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Fermilab Booster beam emittances from quadrupole modes measured by BPMs*

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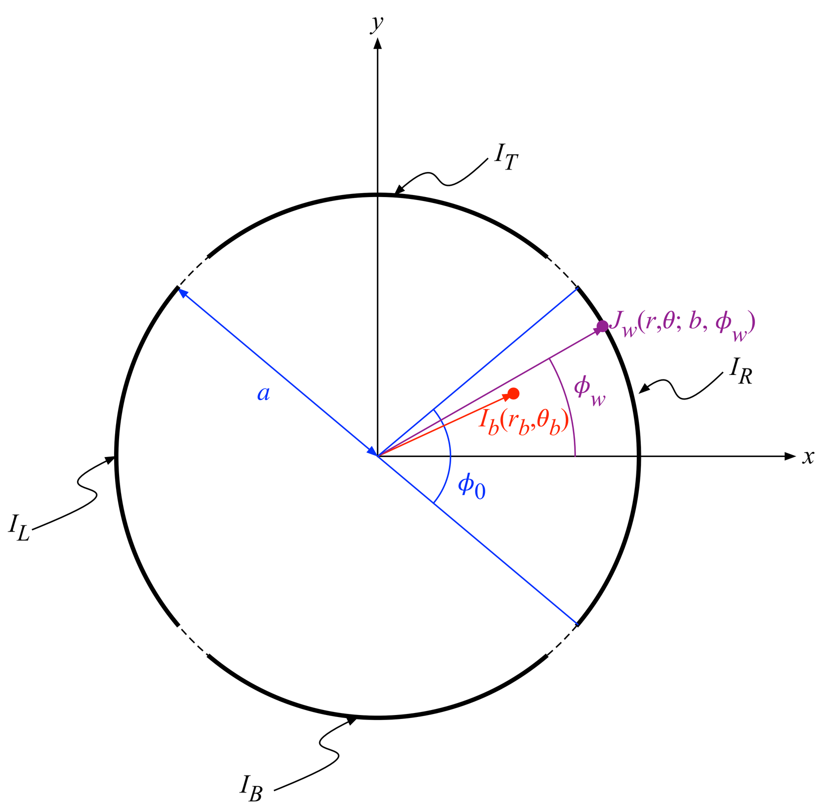
Abstract

The measurement of beam emittances by extracting the quadrupole mode signal from a 4 plate beam position monitor (BPM) was published at least 40 years ago. Unfortunately, in practice, this method suffers from poor signal to noise ratio and requires a lot of tuning to extract out the emittances. In this paper, an improved method where multiple BPMs are used together with better mathematical analysis is described. The BPM derived emittances are then compared with those measured by the Ion Profile Monitor (IPM). Surprisingly, the BPM measured emittances behave very well and are more realistic than those measured by the IPM.



The cross section of the 4 plate BPM is shown in Fig. 1. The image current density, J_w, that is induced by a pencil current, I_b, at (r_b, θ_b) is given by

$$J_w(r_b,\theta_b;b,\phi_w) = -\frac{I_b(r_b,\theta_b)}{2\pi a}\left[1+2\sum_{n=1}^{\infty}\left(\frac{r_b}{a}\right)^n \cos n\,(\phi_w-\theta_b)\right]. \quad (1)$$

We can integrate the above to obtain the current on each plate R, L, T and B. For example the current, I_R, on the R plate is simply $I_R = \int_{-\phi_0/2}^{\phi_0/2} d\phi\; aJ_w$ to give

$$I_R = -\frac{I_b(x_b,y_b)}{2\pi}\left\{\phi_0 + 2\left[2\left(\frac{x_b}{a}\right)\sin\frac{\phi_0}{2} + \left(\frac{x_b^2-y_b^2}{a^2}\right)\sin\phi_0\right]\right\}. \quad (2)$$

where we have only kept terms lower than $(r_b/a)^3$. We can obtain similar equations for I_L, I_T and I_B.

And if the transverse distribution of the beam is a bi-gaussian distribution centred at $(\bar{x},\bar{y})$ with standard deviations σ_x and σ_y in the x and y directions respectively, then the normalized gaussian distribution function is

$$\rho(x,y) = \frac{1}{2\pi\sigma_x\sigma_y}\exp\left[-\frac{(x-\bar{x})^2}{2\sigma_x^2}\right]\exp\left[-\frac{(y-\bar{y})^2}{2\sigma_y^2}\right]. \quad (3)$$

With the above density function, the new current distribution on the R plate is

$$R = \int_{-\infty}^{\infty} dx\, dy\, \rho I_R, = -\frac{I_b}{2\pi}\left\{\phi_0 + 2\left[2\left(\frac{\bar{x}}{a}\right)\sin\frac{\phi_0}{2} + \left(\frac{\sigma_x^2-\sigma_y^2}{a^2}+\frac{\bar{x}^2-\bar{y}^2}{a^2}\right)\sin\phi_0\right]\right\}. \quad (4)$$

The remaining plates will also have a similar current distributions L, T, B.

