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HELIUM COOLING OF THE SHELLS OF A DIPOLE MAGNET IN THE MODEL 135 CRYOSTAT

Prepared Under Fermilab Subcontract No. 94199
By Cryogenic Consultants, Inc.
Allentown, Pa.

For

Fermi National Accelerator Laboratory, Batavia, Illinois

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CONTENTS

1. Introduction

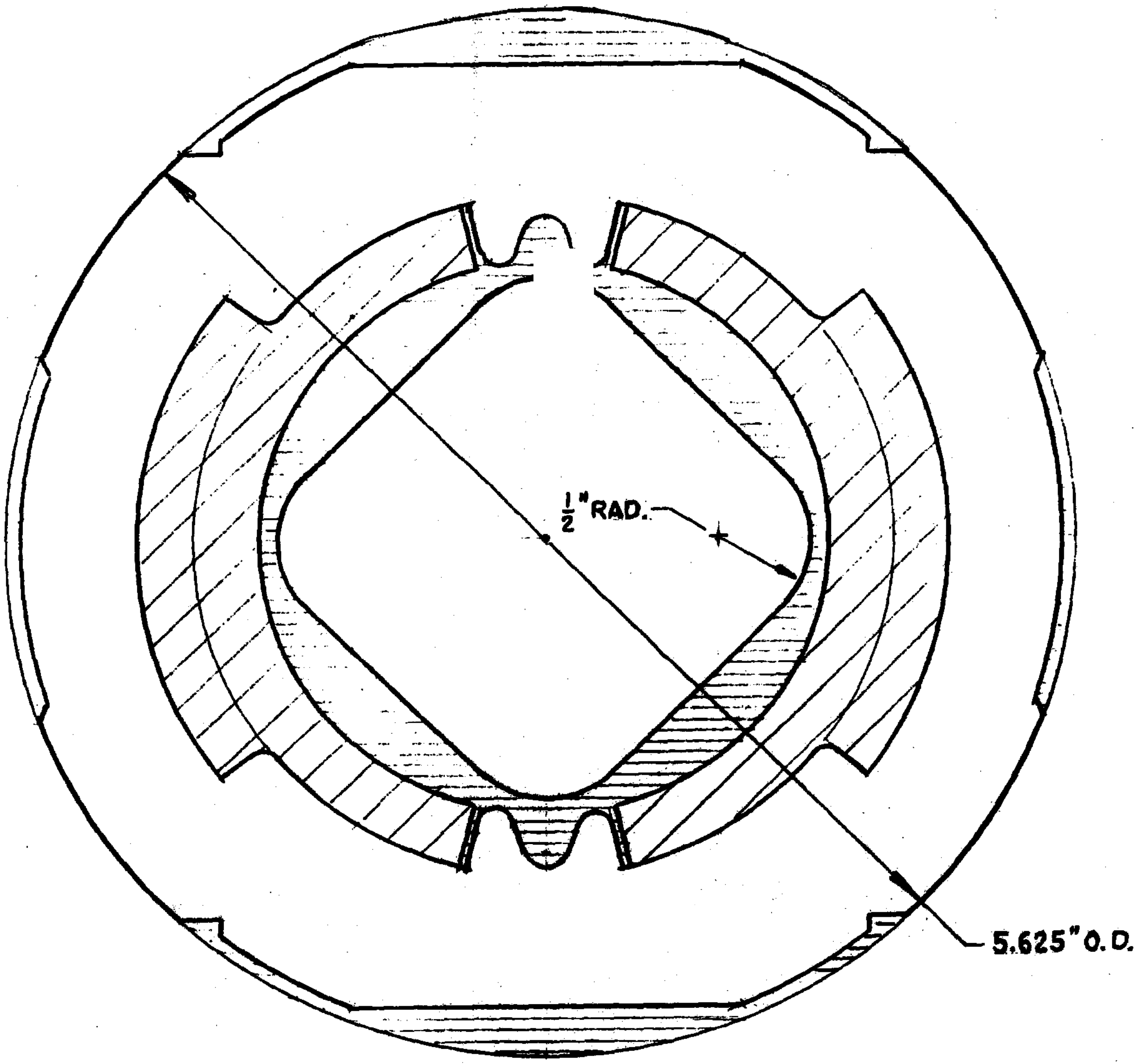
Some calculations have been made in an attempt to determine the temperature of the inner and outer shells of the 20 ft dipoles equipped with Type 5 collars. The elimination of a continuous circular path, which allows warm fluid to rise between shells 1 and 2 is the most significant modification from a cryogenic viewpoint.

It has been postulated that the inner shell is cooled directly by the single-phase liquid helium flowing between bore tube and shell, and that the outer shell is cooled by conduction of heat through the stainless steel collar to the single-phase fluid flowing through the narrow channel between cryostat wall and collar. The contact resistance between outer shell and collar has been assumed to be zero, because of the large force which presses the shell against the collar.

Heat generated from A-C losses due to ramping have been assumed to be 600 joules per cycle (60 sec) with up and down ramps of 18 sec. 500 joules are generated in the inner shell and 100 joules in the outer shell.

Dimensional data of the flow cross section have been based on drawings of cryostat and verbal information from G. Biallas to CCI. Figure 1 summarizes this information.

The single-phase liquid between bore tube and inner shell rises in temperature in flowing through the magnet. This liquid is only cooled by mixing with the other streams. These streams, in turn, are cooled very effectively by the two-phase liquid flowing in counterflow around the cryostat.



ID of Helium 1 Ø - 2 Ø Wall = 5.649"
OD of St.Stl. Collar = 5.625"

Dimensions of Cross Section of Dipole Magnet

FIGURE 1

2. Conclusions

1. The first magnet of a string has the lowest temperature.
2. Subsequent magnets operate at higher temperatures for the inner shell, because of cooling of the single-phase fluid by a mixing process only.
3. The temperature of the shells is sensitive to flow rate of single and two-phase liquid through the magnet. Higher flow rates will lower the shell temperatures.
4. The temperature of the shells is ramp rate sensitive.
5. The magnet test facility does not reflect the operating conditions of the dipoles in doubler operation, because of high flow rate and low temperature of the single-phase fluid stream.
6. It will be instructive to test a magnet in the magnet test facility at somewhat higher fluid temperature and lower flow rate to determine maximum current and ramp rate as a function of these variables.
7. It will be instructive to measure the temperature of the single-phase liquid at the point where it enters magnets 1 through 5.

3. Assumptions

1. The passages permitting circumferential flow between shells 1 and 2 have been blocked.
2. Energy distribution in the mass of the superconductor due to ramping is 100 joules in outer shell; 500 joules in inner shell.

