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# Primordial black holes in phantom cosmology

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**Abstract** We investigate the effects of accretion of phantom energy onto primordial black holes. Since Hawking radiation and phantom energy accretion contribute to a *decrease* of the mass of the black hole, the primordial black hole that would be expected to decay now due to the Hawking process would decay *earlier* due to the inclusion of the phantom energy. Equivalently, to have the primordial black hole decay now it would have to be more massive initially. We find that the effect of the phantom energy is substantial and the black holes decaying now would be *much* more massive—over ten orders of magnitude! This effect will be relevant for determining the time of production and hence the number of evaporating black holes expected in a universe accelerating due to phantom energy.

**Keywords** Black hole, Dark energy, Phantom energy, Hawking radiation

## 1 Introduction

Numerous astrophysical observations are consistent with the standard cold dark matter model with the inclusion of an effective cosmological constant. A classical cosmological constant is generally avoided as quantum gravity attempts lead to a natural expectation of a cosmological term of Planck scale, which is totally at odds with the value required by observation (see for example [1]). As such, it is generally assumed that there is some physical field that comes into play *after the Planck era* and there is some principle that excludes the induced Planck energy cosmological term (see for example [2]). This exotic field is often called “dark energy”. According to the generally accepted modeling, the latter constitutes more than 70% of the total energy density of the universe while the matter component

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carries most of the remaining part [3; 4; 5; 6; 7]. It is not clear how seriously to take the ‘‘quantum gravity’’ requirements considering that there is no viable theory of quantum gravity to date [1; 8]. Further, there is no clear evidence that the onset of quantum gravity will be at Planck scale and not orders of magnitude away from it [9]. There are alternate (classical) explanations of the observations available in the literature (for example by Wiltshire [10; 11; 12; 13]) but here we shall follow the generally accepted view of some form of dark energy providing the observed acceleration of the Universe.

The observable universe locally appears to be spatially flat with an equation of state (EoS) parameter (the ratio of pressure to the energy density)  $\omega(\equiv p_x/\rho_x) \simeq -1$ . The dark energy is an exotic vacuum energy with negative pressure and positive energy density which arises due to quantum vacuum fluctuations in spacetime. Caldwell and co-workers [14; 15] considered the possibility of dark energy with the super-negative EoS parameter  $\omega < -1$ , which they called ‘phantom energy’. It also gives negative pressure. The motivation to consider phantom energy as the candidate for dark energy arises from the observational data of the cosmic microwave background power spectrum and supernovae of type Ia. The phantom energy violates the general relativistic energy conditions including the null and dominant ones [16]. Its implications in cosmology give rise to exotic phenomena like an imaginary value of the sound speed, negative temperature, the divergence of the scale factor  $a(t)$  and the energy density  $\rho_x \sim a^{-3(1+\omega)}$ , at a finite time resulting in a ‘big rip’, an epoch when the spacetime is torn apart [17; 18; 19; 20; 21] (see [22] for a review on the big rip singularity). However there are some attempts made recently in which the occurrence of a big rip singularity is avoided by phantom energy decay into matter [23; 24; 25]. Another attempt is the ‘big trip’, in which a wormhole accretes phantom energy and grows so large that it engulfs the whole universe [26]. A similar scenario is also proposed for black holes whereby the black hole event horizon inflates to swallow up the cosmological horizon, resulting in a naked singularity [27]. Moreover, quantum gravitational effects (if they exist) may avoid the big rip singularity. If the big rip cannot be avoided, the smaller the parameter  $\omega$ , the closer the big rip will be to the present time.

