solved, but it is possible to build solutions with no supersymmetry but equally solving tadpoles.

Our results shed new light on several features observed in specific examples of classical solutions to dynamical tadpoles, and provide a deeper understanding of the appearance of singularities, and the stringy mechanism smoothing them out and capping off dimensions to yield dynamical compactification. In particular, we emphasize that our discussion unifies several known phenomena and sheds new light on the strong coupling singularities of type I' in [17] and in heterotic M-theory [18] (and its lower bound on the 4d Newton's constant). There are several directions which we leave for future work, for instance:

- As is clear from our explicit examples, many constructions of this kind can be obtained via a reinterpretation of known compactifications. This strongly suggests that our lessons have a general validity in string theory. It would be interesting to explore the discussion of tadpoles, cobordism and spontaneous compactifications in general setups beyond tori.

- A general consequence of (1.1) is a non-decoupling of scales between the geometric scales controlling the order parameter of the dynamical tadpole and the geometric size of the spontaneously compactified dimensions. This is reminiscent of the swampland AdS distance conjecture [19]. It would be interesting to explore the generation of hierarchies between the two scales, possibly based on discrete $\mathbf{Z}_k$ gauge symmetries as in [20].

- Our picture can be regarded as belonging to the rich field of swampland constraints on quantum gravity [21] (see [22–24] for reviews). It would be interesting to study the interplay with other swampland constraints. In particular, the relation between the strength of the dynamical tadpole and the size of the spacetime dimensions is tantalizingly reminiscent of the first condition on $|\nabla V|/V$ of the de Sitter conjecture [25–27], with $\mathcal{T} = |\nabla V|$ and if we interpret $V$ as the inverse Hubble volume and hence a measure of size or length scale in the spacetime dimensions. It would be interesting to explore cosmological setups and a possible role of horizons as alternative mechanisms to cut off spacetime. Also, the inequality admittedly works in different directions in the two setups, thus suggesting they are not equivalent, but complementary relations.



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- It would be interesting to apply our ideas to the study of other setups in which spacetime is effectively cut off, such as the capping off of the throat in near horizon NS5-branes due to strong coupling effects, or the truncation in [28] of throats of the euclidean wormholes in pure Einstein+axion theories [29].

- Finally, we have not discussed time-dependent backgrounds.[2] These are obviously highly interesting, but their proper understanding is likely to require new ingredients, such as *end (or beginning) of time* defects (possibly as generalization of the spacelike S-branes [30, 31]).

Until we come back to these questions in future work, the present paper is organized as follows. In section 2 we reinterpret the Klebanov-Strassler (KS) warped throat supported by 3-form fluxes as a template illustrating our two tadpole lessons. Section 2.1 explains that the introduction of RR 3-form flux in type IIB theory on $\mathrm{AdS}_5 \times T^{1,1}$ produces a tadpole. The varying field configuration is the Klebanov-Tseytlin solution, which leads to a metric singularity at a finite distance scaling as (1.1), as we show in section 2.2. In section 2.3 we relate the KS smoothing of this singularity with cobordism defects. In section 2.4 we extend the discussion to other warped throats. In section 3 we present a similar discussion in toroidal compactifications with fluxes. Section 3.1 introduces a $\mathbf{T}_5$ compactification with RR 3-form flux, whose tadpole backreacts producing singularites at finite distance as we show in section 3.2. In section 3.3 we argue they are smoothed out by capping off dimensions and triggering spontaneous compactification. In section 4 we build examples in the context of magnetized D-branes. In section 4.1 we describe the tadpole backreaction and its singularities, which are removed by spontaneous compactification in section 4.2. In section 5 we turn to the dilaton tadpole of several 10d strings. In section 5.1 we consider massive type IIA theory, where the running dilaton solutions produce dynamical cobordisms by introduction of O8-planes as cobordism defects of the IIA theory, eventually closely related to type I' compactifications. In section 5.2 we discuss a similar picture for M-theory on K3 with $G_4$ flux, and a Horava-Witten wall as its cobordism defect. In section 5.3 we consider the 10d non-supersymmetric USp(32) theory, in two different approaches. In section 5.3.1 we build on the classical solution in [9] and discuss its singularities in the light of the cobordism conjecture. In section 5.3.2 we describe an explicit (and remarkably, supersymmetry preserving) configuration solving its tadpole via magnetization and spontaneous compactification on $\mathbf{T}^6$. In section 6 we discuss an interesting application, describing a 6d model with tadpoles, which upon spontaneous compactification reproduces a semi-realistic MSSM-like brane model. Finally, appendix A discusses the violation of swampland constraints of type IIB on $\mathrm{AdS}_5 \times T^{1,1}$ when its tadpole is not duly backreacted, in a new example of the mechanism in [3].

## 2 The fluxed conifold: KS solution as spontaneous cobordism

In this section we consider the question of dynamical tadpoles and their consequences in a particular setup, based on the gravity dual of the field theory of D3-branes at a coni-

[2] For classical solutions of tadpoles involving time dependence, see e.g. [11].

fold singularity. The discussion is a reinterpretation, in terms useful for our purposes, of the construction of the Klebanov-Tseytlin (KT) solution [32] and its deformed avatar, the Klebanov-Strassler (KS) solution [33]. This reinterpretation however provides an illuminating template to discuss dynamical tadpoles in other setups in later sections.

We consider type IIB on $\mathrm{AdS}_5 \times T^{1,1}$, where $T^{1,1}$ is topologically $\mathbf{S}^2 \times \mathbf{S}^3$ [34]. This is the near horizon geometry of D3-branes at the conifold singularity [34] (see also [35–37]), which has been widely exploited in the context of holographic dualities. The vacuum is characterized by the IIB string coupling $e^{\phi} = g_s$ and the RR 5-form flux $N$. The model has no scale separation, since the $T^{1,1}$ and $\mathrm{AdS}_5$ have a common scale $R$, given by

$$R^4 = 4\pi\, g_s N \alpha'^2\,. \tag{2.1}$$

In any event, we will find useful to discuss the model, and its modifications, in terms of the (KT) 5d effective theory introduced in [32]. This is an effective theory not in the Wilsonian sense but in the sense of encoding the degrees of freedom surviving a consistent truncation. In particular, it includes the dilaton $\phi$ (we take vanishing RR axion for simplicity), the NSNS axion $\Phi = \int_{\mathbf{S}^2} B_2$ and the $T^{1,1}$ breathing mode $q$ (actually, stabilized by a potential arising from the curvature and the 5-form flux), which in the Einstein frame enters the metric as

$$ds_{10}^2 = R^2 \left( e^{-5q}\, ds_5^2 + e^{3q} ds^2_{T^{1,1}} \right)\,. \tag{2.2}$$

This approach proved useful in [38] in the discussion of the swampland distance conjecture [39] in configurations with spacetime-dependent field configurations (see [19] for a related subsequent development, and [40, 41]).

### 2.1 The 5d tadpole and its solution

Let us introduce $M$ units of RR 3-form flux in the $\mathbf{S}^3$, namely

$$F_3 = M\,\omega_3\,, \tag{2.3}$$

where $\omega_3$ is defined in eq. (27) in [33]. We do not need its explicit expression, it suffices to say that it describes a constant field strength density over the $\mathbf{S}^3$. The introduction of this flux sources a backreaction on the dilaton and the metric, namely a dynamical tadpole for $\phi$ and $q$. In addition, as noticed in [38], it leads to an axion monodromy potential for $\Phi$ [42–45]. The situation is captured by the KT effective action (with small notation changes) for the 5d scalars $\phi$, $\Phi$ and $q$, collectively denoted by $\varphi^a$

$$S_5 = -\frac{2}{\kappa_5^2}\int d^5x\, \sqrt{-g_5}\Big[\frac{1}{4}R_5 - \frac{1}{2}G_{ab}(\varphi)\partial\varphi^a\partial\varphi^b - V(\varphi)\Big], \tag{2.4}$$

with the kinetic terms and potential given by

$$G_{ab}(\varphi)\partial\varphi^a\partial\varphi^b = 15(\partial q)^2 + \frac{1}{4}(\partial\phi)^2 + \frac{1}{4}e^{-\phi-6q}(\partial\Phi)^2\,, \tag{2.5}$$

$$V(\varphi) = -5e^{-8q} + \frac{1}{8}M^2\, e^{\phi-14q} + \frac{1}{8}(N + M\Phi)^2 e^{-20q}\,. \tag{2.6}$$



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Clearly $g_s M^2$ is an order parameter of the corresponding dynamical tadpole. In the following we focus on the case[3] of $N$ being a multiple of $M$.

Ignoring the backreaction of the dynamical tadpole (i.e. considering constant profiles for the scalars over the 5d spacetime) is clearly incompatible with the equations of motion. Furthermore, as argued in [3], it can lead to violations of swampland constraints. In particular, since the introduction of $F_3$ breaks supersymmetry, if the resulting configuration was assumed to define a stable vacuum, it would violate the non-susy AdS conjecture [46]; also, as we discuss in appendix A, it potentially violates the Weak Gravity Conjecture [47].

Hence, we are forced to consider spacetime-dependent scalar profiles to solve the equations of motion. Actually, this problem was tackled in [33], with the scalars running with $r$, as we now review in the interpretation in [38]. There is a non-trivial profile for the axion $\Phi$, given by

$$\Phi = 3 g_s M \, \log(r/r_0) \,. \tag{2.7}$$

This implies the cancellation of the dilaton tadpole, which can be kept constant $e^\phi = g_s$, as follows from its equation of motion from (2.5), (2.6)

$$\nabla\phi \sim -e^{-6q-\phi}(\partial\Phi)^2 + e^{-14q+\phi} M^2 \,. \tag{2.8}$$

### 2.2 Singularity at finite distance

The varying $\Phi$ corresponds to the introduction of an NSNS 3-form flux in the configuration

$$H_3 = -g_s \, *_{6d} \, F_3 \,, \tag{2.9}$$

where the 6d refers to $T^{1,1}$ and the $\mathrm{AdS}_5$ radial coordinate $r$, and the Hodge duality is with the $\mathrm{AdS}_5 \times T^{1,1}$ metric. This is precisely such that the complexified flux combination $G_3 = F_3 - \tau H_3$ satisfies the imaginary self duality (ISD) constraint making it compatible with 4d Poincaré invariance in the remaining 4d coordinates (and in fact, it also preserves supersymmetry). The backreaction on the metric thus has the structure in [48, 49]. The metric (2.2) takes the form

$$ds_{10}^2 = Z^{-\frac{1}{2}} \eta_{\mu\nu} dx^\mu dx^\nu + Z^{\frac{1}{2}} \left( dr^2 + r^2 ds^2_{T^{1,1}} \right) , \tag{2.10}$$

where $Z$ obeys a Laplace equation in $\mathrm{AdS}_5$, sourced by the fluxes, and reads

$$Z(r) = \frac{1}{4r^4} \left(g_s M\right)^2 \, \log(r/r_0) \,. \tag{2.11}$$

The warp factor also enters in the RR 5-form flux, which decreases with $r$ as

$$N(r) = \int_{\mathbf{S}^5} F_5 = g_s M^2 \log(r/r_0) \,. \tag{2.12}$$

This matches nicely with the monodromy for the axion $\Phi$ as it runs with $r$ [38]. These features (as well as some other upcoming ones) were nicely explained as the gravity dual of a Seiberg duality cascade in [33].

[3] This implies that the configuration is uncharged under a discrete $\mathbf{Z}_M$ symmetry, measured by $N$ mod $M$, and associated to the redundancy generated by transformation $\phi \to \phi + 1$, $N \to N - M$, see footnote 5.



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This 5d running solution in [32] solves the dynamical tadpole, but is not complete, as it develops a metric singularity at $r = r_0$. This is a physical singularity at finite distance in spacetime, whose parametric dependence on the parameters of the initial model is as follows

$$\begin{aligned}\Delta(r) &= \int_{r_0}^{r} Z(r)^{\frac{1}{4}}\, dr \sim \int_{r_0}^{r} (g_s M)^{\frac{1}{2}} \big[\log(r/r_0)\big]^{\frac{1}{4}} \frac{dr}{r} \\ &\sim (g_s M)^{\frac{1}{2}} \big[\log(r/r_0)\big]^{\frac{5}{4}} = (g_s N)^{\frac{1}{4}} \frac{N}{g_s M^2} \sim R\, \frac{N}{g_s M^2}\,. \end{aligned} \tag{2.13}$$

In the last equalities we used (2.12), (2.1). Hence, starting with an $\mathrm{AdS}_5 \times T^{1,1}$ theory with $N$ units of RR 5-form flux, the introduction of $M$ units of RR 3-form flux leads to a breakdown of the corresponding spacetime-dependent solution at a distance scaling as $\Delta \sim M^{-2}$. Recalling that the dynamical tadpole is controlled by an order parameter $\mathcal{T} = g_s M^2$, this precisely matches the scaling relation (1.1) of the Finite Distance Lesson.

### 2.3 Dynamical cobordism and the KS solution

As is well known, the singularity in the KT solution is smoothed out in the KS solution [33]. This is given by a warped version of the deformed conifold metric, instead of the conical conifold singularity, with warp factor again sourced by an ISD combination of RR 3-form flux on $\mathbf{S}^3$ and NSNS 3-form flux on $\mathbf{S}^2$ times the radial coordinate. At large $r$ the KS solution asymptotes to the KT solution, but near $r \sim r_0$, the solutions differ and the KT singularity is replaced by the finite size $\mathbf{S}^3$ of the deformed conifold.

Hence, the Finite Distance Lesson still applies even when the singularity is removed, and the impossibility to extend the coordinate $r$ to arbitrary distances is implemented by a smooth physical end of spacetime. The purpose of this section is to highlight a novel insight on the KS solution, as a non-trivial realization of the swampland cobordism conjecture [14, 15]. The latter establishes that any consistent quantum gravity theory must be trivial in (a suitably defined version of) cobordism. Namely in an initial theory given by an $n$-dimensional internal compactification space (possibly decorated with additional ingredients, like branes or fluxes), there must exist configurations describing an $(n+1)$-dimensional (possibly decorated) geometry whose boundary is the initial one. The latter describes an end of the world defect (which we will refer to as the 'cobordism defect') for the spacetime of the initial theory. Since the arguments about the swampland cobordism conjecture are topological, there is no claim about the unprotected properties of the cobordism defect, although in concrete examples it can preserve supersymmetry; for instance, in maximal dimensions, the Horava-Witten boundary is the cobordism defect for 11d M-theory, and similarly the O8-plane is the cobordism defect of type IIA theory.[4]

In our setup, the initial theory is $\mathrm{AdS}_5 \times T^{1,1}$ with $N$ units of RR 5-form flux and $M$ units of RR 3-form flux on $\mathbf{S}^3$. From the above discussion, it is clear that the KS solution

[4]Other 10d theories are conjectured to admit cobordism branes, but they cannot be supersymmetric and their nature is expected to be fairly exotic, and remains largely unknown. We will come back to this point in section 5.3.1.



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is just the cobordism defect of this theory.[5] The remarkable feature is that the end of spacetime is triggered dynamically by the requirement of solving the equations of motion after the introduction of the RR 3-form flux, hence it is fair to dub it dynamical cobordism. Hence, this is a very explicit illustration of the Dynamical Cobordism Lesson.

This powerful statement will be realized in many subsequent examples in later sections, and will underlie the phenomenon of spontaneous compactification, when the cobordisms close off the spacetime directions bounding them into a compact variety.

### 2.4 More general throats

A natural question is the extension of the above discussion to other $\mathrm{AdS}_5 \times \mathbf{X}_5$ vacua with 3-form fluxes. This question is closely related to the search for general classes of gravity duals to Seiberg duality cascades and their infrared deformations, for which there is a concrete answer if $\mathbf{X}_5$ is the real base of a non-compact toric CY threefold singularity $\mathbf{Y}_6$, which are very tractable using dimer diagrams [50, 51] (see [52] for a review).

From our perspective, the result in [53] is that the $\mathbf{X}_5$ compactification with 3-form flux $F_3$ admits a KS-like end of the world (cobordism defect[6]) if $\mathbf{Y}_6$ admits a complex deformation which replaces its conical singularity by a finite-size 3-cycle corresponding to the homology dual of the class $[F_3]$. In cobordism conjecture terms, in these configurations the corresponding global symmetry is broken, and spacetime may close off without further ado (as the axion monodromy due to the 3-form fluxes allows to eat up the RR 5-form flux before reaching the end of the world). Such complex deformations are easily discussed in terms of the web diagram for the toric threefold, as the splitting of the web diagram into consistent sub-diagrams [53]. Simple examples include the deformation of the complex cone over $\mathrm{dP}_2$ to a smooth geometry, or the deformation of the complex cone over $\mathrm{dP}_3$ to a conifold, or to a smooth geometry.

There are however singularities (or 3-form flux assignments), for which the complex deformations are simply not available. One may then wonder about how our Dynamical Cobordism lesson applies. The answer was provided in particular examples in [54–56]: the infrared end of the throat contains an explicit system of fractional D-branes, which in the language of the cobordism conjecture kill the corresponding cobordism classes, and allow the spacetime to end. As noticed in these references, the system breaks supersymmetry, and in [55] it was moreover noticed (as later revisited in [57]) to be unstable and lead to a runaway behaviour for the field blowing up the singularity. Hence, this corresponds to an additional dynamical tadpole, requiring additional spacetime dependence, to be solved. Simple examples include the complex cone over $\mathrm{dP}_1$, and the generic $Y^{p,q}$ theories. We will not enter the discussion of possible mechanisms to stabilize these models, since following [58] they are likely to require asymptotic modifications of the warped throat ansatz (i.e. at all positions in the radial direction, including the initial one).

[5] Recalling footnote 3, the case of $N$ multiple of $M$ implies the vanishing of a $\mathbf{Z}_M$ charge, and allows the cobordism defect to be purely geometrical; otherwise the cobordism defect ending spacetime must include explicit D3-branes, which are the defect killing the corresponding cobordism class [14].

[6] We note in passing that the regions between different throats in the multi-throat configurations [53] can be regarded as domain walls interpolating between two different, but bordant, type IIB vacua.

## 3 Type IIB fluxes and spontaneous compactification

In this section we construct an explicit 5d type IIB model with a tunable dynamical tadpole, and describe the spacetime-dependent solution solving its equations of motion, which is in fact supersymmetry preserving. The configuration displays dynamical cobordism resulting in spontaneous compactification to 4d. The resulting model is a simple toroidal compactification with ISD NSNS and RR 3-form fluxes [48, 49], in particular it appeared in [59, 60]. With this perspective in hindsight, one can regard this section as a reinterpretation of the latter flux compactification. Our emphasis is however in showing the interplay of the dynamical tadpoles in the 5d theory and the consequences in the spacetime configuration solving them.

### 3.1 The 5d tadpole and its solution

Consider type IIB on $\mathbf{T}^5$, which for simplicity we consider split as $\mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{S}^1$. We label the coordinates of the $\mathbf{T}^2$'s as $(x^1, y^1)$ and $(x^2, y^2)$, with periodicity 1, and introduce complex coordinates as $z^1 = x^1 + \tau_1 y^1$, $z^2 = x^2 + \tau_2 y^2$. We also use a periodic coordinate $x^3 \simeq x^3 + 1$ to parametrize the $\mathbf{S}^1$. For simplicity, we do not consider moduli deviating from this rectangular structure,[7] and also take the $\mathbf{T}^5$ to have an overall radius $R$,

$$ds^2 = R^2[(dz^1)^2 + (dz^2)^2 + (dx^3)^2]\,. \tag{3.1}$$

The result so far is a standard 5d supersymmetric $\mathbf{T}^5$ compactification.

We introduce a non-trivial dynamical tadpole source by turning on an RR 3-form flux (using conventions in [49])

$$F_3 = (2\pi)^2 \alpha' \, N \, dx^1 \, dx^2 \, dx^3\,. \tag{3.2}$$

The introduction of this flux does not lead to RR topological tadpoles, but induces dynamical tadpoles for diverse fields. In the following we focus on the dynamics of the 5d light fields $R$, $\tau_1$, $\tau_2$, the dilaton $\phi$ and the NSNS axion $\Phi$ defined by

$$B_2 = \Phi \, dy^1 \, dy^2\,. \tag{3.3}$$

The discussion of the dynamical tadpole is similar to the $T^{1,1}$ example in section 2, so we sketch the result. There is a dilaton tadpole, arising from the dimensional reduction of the 10d kinetic term for the 3-form flux,

$$\nabla^2 \phi = \frac{1}{12} \, e^{\phi} \, (F_3)^2\,. \tag{3.4}$$

Since $(F_3)^2$ is a constant source density, which does not integrate to zero over $\mathbf{T}^5$, there is no solution for this Laplace equation if we assume the solution to be independent of the 5d spacetime coordinates. One possibility would be to allow for 5d spacetime dependence of $\phi$ (at least in one extra coordinate, as in [9]). Here we consider a different possibility, which

[7] As usual, they can be removed in orbifold models, although we will not focus on this possibility.