3. Single and two-phase helium flow rate is 20 g/sec. Single-phase divides between inside and outside of coil structure in non-equal amounts.
4. Heat leak from environment is taken up directly by the two-phase flow.
5. Liquid helium inventory in a dipole magnet is 12 liters of which 5.5 liters is located inside the shells.
6. Inventory outside the collars is 6.5 liters.
7. Liquid inventory in ends of magnet has not been determined.

4. Dimensional Data

	Cross Section (cm^2)	Volume (cm^3)	d_h (cm)
Flow Area:			
Inside Shell I:	9.08	5,535	.780
Outside Shell II:	1.60	975	.318
Top / Bottom of Collar:	9.12	5,560	.859

Assume a flow rate of 20 g/sec (one-half satellite) through the single-phase system. All of this flow returns through the two-phase system.

Flow Division:	Rate (g/sec)	Inventory (grams)
Inside Shell I:	9.23	683
Outside Shell II:	.89	120
Top / Bottom of Collar:	9.88	685

Flow Area Inside Shell I:

Bore Tube

Flat Sides:	2-7/16"
Radius:	1/2"
Enclosed Area:	2-7/16 x 2-7/16 - (1 - .785) =
	= 5.726 sq in. = 36.94 cm^2
ID Shell I:	3"
Enclosed Area:	7.065 sq in. = 45.57 cm^2
Triangle Top/Bottom:	3/16 x 6/16 = .070 sq in. = .45 cm^2
Cross Section of Flow Area Inside Shell I:	45.57 + .45 - 36.94 = 9.08 cm^2
Volume:	240 x 2.54 x 9.08 = 5,535 cc

$$d_h = \frac{4 \times 9.08}{(\pi \times 3 + 4 \times 1\text{-}7/16 + \pi)\ 2.54} = .780 \text{ cm}$$

$$= .0256 \text{ ft}$$

Flow Area Outside Collars:

Assume that the helium cryostat remains round and that the extra volume between D = 5.649" and D = 5.625" is evenly divided between top and bottom.

ID - Helium Cryostat:

$$A_1 = .785\ (5.649)^2 \times 6.45 = 161.57 \text{ cm}^2$$

OD - Collar:

$$A_2 = .785\ (5.625)^2 \times 6.45 = 160.20 \text{ cm}^2$$

ID - Collar:

$$A_3 = .785\ (5.5)^2 \times 6.45 = 153.16 \text{ cm}^2$$

Sides:

Enclosed angle is 41°.

Flow Area:

$$2 \times \frac{41}{360} \times (160.20 - 153.16) = 1.60 \text{ cm}^2$$

$$d_h = .125'' = .01 \text{ ft}$$

Volume = 160 x 240 x 2.54 = 975 cc

Top / Bottom:

Enclosed angle is 85°.

Flow Area:

$$2 \times \frac{85}{360} \times (161.57 - 153.16) + 2 \times 1/4\,\pi\,(5.625 - .125)^2 \times$$

$$\times \frac{44}{360} - 1 \times 2\text{-}9/16) \times 6.45 \approx 4.44 + 4.37 = 8.81 \text{ cm}^2$$

Extra from out of roundness-of-cryostat to be added. This is:

$$2 \times \frac{41}{360} \times (161.57 - 160.20) = .31 \text{ cm}^2$$

Total flow area outside collars is then: 9.12 cm^2

Volume per 20 ft dipole is: 5,560 cc

Hydraulic diameter of flow area of top and bottom:

$$d_h = \frac{4 \times \frac{9.12}{2}}{\pi\,(5.649 + 5.625) \times 2.54 \times \frac{85}{360}} = .859 \text{ cm}$$

$$= .028 \text{ ft}$$

5. Division of Single-Phase Flow between Flow Areas

Pressure drop is the same.

$$\frac{\Delta P}{L} = \frac{f_1 G_1^2}{193 \rho\, d_{h_1}} = \frac{f_2 G_2^2}{193 \rho\, d_{h_2}} = \frac{f_3 G_3^2}{193 \rho\, d_{h_3}}$$

$$f = \frac{.046}{Re^{.2}} = \frac{.046\, \mu^{.2}}{G^{.2}\, d_h^{\ .2}}$$

Substitute:

Then:

$$\frac{G_1^{\ 1.8}}{d_{h_1}^{\ 1.2}} = \frac{G_2^{\ 1.8}}{d_{h_2}^{\ 1.2}} = \frac{G_3^{\ 1.8}}{d_{h_3}^{\ 1.2}}$$

Total mass flow rate (lb/hr) M equals:

$$M = G_1 \frac{9.08}{930} + G_2 \times \frac{1.6}{930} + G_3 \times \frac{9.12}{930}$$

Where:

G_1 = mass flow rate inside coil I

G_2 = mass flow rate in narrow channels outside collar

G_3 = mass flow rate in top and bottom channels outside the collar

Where G_1, G_2, and G_3 in lb/hr ft^2

d_{h_1} = .0256 ft

d_{h_2} = .0104 ft

d_{h_3} = .0282 ft

Then:

$$81.3\ G_1^{\ 1.8} = 239.6\ G_2^{\ 1.8} = 72.4\ G_3^{\ 1.8}$$

$$G_1 = 1.822 \qquad G_2 = .938\ G_3$$

M - 20 g/sec = 158.59 lb/hr

$158.59 = .00976\ G_1 + .00172\ G_2 + .00981\ G_3 =$

$= (.00976 + .00094 + .01046)\ G_1$

$G_1 = 7{,}495$ lb/hr ft^2

$G_2 = 4{,}114$

$G_3 = 7{,}990$

$M_1 = 9.23$ g/sec

$M_2 = .89$ g/sec

$M_3 = 9.88$ g/sec

Total = 20.00 g/sec

6. HEAT TRANSFER COEFFICIENTS

a) Inside Shell I:

$G = 7{,}495$ lb/hr ft^2

$d_h = .0156$ ft

$\mu = .0077$

$Re = 24{,}918$

$j = .0030$

$C_p = 1.31$ Btu/lb °F

$Pr = .76 \qquad Pr^{2/3} = .904$

$$h = \frac{.0030 \times 7495 \times 1.31}{.904} = 33.0 \text{ Btu/hr ft}^2\ {}^\circ\text{F}$$

$$= 18.7 \times 10^{-3} \text{ W/cm}^2\ {}^\circ\text{K}$$

b) Outside Shell II:

$G = 4,114$

$d_h = .0104$

$\mu = .0077$

$Re = 5,570$

$j = .00409$

$$h = \frac{.00409 \times 4114 \times 1.31}{.904} = 24.4 \text{ Btu/hr ft}^2 \text{ °F}$$

$$= 13.9 \times 10^{-3} \text{ W/cm}^2 \text{ °K}$$

c) Top / Bottom Outside Collar:

$G = 7,990$

$d_h = .028$ ft

$Re = 29,054$

$j = .00295$

$$h = \frac{.00295 \times 7990 \times 1.31}{.904} = 34.1 \text{ Btu/hr ft}^2 \text{ °F}$$

$$= 19.2 \times 10^{-3} \text{ W/cm}^2 \text{ °K}$$

d) Two-Phase Channel:

Two-phase channel heat transfer coefficient from Cryogenics, Vol. 14, 1974: p. 375:

$$h = h_L \left[\frac{1}{X_{tt}^{.66}} + 1.5 \times 10^{-3} \times B_o^{.8} \right]$$

$$X_{tt} = \left(\frac{1 - X}{X}\right)^{.9} \left(\frac{\rho_V}{\rho_L}\right)^{1/2} \left(\frac{\mu_L}{\mu_V}\right)^{.1}$$

$$X = \text{ratio } \frac{\text{mass of vapor}}{\text{total mass}}$$

h_L = heat transfer coefficient for 100% liquid flow

ρ_V = 1.25 lb/cft

ρ_L = 7.8 lb/cft

μ_L = 30.6×10^{-6} g/cm sec

μ_V = 13.4×10^{-6} g/cm sec

B_o = $\frac{q}{rG}$

q = heat flux through wall to two-phase fluid in W/cm^2

r = latent heat of vaporization (19.14 J/gr)

G = 1.64 g/cm^2 sec (12,094 lb/hr ft^2)

Surface for heat transfer is 6,092 cm^2. Heat flux is 10 W.
q = $.27 \times 10^{-3}$ W/cm^2.

We find:

B_o = $.086 \times 10^{-4}$

X_{tt} = .434 for 50% vapor

h = h_L (1.73 + .01) = 1.74 h_L

Re = 28,716

j = .00295

h_L = 51 Btu/hr ft^2 °F

h_L = 30×10^{-3} W/cm^2 °K

h = 52×10^{-3} W/cm^2 °K

7. COOLING OF THE SHELLS OF FIRST MAGNET

a. Inner Shell:

Surface area for heat transfer is:

$$A = \pi \times 3 \times 2.54 \times \frac{304}{360} \times 20 \times 30.5 = 12{,}331 \text{ cm}^2$$

Heat flux from ramping is 500 joules in 60 sec or 8.33 W.

Then:

$$\Delta T_m = \frac{Q}{A}\,\frac{1}{h} = \frac{8.33}{12{,}331}\,\frac{10^3}{18.7} = .036°K$$

Temperature of wall to be determined from:

$$\Delta T_m = \frac{(T_{wall} - T_{in}) - (T_{wall} - T_{out})}{\ln \dfrac{T_{wall} - T_{in}}{T_{wall} - T_{out}}}$$

$$\ln K = \frac{T_{out} - T_{in}}{\Delta T_m}$$

Temperature rise ΔT_r of fluid flowing through the channel is:

$$Q = C_p\, \dot{M}\, \Delta T_r$$

$$C_p = 6.5 \text{ J/g °K at 1.8 atm and 4.6°K}$$

$$8.33 = 6.5 \times 9.23 \times \Delta T$$

$$\Delta T_r = .139°K$$

Then $\ln K = \frac{.139}{.036} = 3.857$

$$K = 47.31$$

$$T_{wall} - T_{in} = (47.31)(T_{wall} - T_{out})$$

$$T_{wall} = \frac{47.31\, T_{out} - T_{in}}{46.31} = \frac{\Delta T_r}{46.31} + T_{out} = .0030 + T_{out}$$

For the first magnet, $T_{in} = 4.5°K$

Then $T_{out} = 4.639°K$

$T_{wall} = 4.642°K$

For the second magnet, we need to determine the inlet temperature of the fluid from the mixture of three streams.

b) <u>Cooling of Outer Shell</u>:

With current, the shells are forced outward with great pressure against the collars. We will assume that the heat generated in the outer shell will be conducted through the stainless steel collar into the liquid flowing outside the collar. Heat is conducted through a layer of 5/8" of stainless steel.

Surface area for heat transfer is (Cu → St.Stl)

$$\frac{2 \times 76}{360} \times \pi \times 4.375 \times 240 \times 6.45 = 8{,}983 \text{ cm}^2$$

Surface area for heat transfer between st.stl. and he is:

$$2 \times \frac{41}{360} \times \pi \times 5.5 \times 240 \times 6.45 = 6{,}092 \text{ cm}^2$$

Calculate ΔT in st.stl. collar by assuming an average area of 7,500 cm^2.

Then, for steady state:

$$\frac{Q}{A} = \frac{100}{60 \times 7500} = .222 \times 10^{-3} \text{ W/cm}^2$$

$$\Delta T = \frac{Q}{A}\frac{d}{k} = \frac{.222 \times 10^{-3} \times .625 \times 2.54}{.0027} = .13°K$$

Temperature difference between he and collar is:

$$\Delta T_m = \frac{Q}{A}\frac{1}{h} = \frac{100}{60 \times 6092}\,\frac{1}{13.9 \times 10^{-3}}$$

$$\Delta T_m = .0197\ °K$$

The liquid helium is cooled by two-phase fluid. The temperature difference betwen helium and helium cryostat wall is also .0197°K. Temperature difference between inner and outer wall of the cryostat is: (wall = .049")

$$Q = \frac{K\,A\Delta T}{d}$$

$$dT = \frac{100}{60}\ \frac{1}{6092}\ \frac{.049 \times 2.54}{.0027} = .013°K$$

Finally, there is a temperature difference between two-phase fluid and wall. The heat transfer coefficient is 52×10^{-3} W/cm^2 °K (p.10).

$$\Delta T = \frac{100}{60}\ \frac{1}{6092}\ \frac{1}{52 \times 10^{-3}} = .0053°K$$

With all increments added, total temperature difference between outer shell and two-phase fluid temperature (T_o) is then:

$(T_o + .1877)$ °K

If two-phase fluid is at 4.5°K, then T_{coil} = 4.6877°K.