Another weird property of phantom energy is that its accretion onto gravitationally bound structures results in their dissociation and disintegration in a rather slow process. It was first analyzed in [28] for several gravitational systems like the solar system and the Milky Way galaxy. Initially, this possibility was investigated for a Schwarzschild black hole by Babichev et al. [29], who showed that the black hole mass goes to zero near the big rip. Interestingly, in this scenario larger black holes lose mass more rapidly than smaller ones. Later on, their model was extended to the Reissner-Nordstrom [31], Schwarzschild de Sitter [32] and Kerr-Newmann [33; 34] black holes. It should be mentioned that it has been argued that the mechanism of accretion followed by Babichev et al. is stationary and does not possess the shift symmetry [30] and hence that the mechanism of dark energy accretion is not realistic and consistent. Nevertheless, we shall follow the Babichev et al. analysis, leaving the detailed analysis for subsequent work, as the effect will be technically difficult to apply and we believe will not make a substantial difference for phantom energy in the neighbourhood of a primordial black hole. It has also been argued that when the back-reaction effects of the accretion process are included in the analysis of Babichev et al. [29], the black hole mass may increase

instead of decreasing [35; 36], thus avoiding the big rip. Also in cyclic cosmological models, black holes do not tear apart near the turnaround but preserve some nonzero mass [37; 38]. We shall ignore the big rip issue here.

## 2 Hawking evaporation of black holes

We are interested in studying the effects of accretion of phantom energy on a static primordial black hole. Carr and Hawking [39] in 1974 considered the formation of black holes of mass  $10^2$  kg and upwards in the early evolution of the universe. After their attempt, several authors investigated various scenarios of PBH formation [40; 41; 42; 43]. The existence of these small mass black holes was based on the assumption that the early universe was not entirely spatially smooth but there were density fluctuations or inhomogeneities in the primordial plasma which gravitationally collapsed to form these black holes. Unlike the conventional black holes that are formed by the gravitational collapse of stars or mergers of neutron stars, the primordial black holes (PBH) are formed due to the gravitational collapse of matter without forming any initial stellar object. The mass of a PBH can be of the order of the particle horizon mass at the time of its formation [44; 45]

$$M_{\text{PBH}} \approx \frac{c^3 t}{G} \approx 10^{12} \left( \frac{t}{10^{-23} \text{ s}} \right) \text{ kg.} \quad (1)$$

Therefore PBHs that formed in the early history of the universe must be less massive while those that formed later must be more massive. Black holes formed at Planck time  $10^{-43}$  s would have Planck mass  $10^{-8}$  kg.

Using classical arguments, Penrose and Floyd showed that one can extract rotational energy from a rotating black hole [46]. Penrose went on to argue (see [8] and references therein) that one could take thermal energy from the environs of a black hole and throw it into the black hole to get usable energy out. This would apparently reduce the entropy around the black hole. As such, he had argued that *there must be an entropy of the black hole that increases at least as much as that of its environs decreases*. Hawking had pointed out that in any physical process the area of a black hole always increases [47] just as entropy always increases. This led Bekenstein [48] to propose a linear relationship between the area and entropy of a black hole. Thus Bekenstein [49; 50] generalized the second law of thermodynamics to state that the sum of the entropy of the black hole and its environs never decreases. However, at this stage it seemed that the connection between black holes and thermodynamics was purely formal. At this stage Fulling pointed out that quantization of scalar fields in accelerated frames gives an ambiguous result [51], which seemed to yield radiation seen in the accelerated with a fractional number of particles. Hawking repeated the calculation for an observer near a black hole and obtained the same result by various methods and found that the radiation had a thermal spectrum [52]. This led him to propose that mini-PBHs would evaporate away in a finite time [53].