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is to let the NSNS axion $\Phi$ acquire a dependence on one of the 5d coordinates, which we denote by $y$, as follows

$$\Phi = -(2\pi)^2\alpha' \, \frac{N}{t_3}\, y \quad \Rightarrow \quad H_3 = -(2\pi)^2\alpha' \, \frac{N}{t_3}\, dy^1\, dy^2\, dy\,. \tag{3.5}$$

We have thus turned on NSNS 3-form field strength in the directions $y^1$, $y^2$ in $\mathbf{T}^5$ and the 5d spacetime coordinate $y$. Here the sign has been introduced for later convenience, and $t_3$ is a positive real parameter allowing to tune the field strength density, whose meaning will become clear later on.

Including this new source, the dilaton equation of motion becomes

$$\nabla^2\, \phi = \frac{1}{12}\left[ e^{\phi}\,(F_3)^2 - e^{-\phi}\,(H_3)^2 \right]. \tag{3.6}$$

Hence, the spacetime-dependent profile (3.5) can cancel the right hand side and solve the dilaton tadpole when

$$e^{2\phi}\,(F_3)^2 = (H_3)^2\,. \tag{3.7}$$

We can thus keep the dilaton constant $e^{\phi} = g_s$. Taking for simplicity purely imaginary $\tau_1 = it_1$ and $\tau_2 = it_2$, the condition (3.7) is simply

$$g_s\, t_1\, t_2\, t_3 = 1\,. \tag{3.8}$$

In addition to the dilaton, the 3-form fluxes backreact on the metric and other fields, which we discuss next.

### 3.2 The singularities

We now discuss the backreaction on the metric and other fields. For convenience, we use the complex coordinates $z^1$, $z^2$ and $z^3 = x^3 + iy$. In terms of these, we can write the combination

$$G_3 = F_3 - \tau H_3 = \frac{(2\pi)^2}{4}\alpha'\, N(d\bar{z}_1\, dz_2\, dz_3 + dz_1 d\bar{z}_2 dz_3 + dz_1 dz_2 d\bar{z}_3 + d\bar{z}_1 d\bar{z}_2 d\bar{z}_3)\,. \tag{3.9}$$

Regarding $\mathbf{T}^5 \times \mathbf{R}^1_y$ as a (non-compact) CY, this is a combination of $(2,1)$ and $(0,3)$ components, which is thus ISD. There is a backreaction on the metric and RR 4-form of the familiar black 3-brane kind. In particular, the metric includes a warp factor $Z$

$$ds^2_{10} = Z^{-\frac{1}{2}}\eta_{\mu\nu}dx^{\mu}dx^{\nu} + Z^{\frac{1}{2}}\, R^2[dz^1 d\bar{z}^1 + dz^2 d\bar{z}^2 + dz^3 d\bar{z}^3]\,, \tag{3.10}$$

where $x^{\mu}$ runs through the four Poincaré invariant spacetime coordinates. The warp factor is determined by the Laplace equation

$$-\tilde{\nabla}^2 Z = \frac{g_s}{12} G_3 \cdot \overline{G}_3 = \frac{g_s}{6}\,(F_3)^2\,, \tag{3.11}$$

with the tilde indicating the Laplacian is computed with respect to the unwarped, flat metric, and in the last equation we used (3.7).

Note that, since $y$ parametrizes a non-compact dimension, there is no tadpole problem in solving (3.11) i.e. we need not add background charge. One may then be tempted to conclude that this provides a 5d spacetime-dependent configuration solving the 5d tadpole. However, the solution is valid locally in $y$, but cannot be extended to arbitrary distances in this direction. Since the local flux density in $\mathbf{T}^5$ is constant, we can take $Z$ to depend only on[8] $y$, hence leading to a solution

$$-\frac{d^2Z}{dy^2} = \frac{g_s}{6}(F_3)^2 \quad \Rightarrow \quad Z = 1 - \frac{g_s}{12}(F_3)^2\, y^2\,, \tag{3.12}$$

where we have set an integration constant to 1. The solution hits metric singularities at

$$y^{-2} = \frac{1}{12} g_s (F_3)^2\,, \tag{3.13}$$

showing there is a maximal extent in the direction $y$. Let us introduce the quantity $\mathcal{T} = \frac{1}{12} g_s (F_3)^2$, which controls the parametric dependence of the tadpole. Then, the distance between the singularities is

$$\Delta = \int_{-\mathcal{T}^{-1/2}}^{\mathcal{T}^{-1/2}} Z^{\frac{1}{4}}\, dy = \frac{2}{\sqrt{\mathcal{T}}} \int_0^1 (1-t^2)^{\frac{1}{4}}\, dt\,, \tag{3.14}$$

with $t = \sqrt{\mathcal{T}} y$. We thus recover the scaling (1.1) with $n = 2$,

$$\Delta^{-2} \sim \mathcal{T}\,. \tag{3.15}$$

Hence the appearance of the singularities as a consequence of the dynamical tadpole is as explained in the introduction.

### 3.3 Cobordism and spontaneous compactification

The appearance of singularities is a familiar phenomenon. In this section we argue that they must be smoothed out, somewhat analogously to the KS solution in section 2. The fact that it is possible follows from the swampland cobordism conjecture [14, 15], namely there must exist an appropriate cobordism defect closing off the extra dimension into nothing. Since there are two singularities, the formerly non-compact dimension becomes compact, in an explicit realization of spontaneous compactification.[9]

In the following, we directly describe the resulting geometry, which turns out to be a familiar $\mathbf{T}^6$ (orientifold) compactification with ISD 3-form fluxes. Consider type IIB theory on $\mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{T}^2$, with

$$F_3 = (2\pi)^2 \alpha'\, N\, dx^1\, dx^2\, dx^3\,, \quad H_3 = (2\pi)^2 \alpha'\, N\, dy^1\, dy^2\, dy^3\,. \tag{3.16}$$

We use $z^i = x^i + i t_i y^i$, hence the above defined $t_3$ is the complex structure modulus for the $\mathbf{T}^2$ involving the newly compact dimension. For moduli satisfying (3.8) the $\mathbf{T}^6$ flux combination $G_3$ is given by (3.9), which is ISD and indeed compatible with 4d Poincaré

[8]In fact, this is the leading behaviour at long distances, compared with the $\mathbf{T}^5$ size scale $R$.
[9]Spontaneous compactification has been discussed in the context of dynamical tadpoles in [9].



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invariance as usual. Notice that in this case, it is possible to achieve a large size for the new compact dimension $t_3 \gg 1$ by simply e.g. taking small $g_s$. This corresponds to the regime of small 5d tadpole, with the relation

$$t_3^{-2} \sim g_s^2 \sim \mathcal{T}^2 \,, \tag{3.17}$$

in agreement with the maximal distance relation in the previous section.

Consistency, in the form of $C_4$ RR tadpole cancellation, requires the introduction of O3-planes at fixed points of the involution $\mathcal{R} : z^i \to -z^i$ (together with mobile D3-branes). From the perspective of the 5d theory, the additional dimension is compactified on an interval, with two end of the world defects given by the O3-planes, which constitute the cobordism defects of the configuration (possibly decorated with explicit D3-branes if needed).

## 4 Solving dynamical tadpoles via magnetization

In this section we consider a further setup displaying dynamical tadpoles, based on compactifications with magnetized D-branes [61–65]. In toroidal setups, these have been (either directly or via their T-dual intersecting brane world picture) widely used to realize semi-realistic particle physics models in string theory. In more general setups, magnetized 7-branes are a key ingredient in the F-theory realization of particle physics models [66–68].

### 4.1 Solving dynamical tadpoles of magnetized branes

We consider a simple illustrative example. Consider type IIB theory compactified on $\mathbf{T}^2 \times \mathbf{T}^2$ (labelled 1 and 2, respectively) and mod out by $\Omega\mathcal{R}_1(-1)^{F_L}$, where $\mathcal{R}_1 : z_1 \to -z_1$. This introduces 4 $\mathrm{O7}_1$-planes spanning $(\mathbf{T}^2)_2$ and localized at the fixed points on $(\mathbf{T}^2)_1$. We also have 32 D7-branes (as counted in the covering space), split as 16 D7-branes (taken at generic points) and their 16 orientifold images. This model is related by T-duality on $(\mathbf{T}^2)_1$ to a type I toroidal compactification, but we proceed with the D7-brane picture.

We introduce $M$ units of worldvolume magnetic flux along $(\mathbf{T}^2)_2$ for the U(1) of a D7-brane[10]

$$\frac{1}{2\pi\alpha'} \int_{\mathbf{T}^2} F_2 = M \,. \tag{4.1}$$

The orientifold requires we introduce $-M$ units of flux on the image D7-brane.[11] This also ensures that there is no net induced $\mathbf{Z}$-valued D5-brane charge in the model, and hence no associated RR tadpole, in agreement with the fact that the RR 6-form is projected out. In addition, there is a $\mathbf{Z}_2$ K-theory charge [70] which is cancelled as long as $M \in 2\mathbf{Z}$.

The introduction of the worldvolume flux leads to breaking of supersymmetry. As is familiar in the discussion of supersymmetries preserved by different branes [71], we introduce the angle

$$\theta_2 = \arctan(2\pi\alpha' F) = \arctan(M\chi) \,, \tag{4.2}$$

where $F$ is the field strength and $\chi$ is the inverse of the $\mathbf{T}^2$ area, in string units.

[10]If $N$ the D7-branes are coincident, it is also possible to use the overall $\mathrm{U}(1) \subset \mathrm{U}(N)$. We will stick to the single D7-brane for the moment, but such generalization will arise in later examples.

[11]For simplicity we consider vanishing discrete NSNS 2-form flux [69], although such generalization will arise in later examples.



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This non-supersymmetric configuration introduces dynamical tadpoles. For small $\theta_2$, the extra tension can be described in effective field theory as an FI term controlled by $\theta$ [72–74]. In fact, in [75] a similar parametrization was proposed for arbitrary angles. By using the DBI action, the extra tension has the structure

$$V \sim \frac{1}{g_s}\left(\sqrt{1+(\tan\theta_2)^2}-1\right). \tag{4.3}$$

This leads to a tadpole for the dilaton and the $(\mathbf{T}^2)_2$ Kähler modulus.

We now consider solving the tadpole by allowing for some spacetime-dependent background. Concretely, we allow for a non-trivial magnetic field $-F$ on two of the non-compact space coordinates, parametrized by the (for the moment, non-compact) coordinate $z_3$. In fact this leads to a configuration preserving supersymmetry since, defining the angle $\theta_3$ in analogy with (4.2), we satisfy the SU(2) rotation relation $\theta_3+\theta_2=0$ [71]. In other words, the field strength flux has the structure

$$F_2 = F(dz_2 d\bar{z}_2 - dz_3 d\bar{z}_3)\,, \tag{4.4}$$

which is $(1,1)$ and primitive (i.e. $J\wedge F_2=0$), which are the supersymmetry conditions for a D-brane worldvolume flux.

Hence, it is straightforward to find spacetime-dependent solutions to the tadpole of the higher-dimensional theory, at the price of breaking part of the symmetry of the lower-dimensional spacetime. In the following we show that, as in earlier examples, this eventually also leads to spontaneous compactification.

### 4.2 Backreaction and spontaneous compactification

The spacetime field strength we have just introduced couples to gravity and other fields, so we need to discuss its backreaction.

In fact, this is a particular instance of earlier discussions, by considering the F-theory lift of the D7-brane construction. This can be done very explicitly by taking the configuration near the $\mathrm{SO}(8)^4$ weak coupling regime [76]. The configuration without magnetic flux $M=0$ simply lifts to F-theory on K3$\times\mathbf{T}^2\times\mathbf{R}^2$, where the $(\mathbf{T}^2)_1$ (modulo the $\mathbf{Z}_2$ orientifold action) is the $\mathbf{P}_1$ base of K3, and the $\mathbf{T}^2$ and $\mathbf{R}^2$ explicit factors correspond to the directions $z_2$ and $z_3$, respectively. As is familiar, the 24 degenerate fibers of the K3 elliptic fibration form 4 pairs, reproducing the 4 orientifold planes, and 16 D7-branes in the orientifold quotient. Actually, the discussion below may be carried out for F-theory on K3 at generic points in moduli space, even not close to the weak coupling point.

The introduction of magnetization for one 7-brane corresponds to the introduction of a $G_4$ flux along the local harmonic $(1,1)$-form supported at an $I_1$ degeneration (or enhanced versions thereof, for coincident objects), of the form

$$G_4 = \omega_2\wedge F(dz_2 d\bar{z}_2 - dz_3 d\bar{z}_3)\,. \tag{4.5}$$

This flux is self-dual, and in fact $(2,2)$ and primitive, which is the supersymmetry preserving condition for 4-form fluxes in M/F-theory [48, 77]. The backreacted metric is described by a warp factor satisfying a Laplace equation sourced by the fluxes, similar to (3.11). Considering the regime in which the warp factor is taken independent of the internal space and depends only on the coordinates in the $\mathbf{R}^2$ parametrized by $z_3$, the constant flux density leads to singularities at a maximal length scale $\Delta$

$$\Delta^{-2} \sim F^2 \,. \tag{4.6}$$

This is another instance of the universal relation (1.1) with $\mathcal{T} \sim F^2$, hence $n = 2$.

This is in complete analogy with earlier examples. Hence, we are led to propose that the smoothing out of these singularities is provided by the compactification of the corresponding coordinates, e.g. on a $\mathbf{T}^2$, with the addition of the necessary cobordism defects, namely orientifold planes and D-branes.[12]

To provide an explicit solution, we introduce the standard notation (see e.g. [63, 64]) of $(n, m)$ for the wrapping numbers and the magnetic flux quanta on the $(\mathbf{T}^2)_i$'s for the directions $i = 1, 2, 3$. In this notation, the $O7_1$-planes and unmagnetized $D7_1$-branes are associated to $(0, 1) \times (1, 0) \times (1, 0)$, while the magnetized $D7_1$-branes[13] correspond to $(0, 1) \times (1, M) \times (1, -M)$, and $(0, 1) \times (1, -M) \times (1, M)$ for the orientifold images. In other words, we require a flux quantization condition on $(\mathbf{T}^2)_3$ as in (4.1), up to a sign flip.

Since now the last complex dimension is compact, there is an extra RR tadpole cancellation condition, which requires the introduction of 16 $O7_3$-planes, wrapped on $(\mathbf{T}^2)_1 \times (\mathbf{T}^2)_2$ and localized at fixed points in $(\mathbf{T}^2)_1$, namely with wrapping numbers $(1, 0) \times (1, 0) \times (0, 1)$. This introduces an extra orbifold action generated by $(z_1, z_2, z_3) \to (z_1, -z_2, -z_3)$, so the model can be regarded as a (T-dual of a) magnetized version of the D9/D5-brane $\mathbf{T}^4/\mathbf{Z}_2$ orientifolds in [78, 79]. Allowing for $n$ additional mobile $D7_3$-branes (as counted in the covering space, and arranged in orbifold and orientifold invariant sets), the RR tadpole cancellation conditions is

$$2M^2 + n = 32 \,. \tag{4.7}$$

The supersymmetry condition is simply that the $\mathbf{T}^2$ parameters satisfy $\chi_2 = \chi_3$.

From the perspective of the original 6d configuration, the tadpole in the initial $\mathbf{T}^2 \times \mathbf{T}^2$ configuration has triggered a spontaneous compactification. Since the additional O-planes and D-branes required to cancel the new RR tadpoles are localized in $z_3$, they can be interpreted as the addition of I-branes to cancel the cobordism charge of the original model.

It should be possible to generalize the above kind of construction to global K3-fibered CY threefolds with O7-planes. The local fibration in a small neighbourhood of a generic point of the base provides a local 6d model essentially identical to our previous one. On

[12]To be precise, the cobordism defects of an $\mathbf{S}^1$ compactification of the model. This is analogous to the mechanism by which F-theory on half a $\mathbf{P}_1$ provides the cobordism defect for type IIB on $\mathbf{S}^1$ with $SL(2, \mathbf{Z})$ monodromy [14] (see also [16]). In fact, since magnetized branes often lead to chiral theories in the bulk, this extra circle compactification allows them to become non-chiral and admit an end of the world describable at weak coupling, see the discussion below (5.14) in section 5.3. We will nevertheless abuse language and refer as cobordism defect to the structures involved in the final spontaneous compactification under discussion.

[13]If the magnetization is in the $U(1) \subset U(N)$ of a stack of $N$ coincident branes, see footnote 10, the corresponding wrapping goes as $(N, M)$.

the other hand, the global geometry defining how the two extra dimensions compactify would correspond to another possible spontaneous compactification, with the ingredients required for the cancellation of the new RR tadpoles.

However, a general drawback of this class of models is that the scales of the compact spaces in the directions 2 and 3 are of the same order.[14] Thus, there is no separation of scales, and no reliable regime in which the dynamics becomes that of a 6d model. This is easily avoided in more involved models, as we will see in the examples in coming sections.

## 5 Solving tadpoles in 10d strings

In this section we consider dynamical tadpoles arising in several 10d string theories, and confirm the general picture. We illustrate this with various examples, with superymmetry (massive type IIA and M-theory on K3), and without it (non-supersymmetric 10d USp(32) theory).

### 5.1 Massive IIA theory

We consider 10d massive type IIA theory [80]. This can be regarded as the usual type IIA string theory in the presence of an additional RR 0-form field strength $F_0 \equiv m$. The string frame effective action for the relevant fields is

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi}[R + 4(\partial\phi)^2] - \frac{1}{2}(F_0)^2 - \frac{1}{2}(F_4)^2 \right\} + S_{\text{top}} \,, \tag{5.1}$$

where $S_{\text{top}}$ includes the Chern-Simons terms. In the Einstein frame $G_E = e^{-\frac{\phi}{2}}G$, we have

$$S_{10,E} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G_E} \left\{ \left[ R - \frac{1}{2}(\partial\phi)^2 \right] - \frac{1}{2} e^{\frac{5}{2}\phi} m^2 - \frac{1}{2} e^{\frac{1}{2}\phi}(F_4)^2 \right\} . \tag{5.2}$$

Here we have used $m$ to emphasize this quantity is constant. This theory is supersymmetric, but at a given value of $\phi$, it has a tadpole controlled by

$$\mathcal{T} \sim e^{\frac{5}{2}\phi} m^2 \,. \tag{5.3}$$

This is in particular why the massive IIA theory does not admit 10d maximally symmetric solutions. In the following we discuss two different ways of solving it, leading to Minkowski or AdS-like configurations.

#### 5.1.1 Solution in 9d and type I' as cobordism

To solve the tadpole (5.3) we can consider a well-known 1/2 BPS solution with the dilaton depending on one coordinate $x^9$. Since the flux $m$ can be regarded as generated by a set of $m$ distant D8-branes, this is closely related to the solution in [81]. We describe it in

[14]In the toroidal example, if the magnetization along $(\mathbf{T}^2)_2$ is on the overall $\mathrm{U}(1) \subset \mathrm{U}(16)$ of 16 coincident D7-branes, the magnetic field along $z_2$ is $F \sim M/16$; this weakened tadpole implies an increase of the critical size of the spontaneously compactified dimensions by a factor of 4.