In summary, we find for the first magnet in the string:

$T_{inner\ shell}$	=	4.642°K
$T_{outer\ shell}$	=	4.688°K
$T_{two\text{-}phase}$	=	4.50°K
$T_{single\text{-}phase\ in}$	=	4.50°K
Heat dissipated	=	600 joules/cycle
Cycle frequence	=	1 per min

8. COOLING OF THE SHELLS OF SECOND MAGNET

Fluid entering magnet #2 has a different temperature, as follows:

1) 9.88 gr/sec at 4.50°K
2) .89 gr/sec at 4.520°K
3) 9.23 gr/sec at 4.639°K

We find H_1 = 11.20 J/gr

H_2 = 11.31 J/gr

H_3 = 12.10 J/gr

We find H_{mixt} = 11.62 J/gr

Temperature of fluid entering magnet #2 is 4.576°K.

The inner shell of magnet #2 now will be somewhat warmer. We will add the increment of .076°K to all numbers and find $T_{inner\ shell}$ = 4.718°K for the second magnet. The temperature of the fluid leaving the inner channel from magnet #2 is then 4.718 - .003 = 4.715°K. Enthalpy of this fluid is 11.62 + .90 = 12.52 J/gr.

The fluid flowing through top and bottom channel outside the collar will enter at T = 4.576°K and will be cooled by the two-phase fluid. The heat transfer coefficient in this channel is .0192 W/cm^2 °K.

Surface area of the wall separting single and two-phase fluid is 12,380 cm^2. To cool the fluid from 4.576 to 4.5°K requires removal of 9.88 x .42 = 4.15 W.

The overall heat transfer coefficient may be determined from:

$$\frac{1}{U} = \frac{1}{h_1} + \frac{d}{k} + \frac{1}{h_2} = \frac{1}{.0192} + \frac{.0914}{.0027} + \frac{1}{.053} = 98.79$$

$$U = .0095 \text{ W/cm}^2 \text{ °K}$$

We find:

$$\triangle T_m = \frac{4.15}{12380} \quad \frac{1}{.0095} = .035\text{°K}$$

Heat transferred is:

$$Q = UA \triangle T_m$$

Also: $\triangle T_m = \frac{T_{in} - T_{out}}{\ln K}$

Also: $Q = C_p M (T_{in} - T_{out})$

We find: $UA = \frac{Q}{\triangle T_m} = C_p M \ln K$

or $\ln K = \frac{U A}{C_p M}$

We find: $\ln K = \frac{.0095 \times 12380}{5.5 \times 9.88} = 2.16$

$$K = 8.71 = \frac{T_{in} - T_w}{T_{out} - T_w}$$

$$T_{out} - T_w = \frac{T_{in} - T_w}{8.71} = \frac{4.576 - 4.50}{8.71} = .0087$$

$$T_{out} = 4.509\text{°K}$$

Heat transferred is then: 9.88 x .067 x 5.5 = 3.64 W

The fluid flowing through the side channels between collar and he cryostat enters with T = 4.576°K. The overall coefficient for heat transfer is determined from:

$$\frac{1}{U} = \frac{1}{.0139} + \frac{.0914}{.0027} + \frac{1}{.053} = 124.66$$

$$U = .0080 \text{ W/cm}^2 \text{ °K}$$

We find: $\ln K = \frac{U\,A}{C_p\,M} = \frac{.0080 \times 6092}{5.50 \times .89} = 9.98$

$K = 21{,}664$

We found in magnet #1 that the temperature difference for removal of 1.67 W is of the order of .018°K. This increment is added to the one required to reduce the initial difference from .076 to zero. It appears that the fluid flowing through the side channels will be cooled to a temperature of 4.518°K and essentially will stay at this temperature level at the exit of each magnet.

Fluid entering magnet #3 temperature and enthalpy will be determined from:

1) 9.88 g/sec at H = 11.25 (T = 4.509)

2) .89 g/sec at H = 11.30 (T = 4.520)

3) 9.23 g/sec at H = 12.52 (T = 4.715)

$H_{mixt} = 11.84$ J/gr

$T_{mixt} = 4.615$°K

Temperature of shells of magnet #2 are then:

$T_{inner\ shell} = 4.718$°K

$T_{outer\ shell} = 4.688$°K

Temperature of liquid entering third magnet is T = 4.615°K

9. COOLING OF THE SHELL OF THIRD MAGNET

Inner Shell:

Temperature rise of liquid is .139°K.

Temperature difference between wall and fluid: ΔT_m = .036°K

Wall temperature is then: .003°K + $T_{fluid\ out}$ =

= 4.615 + .139 + .003 = 4.757°K.

Outer Shell:

The single-phase liquid enters the cooling passage at T = 4.615°K. This liquid quickly reaches the equilibrium temperature of 4.538°K, as in the first magnet, because:

$$\ln K = \frac{.0139 \times 6092}{5.5 \times .89} = 17.3$$ for the full length of cooling channel.

At 1 ft from the entrance:

$$\ln K = .865$$

$$K = 2.38$$

The original temperature difference of .115°K has been reduced to .048°K. It appears that the first one to two ft of the outer shell are somewhat warmer.

Summary of Shell Temperatures of Magnets:

Magnet:	1	2	3	4	5
Temp. (°K) of:	4.500	4.576	4.615	4.635	4.645
Single-Phase In:					
Inner Shell:	4.642	4.718	4.757	4.777	4.787
Outer Shell:	4.688	4.688	4.688	4.688	4.688

10. PRESSURE DROP - SINGLE-PHASE FLUID

Calculate the pressure drop of the single-phase flow between inner shell and bore tube.

a) $G = 7{,}495 \text{ lb/hr ft}^2$

$Re = 24{,}918$

$d_h = .0256 \text{ ft}$

$f = 2\,j = .0060$

$\rho = 7.6 \text{ lb/cft}$

$$\frac{\Delta P}{L} = \frac{f\,(G^1)^2}{193\,\rho \times d_h}$$

$$G^1 = \frac{G}{3600} = 2.082 \text{ lb/sec ft}^2$$

$d_h = .0256 \times 12 = .307 \text{ in.}$

$$\Delta P = \frac{.006 \times (2.082)^2}{193 \times 7.6 \times .307} \times 20 = .00115 \text{ psig}$$

b) At exit from the magnet, the liquid enters the connecting tube between magnets. This tube carries the conductors. Pressure drop is difficult to calculate. We will assume that the bellows of the tube is lined on the inside with a straight tube. The diameter of this tube is assumed to be 1-1/8" (ID). Cross section of a pair of conductors with supports is assumed to be .5 sq in. Flow area is then .49 sq in. Inlet and discharge will have some contraction for the stream. Velocity at inlet and discharge will be based on a flow area of .3 sq in. Pressure drop for the connecting tube is then:

$$\Delta P = \rho v^2$$

$$\rho = .122 \text{ g/cc}$$

$$V = 85 \text{ cm/sec}$$

$$\Delta P = .876 \text{ cm } H_2O = .013 \text{ psig}$$

11. PRESSURE DROP - TWO-PHASE FLUID

Area for flow is 12.22 cm^2.