The corresponding Hawking evaporation process reduces the mass of the black hole by [54]

$$\left. \frac{dM}{dt} \right|_{\text{hr}} = - \frac{\hbar c^4}{G^2} \frac{\alpha}{M^2}, \quad (2)$$

where  $\alpha$  is the spin parameter of the emitting particles. Integration of Eq. (2) gives the evolution of PBH mass as

$$M_{\text{hr}} = M_i \left(1 - \frac{t}{t_{\text{hr}}}\right)^{1/3}, \quad (3)$$

where the Hawking evaporation time scale  $t_{\text{hr}}$  is

$$t_{\text{hr}} = \frac{G^2 M_i^3}{\hbar c^4 3\alpha}. \quad (4)$$

It is obvious from Eq. (3) that as  $t \rightarrow t_{\text{hr}}$ , the mass  $M_{\text{hr}} \rightarrow 0$ . Plugging in  $t_{\text{hr}} = t_o$  (the current age of the universe) in Eq. (4) gives the mass  $10^{12}$  kg of the PBH that should have been evaporating now. Hence from Eq. (1), it can be estimated that these PBHs were formed before about  $10^{-23}$  s. For  $M_i \gg 10^{14}$  kg  $\alpha = 2.011 \times 10^{-4}$ , hence Eq. (4) implies  $t_{\text{hr}} \simeq 2.16 \times 10^{-18} \left(\frac{M}{\text{kg}}\right)^3$  s. While for  $5 \times 10^{11}$  kg  $\ll M_i \ll 10^{14}$  kg,  $\alpha = 3.6 \times 10^{-4}$  then Eq. (4) gives  $t_{\text{hr}} \simeq 4.8 \times 10^{-18} \left(\frac{M}{\text{kg}}\right)^3$  s. Therefore detecting PBHs would be a good tool to probe the very early universe (closer to the Planck time). The evaporation of PBHs could still have interesting cosmological implications: they might generate the microwave background [55] or modify the standard cosmological nucleosynthesis scenario [56] or contribute to the cosmic baryon asymmetry [57]. Some authors have also considered the possibility of the accretion of matter and dust onto the seed PBH resulting in the formation of super-massive black holes which reside in the centers of giant spiral and elliptical galaxies [58].

### 3 Phantom energy accretion onto black hole

The FRW equations governing the dynamics of our gravitational system are given by

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_x), \quad (5)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}[\rho_m + \rho_x(1 + \omega)]. \quad (6)$$

Here  $\rho_m$  and  $\rho_x$  denote the energy densities of matter and the exotic energy densities respectively. The scale factor  $a(t)$  goes like [20]

$$a(t) = \frac{a(t_0)}{[-\omega + (1 + \omega)t/t_0]^{-\frac{2}{3(1+\omega)}}} \quad (t > t_0), \quad (7)$$

where  $t_0$  is the time when the universe transits from matter to exotic energy domination (which is roughly equal to the age of the universe). Notice that the scale factor  $a(t)$  diverges when the quantity in the square brackets in Eq. (7) vanishes identically i.e.

$$t^* = \frac{\omega}{1 + \omega} t_0. \quad (8)$$

Subtracting  $t_0$  from Eq. (8), we get

$$t^* - t_0 = \frac{1}{1 + \omega} t_0. \quad (9)$$

The evolution of energy density of the exotic energy is given by

$$\rho_x^{-1} = 6\pi G(1 + \omega)^2 (t^* - t)^2. \quad (10)$$

A black hole accreting only the exotic energy has the following rate of change in mass [29]

$$\left. \frac{dM(t)}{dt} \right|_x = \frac{16\pi G^2}{c^5} M^2 (\rho_x + p_x). \quad (11)$$

It is clear that when  $\rho_x + p_x < 0$ , the mass of the black hole will decrease. We are particularly interested in the evolution of black holes about and after  $t = t_0$  since the dark energy is presumably negligible before that time and may not have any noticeable effects on the black hole. Using Eqs. (9) and (10) in (11), we get

$$\left. \frac{dM(t)}{dt} \right|_x = \frac{8G}{3c^3} \frac{M^2}{t_0^2} (1 + \omega). \quad (12)$$