Finally, we can take the appropriate sum and difference combinations of R, L, T and B to create the dipole modes, d_x and d_y and quadrupole mode q

$$\left.\begin{aligned} d_x &= \frac{R-L}{R+L+T+B} = \frac{\sin\frac{\phi_0}{2}}{\frac{\phi_0}{2}}\left(\frac{\bar{x}}{a}\right) \\ d_y &= \frac{T-B}{R+L+T+B} = \frac{\sin\frac{\phi_0}{2}}{\frac{\phi_0}{2}}\left(\frac{\bar{y}}{a}\right) \\ q &= \frac{2\sin\phi_0}{\phi_0}\left(\frac{\sigma_x^2-\sigma_y^2}{a^2}+\frac{\bar{x}^2-\bar{y}^2}{a^2}\right)\end{aligned}\right\}. \quad (5)$$

q can be written in terms of d_x and d_y to give

$$q - (d_x^2 - d_y^2)\phi_0\cot\frac{\phi_0}{2} = \frac{2\sin\phi_0}{\phi_0}\left(\frac{\sigma_x^2-\sigma_y^2}{a^2}\right). \quad (6)$$

The above can be written In terms of lattice functions by using the following relationship

$$\sigma_{x,y}^2 = \frac{\beta_{x,y}\epsilon_{x,y}}{\pi} + (D_{x,y}\sigma_p)^2 \quad (7)$$

where $\epsilon_{x,y}$ are the emittances, $\beta_{x,y}$ are the beta functions, $D_{x,y}$ are the dispersions of the beam in the x and y directions respectively; and $\sigma_p^2 = \langle (dp/p)^2\rangle$ is the standard deviation of the relative momentum spread of the beam.

Thus, Eq. 6 becomes

$$\Delta_q \equiv \frac{2}{a^2}\frac{\sin\phi_0}{\phi_0}\left[\frac{\beta_x\epsilon_x}{\pi} - \frac{\beta_y\epsilon_y}{\pi} + (D_x\sigma_p)^2\right] \quad (8)$$

where Δ_q is as defined above and we have set $D_y = 0$ because the vertical dispersion is small in Booster.

We can write down a matrix equation using the above from the measurements of 1 to n BPMs or 1 to n samples from m BPMs

$$\begin{pmatrix} \beta_x(1) & -\beta_y(1) & D_x^2(1) \\ \beta_x(2) & -\beta_y(2) & D_x^2(2) \\ \vdots & \vdots & \vdots \\ \beta_x(j) & -\beta_y(j) & D_x^2(j) \\ \vdots & \vdots & \vdots \\ \beta_x(n) & -\beta_y(n) & D_x^2(n)\end{pmatrix}\begin{pmatrix}\epsilon_x/\pi \\ \epsilon_y/\pi \\ \sigma_p^2\end{pmatrix} = \frac{a^2\phi_0}{2\sin\phi_0}\begin{pmatrix}\Delta_q(1) \\ \Delta_q(2) \\ \vdots \\ \Delta_q(j) \\ \vdots \\ \Delta_q(n)\end{pmatrix} \quad (9)$$

where the jth row comes from the jth pickup or the jth sample. If $n > 3$ then we have a non-square matrix on the lhs, which means that we have an over-determined set of equations. This non-square matrix is easily inverted using SVD methods. See for example, *Mathematica*'s `PseudoInverse[]` function.

Simply inverting the above with the raw BPM data will give garbage emittances.