Flow rate is 20 g/sec.

Then: $G = 1.64 \text{ g/cm}^2 \text{ sec} = 12{,}067 \text{ lb/hr ft}^2$

$$d_h = 2 \times .104 = .208''$$

For two-phase flow, properties of fluid are as follows:

1.2 atm $\rho_g = 1.25$ lb/cft

$\rho_L = 7.55$ lb/cft

$\mu_L = 30.6 \times 10^{-4}$ centipoise

$j = .07$ dynes/cm

$$\lambda = \left[\frac{\rho_g}{.075} \quad \frac{\rho_L}{62.3} \right]^{1/2} = 1.42$$

$$\psi = \frac{73}{j} \left[\mu_L \frac{(6.23 \times 10)^2}{\rho_L} \right]^{1/3} = 1{,}213$$

For 50-50 mixture of liquid and gas:

$$\frac{L}{G} \lambda \psi = .5 \times 1.42 \times 1213 = 861$$

$$\frac{G}{\lambda} = \frac{6035}{1.42} = 4{,}250$$

Flow Regime Maps

The boundaries between flow regimes are not sharp and the pictures used to describe them represent idealized descriptions of a very complex distribution of phases.

Figure 9 shows the flow regimes which have been identified by Baker [6] in a horizontal pipe, and Fig. 10 is the flow regime map. The slug and plug regimes are