Therefore the mass change rate for a black hole accreting pure exotic energy is determined by Eq. (12). For the phantom energy accretion, the time scale is obtained by integrating Eq. (12) to get

$$M(t) = M_i \left( 1 - \frac{t}{t_x} \right)^{-1}, \quad (13)$$

where  $t_x$  is the characteristic accretion time scale given by

$$t_x^{-1} = \frac{16\pi G^2}{c^5} M_i (\rho_x + p_x). \quad (14)$$

Using Eqs. (9) and (10) in (14), we get

$$t_x = \frac{3c^3}{8G M_i (1 + \omega)} \frac{t_0^2}{t_0}. \quad (15)$$

#### 4 Evolution of mass due to phantom energy accretion and Hawking evaporation

The expression determining the cumulative evolution of the black hole is obtained by adding Eqs. (2) and (12) i.e.

$$\left. \frac{dM(t)}{dt} \right|_{\text{Total}} = \left. \frac{dM}{dt} \right|_{\text{hr}} + \left. \frac{dM}{dt} \right|_x, \quad (16)$$

$$= -\frac{\hbar c^4}{G^2} \frac{\alpha}{M^2} + \frac{8G}{3c^3} \frac{M^2}{t_0^2} (1 + \omega). \quad (17)$$

We write the above equation as

$$\frac{dM}{dt} = -aM^2 - \frac{b}{M^2}, \quad (18)$$

where

$$a = \frac{8G}{3c^3} \frac{\varepsilon}{t_0^2}, \quad b = \frac{\hbar c^4 \alpha}{G^2}. \quad (19)$$

Here  $\varepsilon = -\omega - 1$ . Thus (18) can be written in the form

$$-\int dt = \frac{1}{b} \int \frac{M^2 dM}{1 + \frac{a}{b} M^4}$$

To integrate above equation, we assume

$$x = \left( \frac{a}{b} \right)^{1/4} M, \quad (20)$$

which yields

$$-\int dt = \frac{1}{(a^3 b)^{1/4}} \int \frac{x^2 dx}{1 + x^4}, \quad (21)$$

We note that [59]

$$\begin{aligned} \int \frac{x^{m-1} dx}{1+x^{2n}} &= -\frac{1}{2n} \sum_{k=1}^n \cos\left(\frac{m\pi(2k-1)}{2n}\right) \ln \left| 1 - 2x \cos\left(\frac{2k-1}{2n}\pi\right) + x^2 \right| \\ &+ \frac{1}{n} \sum_{k=1}^n \sin\left(\frac{m\pi(2k-1)}{2n}\right) \tan^{-1} \left[ \frac{x - \cos\left(\frac{2k-1}{2n}\pi\right)}{\sin\left(\frac{2k-1}{2n}\pi\right)} \right], \quad m < 2n. \end{aligned} \quad (22)$$

In our case,  $m = 3$  and  $n = 2$ , hence the above equation yields

$$\int \frac{x^2 dx}{1+x^4} = \frac{1}{4\sqrt{2}} \ln \left| \frac{1 - \sqrt{2}x + x^2}{1 + \sqrt{2}x + x^2} \right| + \frac{1}{2\sqrt{2}} \tan^{-1} \left( \frac{\sqrt{2}x}{1+x^2} \right). \quad (23)$$

On substituting the value of  $x$  above, we obtain

$$t = t_0 + \frac{1}{4\sqrt{2}} \ln \left| \frac{1 - \sqrt{2} \left(\frac{a}{b}\right)^{1/4} M + \left(\frac{a}{b}\right)^{1/2} M^2}{1 + \sqrt{2} \left(\frac{a}{b}\right)^{1/4} M + \left(\frac{a}{b}\right)^{1/2} M^2} \right| + \frac{1}{2\sqrt{2}} \tan^{-1} \left[ \frac{\sqrt{2} \left(\frac{a}{b}\right)^{1/4} M}{1 + \left(\frac{a}{b}\right)^{1/2} M^2} \right]. \quad (24)$$