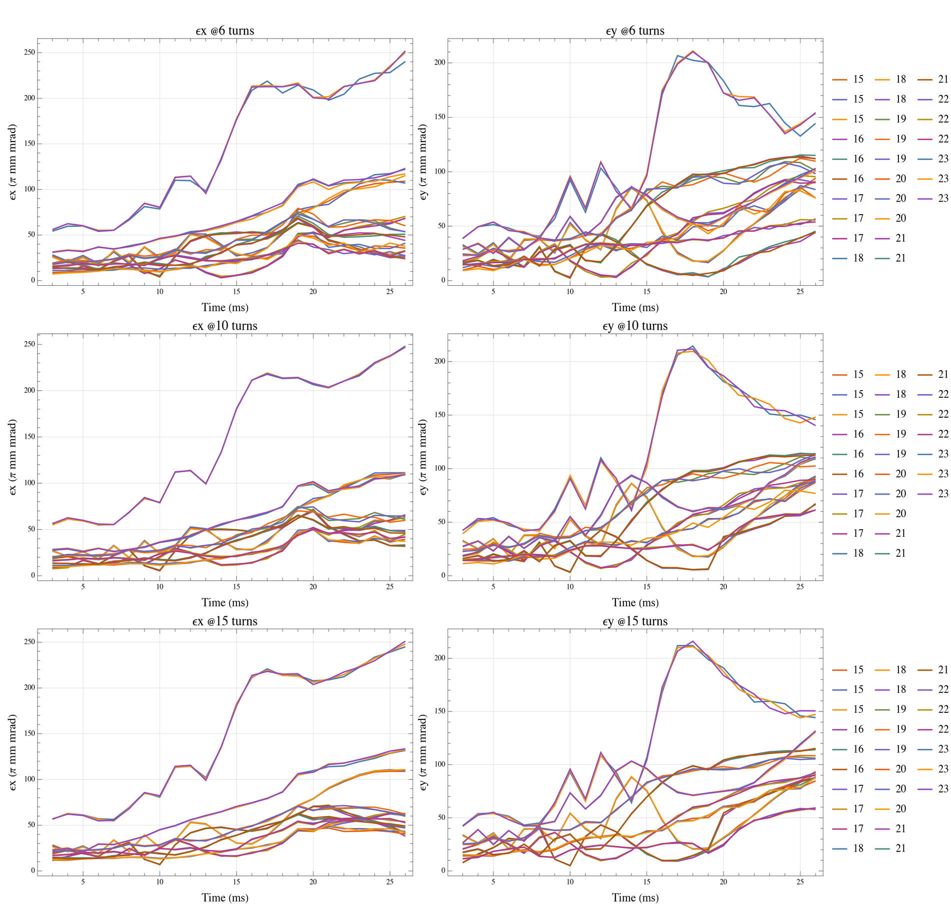
BPM data will have to be manipulated:

1. Theoretically, the gain on each BPM plate is required to be the same for Eq. 9 to work. Therefore, we have to correct the signal from the BPM plates to ensure that this condition is satisfied. Note: The choice of gain is critical for ensuring the emittance solutions are real and not complex. Details about our method is discussed in Ref. [6].
2. Any DC offsets from each BPM plate when there is no beam has to be removed.
3. The most important observation of Eq. 9 is that on the rhs, there is the difference $\beta_x\epsilon_x/\pi - \beta_y\epsilon_y/\pi$. This means that if we had a round beam and $\beta_x \approx \beta_y$ then this difference is close to zero. So, in this configuration, it may not be possible to actually extract out the emittances. The ideal situation for extracting out the emittances would be to have one of the β's a lot larger than the other, i.e. either $\beta_x \gg \beta_y$ or $\beta_x \ll \beta_y$.

For Booster, the BPMs in the S locations are good for extracting ϵ_x because $\beta_x(30\,\text{m}) \gg \beta_y(5\,\text{m})$ while the BPMs at the L locations are good for extracting ϵ_y because $\beta_x(7\,\text{m}) \ll \beta_y(20\,\text{m})$.[1] Therefore, Eq. 9 must contain data from both L and S BPMs to get good $\epsilon_{x,y}$ solutions. In this paper, we have paired the data from S01 and L01, S02 and L02, etc. for solving Eq. 9.

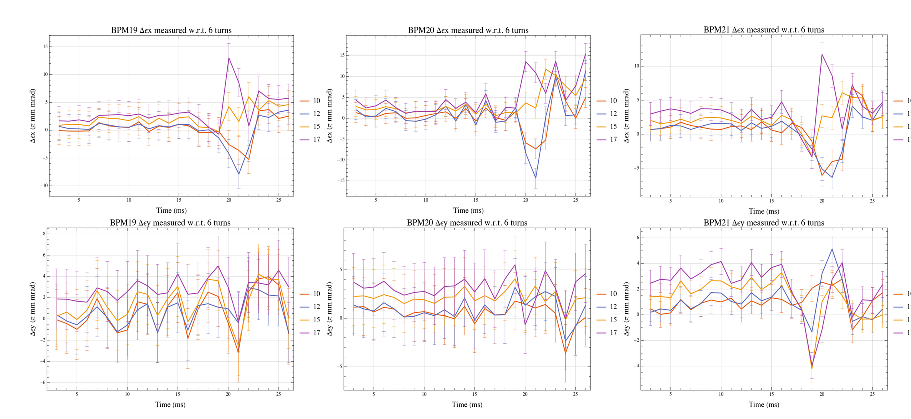
Use BPM pairs (L01, S01), (L02, S02) …, (L24, S24) to calculate emittances.

Systematic error removal



Systematic error of emittances calculated from each BPM pair are reproducible. Therefore, can remove by taking difference w.r.t. reference emittance at 6 turns, for example.