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conventions closer to [17], for later use. In the Einstein frame, the metric and dilaton background have the structure

$$(G_E)_{MN} = Z(x^9)^{\frac{1}{12}}\,\eta_{MN}\,, \quad e^{\phi} = Z(x^9)^{-\frac{5}{6}}\,, \quad \text{with } Z(x^9) \sim B - m x^9\,, \tag{5.4}$$

where $B$ is some constant (in the picture of flux generated by distant D8-branes, it relates to the D8-brane tensions). The solution hits a singularity at $x^9 = B/m$. Starting at a general position $x^9$, the distance to the singularity is

$$\Delta = \int_{x^9}^{\frac{B}{m}} Z(x^9)^{\frac{1}{24}}\,dx^9 \sim Z(x^9)^{\frac{25}{24}}\,m^{-1} \sim m^{-1} e^{-\frac{5}{4}\phi}\,, \tag{5.5}$$

where in the last equality we have traded the position for the value the dilaton takes there. Recalling (5.3), this reproduces the Finite Distance scaling relation (1.1) with $n = 2$,

$$\Delta^{-2} \sim \mathcal{T}\,. \tag{5.6}$$

It is easy to propose the stringy mechanism capping off spacetime before or upon reaching this singularity, according to the Dynamical Cobordism lesson. This should be the cobordism defect of type IIA theory, which following [14] is an O8-plane, possibly with D8-branes.

In fact, this picture is implicitly already present in [17], which studies type I' theory, namely type IIA on an interval, namely IIA on $\mathbf{S}^1$ modded out by $\Omega\mathcal{R}$ with $\mathcal{R} : x^9 \to -x^9$, which introduces two O8$^-$-planes which constitute the interval boundaries. There are 32 D8-branes (in the covering space), distributed on the interval, which act as domain walls for the flux $F_0 = m$, which is piecewise constant in the interval. The metric and the dilaton profile are controlled by a piecewise linear function $Z(x^9)$. The location of the boundaries at points of strong coupling was crucial to prevent contradiction with the appearance of certain enhanced symmetries in the dual heterotic string (the role of strong coupling at the boundaries for the enhancements was also emphasized from a different perspective in [82, 83]). In our setup, we interpret the presence of (at least, one) O8-plane as the cobordism defect triggered by the presence of a dynamical tadpole in the bulk theory.

#### 5.1.2 A non-supersymmetric Freund-Rubin solution

We now consider for illustration a different mechanism to cancel the dynamical tadpole, which in fact underlies the spontaneous compactification to (non-supersymmetric[15]) AdS$_4\times$ $\mathbf{S}^6$ in [80]). The idea is that, rather than solving for the dilaton directly, one can introduce an additional flux $F_4$ along three space dimensions and time (or its dual $F_6$ on six space dimensions) to balance off the dilaton sourced by $F_0$. This can be used to fix $\phi$ to a constant, and following [80] leads to a scaling

$$F_4 \sim m^2\, d(\text{vol})_4\,, \tag{5.7}$$

[15] Thus, it should be unstable according to [46]. However, being at a maximum of a potential is sufficient to avoid dynamical tadpoles, so the solution suffices for our present purposes.



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where $d(\mathrm{vol})_4$ is the volume form in the corresponding 4d. Using arguments familiar by now, the constant $F_4$ backreaction on the metric is encoded in a solution of the 4d Laplace equation with a constant source, leading to a solution quadratic in the coordinates (to avoid subtleties, we take solutions depending only on the space coordinates). This develops a singularity at a distance scaling as

$$\Delta^2 \sim |F_4|^{-2} \sim m^{-4} \sim \mathcal{T}\,, \tag{5.8}$$

where in comparison with (5.3) we have taken constant dilaton.

The singularities are avoided by an $\mathrm{AdS}_4 \times \mathbf{S}^6$ compactification, whose curvature radius is $R \sim m^2$, in agreement with the above scaling. From our perspective, the compactification should be regarded as a dynamical cobordism (where the cobordism is actually that of the 10d theory on an $\mathbf{S}^5$ (i.e. equator of $\mathbf{S}^5$)).

### 5.2 An aside on M-theory on K3

In this section we relate the above system to certain compactifications of M-theory and to the Horava-Witten end of the world branes as its cobordism defect. Although the results can be obtained by direct use of M-theory effective actions, we illustrate how they can be recovered by applying simple dualities to the above system.

Consider the above massive IIA theory with mass parameter $m$, and compactify on $\mathbf{T}^4/\mathbf{Z}_2$. This introduces O4-planes, and requires including 32 D4-branes in the configuration, either as localized sources, or dissolved as instantons on the D8-branes. Actually this can be considered as a simple model of K3 compactifications, where in the general K3 the O4-plane charge is replaced by the contribution to the RR $C_5$ tadpole arising from the CS couplings of D8-branes and O8-planes to $\mathrm{tr}\, R^2$.

We now perform a T-duality in all the directions of the $\mathbf{T}^4/\mathbf{Z}_2$ (Fourier-Mukai transform in the case of general K3). We obtain a similar model of type I' on $\mathbf{T}^4/\mathbf{Z}_2$, but now with the tadpole being associated to the presence of $m$ units of non-trivial flux of the RR 4-form field-strength over $\mathbf{T}^4$ (namely, K3). Also, the dilaton of the original picture becomes related to the overall Kähler modulus of K3. Finally, we lift the configuration to M-theory by growing an extra $\mathbf{S}^1$ and decompactifying it. We thus end up with a 7d compactification of M-theory on K3, with $m$ units of $G_4$ flux,

$$\int_{\mathrm{K3}} G_4 = m\,. \tag{5.9}$$

This leads to a dynamical tadpole, cancelled by the variation of the overall Kähler modulus (i.e. the K3 volume) along one the 7d space dimensions, which we denote by $x^{11}$. As in previous sections, this will trigger a singularity at a finite distance in $x^{11}$, related to the tadpole by $\Delta^{-2} \sim \mathcal{T}$. The singularity is avoided by the physical appearance of a cobordism defect, which for M-theory is a Horava-Witten (HW) boundary [84]. This indeed can support the degrees of freedom to kill the $G_4$ flux, as follows. From [85], the 11d $G_4$ is sourced by the boundary as

$$dG_4 = \delta(x^{11}) \left( \mathrm{tr}\, F^2 - \frac{1}{2} \mathrm{tr}\, R^2 \right), \tag{5.10}$$

where $\delta(x^{11})$ is a bump 1-form for the HW brane, and $F$ is the field-strength for the $E_8$ gauge fields in the boundary. Hence, the $m$ units of $G_4$ in the K3 compactification can be absorbed by a HW boundary with an $E_8$ bundle with instanton number $12+m$ (the 12 coming from half the Euler characteristic of K3 $\int_{\rm K3} \operatorname{tr} R^2 = 24$).

The above discussion is closely related to the picture in [18], which discusses compactification of HW theory (namely, M-theory on an interval with two HW boundaries) on K3 and on a CY threefold. It includes a Kähler modulus varying over the interval according to a linear function[16] and the appearance of a singularity at finite distance. In that case, the HW brane was located at the strong coupling point, based on heuristic arguments, and this led, in the $\mathrm{CY}_3$ case, to a lower bound on the value of the 4d Newton's constant.

Our perspective remarkably explains that the location of the HW wall is not an arbitrary choice, but follows our physical principle of Dynamical Cobordism, and the bound on the Newton's constant is a consequence of that of Finite Distance!

### 5.3 Solving tadpoles in the non-supersymmetric 10d USp(32) theory

The previous examples were based on an underlying supersymmetric vacuum, on top of which the dynamical tadpole is generated via the introduction of fluxes or other ingredients. In this section we consider the opposite situation, in which the initial theory is strongly non-supersymmetric and displays a dynamical tadpole from the start. In particular we consider the non-supersymmetric 10d USp(32) theory constructed in [4], in two different ways: first, we use our new insights to revisit the spacetime-dependent solution proposed in [9] (see also [10] for other proposals); then we present a far more tractable solution involving magnetization, which in fact provides a supersymmetric compactification of this non-supersymmetric 10d string theory.

#### 5.3.1 The Dudas-Mourad solution and cobordism

The non-supersymmetric 10d USp(32) theory in [4] is obtained as an $\Omega$ orientifold of type IIB theory. The closed string sector is as in type I theory, except that the $\mathrm{O9}^-$-plane is replace by an $\mathrm{O9}^+$-plane. Cancellation of RR tadpoles requires the introduction of open strings, which must be associated to 32 $\overline{\mathrm{D9}}$-branes. The closed string sector is a 10d $\mathcal{N}=1$ supergravity multiplet; the orientifold action on the D9-branes breaks supersymmetry, resulting in an open string sector with USp(32) gauge bosons and gauginos in the two-index antisymmetric representation. All anomalies cancel, a remarkable feat from the field theory viewpoint, which is just a consequence of RR tadpole cancellation from the string viewpoint.

Although the RR tadpoles cancel, the NSNS tadpoles do not, implying that there is no maximally symmetric 10d solution to the equations of motion. In particular there is a dynamical dilaton tadpole of order the string scale, as follows from the terms in the 10d (Einstein frame) action

$$S_E = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G}\left[R-\frac{1}{2}(\partial\phi)^2\right] - T_9^E\int d^{10}x\sqrt{-G}\,64\,e^{\frac{3\phi}{2}}\,, \tag{5.11}$$

[16] In the presence of explicit M5-branes, it is a piecewise linear function. It is straightforward to include them in our cobordism description if wished, with explicit branes considered as part of the cobordism defect.

where $T_9^E$ is the (anti)D9-brane tension. The tadpole scales as $\mathcal{T} \sim T_9^E g_s^{3/2}$, with the dilaton dependence arising from the fact that the supersymmetry breaking arises from the Moebius strip worldsheet topology, with $\chi = 3/2$.

Ref. [9] proposed solutions of this dynamical tadpole with 9d Poincaré invariance, an d the dilaton varying over one spacetime dimension (see also [86, 87] for more recent, related work). In the following we revisit the solution with dependence on one spatial coordinate $y$, from the vantage point of our Lessons.

The 10d solution is, in the Einstein frame,

$$\begin{aligned}\phi &= \frac{3}{4}\alpha_E y^2 + \frac{2}{3}\log|\sqrt{\alpha_E}y| + \phi_0\,,\\ ds_E^2 &= |\sqrt{\alpha_E}y|^{\frac{1}{9}}\, e^{-\frac{\alpha_E y^2}{8}}\eta_{\mu\nu}dx^\mu dx^\nu + |\sqrt{\alpha_E}y|^{-1}e^{-\frac{3\phi_0}{2}}\, e^{-\frac{9\alpha_E y^2}{8}}\, dy^2\,,\end{aligned} \tag{5.12}$$

where $\alpha_E = 64k^2T_9$. There are two singularities, at $y = 0$ and $y \to \infty$, which despite appearances are separated by a finite distance

$$\Delta \sim \int_0^\infty |\sqrt{\alpha_E}y|^{-\frac{1}{2}}e^{-\frac{3\phi_0}{4}}\, e^{-\frac{9\alpha_E y^2}{16}}\, dy \sim e^{-\frac{3\phi_0}{4}}\, \alpha_E^{-\frac{1}{2}}\,. \tag{5.13}$$

The fact that the solution has finite extent in the spatial dimension on which the fields vary is in agreement with the Finite Distance Lesson, and in fact satisfying its quantitative bound (1.1)

$$\Delta^{-2} \sim \mathcal{T}\,. \tag{5.14}$$

We can now consider how the Dynamical Cobordism Lesson applies in the present context. Following it, we expect the finite extent in the spatial dimensions to be physically implemented via the cobordism defect corresponding to the 10d USp(32) theory. In general the cobordism defect of bulk chiral 10d theories are expected to be non-supersymmetric, and in fact rather exotic, as their worldvolume dynamics must gap a (non-anomalous) set of chiral degrees of freedom. In fact, on general grounds they can be expected to involve strong coupling.[17] An end of the world defect imposes boundary conditions on bulk supergravity fields, which at weak coupling should be at most linear in the fields, to be compatible with the superposition principle. A typical example are boundary conditions that pair up bulk fermions of opposite chiralities. However, the anomaly cancellation in the 10d USp(32) theory involves fields of different spins, which cannot be gapped by this simple mechanism, and should require strong coupling dynamics (a similar phenomenon in a different context occurs in [88]).

This strong coupling fits nicely with the singularity at $y \to \infty$, but the singularity at $y = 0$ lies at weak coupling. The simplest way out of this is to propose that the singularity at $y = 0$ is actually smoothed out by perturbative string theory (namely, $\alpha'$ corrections, just like orbifold singularities are not singular in string theory), and does not turn into an end of the world defect. Hence the solution (5.12) extends to $y < 0$, and, since the background is even in $y$, develops a singularity at $y \to -\infty$. This is still at finite distance $\Delta$ scaling as (5.14), and lies at strong coupling, thus allowing for the possibility that the singularity is turned into the cobordism defect of the 10d USp(32) theory.

[17] We are indebted to Miguel Montero for this argument, and for general discussions on this section.

It would be interesting to explore this improved understanding of this solution to the dilaton tadpole. Leaving this for future work, we turn to a more tractable solution in the next section.

#### 5.3.2 Solving the tadpole via magnetization

We now discuss a more tractable alternative to solve the dynamical tadpole via magnetization, following section 4.

Stabilizing the tadpole via magnetization is, ultimately, equivalent to finding a compactification (on a product of $\mathbf{T}^2$'s) which is free of tadpoles, for instance by demanding it to be supersymmetric. Hence we need to construct a supersymmetric compactification of the non-supersymmetric 10d USp(32) theory [4].

As explained above, the 10d model is constructed with an O9$^+$-plane and 32 $\overline{\text{D9}}$-branes. Hence, we need to introduce worldvolume magnetic fields in different 2-planes, in such a way that the corresponding angles add up to 0 mod $2\pi$. It is easy to convince oneself that this requires magnetization in at least three complex planes, ultimately triggering a $\mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{T}^2$ compactification. In order to preserve supersymmetry, we need the magnetization to induce D5-brane charges, rather than $\overline{\text{D5}}$-brane charge, hence we need the presence of three independent kinds of negatively charged O5$_i^-$-planes, where $i = 1, 2, 3$ denotes the $\mathbf{T}^2$ wrapped by the corresponding O5-plane. We are thus considering an orientifold of $\mathbf{T}^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ with an O9$^+$-plane, and 8 O5$_i^-$-planes.[18]

The wrapping numbers for the O-planes, and for one simple solution of all constraints for the D9-branes (and their explicitly included orientifold image D9-branes), are

| Object | $N_\alpha$ | $(n_\alpha^1, m_\alpha^1)$ | $(n_\alpha^2, m_\alpha^2)$ | $(n_\alpha^3, m_\alpha^3)$ |
|---|---|---|---|---|
| O9$^+$ | 32 | $(1,0)$ | $(1,0)$ | $(1,0)$ |
| O5$_1^-$ | $-32$ | $(1,0)$ | $(0,1)$ | $(0,-1)$ |
| O5$_2^-$ | $-32$ | $(0,1)$ | $(1,0)$ | $(0,-1)$ |
| O5$_3^-$ | $-32$ | $(0,1)$ | $(0,-1)$ | $(1,0)$ |
| D9 | 16 | $(-1,1)$ | $(-1,1)$ | $(-1,1)$ |
| D9$'$ | 16 | $(-1,-1)$ | $(-1,-1)$ | $(-1,-1)$ |

It obeys the RR tadpole conditions for the $\mathbf{Z}$-valued D9- and D5-brane charges, and the discrete $\mathbf{Z}_2$ RR tadpole conditions for D3- and D7$_i$-brane charges [70].

The supersymmetry condition determined by the O-plane wrappings is

$$\sum_i \arctan(-\chi_i) \equiv \theta_1 + \theta_2 + \theta_3 = 0 \text{ mod } 2\pi \,. \tag{5.15}$$

The model is in fact T-dual (in all $\mathbf{T}^6$ directions) to that in section 5 of [59].

It is easy to see that the above condition forces at least one of the $\mathbf{T}^2$ to have $\mathcal{O}(1)$ area in $\alpha'$ units. From our perspective, this a mere reflection of the fact that the 10d dynamical

[18] For such combinations of orientifold plane signs, see the analysis in [89], in particular its table 6. We will not need its detailed construction for our purposes.

tadpole to be canceled is of order the string scale, hence it agrees with the scaling $\Delta^{-2} \sim \mathcal{T}$. Happily, the use of an $\alpha'$ exact configuration, which is moreover supersymmetric, makes our solution reliable. This is an improvement over other approaches e.g. as in section 5.3.1.

Although we have discussed the compactification on (an orientifold of) $\mathbf{T}^6$ directly, we would like to point out that it is easy to describe it as a sequence of $\mathbf{T}^2$ spontaneous compactifications, each eating up a fraction of the initial 10d tadpole until it is ultimately cancelled upon reaching $\mathbf{T}^6$. However, this picture does not really correspond to a physical situation, given the absence of decoupling of scales. This is true even in setups which seemingly allow for one $\mathbf{T}^2$ of parametrically large area. Indeed, consider for instance the regime $\chi_3 \sim 2\lambda$ and $\chi_1, \chi_2 \sim \lambda^{-1}$, for $0 < \lambda \ll 1$, which corresponds to $\theta_1, \theta_2 \sim \frac{\pi}{2} + \lambda$, $\theta_3 \sim \pi - 2\lambda$. This corresponds to a compactification on substringy size $(\mathbf{T}^2)_1 \times (\mathbf{T}^2)_2$ and a parametrically large $(\mathbf{T}^2)_3$. However, the fact that the $(\mathbf{T}^2)_1$, $(\mathbf{T}^2)_2$ can be T-dualized into large area geometries shows that there is not true decoupling of scales: in the original picture, the small sizes imply that there are towers of light winding modes, whose scale is comparable with the KK modes of $(\mathbf{T}^2)_3$. Hence, the lack of decoupling is still present, as expected from our general considerations in the introduction.

## 6 The SM from spontaneous compactification

In this section we explore an interesting application of the above mechanism, and provide an explicit example of a 6d theory with brane-antibrane pairs, and a dynamical tadpole triggering spontaneous compactification to a 4d (MS)SM-like particle physics model. Interestingly, the complete chiral matter and electroweak sector, including the Higgs multiplets, are generated as degrees of freedom on cobordism branes. Only the gluons are present in some form in the original 6d models.

Consider the type IIB orientifold of $\mathbf{T}^4/\mathbf{Z}_2$ with orientifold action $\Omega$ constructed in [78, 79], possibly with magnetization. To describe it, we introduce the notation in [69, 90] of wrapping numbers $(n^i_\alpha, m^i_\alpha)$, where $n^i_\alpha$ and $m^i_\alpha$ provide the wrapping number and magnetic flux quantum of the D-brane $\alpha$ on the $i^{\text{th}}$ $\mathbf{T}^2$, respectively. We consider the following stacks of D-branes (and their orientifold images, not displayed explicitly)

| $N_\alpha$ | $(n^1_\alpha, m^1_\alpha)$ | $(n^2_\alpha, m^2_\alpha)$ |
|---|---|---|
| $N_{a+d} = 6 + 2$ | $(1, 3)$ | $(1, -3)$ |
| $N_{h_1} = 4$ | $(1, -3)$ | $(1, -4)$ |
| $N_{h_2} = 4$ | $(1, -4)$ | $(1, -3)$ |
| 40 | $(0, 1)$ | $(0, -1)$ |

The O9- and O5-planes correspond to the wrapping numbers $(1, 0) \times (1, 0)$ and $(0, 1) \times (0, -1)$ respectively. The stacks $a$ and $d$ are taken different and separated by Wilson lines, but they can be discussed jointly for the time being. They correspond to 8 D9-branes with worldvolume magnetic fluxes 72 units of D5-brane charge. The stacks $h_1$ and $h_2$ correspond to 8 additional D9-branes, with 96 with units of induced $\overline{\text{D5}}$-branes charge.

The addition of 40 explicit D5-branes leads to RR tadpole cancellation (once orientifold images are included). In terms of the wrapping numbers, we have

$$\sum_\alpha N_\alpha n_\alpha^2 n_\alpha^3 = 16\,, \quad \sum_\alpha N_\alpha m_\alpha^2 m_\alpha^3 = -16\,. \tag{6.1}$$

The model is far from supersymmetric due to the presence of D5-$\overline{\text{D5}}$ pairs, and in fact has a decay channel to supersymmetric model by their annihilation. On the other hand, even at the top of the tachyon potential, the theory is not at a critical point of its potential due to dynamical tadpole for the closed string moduli, namely the area moduli of the $\mathbf{T}^2$'s. In other words, the excess tension depends on these, as they enter the angles determining the deviation from the supersymmetry condition

$$\arctan\left(\frac{m_\alpha^1}{n_\alpha^1}\chi_1\right) + \arctan\left(\frac{m_\alpha^2}{n_\alpha^2}\chi_2\right) = 0\,. \tag{6.2}$$

For instance, we can make the stacks $a$, $d$ supersymmetric, by choosing

$$\chi_1 = \chi_2\,, \tag{6.3}$$

but the D-branes $h_1$ and $h_2$ break supersymmetry. Hence, there is a dynamical tadpole associated to the excess tension of these latter objects.