We now redefine the values of  $a$  and  $b$  by assuming  $M = mM_i$ , where  $m$  is a dimensionless parameter and  $M_i$  is the initial mass of the black hole. Thus (18) becomes

$$\frac{dm}{dt} = -a'm^2 - \frac{b'}{m^2}, \quad (25)$$

where  $a' = aM_i$  and  $b' = b/M_i^3$ . For the terms to be equal strength, we require  $a' \approx b'$ . Thus

$$M_i \approx \left(\frac{b}{a}\right)^{1/4}. \quad (26)$$

Now

$$\frac{b}{a} = \frac{3\hbar c^7 t_0^2 \alpha}{8G^3 \varepsilon}, \quad \text{or} \quad \varepsilon = \frac{3\hbar c^7 t_0^2 \alpha}{8G^3 M_i^4}. \quad (27)$$

We can normalize

$$t = t_0 \left[ 1 - \frac{\frac{1}{4\sqrt{2}} \ln \left| \frac{1 - \sqrt{2} \left(\frac{a'}{b'}\right)^{1/4} m + \left(\frac{a'}{b'}\right)^{1/2} m^2}{1 + \sqrt{2} \left(\frac{a'}{b'}\right)^{1/4} m + \left(\frac{a'}{b'}\right)^{1/2} m^2} \right| + \frac{1}{2\sqrt{2}} \tan^{-1} \left[ \frac{\sqrt{2} \left(\frac{a'}{b'}\right)^{1/4} m}{1 + \left(\frac{a'}{b'}\right)^{1/2} m^2} \right]}{\frac{1}{4\sqrt{2}} \ln \left| \frac{1 - \sqrt{2} \left(\frac{a'}{b'}\right)^{1/4} + \left(\frac{a'}{b'}\right)^{1/2}}{1 + \sqrt{2} \left(\frac{a'}{b'}\right)^{1/4} + \left(\frac{a'}{b'}\right)^{1/2}} \right| + \frac{1}{2\sqrt{2}} \tan^{-1} \left[ \frac{\sqrt{2} \left(\frac{a'}{b'}\right)^{1/4}}{1 + \left(\frac{a'}{b'}\right)^{1/2}} \right]} \right]. \quad (28)$$

Replacing  $p = a'/b' = \frac{8\varepsilon G^3}{3\alpha\hbar c^7 t_0^2} M_i^4 \sim M_i^4$  (the ratio of the phantom component to the Hawking component, in the energy radiated) the above equation becomes

$$t = t_0 \left[ 1 - \frac{\frac{1}{4\sqrt{2}} \ln \left| \frac{1 - \sqrt{2} p^{1/4} m + p^{1/2} m^2}{1 + \sqrt{2} p^{1/4} m + p^{1/2} m^2} \right| + \frac{1}{2\sqrt{2}} \tan^{-1} \left( \frac{\sqrt{2} p^{1/4} m}{1 + p^{1/2} m^2} \right)}{\frac{1}{4\sqrt{2}} \ln \left| \frac{1 - \sqrt{2} p^{1/4} + p^{1/2}}{1 + \sqrt{2} p^{1/4} + p^{1/2}} \right| + \frac{1}{2\sqrt{2}} \tan^{-1} \left( \frac{\sqrt{2} p^{1/4}}{1 + p^{1/2}} \right)} \right]. \quad (29)$$

Moreover, the power emission due to Hawking evaporation from the stationary black hole of mass  $M \gg 10^{17}$  g [54]

$$P = 3.458 \times 10^{46} (M/\text{g})^{-2} \text{erg s}^{-1}, \quad (30)$$

and for mass  $5 \times 10^{14}$  g  $\ll M \ll 10^{17}$  g,

$$P \approx 3.6 \times 10^{16} (M/10^{15} \text{ g})^{-2} \text{erg s}^{-1}. \quad (31)$$

**Fig. 1** The normalized time is plotted against the mass parameter for  $p = 0.1$

**Fig. 2** The mass parameter is plotted against the normalized time for  $p = 0.1$

**Fig. 3** The normalized time is plotted against the mass parameter for  $p = 0.5$

**Fig. 4** The mass parameter is plotted against the normalized time for  $p = 0.5$

In our analysis, the mass in the above two expressions is replaced by

$$M = \left( \frac{3\hbar c^7 t_0^2 \alpha}{8G^3 \varepsilon} \right)^{1/4} \text{ g.} \quad (32)$$