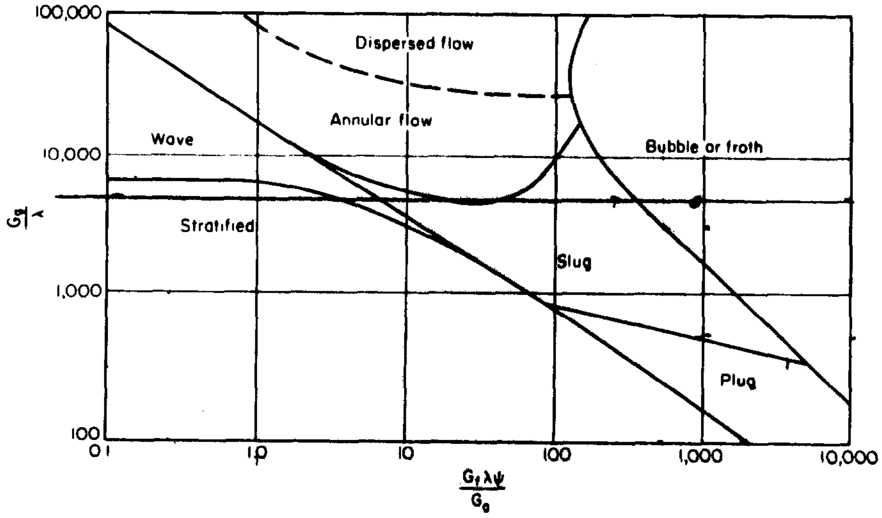


FIGURE 2

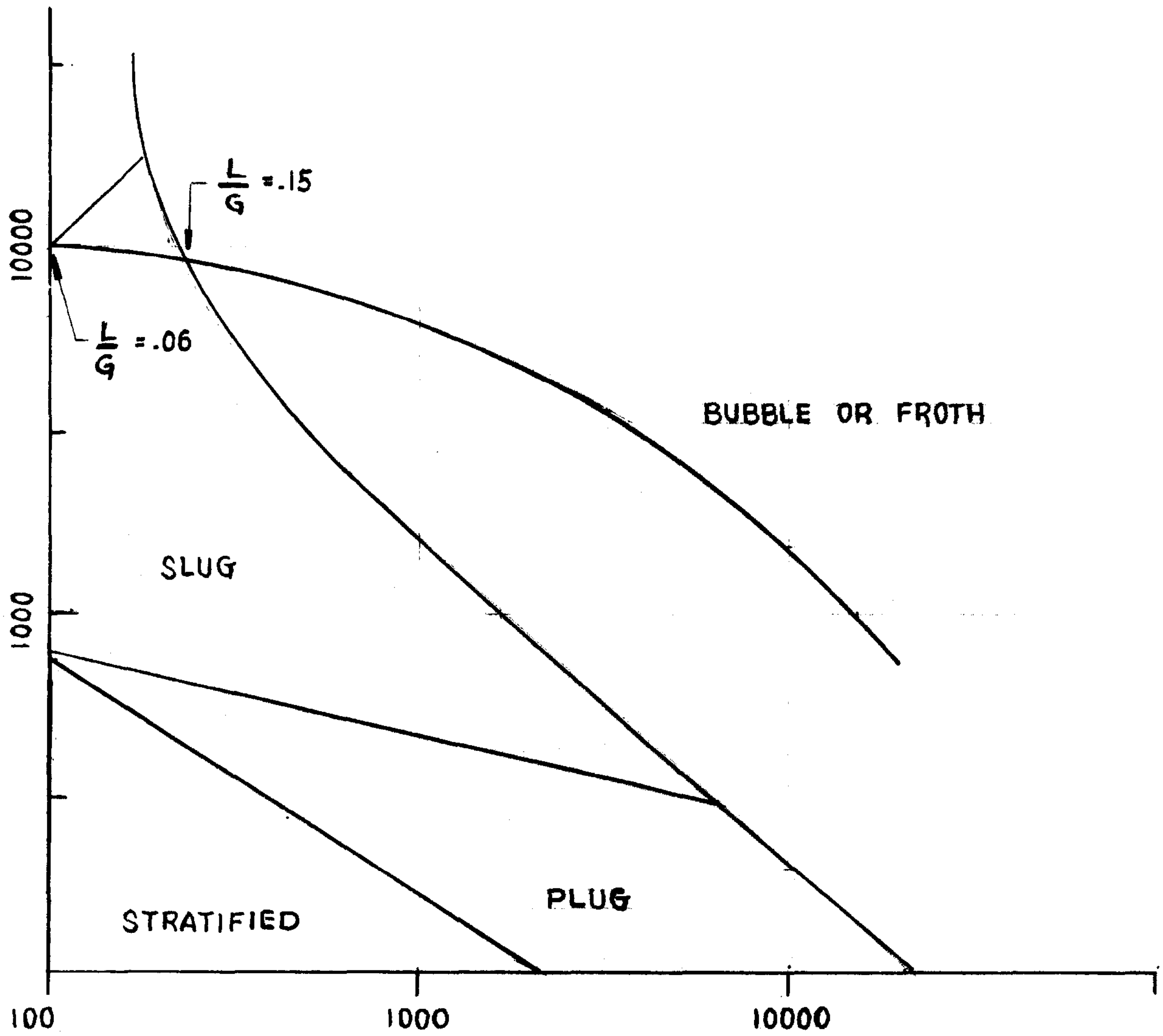

L/G = .15
L/G = .06
10000
1000
BUBBLE OR FROTH
SLUG
PLUG
STRATIFIED
100
1000
10000

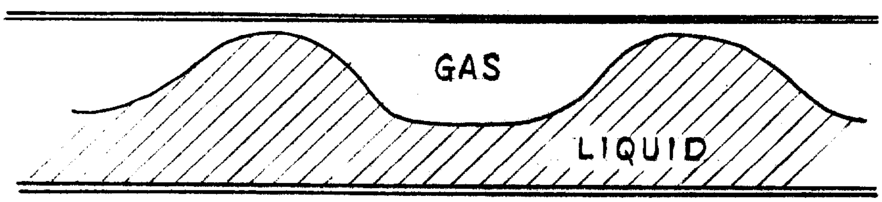
FIGURE 3

<u>Flow Regime as a Function of Vapor Fraction</u>:

Total mass flow rate - 24.6 g/sec (195 lb/hr)

L/G	10	5	1	.5	.25	.1
L	10,699	10,055	6,034	4,022	3,017	1,097
G	1,097	2,011	6,034	8,044	9,049	10,699
L/G $\lambda \psi$	17,225	8,612	1,723	861	431	172
G/λ	772	1,416	4,249	5,665	6,373	7,725
log L/G ψ/λ	4.236	3.935	3.236	2.935	2.634	2.336
log G/λ	2.888	3.151	3.628	3.753	3.804	3.888

The curve shows that somewhere in the quality range of L/G = .13, the flow regime changes from froth to slug. If this occurs in a round pipe, the flow looks like this:



<u>FIGURE 4</u>

It is conceivable that in the configuration of the two-phase channel, stratification occurs. Also, at lower ratios of L/G $\lesssim$.06 the flow regime enters the annular flow regime. In this area the liquid flows along the wall with the gas in the center. Again, the configuration of the two-phase channel may lead to separation of liquid and gas.

Vapor velocity at L/G = .1

Volume flow rate of gas is: 1,089 cc/sec

Volume flow rate of liquid is: 18.5 cc/sec

Velocity of vapor is: 89.2 cm/sec

We will calculate the pressure drop of the two-phase flow as a function of fraction of vapor. At the J-T valve, the single-phase fluid will be assumed to be 4.64°K in temperature with an enthalpy of 12.0 Joules/gr. When this fluid expands to 1.2 atm, fraction of vapor will be X:

$$(1)\ (12.0) = (X)\ (29.94) + (1 - X)\ (10.80)$$

$$X = \frac{1.20}{19.94} = .060$$

This will be the initial fraction when the two-phase fluid enters the first magnet.

Pressure drop is calculated from:

a) Calculate liquid only flowing through tube.

b) Determine parameter X from:

$$X^2 = \frac{Re_g{}^m}{Re_L{}^n} \quad \frac{C_L}{C_g} \left(\frac{W_L}{W_g}\right)^2 \quad \frac{\rho_g}{\rho_L}$$

Where m = n = .2 for turbulent flow of gas and liquid (Re_g and $Re_L > 2300$)

$C_L = C_g$

W_L = flow rate of liquid in lb/hr

W_g = flow rate of gas in lb/hr

ρ_g = density of gas in lb/cft

ρ_L = density of liquid in lb/cft

c) Determine value of $\emptyset_{tt}$ from Figure 5.

d) $\triangle P \text{ mixture} = \emptyset_{tt}^2 \triangle P_L$

Tables 1 and 2 show the calculated values of the pressure drop as a function of vapor fraction.

Determine the Reynolds numbers for the liquid flowing alone and for the gas flowing alone in the pipe, each at its own mass velocity. These Reynolds numbers are defined as

$$N_{\mathrm{Ref}} = \frac{G_f D}{\mu_f} \tag{16}$$

$$N_{\mathrm{Reg}} = \frac{G_g D}{\mu_g} \tag{17}$$

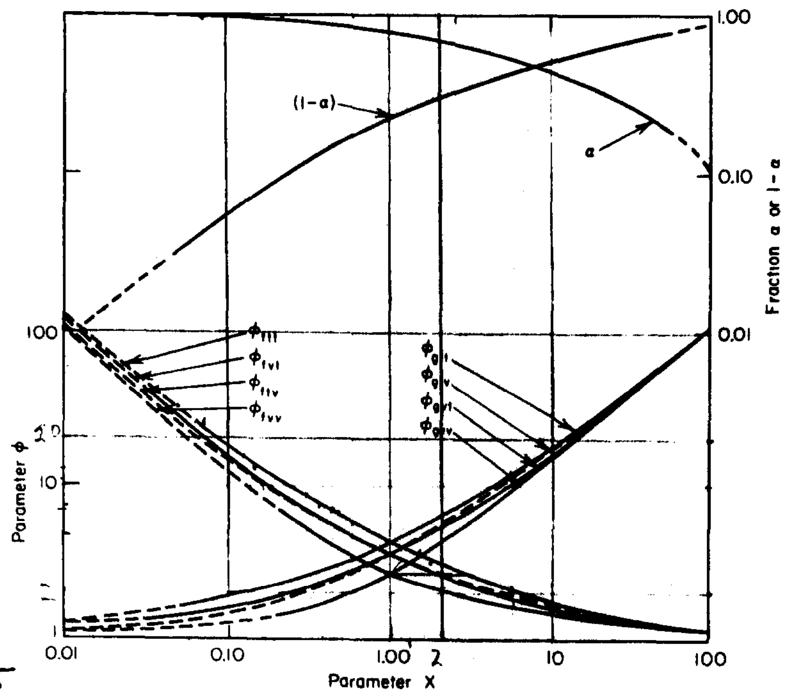


Fig. ~~6~~ 5 Void fraction and adiabatic friction multipliers for all fluids at about one atmosphere pressure [2].