The dynamical tadpole can be solved by introducing magnetization along two of the 6d spacetime dimensions. The backreaction of this extra flux forces these two dimensions to be compactified on a $\mathbf{T}^2$, with the addition of cobordism I-branes [15], which in general includes orientifold planes and D-branes, as in the examples above. We take these extra branes to be arranged in two new stacks $b$ and $c$. Overall, we end up with an orientifold of $\mathbf{T}^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$, with D-brane stacks and topological numbers given by

| $N_\alpha$ | $(n_\alpha^1, m_\alpha^1)$ | $(n_\alpha^2, m_\alpha^2)$ | $(n_\alpha^3, m_\alpha^3)$ |
|---|---|---|---|
| $N_{a+d} = 6+2$ | $(1,3)$ | $(1,-3)$ | $(1,0)$ |
| $N_b = 2$ | $(0,1)$ | $(1,0)$ | $(0,1)$ |
| $N_c = 2$ | $(-1,0)$ | $(0,-1)$ | $(0,1)$ |
| $N_{h_1} = 2$ | $(1,-3)$ | $(1,-4)$ | $(2,-1)$ |
| $N_{h_2} = 2$ | $(1,-4)$ | $(1,-3)$ | $(2,-1)$ |
| 40 | $(0,1)$ | $(0,-1)$ | $(0,1)$ |

The model satisfies the RR tadpole conditions

$$\begin{aligned} \sum_\alpha N_\alpha n_\alpha^1 n_\alpha^2 n_\alpha^3 &= 16\,, \qquad \sum_\alpha N_\alpha n_\alpha^1 m_\alpha^2 m_\alpha^3 = 16\,, \\ \sum_\alpha N_\alpha m_\alpha^1 n_\alpha^2 m_\alpha^3 &= 16\,, \qquad \sum_\alpha N_\alpha m_\alpha^1 m_\alpha^2 n_\alpha^3 = -16\,. \end{aligned} \tag{6.4}$$

This corresponds to O9-planes along $(1,0)\times(1,0)\times(1,0)$, and O5-planes along $(0,1)\times(0,-1)\times(1,0)$, as already present in the 6d theory, and cobordism O5-planes along $(0,1)\times(1,0)\times(0,1)$ and $(1,0)\times(0,1)\times(0,1)$.

The model still contains only 3 stacks of D-branes with non-trivial angles, so that they are just enough to fix the 2 parameters $\chi_i$ of the $\mathbf{T}^2$'s. The O-planes fix the supersymmetry condition signs to

$$\arctan\left(\frac{m_\alpha^1}{n_\alpha^1}\chi_1\right)+\arctan\left(\frac{m_\alpha^2}{n_\alpha^2}\chi_2\right)-\arctan\left(\frac{m_\alpha^3}{n_\alpha^3}\chi_3\right)=0\,. \tag{6.5}$$

Using the branes above, we get

$$\chi_1=\chi_2\,,\quad \chi_3=\frac{14\chi_1}{1-12\chi_1^2}\,. \tag{6.6}$$

The regime of large $(\mathbf{T}^2)_3$ corresponds to small $\chi_3$, which is also attained for small $\chi_1$. Note that in this context the last condition $\chi_1\sim\chi_3$ encodes the relation between the 6d tadpole and the inverse area of the spontaneously compactified $\mathbf{T}^2$.

The model is, up to exchange of directions in the $\mathbf{T}^6$ and overall sign flips, precisely one of the examples of 4d MSSM-like constructions in [59, 60]. The gauge group is $\mathrm{U}(3)_a\times\mathrm{USp}(2)_b\times\mathrm{U}(1)_c\times\mathrm{U}(1)_d$, where we break the naive $\mathrm{USp}(2)_c$ by Wilson lines or shifting off the O-plane for the corresponding D5-branes. Taking into account the massive U(1)'s due to $BF$ couplings, this reproduces the SM gauge group. In addition, open strings between the different brane stacks reproduce a 3-family (MS)SM chiral matter content, and the MSSM Higgs doublet pair. Hence, we have described the spontaneous compactification of a 6d model to a semi-realistic MSSM-like 4d theory.

A fun fact worth emphasizing is that most of the SM spectrum is absent in the original 6d model, and arises only after the spontaneous compactification. In particular, all the MSSM matter and Higgs chiral multiplets, as well as the electroweak gauge sector, arise from open string sectors involving the $b$ and $c$ branes, which arises as cobordism branes. It is remarkable that cobordism entails that spontaneous compactification implies not just the removal of spacetime dimensions, but also the dynamical appearance of novel degrees of freedom. It is tantalizing to speculate on the potential implications of these realizations in cosmological or other dynamical setups.

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## A Dynamical tadpoles and swampland constraints

In this appendix we use the model in section 2 to illustrate the result in [3] that, in theories with a dynamical tadpole which is not duly backreacted on the field configuration, the mistreatment can show up as violations of swampland constraints.

We consider type IIB theory on $\mathrm{AdS}_5 \times T^{1,1}$ and introduce $M$ units of RR 3-form flux. In the coordinates in [33, 34], it reads

$$F_3 = \frac{1}{2} M [\sin\theta_1 (\cos\theta_2 d\theta_1 d\phi_1 d\phi_2 + d\theta_1 d\phi_1 d\psi) + \sin\theta_2 (\cos\theta_1 d\theta_2 d\phi_1 d\phi_2 - d\theta_2 d\phi_2 d\psi)] \,.$$

It has constant coefficients in terms of *fünf*-bein 1-forms $g^i$ in [33] $F_3 = \frac{1}{2} M g^5 (g^1 g^2 + g^3 g^3)$, hence its kinetic term $|F_3|^3$ is constant over the $T^{1,1}$ geometry. This acts as a constant background source for e.g. the Laplace equation for the dilaton, which has no solution over the compact $T^{1,1}$ geometry. This inconsistency of the equations of motion, assuming no backreaction on the underlying geometry, signals the dynamical tadpole in the configuration. In the following we will argue that it moreover can lead to violation of the Weak Gravity Conjecture [47].

For concreteness we focus on the simplest set of states, corresponding to 5d BPS particle states in the original theory ($M = 0$), with the BPS bound corresponding to the WGC bound, for the gauge interaction associated to the KK U(1) dual to the $\mathrm{U}(1)_R$ symmetry of the dual CFT. For small R-charge $n \ll N$, these particle states are dual to chiral primary single-trace mesonic operators of the $\mathrm{SU}(N)^2$ theory, e.g. $\mathrm{tr}\,(A_1 B_1 \ldots A_1 B_1)$; in the AdS side, they correspond to KK gravitons with momentum $n$ on the $\mathbf{S}^1$. For very large R-charge, the KK gravitons polarize due to Myers' effect [91] into giant gravitons [92], and their dual operators are determinant or sub-determinant operators [93]. Note that on $T^{1,1}$ we have D3-branes wrapped on homologically trivial 3-cycles (but sustained as BPS states by their motion on $\mathbf{S}^1$), hence they are different from (di)baryonic operators, which correspond to D3-branes wrapped on the non-trivial $\mathbf{S}^3$ [94].

Our strategy is to consider these states in the presence of $F_3$, but still keeping the geometry as $\mathrm{AdS}_5 \times T^{1,1}$ (i.e. with no backreaction of the dynamical tadpole), and show that the interaction of $F_3$ makes these states non-BPS, hence violating the WGC bound. This analysis will be quite feasible in the giant graviton regime $1 \ll n \sim N$, by using the wrapped D3-brane worldvolume action. Admittedly, proving a full violation of the WGC would require showing the violation of the BPS condition for all values of $n$; we nevertheless consider the large $n$ result as a compelling indication that the WGC is indeed violated in this configuration, thus making its inconsistency manifest.

Supersymmetric 3-cycles for D3-branes are easily obtained from holomorphic 4-cycles in the underlying CY threefold [95, 96] (see also [97]). Describing the conifold as $z^1 z^2 - z^3 z^4 = 0$, any holomorphic function of these coordinates $f(x, y, z, w) = 0$ defines a holomorphic 4-cycle corresponding to a giant graviton D3-branes, i.e. wrapped on a trivial[19] 3-cycle in $T^{1,1}$. We focus on a simple class of D3-branes studied in detail in [98]. They are defined by the 4-cycle $z^1 = \sqrt{1 - \alpha^2}$, with $\alpha \in [0, 1]$ being a real constant, encoding the size of the 3-cycle (with $\alpha = 0, 1$ corresponding to the pointlike KK graviton and the maximal giant graviton, respectively). We will follow the analysis in [98] with the inclusion of the effect of $F_3$ on the D3-brane probe.

[19]Di-baryonic D3-branes are on the other hand associated to non-Cartier divisors in the conifold, i.e. 4-cycles which can be defined in terms of the $a_i$, $b_i$ homogeneous coordinates of the linear sigma model, but cannot be expressed as a single equation $f(z^i) = 0$.

It is convenient to change to new coordinates $\{\chi_1, \chi_2, \chi_3, \alpha, \nu\}$

$$\begin{cases} \chi_1 = \frac{1}{3}(\psi - \phi_1 - \phi_2) \\ \chi_2 = \frac{1}{3}(\psi + 3\phi_1 - \phi_2) \\ \chi_3 = \frac{1}{3}(\psi - \phi_1 + 3\phi_2) \end{cases} \qquad \begin{cases} \sqrt{1-\alpha^2} = \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2} \\ \nu = \frac{2u}{\alpha^2+u^2} \quad \text{with} \quad u = \cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2} \end{cases} \tag{A.1}$$

These are adapted to the D3-brane embedding, which simply reads

$$\sigma^0 = t, \;\; \sigma^1 = \nu \text{ (doubly-covered)}, \;\; \sigma^2 = \chi_2, \;\; \sigma^3 = \chi_3 \,.$$

The double covering is very manifest for the maximal giant graviton, $\alpha = 1$, $z^1 = 0$. It corresponds to the defining equation $z^3 z^4 = 0$, which splits in two components, corresponding to two (oppositely oriented) copies of the non-trivial[20] $\mathbf{S}^3$. The double covering remains even for non-maximal giants, even though they correspond to irreducible 4-cycles.

The RR 3-form field strength in these coordinates is

$$\begin{aligned} F_3 = M(&[a_{12}\, d\chi_1 \wedge d\chi_2 + a_{13}\, d\chi_1 \wedge d\chi_3 + a_{23}\, d\chi_2 \wedge d\chi_3] \wedge d\alpha \\ &+ [v_{12}\, d\chi_1 \wedge d\chi_2 + v_{13}\, d\chi_1 \wedge d\chi_3 + v_{23}\, d\chi_2 \wedge d\chi_3] \wedge d\nu) \,, \end{aligned} \tag{A.2}$$

with

$$\begin{cases} a_{12} = \frac{9}{4}\alpha(1 \pm \frac{\sqrt{1-\nu^2}}{1-c}) \\ a_{13} = \frac{9}{4}\alpha(-1 \pm \frac{\sqrt{1-\nu^2}}{1-c}) \\ a_{23} = \frac{9}{4}\alpha(\frac{-c}{1-c}) \end{cases} \qquad \begin{cases} v_{12} = \mp\frac{9}{4}\frac{c(\nu^2-c)}{\nu^3\sqrt{1-\nu^2}(1-c)} \\ v_{13} = \mp\frac{9}{4}\frac{c(\nu^2-c)}{\nu^3\sqrt{1-\nu^2}(1-c)} \\ v_{23} = -\frac{9}{4}\frac{c^2}{\nu^3(1-c)} \end{cases}$$

where we have introduced $c = 1 - \sqrt{1-\alpha^2\nu^2}$. We can fix a gauge and find the RR 2-form

$$C_2 = M(c_{12}d\chi_1 \wedge d\chi_2 + c_{13}d\chi_1 \wedge d\chi_3 + c_{23}d\chi_2 \wedge d\chi_3) \,, \tag{A.3}$$

with

$$\begin{cases} c_{12} = -\frac{9}{8}(-\alpha^2 \mp \frac{2\sqrt{1-\nu^2}c}{\nu^2}) \\ c_{13} = -\frac{9}{8}(\alpha^2 \mp \frac{2\sqrt{1-\nu^2}c}{\nu^2}) \\ c_{23} = \frac{9}{8}\frac{(\alpha^2\nu^2-2c)}{\nu^2} \end{cases}$$

Its pullback on the D3-brane worldvolume is

$$P[C_2] = M\dot{\chi}_1(c_{12}dt \wedge d\chi_2 + c_{13}dt \wedge d\chi_3) + Mc_{23}d\chi_2 \wedge d\chi_3 \,. \tag{A.4}$$

We can now compute the effect of this background on the D3-brane by using its worldvolume action. This is easy in the S-dual frame, in which the RR 2-form couples to the D3-brane just like the NSNS 2-form in the original DBI+CS D3-brane action.[21] After integrating over $\chi_2$, $\chi_3$, this reads

$$\begin{aligned} &S = S_{\text{BDI}} + S_{\text{CS}} = \frac{64\pi^2}{9}\int dtL \,, \\ &\text{with} \quad L = \int_0^1 d\nu\; 2\Big(-T_3\sqrt{-\det\left(P[G]_{\mu\nu} + P[C_2]_{\mu\nu}\right)} + \mu_3 R^4 c_4 \dot{\chi}_1\Big) \,, \end{aligned} \tag{A.5}$$

[20]In terms of the linear sigma model coordinates we have $z^1 = a_1 b_1$, $z^2 = a_2 b_2$, $z^3 = a_1 b_2$, $z^4 = a_2 b_1$, and the two components correspond to $a_1 = 0$ and $b_1 = 0$, which are non-Cartier divisors.

[21]Related to this, one can check that the above background is neither pure gauge on the D3, nor cannot be removed by a change in the worldvolume gauge field strength flux.

where the factor of 2 of the double-covering of $\nu$ has been added, and the last term arises from the CS coupling to the RR 4-form as in [98].

We are interested in focusing on the angular momentum of the state $P_{\chi_1} = \frac{\partial L}{\partial \dot{\chi}_1}$ conjugate to the angular coordinate $\chi_1$. This reads

$$P_{\chi_1} = \frac{3}{2}\int_0^1 d\nu \left( \frac{\sqrt{3\pi} A g_\nu \, N T_3(\dot{\chi}_1)}{\sqrt{N \, g_\nu (B - A(\dot{\chi}_1)^2)}} + 9\pi c_4 \mu_3 \, N \right), \tag{A.6}$$

with

$$\begin{aligned} A &= \frac{81}{64\nu^4}\{-4\,M^2[(2(1-\alpha^2\nu^2)c - \alpha^2\nu^2)c] - 3\pi N(\alpha^2-1)\nu^2 c^2\} \equiv A_{M^2} M^2 + A_N N \,, \\ B &= \frac{81}{64\nu^4}\{4M^2\big[(4-\alpha^2\nu^2)c^2 - 2\alpha^2\nu^2 c\big] \\ &\qquad + \pi N\big[2\alpha^4\nu^4 + (\alpha^2\nu^2 - 2c)(-3\alpha^2\nu^2 - 3\nu^2 + 8)\big]\} \\ &\equiv B_{M^2} M^2 + B_N N \,. \end{aligned} \tag{A.7}$$

In the last equalities we have highlighted the parametric dependence on $N$ and $M$.

Despite the fact that we have not managed to find a closed form for the result, since $M \ll N$ we can find an expansion for the integrand in the form

$$n = p_0(\alpha, \nu, \dot{\chi}_1)\, N + p_2(\alpha, \nu, \dot{\chi}_1)\, M^2 + \mathcal{O}(M^4)\,, \tag{A.8}$$

where the coefficient functions are computable, but we will not need their explicit expressions.

The coefficient $p_0$ is the survivor for the $M = 0$ case, and leads to an integer momentum. On the other hand, the subleading correction $p_2$ produces a momentum which is not integer. This already signals a problem, since (as the geometry is considered undeformed even after introducing $F_3$) the gauge coupling of the KK U(1) is as in the $M = 0$ case, hence charges under it should be integer in the same units. Hence one can directly claim that the assumption of ignoring the dynamical tadpole backreaction lead to violation of charge quantization, in contradiction with common lore for consistency with quantum gravity [99].

The above discussion however seems to contradict the fact that any quantum excitation on a periodic $\mathbf{S}^1$ direction must have quantized momentum to have a well-defined wavefunction. In fact, an alternative interpretation of the above mismatch is that the D3-brane probe computation assumes a well-defined worldvolume embedding, in particular well-defined (hence classical) trajectories for the 5d particle. It is only for BPS states in supersymmetric vacua that such a computation is guaranteed to end up producing quantized momenta. The fact that our holomorphic embedding ansatz fails to do so is just a reflection that the actual integer-quantized states are *not* described by holomorphic equations. Since the latter condition is the one ensuring the match between the particle mass and charge, it is clear that non-holomorphic embeddings will produce larger masses for the same charge, hence violating the BPS/WGC bound.





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# Dynamical Cobordism and Swampland Distance Conjectures

**Ginevra Buratti, José Calderón-Infante, Matilda Delgado and Angel M. Uranga**

*Instituto de Física Teórica IFT-UAM/CSIC,*
*C/ Nicolás Cabrera 13-15, Campus de Cantoblanco, 28049 Madrid, Spain*
*E-mail:* ginevra.buratti@uam.es, j.calderon.infante@csic.es, matilda.delgado@uam.es, angel.uranga@csic.es

Abstract: We consider spacetime-dependent solutions to string theory models with tadpoles for dynamical fields, arising from non-trivial scalar potentials. The solutions have necessarily finite extent in spacetime, and are capped off by boundaries at a finite distance, in a dynamical realization of the Cobordism Conjecture. We show that as the configuration approaches these cobordism walls of nothing, the scalar fields run off to infinite distance in moduli space, allowing to explore the implications of the Swampland Distance Conjecture. We uncover new interesting scaling relations linking the moduli space distance and the SDC tower scale to spacetime geometric quantities, such as the distance to the wall and the scalar curvature. We show that walls at which scalars remain at finite distance in moduli space correspond to domain walls separating different (but cobordant) theories/vacua; this still applies even if the scalars reach finite distance singularities in moduli space, such as conifold points.

We illustrate our ideas with explicit examples in massive IIA theory, M-theory on CY threefolds, and 10d non-supersymmetric strings. In 4d $\mathcal{N} = 1$ theories, our framework reproduces a recent proposal to explore the SDC using 4d string-like solutions.

Keywords: Flux compactifications, Superstring Vacua, Supersymmetry Breaking

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## Contents





## 1 Introduction and conclusions

A remarkable proposal in the Swampland Program of quantum gravity constraints on effective field theories [1] (see [2–4] for reviews) is the Cobordism Conjecture [5], that is based on the expected absence of exact global symmetries in quantum gravity. In short, it states that any configuration in a consistent theory of quantum gravity should not carry any cobordism charge. In practice, it implies that any configuration in a consistent theory of quantum gravity should admit, at the topological level, the introduction of a boundary ending spacetime into nothing,[1] in the sense of [6] (see [7, 8] for recent related discussions). Accordingly, we will refer to such boundaries as *walls of nothing*. Equivalently, it implies that any two consistent theories of quantum gravity must admit, at the topological level, an interpolating configuration connecting them, as a generalized domain wall separating the two theories. We will refer to such configurations as *interpolating domain walls*.

[1] This boundary may be dressed by additional defects, such as D-branes or O-planes in string setups, to absorb the relevant charges.

The Cobordism Conjecture is topological in nature. However, it can lead to remarkable breakthroughs when supplemented by additional assumptions. For instance, the extra ingredient of supersymmetry of the theory (and possibly of its walls) has led to highly non-trivial constraints in lower dimensional theories, see e.g. [9, 10].