Now choosing  $\varepsilon = 0.1$ , we obtain  $M = 8.74029 \times 10^{22}$  g which will be evaporating now due to the combined effects of phantom energy and Hawking radiation. Then using Eq. (30), the corresponding power emission will be  $P = 4.52661 \text{ erg s}^{-1}$ . We can compare this result with that of a black hole of mass  $M \simeq 1.05 \times 10^{12}$  g evaporating just now due to Hawking radiation only. The corresponding power emission will be  $P \simeq 3.144 \times 10^{22} \text{ erg s}^{-1}$ . Note that the power emission from a black hole decreases when the effects of phantom energy are incorporated. Similarly, for very large values of  $\varepsilon \sim 10^{25}$  would give  $M = 2.763923 \times 10^{16}$  g. Using this mass in (31), the power emitted is  $4.52661 \times 10^{13} \text{ erg s}^{-1}$ . However, such large values would lead to a very early big rip and hence must be excluded. Thus black holes  $\sim 10^{22}$  g are of more interest for observational purposes since these are the ones that should be evaporating now.

## 5 Conclusion

In this paper, we have analyzed the Hawking radiation effects combined with the phantom energy accretion on a stationary black hole. The former process has been thoroughly investigated in the literature. However there is as yet no observational support to it. According to standard theory it is assumed that after the formation of PBHs (of mass  $\sim 10^{12}$  kg with a Hawking temperature  $10^{12}$  K), they would absorb virtually no radiation or matter whatsoever during their evolution and radiate continuously till they evaporate in a burst of gamma rays at the present time. This scenario assumes that the Hawking temperature for such black holes was always larger than the background temperature of the CMB. Strictly speaking, this cannot be true. Consequently PBHs could have accreted the background radiation (and even some matter) and *grown* in mass. Hence there should be no PBH left to be evaporating right now [60]. However, the above scenario is modified when phantom energy comes into play. When phantom energy and the Hawking process are relevant the total life time scale of the PBH is significantly shortened and the formation of the PBH exploding now is delayed.

From Eq. (29) we obtain the time as a function of mass instead of getting mass as a function of time. To make sense of the results we need to obtain the evolution

**Fig. 5** The normalized time is plotted against the mass parameter for  $p = 1$

**Fig. 6** The mass parameter is plotted against the normalized time for  $p = 1$

**Fig. 7** The normalized time is plotted against the mass parameter for  $p = 5$

**Fig. 8** The mass parameter is plotted against the normalized time for  $p = 5$

with time. This is done by inverting the explicit function. We have plotted the normalized time  $\tau = t/t_0$  against the dimensionless mass parameter  $m$  and  $m$  against  $\tau$  for different choices of the parameter  $p$ , in Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10. It is observed that increasing  $p$  increases the steepness of the curve specifying the mass evolution. Therefore the black hole loses mass faster for larger  $p$  till it vanishes at  $\tau = 1$ , the present time. In particular, Figs. 7 and 9 show the same evolution of mass for larger values of  $p$ . It appears that the graphs contain a redundant (or nonphysical) part of the mass evolution and the only physically interesting section is above the horizontal curve crossing  $t = 0$ . Thus in effect, see Figs. 8 and 10, the initial mass of the black hole must be taken  $0.45M_i$  of the value given by for  $p = 5$  and about  $0.315M_i$  for  $p = 10$ . It is obvious that the results are very insensitive to changes of the parameter  $\varepsilon$  for the phantom energy. As such, they can be regarded as fairly robust.

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**Fig. 9** The normalized time is plotted against the mass parameter for  $p = 10$

**Fig. 10** The mass parameter is plotted against the normalized time for  $p = 10$

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