FIGURE 5

TABLE I

L/G	15.67	10	5	1	.5	.25
L	11,343	10,970	10,055	6,033.5	4,022	2,413
G	724	1,087	2,011	6,033.5	8,045	9,654
Re_L	26,569	25,695	23,552	14,132	9,421	5,652
Re_g	3,922	5,942	10,893	32,681	43,577	52,292
W_L (lb/hr)	149.08	144.17	132.16	79.29	52.86	31.72
W_g	9.51	14.42	26.43	79.29	105.73	126.87
ρ_g/ρ_L	.170	.170	.170	.170	.170	.170
X^2	28.49	12.68	3.64	.201	.054	.0166
ϕ_{tt}	2.2	2.6	3.2	6.4	9.15	14.8
f_L	.0060	.0060	.0061	.0068	.0074	.0082
$\left(\frac{\Delta P}{L}\right)_L$	1.97×10^{-4}	1.84×10^{-4}	1.57×10^{-4}	$.63\times10^{-4}$	$.30\times10^{-4}$	$.12\times10^{-4}$
$\left(\frac{\Delta P}{L}\right)_{mixt}$	9.5×10^{-4}	1.24×10^{-3}	1.57×10^{-3}	2.6×10^{-3}	2.51×10^{-3}	2.6×10^{-3}

For all gas: $\frac{\Delta P}{L} = 1.09 \times 10^{-3}$ psig/ft

For all liquid: $\frac{\Delta P}{L} = 2.19 \times 10^{-4}$ psig/ft

TABLE II

L/G	4	3	2	.75	.375	.2
L	9,654	9,050	8,044	5,172	3,291	2,011
G	2,413	3,017	4,022	6,895	8,776	10,056
Re_L	22,597	21,183	18,829	12,106	7,703	4,707
Re_g	12,906	16,137	21,513	36,880	46,941	53,787
W_L	126.87	118.94	105.73	67.97	43.25	26.43
W_g	31.72	39.65	52.86	90.62	115.34	132.16
ρ_g/ρ_L	.170	.170	.170	.170	.170	.170
X^2	2.06	-	.69836	.1195	.02909	.01106
ϕ_{tt}	3.78	-	4.5	7.53	10.92	17.0
f_L	.00619	-	.00642	-	.00768	.00848
$\left(\frac{\Delta P}{L}\right)_L$	1.47×10^{-4}	-	1.058×10^{-4}	$.478\times10^{-4}$	$.211\times10^{-4}$	$.087\times10^{-4}$
$\left(\frac{\Delta P}{L}\right)_{mixt.}$	2.10×10^{-3}	-	2.15×10^{-3}	2.70×10^{-3}	2.53×10^{-3}	2.52×10^{-3}

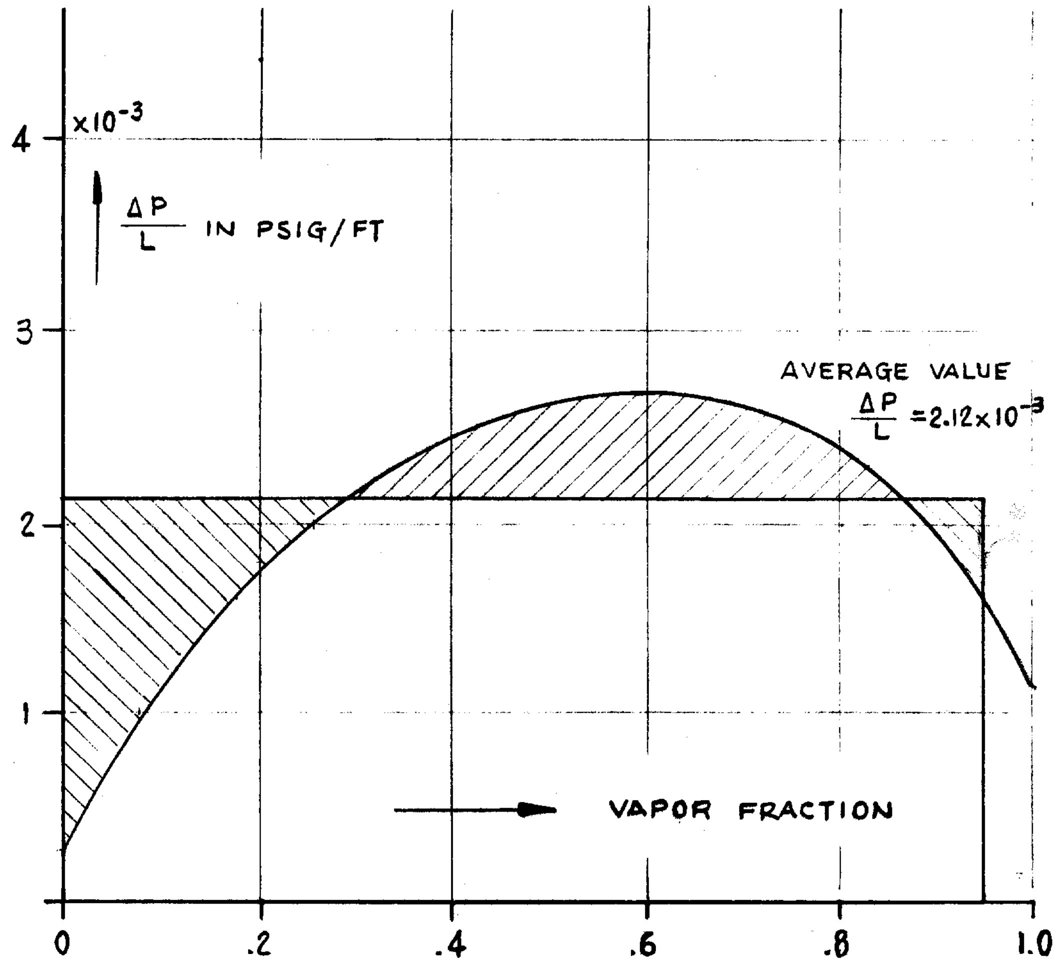
The results of the calculations are shown in Figure 6.

If we assume that all magnets have the same heat load and that 20 magnets are arranged in series, then total pressure drop is (400 ft):

$$\Delta P = 400 \times 2.12 \times 10^{-3} = .85 \text{ psig}$$

This also assumes that the fluid leaving the last magnet still contains approximately 5% liquid.

In addition to the pressure drop in the two-phase channels, there is pressure drop in the crossovers between channels. This pressure drop consists of velocity heads primarily.