An important step forward in endowing cobordism walls with dynamics was taken in [11], in the study of theories with tadpoles for dynamical fields (dubbed *dynamical tadpoles*, as opposed to topological tadpoles, such as RR tadpoles, which lead to topological consistency conditions on the configuration[2]). These are ubiquitious in the presence of scalar potentials, and in particular in non-supersymmetric string models. In theories with dynamical tadpoles the solutions to the equations of motion vary over the non-compact spacetime dimensions. Based on the behaviour of large classes of string models, it was proposed in [11] that such spacetime-dependent running solutions must hit cobordism walls of nothing at a finite distance $\Delta$ in spacetime[3] (as measured in the corresponding Einstein frame metric), scaling as $\Delta^{-n} \sim \mathcal{T}$ with the strength of the tadpole $\mathcal{T}$. These examples included holographic $\mathrm{AdS}_5 \times T^{1,1}$ compactifications with RR 3-form flux, type IIB 3-form flux compactifications, magnetized D-brane models, massive IIA theory, M-theory on K3 with $G_4$ flux, and the 10d non-supersymmetric USp(32) string theory. On the other hand, interpolating cobordism walls connecting different theories were not discussed. One of the motivations of this work is to fill this gap.

We argue that, when a running solution in theories with dynamical tadpoles hits a wall, the behaviour of the configuration across the wall, and in particular the sharp distinction between interpolating domain walls and walls of nothing, is determined by the behaviour of scalar fields as one reaches the wall, via a remarkable correspondence:

- When scalars remain at finite distance points in moduli space as one hits the wall, it corresponds to an interpolating domain wall, and the solution continues across it in spacetime (with jumps in quantities as determined by the wall properties);
- On the other hand, when the scalars run off to infinity in moduli space as one reaches the wall (recall, at a finite distance in spacetime), it corresponds to a wall of nothing, capping off spacetime beyond it.

We also argue that scalars reaching singular points at finite distance in moduli space upon hitting the wall still define interpolating domain walls, rather than walls of nothing; hence, walls of nothing are not a consequence of general singularities in moduli space, but actually to those at infinity in moduli space. This suggests that, in the context of dynamical solutions,[4] the walls of nothing of the Cobordism Conjecture are closely related to the Swampland Distance Conjecture.[5] We indeed find universal scaling relations between the

[2]Note however that dynamical tadpoles were recently argued in [12] to relate to violation of swampland constraints of quantum gravity theories.

[3]For related work on dynamical tadpoles in non-supersymmetric theories, see [13–20].

[4]Note that, in setups with no dynamical tadpole, one can still have e.g. cobordism walls of nothing without scalars running off to infinity: for instance, 11d M-theory, which does not even have scalars, admits walls of nothing defined by Horava-Witten boundaries; similar considerations may apply to potential theories with no moduli (or with all moduli stabilized at high enough scale).

[5]The status of the SDC in spacetime dependent running solutions was addressed in [21].



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(finite) distance to the wall in spacetime and the scale of the SDC tower [22]. In addition, we uncover a universal scaling relation between the curvature scalar in running solutions and the SDC tower scale that is reminiscent of the Anti de Sitter Distance Conjecture (ADC) [23].

We illustrate these ideas in several large classes of string theory models, including massive IIA, and M-theory on CY threefolds. Moreover, we also argue that our framework encompasses the recent discussion of EFT string solutions in 4d $\mathcal{N}=1$ theories in [24] (see also [25]), where saxion moduli were shown to attain infinity in moduli space at the core of strings magnetically charged under the corresponding axion moduli. We show that EFT string solutions are the cobordism walls of nothing of $\mathbf{S}^1$ compactifications of the 4d $\mathcal{N}=1$ theory with certain axion fluxes on the $\mathbf{S}^1$. Our scalings also relate to those between EFT string tensions and the SDC tower scale in [24].

The paper is organized as follows. In section 2 we present the main ideas in the explicit setup of running solutions in massive IIA theory, and their interplay with type I' solutions [26]. In section 3 we carry out a similar discussion for M-theory on CY threefolds with $G_4$ flux (in section 3.1) and their relation to strongly coupled heterotic strings [27]. In section 3.2 we use it to discuss domain walls across singularities at finite distance in moduli space, following [28]. In section 4 we discuss the $\mathbf{S}^1$ compactification of general 4d $\mathcal{N}=1$ theories. In section 4.1 we introduce dynamical tadpoles from axion fluxes, whose running solutions hit walls of nothing at which saxions run off to infinity. In section 4.2 we relate the discussion to the EFT strings of [24]. In section 5 we discuss the moduli space distances in walls of nothing and interpolating walls in 4d $\mathcal{N}=1$ theories with non-trivial superpotentials of the kind arising in flux compactifications. In section 6 we discuss our proposal in non-supersymmetric string theories, in particular the 10d USp(32) string. In section 7 we offer some final remarks and outlook. Appendix A provides some observations on cobordism walls in holographic throats.

## 2 Cobordism walls in massive IIA theory

**Walls of nothing and infinite moduli space distance.** In this section we consider different kinds of cobordism walls in massive IIA theory [29], extending the analysis in [11]. The Einstein frame 10d effective action for the relevant fields is

$$S_{10,E} = \frac{1}{2\kappa^2}\int d^{10}x\,\sqrt{-G_E}\left\{\left[R-\frac{1}{2}(\partial\phi)^2\right]-\frac{1}{2}e^{\frac{5}{2}\phi}F_0^2-\frac{1}{2}e^{\frac{1}{2}\phi}(F_4)^2\right\}, \tag{2.1}$$

where the Romans mass parameter is denoted by $F_0$ to suggest it is a 0-form field strength flux. This theory is supersymmetric, but has a dilaton tadpole

$$\mathcal{T}\sim e^{\frac{5}{2}\phi}F_0^2\,, \tag{2.2}$$

so the theory does not admit 10d maximally symmetric solutions. The solutions with maximal (super)symmetry are 1/2 BPS configurations with the dilaton depending on one coordinate $x^9$, closely related to that in [30]. In conventions closer to [26], the Einstein frame metric and dilaton are

$$(G_E)_{MN} = Z(x^9)^{\frac{1}{12}}\,\eta_{MN}\,,\quad e^{\phi}=Z(x^9)^{-\frac{5}{6}}\,,\quad \text{with}\;\; Z(x^9)\sim -F_0\,x^9\,, \tag{2.3}$$



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where we have set some integration constant to zero. The solution hits a singularity at $x^9 = 0$. The spacetime distance from a general position $x^9$ to the singularity is [11]

$$\Delta = \int_{x^9}^{0} Z(x^9)^{\frac{1}{24}}\, dx^9 \sim Z(x^9)^{\frac{25}{24}}\, F_0^{-1} \sim F_0^{-1} e^{-\frac{5}{4}\phi} \sim \mathcal{T}^{-\frac{1}{2}}\,, \tag{2.4}$$

in agreement with the scaling relation $\Delta^{-2} \sim \mathcal{T}$, that was dubbed Finite Distance lesson in [11]. Following the Dynamical Cobordism proposal therein, the singularity is resolved in string theory into a cobordism wall of nothing, defined by an O8-plane (possibly dressed with D8-branes to match the $F_0$ flux to be absorbed),[6] ending the direction $x^9$ as a boundary.

We now notice that, since $Z \to 0$ implies $\phi \to \infty$ as $x^9 \to 0$, the dilaton runs off to infinity in moduli space as one hits the wall, as befits a wall of nothing from our discussion in the introduction. According to the SDC, there is an infinite tower of states becoming massless in this region, with a scale decaying exponentially with the moduli space distance $D$ as

$$M_{\text{SDC}} \sim e^{-\lambda D}\,, \tag{2.5}$$

with some positive $\mathcal{O}(1)$ coefficient $\lambda$.

It is interesting to find a direct relation between these quantities and the spacetime distance to the wall. The distance in moduli space is given by $\phi = \sqrt{2}\, D$, as can be seen from the kinetic term for $\phi$ in (2.1). From (2.4) we have

$$\Delta \sim e^{-\frac{5}{2\sqrt{2}} D}\,, \quad M_{\text{SDC}} \sim \Delta^{\frac{2\sqrt{2}}{5}\lambda}\,. \tag{2.6}$$

Hence the SDC tower scale goes to zero with the distance to the wall with a power-like scaling.

It is a natural question to ask if this tower of states becomes light in the actual dynamical configuration (rather than in the adiabatic framework of the standard formulation of the SDC). In this particular setup, the SDC tower corresponds to D0-branes which end up triggering the decompactification of the M-theory eleventh dimension. In the dynamical solution, there are a finite number of extra massless states, responsible for the enhancement of the perturbative open string gauge group to the exceptional symmetries which are known to arise from the heterotic dual theory [26] (see also [31]). On the other hand, there is no signal of an infinite tower of states becoming massless simultaneously. The appearance of the SDC in the dynamical context has thus different implications as compared with the usual adiabatic formulation.

Let us now turn to another novel, and tantalizing, scaling. The scalar curvature for the running solution reads

$$|R| \sim (-x^9)^{-\frac{25}{12}} \sim e^{\frac{5}{\sqrt{2}} D}\,. \tag{2.7}$$

Using this, we can write the SDC tower scale in terms of the scalar curvature as

$$M_{\text{SDC}} \sim e^{-\lambda D} \sim |R|^{-\frac{\sqrt{2}}{5}\lambda}\,. \tag{2.8}$$

[6] This imposes a swampland bound on the possible values of $F_0$ that are consistent in string theory.

This scaling is highly reminiscent of the Anti de Sitter Distance Conjecture (ADC) of [23],[7] even though the setup under consideration is very different.[8] Note however that, as in the ADC, it signals a failure of the decoupling of scales, and hence a breakdown of the effective field theory near the wall of nothing. This fits nicely with our observation that the wall can only be microscopically defined in the UV complete theory, and works as a boundary condition defect at the level of the effective theory.

**Interpolating domain walls.** There is a well known generalization of the above solutions, which involves the inclusion of D8-branes acting as interpolating domain walls across which $F_0$ jumps by one unit. The general solution of this kind is provided by (2.3) with a piecewise constant $F_0$ and a piecewise continuous function $Z$ [26].

The D8-brane domain walls are thus (a very simple realization of) cobordism domain walls interpolating between different Romans IIA theories (differing just in their mass parameter). The point we would like to emphasize is that, since $Z$ remains finite across them, the dilaton remains at finite distance in moduli space, as befits interpolating domain walls from our discussion in the introduction.

## 3 Cobordism walls in M-theory on CY3

In this section we recall results from the literature on the strong coupling limit of the heterotic string, also known as heterotic M-theory [27, 32–34] (see [35, 36] for review and additional references). They provide straightforward realizations of the different kinds of cobordism walls in M-theory compactifications on CY threefolds. The discussion generalizes that in [11], and allows to study the behaviour at singular points at finite distance in moduli space, in particular flops at conifold points.

### 3.1 M-theory on CY3 with $G_4$ flux

We consider M-theory on a CY threefold $\mathbf{X}$, with $G_4$ field strength fluxes on 4-cycles. For later convenience, we follow the presentation in [28]. We introduce dual basis of 2- and 4-cycles $C^i \in H_2(\mathbf{X})$ and $D_i \in H_4(\mathbf{X})$, and define

$$\int_{D_i} G_4 = a_i \, , \quad \int_{C^i} C_6 = \tilde{\lambda}^i \, . \tag{3.1}$$

We also denote by $b_i$ the 5d vector multiplet of real Kähler moduli, with the usual Kähler metric and the 5d $\mathcal{N} = 1$ prepotential

$$G_{ij} = -\frac{1}{2} \frac{\partial^2}{\partial b_i \partial b_j} \ln \mathcal{K} \, , \qquad \mathcal{K} \equiv \frac{1}{3!} d_{ijk} b^i b^j b^k \, , \tag{3.2}$$

with $d_{ijk}$ being the triple intersection numbers of $\mathbf{X}$. We have the familiar constraint $\mathcal{K} = 1$ removing the overall modulus $V$, which lies in a hypermultiplet.

[7] It is possible that the result is ultimately linked to the generalized distance conjectures in [23]; we leave this as an open question for future work.

[8] In contrast to the ADC, that considers the limit of vanishing curvature of a family of AdS vacua, in our setup the scalar curvature blows up as the singularity is approached. However, we do find a power-like scaling similar to the ADC one.



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The 5d effective action for these fields is

$$S_5 = -\frac{M_{p,11}^9}{2} L^6 \Big[ \int_{M_5} \sqrt{-g_5} \left( R + G_{ij}(b)\partial_M b^i \partial^M b^j + \frac{1}{2V^2}\partial_M V \partial^M V + \lambda(\mathcal{K}-1) \right) \\ + \frac{1}{4V^2} G^{ij}(b) a_i \wedge \star a_j + d\tilde{\lambda}^i \wedge a_i \Big] - \sum_{n=0}^{N+1} \alpha_i^{(n)} \int_{M_4^{(n)}} \left( \tilde{\lambda}^i + \frac{b^i}{V}\sqrt{g_4} \right) . \quad (3.3)$$

Here $\lambda$ is a Lagrange multiplier, and $L$ the reference length scale of the Calabi-Yau. With hindsight, we include 4d localized terms which correspond to different walls in the theory, with induced 4d metric $g_4$.

The $G_4$ fluxes $a_i$ induce dynamical tadpoles for the overall volume and the Kähler moduli $b_i$. There are 1/2 supersymmetric solutions running in one spacetime coordinate, denoted by $y$, with the structure

$$\begin{aligned} ds_5^2 &= e^{2A} ds_4^2 + e^{8A} dy^2 \,, \\ V &= e^{6A} \,, \quad b^i = e^{-A} f^i \,, \\ e^{3A} &= \left( \frac{1}{3!} d_{ijk} f^i f^j f^k \right) \,, \\ (d\tilde{\lambda}^i)_{\mu\nu\rho\sigma} &= \epsilon_{\mu\nu\rho\sigma} e^{-10A} \left( -\partial_{11} b^i + 2 b^i \partial_{11} A \right) . \end{aligned} \quad (3.4)$$

The whole solution is determined by a set of one-dimensional harmonic functions. They are given in terms of the local values of the $G_4$ fluxes,

$$d_{ijk} f^j f^k = H_i \,, \qquad H_i = a_i y + c_i \,. \quad (3.5)$$

Here the $c_i$ are integration constants set to have continuity of the $H_i$, and hence of the $f_i$, across the different interpolating domain walls in the system, which produce jumps as follows. Microscopically, the interpolating domain walls correspond to M5-branes wrapped on 2-cycles $[C] = \sum n_i C^i$, leading to jumps in the fluxes that in units of M5-brane charge are given by

$$\Delta a_i = n_i \,. \quad (3.6)$$

Hence, interpolating domain walls maintain the theory at finite distance in moduli space. This is not the case for cobordism walls of nothing, which arise when $e^A \to 0$, and hence $V \to 0$, which sits at infinity in moduli space. This regime was already discussed (in the simpler setup of K3 compactifications) in [11], where the cobordism domain was argued to be given by a Horava-Witten boundary (dressed with suitable gauge bundle degrees of freedom, as required to absorb the local remaining $G_4$ flux), in agreement with the strong coupling singularity discussed in [27]. The wall appears at a finite spacetime distance $\Delta$ following the scaling $\Delta^{-2} \sim \mathcal{T}$ in [11]. In what follows, we describe the scaling relations of the moduli space distance and the SDC tower at these walls of nothing.

Since they are characterized by the vanishing of the overall volume of $\mathbf{X}$, it is enough to follow the behaviour of $V$ and the discussion simplifies. Restriction to this sector amounts to setting all $f_i \equiv f$ in (3.4), and all $H_i \equiv H$. Also, since the wall of nothing arises when $H \to 0$, we can take this location as $y = 0$ and write

$$e^{2A} \sim H(y) \sim \alpha y \,. \quad (3.7)$$



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Using the metric in (3.4), the spacetime distance from a point $y > 0$ is

$$\Delta = \int_0^y (\alpha y)^2 dy = \frac{1}{3}\alpha^2 y^3 \,. \tag{3.8}$$

We are also interested in the traversed distance in moduli space $D$. Using the kinetic term in (3.3), the relevant integral to compute is

$$D = -\int \frac{1}{\sqrt{2}V}\frac{dV}{dy}dy \,. \tag{3.9}$$

Using $V \sim H^3$, we get as leading behavior near the singularity

$$D \simeq -\frac{3}{\sqrt{2}}\log y = -\frac{1}{\sqrt{2}}\log\frac{3\Delta}{\alpha^2} \,, \tag{3.10}$$

where in the last equality we used (3.8). This implies

$$\Delta \sim e^{-\sqrt{2}D} \,, \tag{3.11}$$

and leads to a power-like scaling of the SDC tower mass

$$M_{\rm SDC} \sim \Delta^{\frac{\lambda}{\sqrt{2}}} \,. \tag{3.12}$$

Computing the curvature scalar from (3.4), we get

$$|R| \sim e^{2\sqrt{2}D} \,. \tag{3.13}$$

So the SDC tower scale can be expressed, in an ADC-like manner, as

$$M_{\rm SDC} \sim |R|^{-\frac{\lambda}{2\sqrt{2}}} \,. \tag{3.14}$$

We thus recover a similar behaviour to the examples in section 2.

### 3.2 Traveling across finite distance singularities in moduli space

The setup of M-theory on a CY3 $\mathbf{X}$ allows to address the question of whether walls of nothing could arise at finite distance in moduli space, if the scalars hit a singular point in moduli space. This is actually not the case, as can be explicitly shown by following the analysis in [28] for flop transitions.

Specifically, they considered the flop transition between two Calabi-Yau manifolds with $(h_{1,1}, h_{2,1}) = (3, 243)$, in the setup of a CY3 compactification of the Horava-Witten theory, namely with two boundaries restricting the coordinate $y$ to an interval. In our more general setup, one may just focus on the dynamics in the bulk near the flop transition as one moves along $y$. Hence we are free to locate the flop transition point at $y = 0$.

In terms of the Kähler moduli $t^i = V^{\frac{1}{3}} b_i$ of $\mathbf{X}$, and changing to a more convenient basis

$$t^1 = U \,, \quad t^2 = T - \frac{1}{2}U - W \,, \quad t^3 = W - U \,, \tag{3.15}$$

and similar (proper transforms under the flop) for $\tilde{\mathbf{X}}$, the Kähler cones of $\mathbf{X}$ and $\tilde{\mathbf{X}}$ are defined by the regions

$$\mathbf{X}: \quad W > U > 0\,, \quad T > \frac{1}{2}U + W\,, \tag{3.16}$$

$$\tilde{\mathbf{X}}: \quad U > W > 0\,, \quad T > \frac{3}{2}U\,. \tag{3.17}$$

This shows that the flop curve is $C_3$, and the area is $W - U$, changing sign across the flop.

Near the flop point $y = 0$, the harmonic functions for the two CYs $\mathbf{X}$ and $\tilde{\mathbf{X}}$ have the form

$$\begin{array}{ll} \mathbf{X} \text{ at } y \leq 0 & \tilde{\mathbf{X}} \text{ at } y \geq 0 \\ H_T = -18y + k_T\,, & \tilde{H}_T = 18y + k_T\,, \\ H_U = -25y + k_0\,, & \tilde{H}_U = 24y + k_0\,, \\ H_W = 6y + k_0\,, & \tilde{H}_W = -5y + k_0\,. \end{array} \tag{3.18}$$

Hence

$$\begin{array}{ll} \mathbf{X} \text{ at } y \leq 0 & \tilde{\mathbf{X}} \text{ at } y \geq 0 \\ H_{W-U} = 31y\,, & \tilde{H}_{W-U} = -29y\,. \end{array} \tag{3.19}$$

Even though the flop point is a singularity in moduli space, and despite the sign flip for $W - U$, the harmonic functions are continuous and the solution remains at finite distance in moduli space. This agrees with the picture that it corresponds to an interpolating domain wall. In fact, as discussed in [28], the discontinuity in their slopes (and the related change in the $G_4$ fluxes) makes the flop point highly analogous to the above described interpolating domain walls associated to M5-branes.

The above example illustrates a further important aspect. It provides an explicit domain wall intepolating between two different (yet cobordant) topologies. It would be extremely interesting to extend this kind of analysis to other topology changing transitions, such as conifold transitions[9] [38]. This would allow for a further leap for the dynamical cobordism proposal, given that moduli spaces of all CY threefolds are expected to be connected by this kind of transitions [39].

We have thus established that physics at finite distance in moduli space gives rise to interpolating domain walls, rather than walls of nothing, even at singular points in moduli space. The implication is that the physics of walls of nothing is closely related to the behaviour near infinity in moduli space and hence to the SDC. In the following section we explore further instances of this correspondence in general 4d $\mathcal{N} = 1$ theories.