Difference after taking 6 turns reference

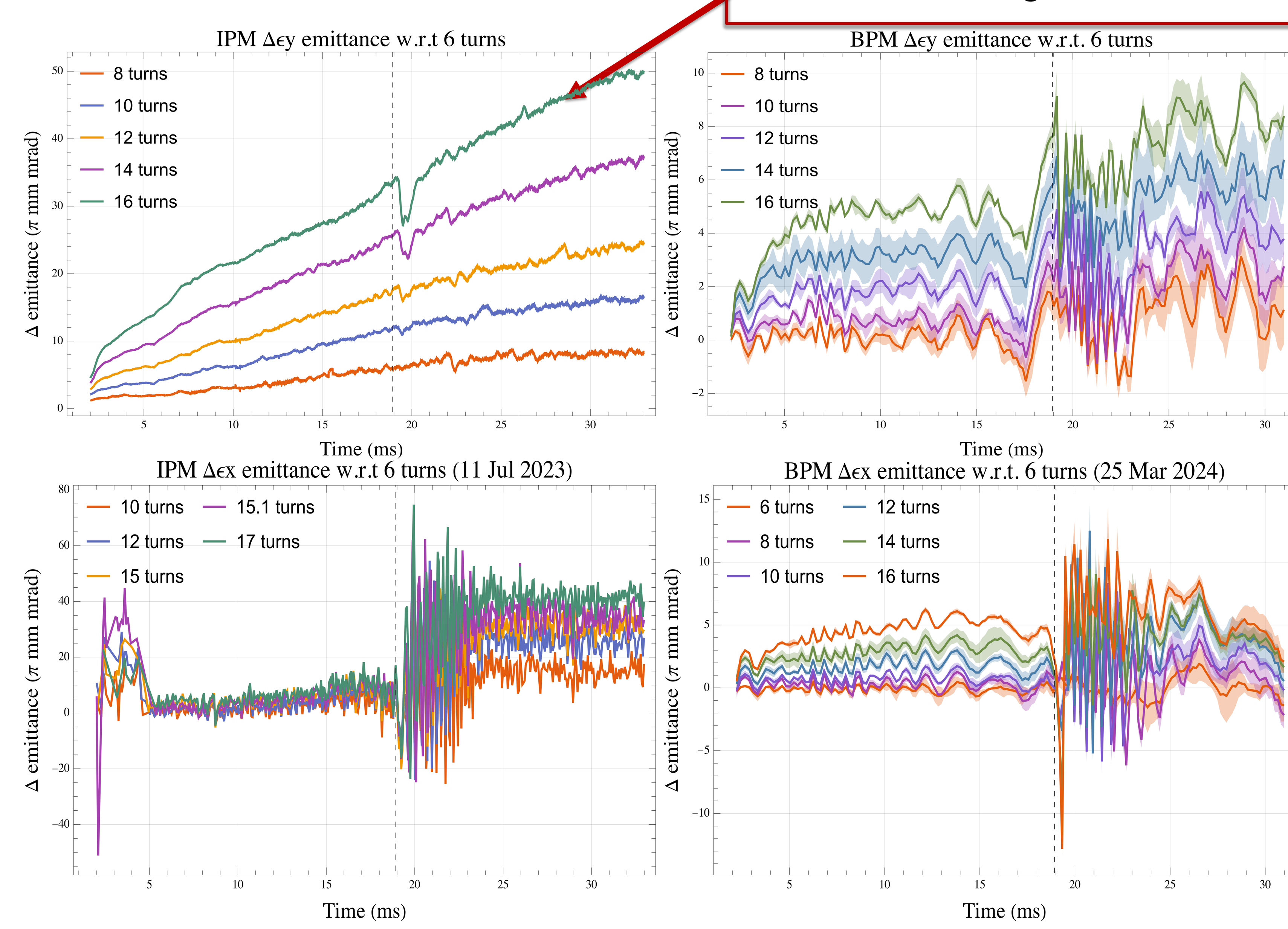


Stare long enough and there are common features. Therefore, average over all BPM pairs.

Vert IPM emittance size and growth is unrealistic. BPM emittance size and growth is more realistic.

Horz IPM emttance is too large. BPM emittance has same oscillatory feature after transition.
Note: sectors 5, 14 & 22 have bad BPMs and were ignored.

Compare with IPM



Conclusion

We have improved the extraction of emittance from the BPM quadrupole mode by using multiple BPMs and with better data analysis. The compromise is that we do not measure absolute emittance but the emittance growth w.r.t. some reference. We have found that the emittance growth extracted from the BPM quadrupole mode looks more sensible than those measured by the IPMs. As part of making this method easy to use and without any expert tuning, we have written a python GUI to interact with the user and a C++ program backend that can calculate Booster's emittance from real time BPM data. If this method proves to be as robust as we believe it to be, it opens up the ability of measuring emittances in nearly all rings that have 4 plate BPMs.