<u>Pressure Drop as a Function of Fluid Quality</u>

FIGURE 6

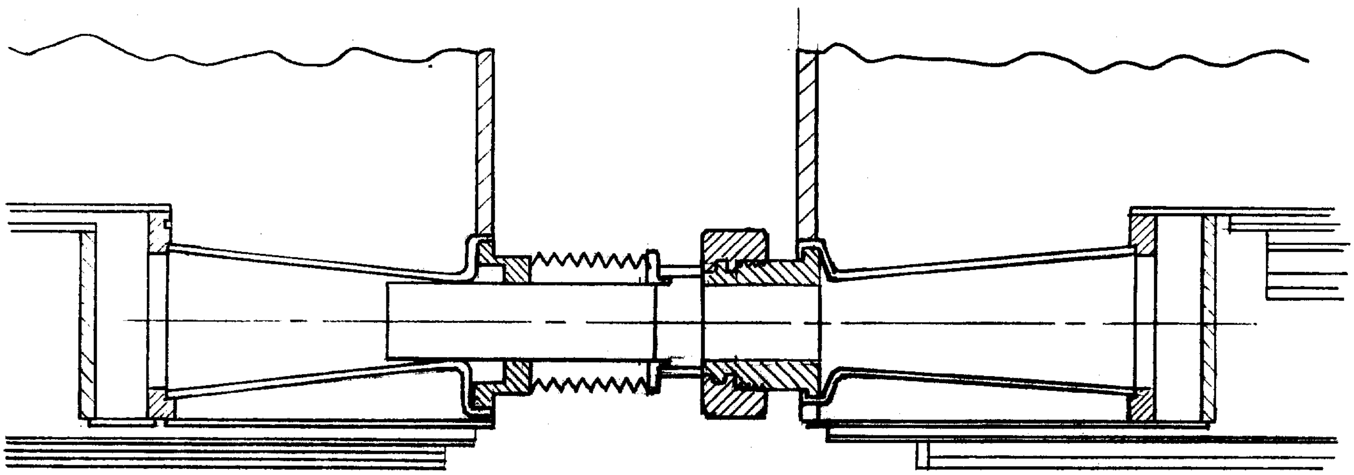


FIGURE 7

Figure 7 shows the arrangement of the crossover. The long cone on either side of the connection acts as a venturi with recovery of velocity head based on area of flow of the narrow section. Pressure drop of the crossover will, therefore, be calculated as follows:

$$\Delta P = 2 \times 1/2 \rho v_1^2 + 2 \times 1/2 \rho v_2^2$$

Where:

V_1 = velocity in annulus around the single-phase tube

V_2 = velocity at entrance of cone

ρ = density of the mixture

The equation can be rewritten as follows:

$$\Delta P = \frac{1}{\rho} (\rho^2 V_1^2 + \rho V_2^2) = \frac{1}{\rho} (G_1^2 + G_2^2)$$

Where:

G_1 and G_2 are mass velocities in annulus and entrance to crossover tube, respectively.

Figure 8 shows the change of the value of $\frac{1}{\rho}$ as a function of vapor fraction in the fluid. At entrance to first magnet after J-T valve with a vapor fraction of .06, we find:

$$\Delta P = 1.25 \times 10^{-4} \text{ psig}$$

At exit of last magnet with a vapor fraction of 0.95, we find:

$$\Delta P = 5.56 \times 10^{-4} \text{ psig.}$$

It appears that the losses in the crossovers are an order of magnitude smaller than those in the annuli.

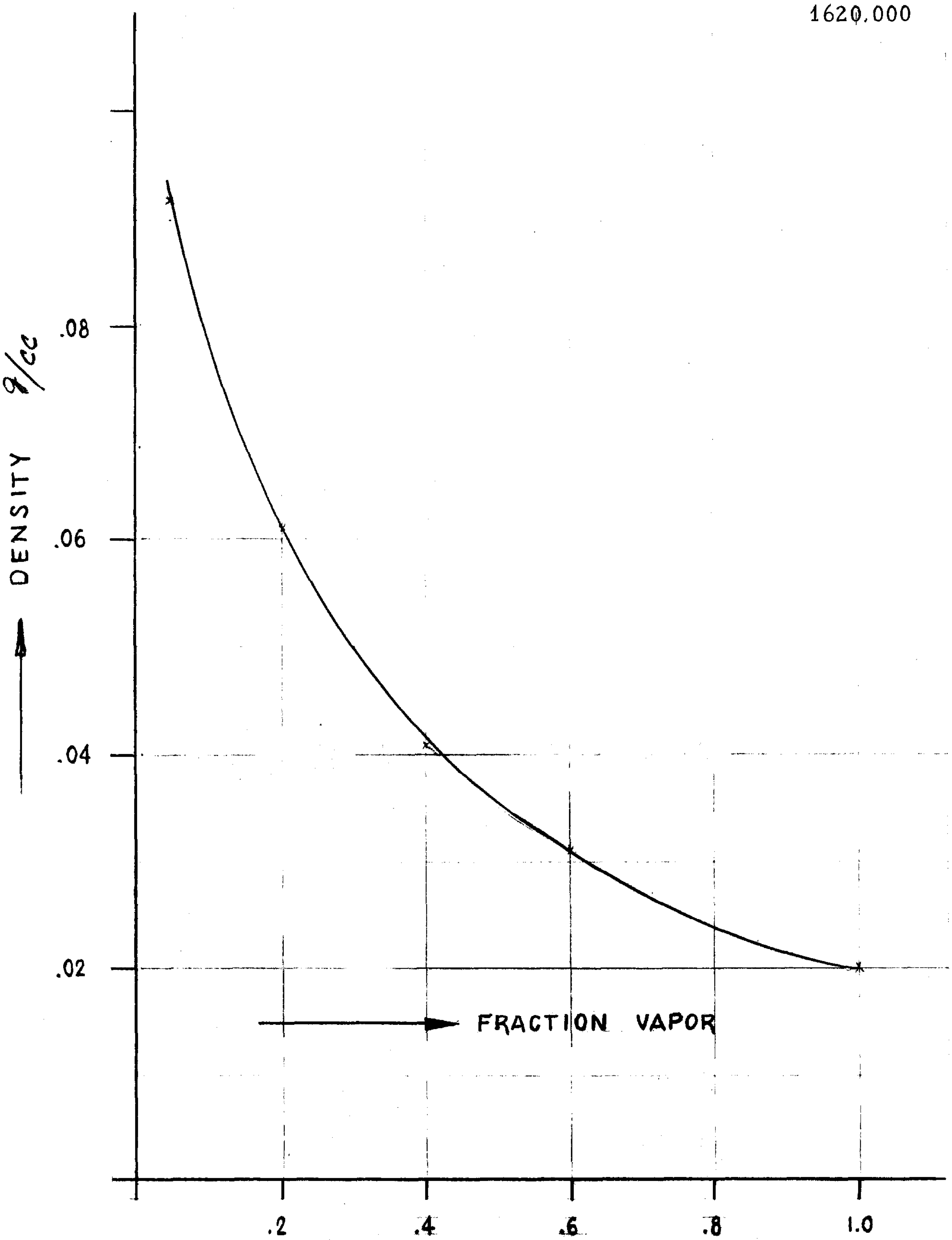


FIGURE 8