## 4 $\mathbf{S^1}$ compactification of 4d $\mathcal{N} = 1$ theories and EFT strings

In this section we study a systematic way to explore infinity in moduli space in general 4d $\mathcal{N} = 1$ theories. This arises in a multitude of string theory constructions, ranging from

[9]For a proposal to realize conifold transitions dynamically in a time-dependent background, see [37].

heterotic CY compactifications to type II orientifolds on CY spaces [40]. Our key tool is an $\mathbf{S}^1$ compactification to 3d with certain axion fluxes. We will show that the procedure secretly matches the construction of EFT strings in [24] (see also [25]). Actually, this correspondence was the original motivation for this paper.

### 4.1 Cobordism walls in 4d $\mathcal{N}=1$ theories on a circle

We want to consider general 4d $\mathcal{N}=1$ theories near infinity in moduli space. According to [41–43], the moduli space in this asymptotic regime is well approximated by a set of axion-saxion complex fields, with metric given by hyperbolic planes. We start discussing the single-field case, and sketch its multi-field generalization at the end of this section.

Consider a 4d $\mathcal{N}=1$ theory with complex modulus $S=s+ia$, where $a$ is an axion of unit periodicity and $s$ its saxionic partner. We take a Kähler potential

$$K=-\frac{2}{n^2}\log(S+\bar{S})\,. \tag{4.1}$$

The 4d effective action is

$$\begin{aligned} S&=\frac{M_{P,4}^2}{2}\int d^4x\sqrt{-g_4}\left\{R_4-\frac{n^{-2}}{s^2}\left[(\partial s)^2+(\partial a)^2\right]\right\}\,,\\ &=\frac{M_{P,4}^2}{2}\int d^4x\sqrt{-g_4}\left\{R_4-(\partial\phi)^2-e^{-2n\phi}\,(\partial a)^2\right\}\,, \end{aligned} \tag{4.2}$$

where in the last equation we have defined $\phi=\frac{1}{n}\log ns$.

We now perform an $\mathbf{S}^1$ compactification to 3d with the following ansatz for the metric[10] and the scalars

$$\begin{aligned} ds_4^2&=e^{-\sqrt{2}\sigma}ds_3^2+e^{\sqrt{2}\sigma}R_0^2d\theta^2\,,\\ \phi&=\phi(x^\mu)\,,\qquad a=\frac{\theta}{2\pi}q+a(x^\mu)\,, \end{aligned} \tag{4.3}$$

where $x^\mu$ denote the 3d coordinates and $\theta\sim\theta+2\pi$ is a periodic coordinate. Regarding the axion as a 0-form gauge field, the ansatz for $a$ introduces $q$ units of its field strength flux (we dub it axion flux) on the $S^1$. We allowed for a general saxion profile to account for its backreaction, as we see next.

The dimensional reduction of the action (4.2) gives (see e.g. [44])

$$S_3=\frac{M_{P,3}}{2}\int d^3x\sqrt{-g_3}\left\{R_3-G_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b-V(\varphi)\right\}\,, \tag{4.4}$$

where

$$G_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b=(\partial\sigma)^2+(\partial\phi)^2+e^{-2n\phi}\,(\partial a)^2\,, \tag{4.5a}$$

$$V(\varphi)=e^{-2\sqrt{2}\sigma-2n\phi}\left(\frac{q}{2\pi R_0}\right)^2\,, \tag{4.5b}$$

and $M_{P,3}=2\pi R_0M_{P,4}^2$ is the 3d Planck mass.

[10] We omit the KK U(1) because it will not be active in our discussion.

The last term in the 3d action corresponds to a dynamical tadpole for a linear combination of the saxion and the radion, induced by the axion flux. We thus look for running solutions of the 3d equations of motion. We focus on solutions with constant axion in 3d $a(x^\mu) = 0$, for which the equations of motion read

$$\frac{1}{\sqrt{-g_3}}\partial_\nu\left(\sqrt{-g_3}g^{\mu\nu}\partial_\mu\sigma\right) = -\sqrt{2}\,e^{-2\sqrt{2}\sigma-2n\phi}\left(\frac{q}{2\pi R_0}\right)^2, \tag{4.6a}$$

$$\frac{1}{\sqrt{-g_3}}\partial_\nu\left(\sqrt{-g_3}g^{\mu\nu}\partial_\mu\phi\right) = -n\,e^{-2\sqrt{2}\sigma-2n\phi}\left(\frac{q}{2\pi R_0}\right)^2. \tag{4.6b}$$

We consider solutions in which the fields run with one of the coordinates $x^3$ (which with hindsight we denote by $r \equiv x^3$). We focus on a particular 3d axion-saxion ansatz

$$s(r) = s_0 - \frac{q}{2\pi}\log\frac{r}{r_0}, \quad a(r) = a_0, \tag{4.7}$$

for which the radion can be solved as

$$\sqrt{2}\sigma = \frac{2}{n}(\phi-\phi_0) + 2\log\frac{r}{R_0} = \frac{2}{n^2}\log\left(1-\frac{q}{2\pi s_0}\log\frac{r}{r_0}\right) + 2\log\frac{r}{R_0}. \tag{4.8}$$

This, together with (4.7), provides the scalar profiles solving the dynamical tadpole. The motivation for this particular solution is that it preserves 1/2 supersymmetry, as we discuss in the next section in the context of its relation with the 4d string solutions in [24].

Note that as $r \to 0$, the radion blows up as $\sigma \to -\infty$, implying that the $\mathbf{S}^1$ shrinks to zero size, and the metric becomes singular. As one hits this singularity, the saxion goes to infinity, so we face a wall at which the scalars run off to infinity in moduli space. According to our arguments, it must correspond to a cobordism wall of nothing, capping off spacetime so that the $r < 0$ region is absent; hence the suggestive notation to regard this coordinate as a radial one, an interpretation which will become more clear in the following section. The finite distance $\Delta$ to the wall can be shown to obey the scaling $\Delta^{-2} \sim \mathcal{T}$ introduced in [11].

Note that the asymptotic regime near infinity in moduli space $s \gg 1$ corresponds to the regime

$$r \ll r_0 e^{\frac{2\pi}{q}(s_0-1)}. \tag{4.9}$$

Hence the exploration of the SDC's implications requires zooming into the region close to the wall of nothing.

Let us emphasize that the microscopic structure of the wall of nothing cannot be determined purely in terms of the effective field theory, and should be regarded as provided by its UV completion.[11] On the other hand, we can use effective field theory to obtain the scaling relations between different quantities, as in the string theory examples in the previous sections.

[11] In particular, possible constraints on $q$ could arise from global consistency of the backreaction.



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**The scaling relations.** We can now study the scaling relations between spacetime and moduli space distances, and the SDC tower scale. From the spacetime profiles for $\sigma$ and $\phi$, it is easy to check that the contribution from the radion dominates in the $r \to 0$ limit. The resulting scaling between the moduli space distance $D$ and $r$ is

$$r \simeq e^{-D}\,, \tag{4.10}$$

showing again that $D \to \infty$ as $r \to 0$. On the other hand, the spacetime distance $\Delta$ in the same limit gives

$$d\Delta \simeq \frac{r}{R_0}\left(-\frac{q}{2\pi s_0}\log\left(\frac{r}{r_0}\right)\right)^{\frac{2}{n^2}} \simeq \frac{1}{R_0}\left(-\frac{q}{2\pi s_0}\right)^{\frac{2}{n^2}} D^{2/n^2} e^{-2D} dD\,. \tag{4.11}$$

Upon integration one gets an incomplete gamma function that, after keeping the leading order in $D \to \infty$, finally gives

$$\Delta \sim e^{-2D+\frac{2}{n^2}\log D}\,. \tag{4.12}$$

This is an exponential behaviour up to logarithmic corrections. It would be interesting to relate this to existing results on log corrections to Swampland conjectures (see [45]), but we skip them for now. The resulting relation allows to write the scalings of the SDC tower scale as

$$M_{\rm SDC} \sim e^{-\lambda D} \sim \Delta^{\frac{\lambda}{2}}\,, \tag{4.13}$$

that is again a power-like relation with $\mathcal{O}(1)$ exponents.

Let us turn to computing the scaling of the SDC scale with the scalar curvature $R$. The general expression for $R$ is rather complicated, but simplifies in the leading order approximation at $r = 0$

$$\log|R| \simeq -4\log r \simeq 4\,D\,. \tag{4.14}$$

Hence, the SDC tower mass scales as

$$M_{\rm SDC} \sim e^{-\lambda D} \sim |R|^{-\frac{1}{4}\lambda}\,. \tag{4.15}$$

Amusingly, we again recover a power-like scaling highly reminiscent of the ADC.

**Multi-field generalization.** Let us end this section by mentioning that the above simple model admits a straightforward generalization to several axion-saxion moduli $a^i$, $s^i$. One simply introduces a vector of axion fluxes $q^i$ and generalizes the above running solution to

$$a^i = a_0^i + \frac{\theta}{2\pi}q^i\,, \quad s^i(r) = s_0^i - \frac{q^i}{2\pi}\log\frac{r}{r_0}\,. \tag{4.16}$$

The corresponding backreaction on $\sigma$ is

$$\sqrt{2}\,\sigma = -K(r) + K_0 + 2\log\frac{r}{R_0}\,. \tag{4.17}$$

We leave this as an exercise for the reader, since the eventual result is more easily recovered by relating our system to the 4d string-like solutions in [24], to which we now turn.

### 4.2 Comparison with EFT strings

The ansatz (4.7) is motivated by the relation of our setup with the string-like solutions to 4d $\mathcal{N}=1$ theories discussed in [24], which we discuss next. This dictionary implies that those results can be regarded as encompassed by our general understanding of cobordism walls of nothing and the SDC.

In a 4d perspective, (4.7) corresponds to a holomorphic profile $z = re^{i\theta}$

$$S = S_0 + \frac{q}{2\pi}\log\frac{z}{z_0}\,. \tag{4.18}$$

The axion flux in (4.3) implies that there is a monodromy $a \to a+q$ around the origin $z=0$. Hence, the configuration describes a BPS string with $q$ units of axion charge. The solution for the metric can easily be matched with that in [24]. The 4d metric takes the form

$$ds_4^2 = -dt^2 + dx^2 + e^{2Z}dzd\bar{z}\,, \tag{4.19}$$

with the warp factor

$$2Z = -K + K_0 = \frac{2}{n^2}\log\frac{s}{s_0}\,. \tag{4.20}$$

This matches the 3d metric (4.19) by writing

$$ds_3^2 = e^{\sqrt{2}\,\sigma}\left(-dt^2+dx^2\right) + e^{2Z+\sqrt{2}\,\sigma}dr^2\,, \tag{4.21}$$

and (4.8) ensures the matching of the $S^1$ radion with the 4d angular coordinate range.

$$\int_0^{2\pi} d\theta e^{\sigma/\sqrt{2}}R_0 = \int_0^{2\pi} d\theta e^{Z} r\,. \tag{4.22}$$

Hence, in 4d $\mathcal{N}=1$ theories there is a clear dictionary between running solutions in $\mathbf{S}^1$ compactifications with axion fluxes and EFT string solutions. The compactification circle maps to the angle around the string; the axion fluxes map to string charges; the coordinate in which fields run (semi-infinite, due to the wall of nothing) maps to the radial coordinate away from the string; the saxion running due to the axion flux induced dynamical tadpole maps to the string backreaction on the saxion, i.e. the string RG flow; the scalars running off to infinity in moduli space as one hits the wall of nothing map to the scalars running off to infinity in moduli space as one reaches the string core. Note that the fact that the wall of nothing is not describable within the effective theory maps to the criterion for an EFT string, i.e. it is regarded as a UV-given defect providing boundary conditions for the effective field theory fields.

This dictionary allows to extend the interesting conclusions in [24] to our context. For instance, the relation between the string tension and its backreaction on the geometry provides a scaling with the spacetime distance. This is the counterpart of the scaling relations we found in our 3d dynamical cobordism discussion in the previous section.

On another line, the Distant Axionic String Conjecture in [24] proposes that every infinite field distance limit of a 4d $\mathcal{N}=1$ effective theory consistent with quantum gravity can be realized as an RG flow UV endpoint of an EFT string. We can thus map it into the proposal that every infinite field distance limit of a 4d $\mathcal{N} = 1$ effective theory consistent with quantum gravity can be realised as the running into a cobordism wall of nothing in some axion fluxed $\mathbf{S}^1$ compactification to 3d. It is thus natural to extend this idea to a general conjecture

**Cobordism Distance Conjecture.** *Every infinite field distance limit of a effective theory consistent with quantum gravity can be realized as the running into a cobordism wall of nothing in (possibly a suitable compactification of) the theory.*

The examples in the previous sections provide additional evidence for this general form of the conjecture, beyond the above 4d $\mathcal{N} = 1$ context.

## 5 4d $\mathcal{N} = 1$ theories with flux-induced superpotentials

In the previous section we discussed cobordism walls in compactifications of 4d $\mathcal{N} = 1$ theories on $\mathbf{S}^1$ with axion fluxes. Actually, it is also possible to study running solutions and walls in these theories without any compactification. This requires additional ingredients to introduce the dynamical tadpoles triggering the running. Happily, there is a ubiquitous mechanism, via the introduction of non-trivial superpotentials, such as those arising in flux compactifications. We discuss those vacua and their corresponding walls in this section. The discussion largely uses the solutions constructed in [46], whose notation we largely follow.

Let us consider a theory with a single axion-saxion complex modulus $\Phi = a + iv$. The 4d effective action, in Planck units, is

$$S = -\int d^4x \sqrt{-g} \left[ \frac{1}{2} R + \frac{|\partial\Phi|^2}{4(\mathrm{Im}\,\Phi)^2} + V(\Phi, \overline{\Phi}) \right] \tag{5.1}$$

with the $\mathcal{N} = 1$ scalar potential

$$V(\Phi, \overline{\Phi}) = e^K \left( K^{\Phi\overline{\Phi}} |D_\Phi W|^2 - 3|W|^2 \right). \tag{5.2}$$

We focus on theories of the kind considered in [46], where the superpotential is induced from a set of fluxes $m^I, e_I$, with $I = 0, 1$, and is given by

$$W = e_I f^I(\Phi) - m^I \mathcal{G}_I(\Phi) \tag{5.3}$$

for $f^I$, $\mathcal{G}_I$ some holomorphic functions whose detailed structure we do not need to specify.

In general, these fluxes induce a dynamical tadpole for $\Phi$, unless it happens to sit at the minimum of the potential. The results in [46] allow to build 1/2 BPS running solutions depending on one space coordinate $y$ with

$$ds^2 = e^{2Z(y)} dx_\mu dx^\mu + dy^2 \,. \tag{5.4}$$

For the profile of the scalar, the solution has constant axion $a$, but varying saxion. Defining the 'central charge' $\mathcal{Z} = e^{\mathcal{K}/2}\, W$ and $\mathcal{Z}_*$ its value at the minimum of the potential (and similarly for other quantities), the profile for the scalar $v$ is

$$v(y) = v_* \coth^2 \left( \frac{1}{2} |\mathcal{Z}_*|\, y \right). \tag{5.5}$$



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Note that in [46] this solution was built as 'the left hand side' of an interpolating domain wall solution (more about it later), but we consider it as the full solution in our setup. Note also that we have shifted the origin in $y$ with respect to the choice in [46].

The backreaction of the scalar profile on the metric is described by

$$Z(y) = d + e^{-\frac{1}{2}\hat{\mathcal{K}}_0} \left[ \log \left( -\sinh \left( \frac{1}{2} |\mathcal{Z}_*|\, y \right) \right) + \log\cosh \left( \frac{1}{2} |\mathcal{Z}_*| y \right) \right] , \tag{5.6}$$

where $d$ is just an integration constant and $\hat{\mathcal{K}}_0$ is an additive constant in the Kähler potential.

The solution exhibits a singularity at $y = 0$, which (since the metric along $y$ is flat) is at finite distance in spacetime from other points. On the other hand it is easy to see that the scalar $v$ runs off to infinity as we hit the wall, since

$$v(y) \to 4\, v_*\, |\mathcal{Z}_*|^{-2}\, y^{-2} \quad \text{as } y \to 0 \, . \tag{5.7}$$

We can obtain the scaling of the moduli space distance with the spacetime distance. Using the kinetic term in (5.1),

$$D = - \int \frac{1}{\sqrt{2} v} \frac{dv}{dy}\, dy \simeq -\sqrt{2} \log y \simeq -\sqrt{2} \log \Delta \, . \tag{5.8}$$

In the last two equalities we have used (5.7) and (5.4) respectively. We thus get a familiar power-like scaling for the SDC scale

$$M_{\text{SDC}} \sim \Delta^{\sqrt{2}\lambda} \, . \tag{5.9}$$

We also recover the ADC-like scaling with the scalar curvature. At leading order in $y \to 0$ one finds

$$\log |R| \simeq -2 \log y \simeq \sqrt{2} D \, , \tag{5.10}$$

which gives

$$M_{\text{SDC}} \sim |R|^{-\frac{1}{\sqrt{2}}\lambda} \, . \tag{5.11}$$

This all fits very nicely with our picture that the solution is describing a cobordism wall of nothing, and that the solution for $y > 0$ is unphysical and not realized. This provides an effective theory description of the cobordism defects for general 4d $\mathcal{N} = 1$ theories, in a dynamical framework. It would be interesting to find explicit microscopic realizations of this setup.

Let us conclude this section by mentioning that it is possible to patch together several solutions of the above kind and build cobordism domain walls interpolating between different flux vacua. In particular in [46] the solution provided 'the left hand side' of one such interpolating domain wall solution whose 'right hand side' was glued before reaching (in our choice of origin) $y = 0$, hence before encountering the wall of nothing. The particular solution on the right hand side was chosen to sit at the minimum of the corresponding potential, for which there is no tadpole and thus the functions $D$ and $v$ are simply set to constants, fixed to guarantee continuity. Consequently, the solutions remain at finite distance in moduli space, in agreement with our picture for interpolating domain walls. In some sense, the flux changing membrane is absorbing the tadpole, thus avoiding the appearance of the wall of nothing. We refer the reader to [46] (see also [25]) for a detailed discussion.

## 6 Walls in 10d non-supersymmetric strings

The above examples all correspond to supersymmetric solutions, and even the resulting running solutions preserve some supersymmetry. This is appropriate to establish our key results, but we would like to illustrate that they are not restricted to supersymmetric setups. In order to illustrate that these ideas can apply more generally, and can serve as useful tools for the study of non-supersymmetric theories, we present a quick discussion of the 10d non-supersymmetric USp(32) theory [47], building on the solution constructed in [13] and revised in [11].[12]

The 10d (Einstein frame) action reads

$$S_E = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G}\left[R - \frac{1}{2}(\partial\phi)^2\right] - T_9^E \int d^{10}x\sqrt{-G}\,64\,e^{\frac{3\phi}{2}}\,, \tag{6.1}$$

where $T_9^E$ is the (anti)D9-brane tension. The theory has a dynamical dilaton tadpole $\mathcal{T} \sim T_9^E g_s^{3/2}$, and does not admit maximally symmetric solutions. The running solution in [13] preserves 9d Poincaré invariance, and reads

$$\begin{aligned} \phi &= \frac{3}{4}\alpha_E y^2 + \frac{2}{3}\log|\sqrt{\alpha_E}y| + \phi_0\,, \\ ds_E^2 &= |\sqrt{\alpha_E}y|^{\frac{1}{9}}\,e^{-\frac{\alpha_E y^2}{8}}\eta_{\mu\nu}dx^\mu dx^\nu + |\sqrt{\alpha_E}y|^{-1}e^{-\frac{3\phi_0}{2}}e^{-\frac{9\alpha_E y^2}{8}}\,dy^2\,, \end{aligned} \tag{6.2}$$

where $\alpha_E = 64k^2T_9$. There are two singularities, at $y = 0$ and $y \to \infty$, which despite appearances are located at finite spacetime distance, satisfying the scaling $\Delta^{-2} \sim \mathcal{T}$ introduced in [11]. In this case, there is no known microscopic description for the underlying cobordism defect, but we can still consider the effective theory solution to study the theory as we hit the walls.

We consider the two singularities at $y = 0, \infty$, and look at the behaviour of the solution near them. The distance from a generic point $y$ to the singularites is given by the integral [11]

$$\Delta \sim \int |\sqrt{\alpha_E}y|^{-\frac{1}{2}}e^{-\frac{3\phi_0}{4}}e^{-\frac{9\alpha_E y^2}{16}}\,dy\,, \tag{6.3}$$

on the intervals $[y, 0]$ when $y \to 0$, and $[y, \infty]$ when $y \to \infty$. They give (lower and upper) incomplete gamma functions

$$\Delta_0 \sim \gamma\left(\frac{1}{4}, \frac{9\alpha_E y^2}{16}\right) \qquad \text{and} \qquad \Delta_\infty \sim \Gamma\left(\frac{1}{4}, \frac{9\alpha_E y^2}{16}\right)\,. \tag{6.4}$$

By expanding at leading order as $y \to 0$ and $y \to \infty$, one gets

$$\Delta_0 \sim y^{\frac{1}{2}} \qquad \text{and} \qquad \Delta_\infty \sim y^{-\frac{3}{2}}e^{-\frac{9\alpha_E y^2}{16}}\,. \tag{6.5}$$

The moduli space distance is $\phi = \sqrt{2}D$. Its leading behavior is $D \simeq -\frac{\sqrt{2}}{3}\ln y$ as $y \to 0^+$ and $D \simeq \frac{3\alpha_E}{4\sqrt{2}}y^2$ as $y \to \infty$. This leads to the scaling relations

$$\begin{aligned} y \to 0^+ :& \quad \Delta_0 \sim e^{-\frac{3}{2\sqrt{2}}D}\,, \\ y \to \infty :& \quad \Delta_\infty \sim D^{-\frac{3}{4}}e^{-\frac{3}{2\sqrt{2}}D} \sim e^{-\frac{3}{2\sqrt{2}}D - \frac{3}{4}\ln D}\,. \end{aligned} \tag{6.6}$$

[12] For other references related to dynamical tadpoles in non-supersymmetric theories, see [14–20].



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In both cases we have the moduli space distance running off to infinity as we approach the wall. This is in agreement with their interpretation as cobordism walls of nothing.[13] Moreover, we recover the a familiar power-like scaling of the SDC mass scale with the same numerical factors in both cases

$$M_{\text{SDC}} \sim e^{-\lambda D} \sim \Delta^{\frac{2\sqrt{2}}{3}\lambda}\,. \tag{6.7}$$

It is interesting to see that one can also recover a standard power-like scaling for both singularities if the moduli space distance $D$ is compared with the spacetime curvature scalar $R$. The latter reads

$$|R| = \sqrt{\alpha_E}\, e^{\frac{3\phi_0}{2}} \left(\frac{2}{9}y^{-1} + \frac{7}{2}\alpha_E y + \frac{9}{8}\alpha_E^2 y^3\right) e^{\frac{9\alpha_E}{8}y^2}\,. \tag{6.8}$$

Let us start with the $y \to 0$ singularity. We can approximate the logarithm of the scalar curvature as

$$\log|R| \simeq -\log y \simeq \frac{3}{\sqrt{2}}D\,. \tag{6.9}$$

This allows to rewrite the SDC scaling in the form of the ADC-like scaling

$$M_{\text{SDC}} \sim e^{-\lambda\Delta} \sim |R|^{-\frac{\sqrt{2}}{3}\lambda}\,. \tag{6.10}$$

Let us now turn to the $y \to \infty$ limit. In this case the logarithm of the scalar curvature is approximated to

$$\log|R| \simeq \frac{9\alpha_E}{8}y^2 \simeq \frac{3}{\sqrt{2}}D\,, \tag{6.11}$$

thus recovering the same behavior as for the other singularity.

As announced, we find a nice power-like scaling, reminiscent as usual of the ADC relations. It is amusing that the precise coefficient arises in both the strong and weak coupling singularities, which may hint towards some universality or duality relation in this non-supersymmetric 10d model.

## 7 Final remarks

In this work we have considered running solutions solving the equations of motion of theories with tadpoles for dynamical fields. These configurations were shown to lead to cobordism walls of nothing at finite distance in spacetime [11], in a dynamical realization of the Cobordism Conjecture. We have also discussed interpolating domain walls across which we change to a different (but cobordant) theory/vacuum. We have shown that the key criterion distinguishing both kinds of walls is related to distance in field space: walls of nothing are characterized by the scalars attaining infinite distance in moduli space, while interpolating domain walls remain at finite distance in moduli space.

[13] The interpretation of the $y \to 0$ singularity as a wall of nothing was deemed unconventional, since it would arise at weak coupling. It is interesting that we get additional support for this interpretation from the moduli space distance behaviour.

Hence, cobordism walls of nothing provide excellent probes of the structure of the effective theory near infinite distance points, and in particular the Swampland Distance Conjectures. This viewpoint encompasses and generalizes that advocated for EFT strings in 4d $\mathcal{N} = 1$ theories in [24]. We have found interesting new general scaling relations linking, for running solutions, the moduli space distance and the SDC tower mass scale to geometric spacetime quantities, such as the distance to the wall or the scalar curvature. The latter takes a form tantalizingly reminiscent of the Anti de Sitter Distance Conjecture (ADC), suggesting it may relate to the generalized distance in [23].

We have illustrated the key ideas in several large classes of string models, most often in supersymmetric setups (yet with nontrivial scalar potentials to produce the dynamical tadpole triggering the running); however, we emphasize that we expect similar behaviours in non-supersymmetric theories, as we have shown explicitly for the 10d non-susy USp(32) theory.

There are several interesting open question that we leave for future work:

- We have mainly focused on space-dependent running solutions. It is clearly interesting to consider time-dependent solutions, extending existing results in the literature [13–20], and exploit them in applications, in particular with an eye on possible implications for inflationary models or quintessence.

- A particular class of time-dependent solutions are dynamical bubbles. In particular, a tantalizing observation is that in the original bubble of nothing in [6], the 4d radion modulus goes to zero size (which lies at infinite distance in moduli space of the $\mathbf{S}^1$ compactification) as one hits the wall. Although the setup is seemingly unrelated, it would be interesting to understand universal features of bubbles of nothing along the lines considered in our work.

- The appearance of ADC-like scaling relations in our running solutions possibly signals an underlying improved understanding of infinite distance limits in dynamical (rather than adiabatic) configurations. For instance, as shown in [21], the $r \to \infty$ limit in the Klebanov-Strassler solution [48] avoids the appearance of a tower of states becoming massless exponentially with the distance. This was related to having a non-geodesic trajectory in moduli space (see [49] for a general discussion about non-geodesics and the SDC). However, as dictated by the lack of separation of scales in this model, an ADC-like scaling is yet respected as the scalar curvature goes to zero in this limit. This could point to a more universal way of writing the SDC in dynamical configurations.

- In all the examples we find precise numbers relating the parameter in the SDC $\lambda$ to the power in the ADC-like scaling. It would certainly be interesting to find a pattern in these values and possibly relate them to properties of the infinite distance limits along the lines of [41–43]. On a similar line of thought, it has been argued that in supersymmetric cases the ADC's scaling parameter should be 1/2 [23], assuming this applies to our setup, it would be interesting to extract the SDC's parameter $\lambda$ from our supersymmetric examples with an ADC-like scaling. It would be remarkable that they match the existing proposals for the value of $\lambda$.



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- The ADC-like scaling may also signal some potential interplay with the Gravitino Distance Conjecture [50, 51]. One expects to find a power relation between the mass of the gravitino and the scalar curvature of the solution, it would be certainly interesting to test this and to look for some pattern in the corresponding powers.
- The trajectory in moduli space in spacetime-dependent solutions has a strong presence in the study of black holes, in particular attractor equations and flows. The attempts to relate them to the SDC (see e.g. [52]) can have an interesting interplay with our general framework.
- We certainly expect interesting new applications of our results to the study of non-supersymmetric strings, and to supersymmetry-breaking configurations in string theory.

We hope to report on these problems in the near future.


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## A Holographic examples

In [11] it was shown that Dynamical Cobordism underlies the structure of the gravity dual of the $\mathrm{SU}(N) \times \mathrm{SU}(N+M)$ conifold theory, namely fractional brane deformation of $\mathrm{AdS}_5 \times T^{1,1}$. This in fact explains the appearance of a singularity at finite radial distance [53] and its smoothing out into a configuration capping of the warped throat [48], as a cobordism wall of nothing. In this appendix we provide some examples of other warped throat configurations which illustrate the appearance of other cobordism walls of nothing, and cobordism domain walls interpolating between theories corresponding to compactification on horizons of different topology. The discussion is strongly inspired by the constructions in [54] (see also [55]).

### A.1 Domain wall to a new vacuum

As a first example we consider a configuration in which a running of the conifold theory hits a wall (given by the tip of a KS throat) interpolating to an $\mathrm{AdS}_5 \times \mathbf{S}^5$ vacuum. The latter is the maximally symmetric solution of a theory at the bottom of its potential, i.e. with no dynamical tadpole. We carry out the discussion in terms of the dual field theory, which translates easily into the just explained gravity picture. The dilaton is constant in the whole configuration, so we skip factors of $g_s$.

Consider the conifold theory with $\mathrm{SU}(N) \times \mathrm{SU}(N+M)$ at some scale, i.e. at some position $r$ there are $N$ units of 5-form flux and $M$ units of 3-form flux. The Klebanov-Tseytlin solution [53] gives a running for the effective flux

$$N(r) = N + M^2 \log(r)\,, \tag{A.1}$$

and we get a singu at a value $r_0$ defined by

$$N + M^2 \log r_0 = 0 \quad \Rightarrow \quad r_0 = e^{-N/M^2}\,. \tag{A.2}$$

Naively, the singularity would seem to be smoothed out into a purely geometric background with a finite size $\mathbf{S}^3$. Indeed, this is the full story if $N$ is multiple of $M$, namely $N = KM$: in the field theory, the $\mathrm{SU}(KM) \times \mathrm{SU}(KM + M)$ theory suffers a cascade of $K$ Seiberg dualities in which $K$ decreases by one unit in each step. Morally, the cascade ends when the effective $K = 0$ and then we just have a pure $\mathrm{SU}(M)$ SYM, which confines and develops a mass gap. This is the end of the RG flow, with no more running, hence the spacetime in capped off in the IR region of the dual throat.

However, as also noticed in [48], the story is slightly different if $N = KM + P$. After the $K$ steps in the duality cascade, one is left over with an $\mathrm{SU}(P)$ gauge theory with three complex scalar degrees of freedom parametrizing a deformed mesonic moduli space corresponding to (the symmetrization of $P$ copies of) the deformed conifold. This gauge theory flows to $\mathcal{N} = 4$ $\mathrm{SU}(P)$ SYM in the infrared, which is a conformal theory. In the parameter range $1 \ll P \ll M \ll N$, the whole configuration admits a weakly coupled supergravity dual given by a KS throat at which infrared region we have a finite size $\mathbf{S}^3$, at which $P$ D3-branes (which we take coincident) would be located; however, since $P$ is large, they backreact and carve out a further $\mathrm{AdS}_5 \times \mathbf{S}^5$ with $P$ units of RR 5-form flux, which continues the radial direction beyond the KS throat endpoint region. Hence, this region acts as an interpolating domain wall between two different (but cobordant) theories, namely the conifold throat (with a dynamical tadpole from the fractional brane charge), and the $\mathrm{AdS}_5 \times \mathbf{S}^5$ vacuum (with no tadpole, and preserving maximal symmetry). The picture is summarized in figure 1.

### A.2 Domain wall to a new running solution

Running can lead to an interpolating domain wall, across which we find not a vacuum, but a different running solution (subsequently hitting a wall of nothing, other interpolating domain walls, or just some AdS vacuum). We now illustrate this idea with an example of a running solution A hitting a domain wall interpolating to a second running solution B, which subsequently hits a wall of nothing. The example is based on the multi-flux throat construction in [54] (whose dimer picture is given in [56]). It is easy to devise other generalizations displaying the different behaviours mentioned above.

Consider the system of D3-branes at the singularity given by the complex cone over $dP_3$. The gauge theory is described by the quiver and dimer diagrams[14] in figure 2.

[14] For references, see [55, 57–59].

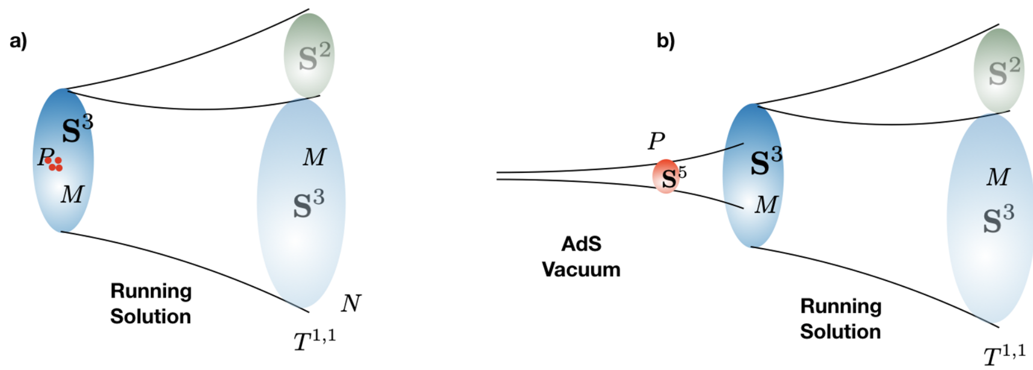


**Figure 1**. Domain wall interpolating between the conifold theory with fractional branes, and an AdS vacuum. Figure a) shows a heuristic intermediate step of a KS solution with a number $P$ of left-over probe D3-branes. If $P$ is large, the appropriate description requires including the backreaction of the D3-branes, which lead to a further AdS throat, to the left of the picture in figure b). Hence the running of the dynamical tadpole in the right hand side ends in a domain wall separating it from an AdS vacuum.

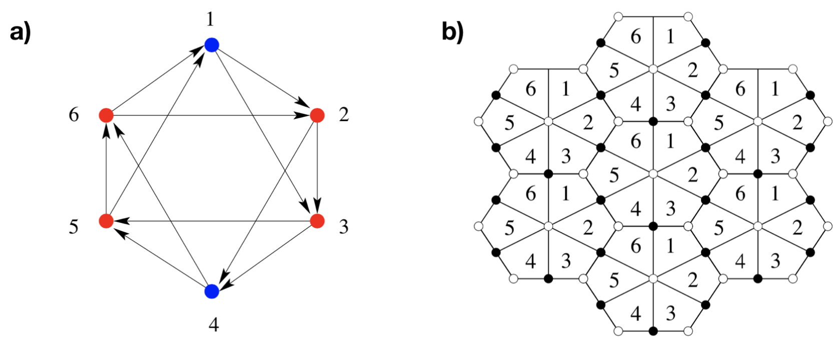


**Figure 2**. The quiver and dimer diagrams describing the gauge theory on D3-branes at the tip of the complex cone over $dP_3$.

We can add fractional branes, i.e. rank assignments compatible with cancellation of non-abelian anomalies. There are several choices, corresponding to different fluxes on the 3-cycles in the dual gravitational theory. Some of them correspond to 3-cycles which can be grown out of the singular origin to provide a complex deformation of the CY. These are described as the splitting of the web diagram into sub-webs in equilibrium, see [56]. In particular we focus on the complex deformation of complex cone over $dP_3$ to a conifold, see the web diagram in figure 3.

There are two kinds of fractional branes, associated to $M$ and $P$. In the gravity dual, these correspond to RR 3-form fluxes on 3-cycles (obtained by an $\mathbf{S}^1$ fibration over a 2-cycle on $dP_3$), and there are NSNS 3-form fluxes in the dual 3-cycles. These are non-compact, namely they span a 2-cycle (dual to the earlier 2-cycle in $dP_3$) and the radial direction. For more details about the quantitative formulas of this kind of solution, see section 5 of [60].

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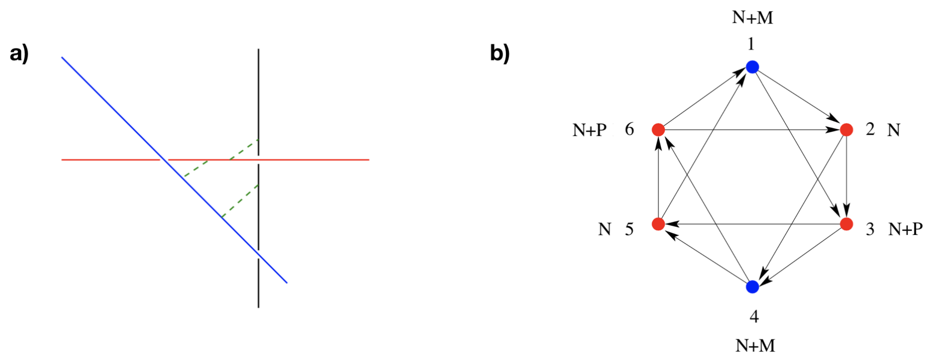


**Figure 3**. a) Web diagram of the complex cone over $dP_3$ splitting into three sub-webs. b) Rank assignment (fractional branes) that trigger those complex deformations.

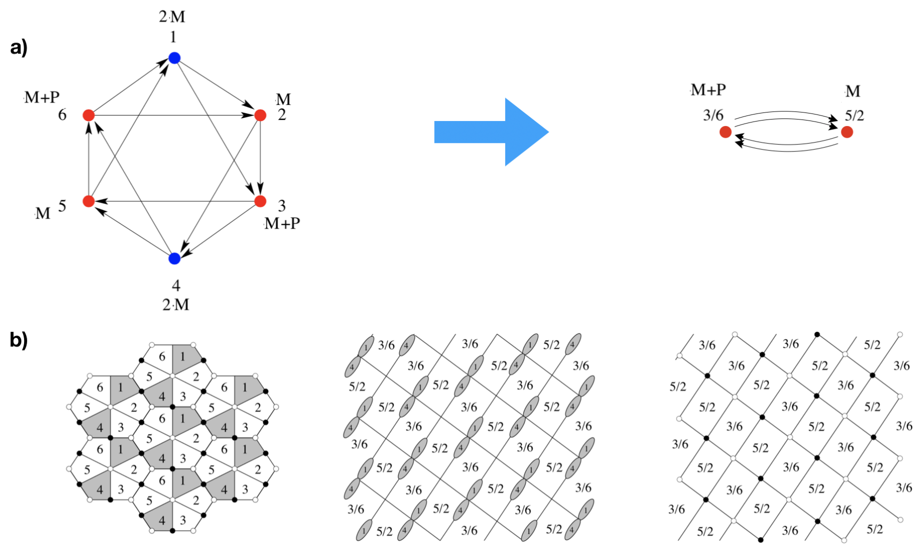


**Figure 4**. a) Quiver of the $dP_3$ theory in the last step of the first cascade, which turns into the conifold upon strong dynamics of the nodes 1 and 4. b) Same story in the dimer picture.

If we focus in the regime[15] $P \ll M$, then the larger flux $M$ implies a larger corresponding component of the $H_3$ flux, which means a faster running of the corresponding 5d NSNS axion. The axion associated to the flux $P$ also runs, but more slowly. In the field theory, the duality cascade is controlled by $M$, so that $N$ is reduced in multiples of $M$ (at leading order in $P/M$). When $N$ is exhausted we are left with a rank assignment as given in figure 4a. The result of the strong dynamics triggered by $M$ can be worked out in field theory as in [54] or using dimers as in [56]. All the info about this last description is in figure 4b.

The result is a conifold theory with $M$ regular branes and $P$ fractional branes. This is the standard KS story (with just different labels for the branes): $M$ decreases in sets of $P$ until it is exhausted, then the running stops due to strong dynamics. In the gravity dual, we

[15] Note that in [54] the regime is the opposite, but both kinds of fractional branes are similar, so the result is the same up to relabeling.

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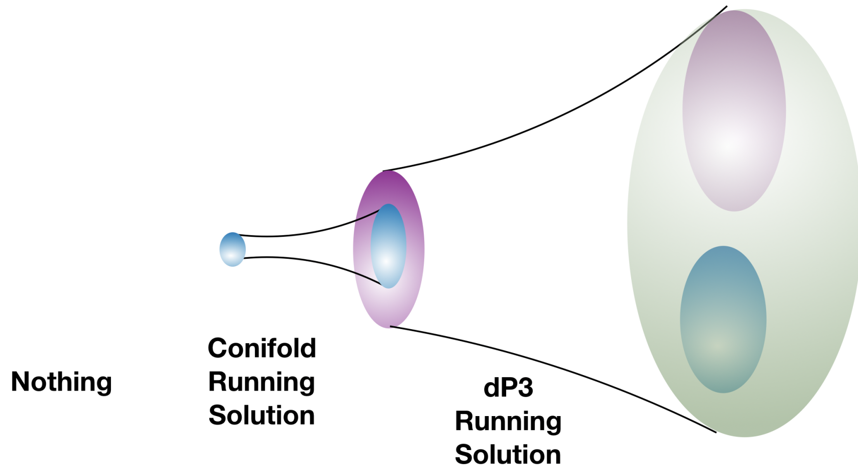


**Figure 5**. Domain wall interpolating between the theory on $dP_3$ with $(M + P)$ fractional branes, and a conifold theory with M regular branes and P fractional ones. The running of one of the dynamical tadpoles in the $dP_3$ theory stops at the wall but the other continues running until it reaches the $\mathbf{S}^3$ at the bottom of the KS throat.

have a KS throat sticking out and spacetime ends on the usual $\mathbf{S}^3$ (alternatively, if $M$ is not a multiple of $P$, there is a number $P$ of leftover D3-branes, which, if large, can trigger a further AdS throat as in section A.1. A sketch of the gravity dual picture is shown in figure 5.

Note that this kind of domain wall interpolates into two topologically different compactifications. As we cross it, the compactification space changes, and the spectrum of light fields changes (at the massless level, one of the axions ceases to exist). In this sense, it is a cobordism domain wall connecting two different quantum gravity theories [5].

### A.3 Cobordism domain walls to disconnected solutions

The construction of singularities admitting complicated patterns of complex deformations (or resolutions) can be carried out systematically for toric singularities, using the techniques in [55]. This can be used to build sequences of domain walls realizing a plethora of possibilities. For our last class of examples, we consider cobordism domain walls to disconnected theories.

This has already been realized in the geometry used in [61] to build a bifid throat, i.e. two throats at the bottom of a throat, see figure 6. These had been proposed in [62] as possible hosts of axion monodromy inflation models (see [63–67] for additional references).

Actually, a far simpler way of getting a running solution with a domain wall to a disconnected set of e.g. vacua is to consider the KS setup in section A.1, with the $P$ leftover D3-branes split into two stacks $P_1$ and $P_2$ of D3-branes at separated locations on the $\mathbf{S}^3$ (with $P_1, P_2 \gg 1$). This corresponds to turning on a vev $v$ for a Higgsing $\mathrm{SU}(P) \to \mathrm{SU}(P_1) \times \mathrm{SU}(P_2)$ (with $P_1 + P_2 = P$) with a scale for $v$ much smaller than the scale of confinement $\Lambda$ of the original $\mathrm{SU}(KM + P) \times \mathrm{SU}(KM + M + P)$ theory. In the gravity dual, we have a running solution in the holographic direction, towards low energies; upon reaching $\Lambda$, we have the $\mathbf{S}^3$ domain wall, out of which we have one $\mathrm{AdS}_5 \times S^5$-like

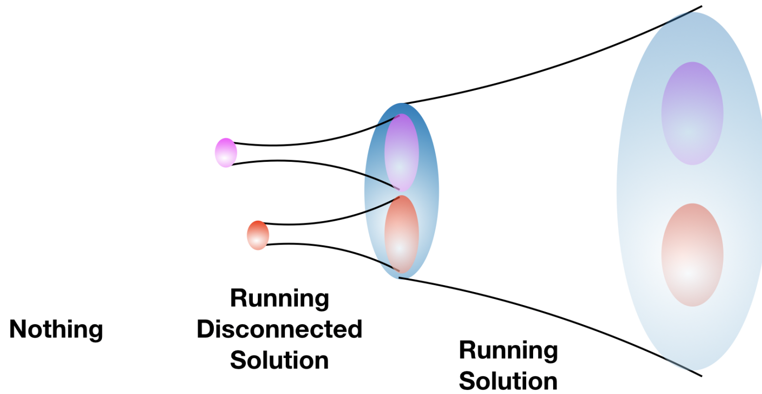


**Figure 6**. Picture of a bifid throat. It represents a domain wall implementing a cobordism between one theory and a disconnected set of two quantum gravity theories.

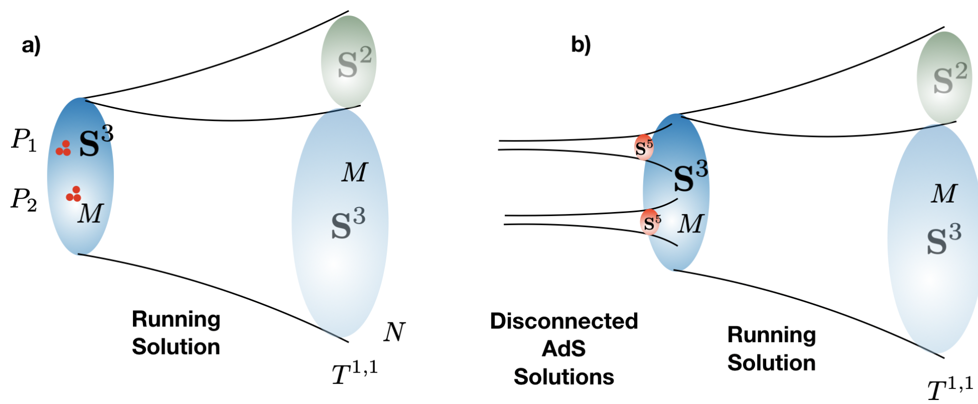


**Figure 7**. Picture of a bifid throat with two AdS tongues. It represents a domain wall implementing a cobordism between one theory and a disconnected set of two AdS theories.

vacuum (with flux $P$), until we hit the scale $v$, and the single throat splits into two $\mathrm{AdS}_5 \times \mathbf{S}^5$ throats (with fluxes $P_1$, $P_2$). If $v \simeq \Lambda$, the splitting of throats happens in the same regime as the domain wall ending the run of the initial solution. This is depicted in figure 7.



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# 6

# Global summary of results and discussion

In this thesis, we have studied different aspects of quantum gravity constraints within the framework of string theory compactifications. The main common thread was provided by running solutions, which played a central role in the articles presented in Chapter 2, 3 and 5. The article presented in Chapter 4 lies somewhat outside this. However, one of its highlights was vacuum energy, and this is not unrelated to the question of dynamical tadpoles, extensively studied in Chapter 5, when both concepts are regarded as features of the scalar potential. Let us briefly summarize the key points and the main results of each of the five articles.

Non compact locally AdS warped throats, and their stability properties, were the object of study of Chapter 2. Motivated by looking at systems of fractional D-branes at singularities, we proposed a new swampland conjecture forbidding stable non-supersymmetric locally AdS warped throats, and thus generalized the analogous statement that non-supersymmetric AdS vacua must be unstable. This allowed us to reinterpret several known facts about warped throats from fractional branes, as well as to derive new results on the instability of large classes of non-supersymmetric throats, with supersymmetry breaking triggered by strong dynamics in the infrared D-brane sectors, or by the presence of stringy sources as orientifold planes. In one of the examples, the instability was associated to the presence of fractional $\mathcal{N} = 2$ branes, and this was later shown to be fully general [4]. As opposed to the pure AdS formulation, our conjecture has no direct bearing on meta-stable non-supersymmetric throats, since in the local case there is no isometry in the radial direction introducing an infinite volume factor multiplying the decay probability, so a finite and potentially small decay amplitude is in principle feasible.

In Chapter 3, we used warped throats as an arena to test the attainability of transplanckian field ranges in string theory. We showed that warped throats of the Klebanov-Strassler kind, regarded as flux compactifications on five-dimensional Sasaki-Einstein manifolds, describe fully backreacted solutions of transplanckian axion monodromy. The key feature of the five-dimensional theory was an axion physically rolling through its dependence on a spatial coordinate, and traversing arbitrarily large distances in field space. The solution included the backreaction on the breathing mode of the compactification space and the vacuum energy, thus yielding a novel form of flattening. We established the description of the system in terms of an effective five-dimensional theory for the axion, and verified its validity in transplanckian regimes. We speculated that similar models in which the axion rolls in the time direction would correspond to embedding the same mechanism in de Sitter vacua, thus providing a natural arena for large field inflation, and potentially linking de Sitter and distance conjectures.

The goal of the article presented in Chapter 4 was to clarify the role of discrete gauge symmetries in certain swampland conjectures. Motivated by black hole arguments combined with the weak gravity conjecture and the species bound, we argued that in theories of quantum gravity with discrete gauge symmetries, namely $\mathbb{Z}_k$, the gauge couplings of U(1) gauge symmetries become weak in the limit of large $k$. We provided explicit examples based on type IIB on $\mathrm{AdS}_5 \times \mathbf{S}^5/\mathbb{Z}_k$ orbifolds, and M-theory on $\mathrm{AdS}_4 \times \mathbf{S}^7/\mathbb{Z}_k$ ABJM orbifolds (and their type IIA reductions). We also studied $\mathrm{AdS}_4$ vacua of type IIA on CY orientifold compactifications, and showed that the parametric scale separation in certain infinite families is controlled by a discrete $\mathbb{Z}_k$ symmetry for domain walls. We accordingly proposed a refined version of the strong AdS distance conjecture, including a parametric dependence on the order of the discrete symmetry for 3-forms. It should be noted that there exist some scale separated $\mathrm{AdS}_3$ models where discrete symmetries of the kind we discussed in the four-dimensional example are absent [3]. This may be a peculiarity of working in three dimensions. Another possibility is that there is some discrete symmetry that protects the scale separation before the orientifolding, and such protection remains after the projection, even though the discrete symmetry is removed.

In Chapter 5, we considered string theory vacua with tadpole sources for dynamical fields. These are dynamical tadpoles (as opposed to topological tadpoles, such as RR tadpoles) and typically indicate the fact that the equations of motion are not obeyed in the proposed configuration, which should be modified by letting the solution acquire some dependence on spacetime coordinates, namely rolling down the slope of a potential. These running solutions share some interesting features. In particular, they often contain metric singularities or strong coupling regimes, which make their physical interpretation difficult.

In the first article of the Chapter, we presented large classes of spacetime-dependent solutions sourced by dynamical tadpoles, which admitted a simple and tractable smoothing out of such singularities. These included $\mathrm{AdS}_5 \times \mathrm{T}_{1,1}$ with 3-form fluxes, 10d massive type IIA, M-theory on K3, 10d non supersymmetric USp(32) string theory, and type IIB compactifications with three-form fluxes and/or magnetized D-branes. We also illustrated a model building application based on a six-dimensional string model whose tadpole triggered spontaneous compactification to four dimensions. Remarkably, this implied not only that some spacetime dimensions were removed, but also that some new degrees of freedom appeared, yielding a semirealistic three-family MSSM-like particle physics model. All these examples revealed some universal features and scaling behaviours, which are most likely to apply to general string theory vacua, and were summed up in two lessons. The Finite Distance lesson implied the appearance of ends of spacetime (or walls of nothing) at a finite spacetime distance, inversely related to the strength of the tadpole. The Dynamical Cobordism lesson identified the physical mechanism cutting off spacetime dimensions as the cobordism defect of the initial theory, predicted by the cobordism conjecture. These results shed new light on several features observed in specific examples of classical solutions to dynamical tadpoles, and provided a deeper understanding of the appearance of singularities, and the stringy mechanism smoothing them out and capping off dimensions to yield dynamical compactification.

In the second article of the Chapter, we showed that as the configuration approaches the cobordism walls of nothing, the scalar fields run off to infinite distance in moduli space, allowing us to explore the implications of the distance conjecture. We uncovered new interesting scaling relations linking the moduli space distance and the scale of the massless tower predicted by the distance conjecture to spacetime geometric quantities, such as the distance to the wall and the scalar curvature. We also considered walls at which scalars remained at finite distance in moduli space and identified them as domain walls interpolating between different (but cobordant) theories. This still applied even if the scalars reached finite distance singularities in moduli space, such as conifold points. We illustrated our ideas with explicit examples in massive IIA theory, M-theory on CY threefolds with 4-form fluxes, and 10d non-supersymmetric strings. In four-dimensional $\mathcal{N} = 1$ theories, our framework reproduced a recent proposal to explore the distance conjecture using four-dimensional axionic string solutions [16].

# 7

# Resumen global de los resultados y discusión

Esta tesis se ha enfocado al estudio de varios aspectos en la comprensión de las restricciones que impone la gravedad cuántica sobre las teorías de campos efectivas a bajas energías en el contexto de compactificaciones de teoría de cuerdas. Soluciones con dependencia espaciotemporal han sido el hilo conductor de los artículos presentados en los Capítulos 2, 3 and 5. El artículo presentado en el Capítulo 4 se encuentra un tanto afuera de esto. Sin embargo, uno de los puntos principales era la energía de vacío, que es relacionada con el tema de tadpoles dinámicos, discutido ampliamente en el Capítulo 5, cuando las dos nociones son consideradas como propriedades del potencial escalar. A continuación resumimos brevemente los puntos clave y los principales resultados de cada uno de los cinco artículos.

Gargantas curvadas con geometría localmente AdS y sus propriedades de estabilidad han sido el objecto de estudio del Capítulo 2. Motivados por considerar sistemas de D-branas fraccionarias en singularidades, hemos propuesto una nueva conjetura de swampland que prohíbe la estabilidad de estas geometrías si supersimetría está rota, generalizando el criterio análogo que excluye vacíos non-supersimétricos estables de tipo AdS. Esto nos ha permitido reinterpretar varios resultados conocidos en la literatura sobre gargantas curvadas obtenidas de branas fraccionarias, así como derivar resultados originales sobre la inestabilidad de largas clases de gargantas non supersimétricas, donde la rotura de la supersimetría es provocada por la dinámica fuerte en el infrarrojo, o por la presencia de ingredientes cuerdosos como planos anti-O3. En un ejemplo particular, la inestabilidad estaba asociada a la presencia de branas fraccionarias $\mathcal{N} = 2$, que fue demostrada posteriormente para ser completamente general [4]. A diferencia de la formulación puramente AdS, la nuestra conjetura no guarda relación directa con gargantas metaestables non-supersimétricas, ya que en el caso local no hay alguna isometría en la dirección radial que introduzca un factor infinito de volumen que multiplica la probabilidad de decaimiento, y por lo tanto una amplitud de decaimiento finita y posiblemente pequeña es en principio realizable.

En el Capítulo 3 hemos utilizado gargantas curvadas como un escenario para probar la obtenibilidad de excursiones de campos transplanckianas en teoría de cuerdas. Hemos demostrado que gargantas curvadas de tipo Klebanov–Strassler, consideradas como compactificaciones con flujos en espacios Sasaki–Einstein cinco dimensionales, definen un modelo de monodromía axionica plenamente backreacted. La característica principal de la teoría efectiva en cinco dimensiones ha sido la presencia de un axion dependiente en la coordenada radial, que recorre distancias arbitrariamente largas en lo espacio de campos. La solución incluía la backreaction sobre el breathing mode del espacio de compactificación y la energía de vacío, generando así una nueva forma de aplanamiento. Especulamos que modelos similares en que el axion se deplaza en la coordenada temporal correspondería a incorporar el mismo mecanismo en vacíos de Sitter, generando así un escenario natural para modelos inflacionarios de campos grandes, y potencialmente relacionando las conjeturas de de Sitter y de la distancia.

El objetivo del Capítulo 4 ha sido analizar el papel de simetrías discretas de gauge en varias conjeturas de swampland. Motivados por argumentos basados en agujeros negros juntos a la conjetura de gravedad débil y la restricción de especies, hemos argumentado que en teorías de gravedad cuántica con simetrías discretas de gauge, en concreto $\mathbb{Z}_k$, las constantes de acoplamiento de las simetrías U(1) de gauge se hacen débiles en el limite de $k$ grande. Hemos analizados ejemplos explícitos basados en tipo IIB en orbifolds $\mathrm{AdS}_5 \times \mathbf{S}^5/\mathbb{Z}_k$, y teoría M en ABJM orbifolds $\mathrm{AdS}_4 \times \mathbf{S}^7/\mathbb{Z}_k$ (y sus reducciones tipo IIA). Además, hemos estudiado vacíos $\mathrm{AdS}_4$ de tipo IIA en compactificaciones orbifold de Calabi-Yau, y hemos mostrado que la separación paramétrica de escalas en algunas familias infinitas es controlada por una simetría discreta $\mathbb{Z}_k$ para paredes de dominio. Por lo tanto, hemos propuesto un refinamiento de la conjetura fuerte de la distancia en Anti-de Sitter, que incluye una dependencia paramétrica del orden de la simetría discreta para 3-formas. Cabe señalar que existen modelos $\mathrm{AdS}_3$ con separación de escalas en que las simetrías discretas del tipo que hemos discutido en el ejemplo cuatro dimensional son ausentes [3]. Esto podría ser una peculiaridad de tres dimensiones. Otra posibilidad es que existe algúna simetría discreta que protege la separación de escalas antes del orientifold, y tal protección permanece después de la proyección, aunque la simetría discreta sea eliminada.

En el Capítulo 5, se ha completado el estudio de potenciales de campos escalares incluyendo tadpoles dinámicos (en oposición a tadpoles topológicos, en concreto RR), que describen inestabilidades que prohíben soluciones máximamente simétricas de las ecuaciones de movimiento, y que por tanto corresponden a procesos de evolución espaciotemporal no trivial. Estas soluciones comparten varias características interesantes. En concreto, suelen contener singularidades de la métrica o regímenes de acoplamiento fuerte, que dificultan la interpretación física.

En el primer artículo del Capítulo, hemos presentado largas clases de soluciones con dependencia espaciotemporal asociadas a tadpoles dinámicos, que admitían una resolución simple y tratable de estas singularidades. Los modelos tratados incluían $\mathrm{AdS}_5{\times}\mathrm{T}_{1,1}$ con flujos de 3-formas, teoría tipo IIA masiva en diez dimensiones, teoría M en K3, teoría de cuerdas USp(32) non supersimétrica en diez dimensiones, y compactificaciones de tipo IIB con flujos de 3-formas y/o D-branas magnetizadas. Además, hemos discutido una aplicación de estas ideas a un modelo seis dimensional cuyo tadpole induce una compactificación espontánea en cuatro dimensiones. Notablemente, esto implicaba que no sólo algunas dimensiones se eliminaron, sino también algunos nuevos grados de libertad aparecían, produciendo un modelo de física de partículas tipo MSSM semi-realista con tres familias. Estos ejemplos han revelados algunas características universales que posiblemente se podrán aplicar a soluciones mas generales de teoría de cuerdas, y que hemos sintetizado en dos lecciones. Con la lección de Distancia Finita hemos establecido la aparición de muros de dominio que constituyen fronteras del espaciotiempo a una distancia finita relacionada inversamente con la intensidad del tadpole. Con la lección de Cobordismo Dinámico hemos identificado el mecanismo que corta las dimensiones espaciotemporal como el defecto de cobordismo de la teoría inicial, demostrando una realización dinámica de la conjetura de cobordismo. Estos resultados han arrojado nueva luz on varias características de soluciones clásicas asociadas a tadpoles dinámicos en ejemplos específicos, y han permitido comprender mejor la aparición de singularidades, así como el mecanismo de sus resolucción con la terminación de dimensiones espaciotemporales que produce compactificaciones dinámicas.

En el segundo artículo del Capítulo, hemos mostrado que cuando la configuración se acerca al muro de dominio, los campos escalares huyen a una distancia infinita en el espacio de módulos, permitiéndonos explorar las implicaciones de la conjetura de la distancia. Hemos desvelado nuevas relaciones entre la distancia en el espacio de módulos y la escala de la torre sin masa de la conjetura de la distancia con cantidades geométricas en el espaciotempo, en concreto la distancia con el muro y la curvatura escalar. Además, hemos considerado muros en que los campos escalares se mantenían en un distancia finita en el espacio de módulos y los hemos identificado como muros de dominio que interpolan entre teorías distintas (pero cobordantes). Esto se aplicaba también cuando los campos escalares alcanzaban singularidades a distancia finita en el espacio de módulos, como puntos de conifold. Hemos ilustrado estas ideas con ejemplos explícitos en teoría masiva IIA, teoría M en $CY_3$ con flujos de 4-formas, y cuerdas non supersimétricas en diez dimensiones. En teorías $\mathcal{N} = 1$ cuadridimensionales, el nuestro esquema ha reproducido una propuesta reciente para explorar la conjetura de la distancia utilizando soluciones de cuerdas axionicas [16].