THEORETICAL DERIVATION OF MASS SPECTRA IN LATTICE GAUGE THEORIES

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ABSTRACT A short review of the theoretical possibilities for determining the particle spectra in lattice gauge theories is done. As an example, the determination of QCD spectrum at strong coupling in a random lattice model which is rotationnally invariant, is presented.

1 - INTRODUCTION

In this paper, we give a very short review of the problems encountered in the theoretical calculation of mass spectra; more details can be found in the literature, e.g.[1]. We also present a recent calculation performed on a random lattice at strong coupling[2].

Let us first recall that lattice gauge theories deal with dynamical gauge field variables U_l located on the links of a lattice and belonging to a gauge group G (typically SU(N)). The pure gauge action

$$S = \beta \sum_p \frac{1}{N} \mathrm{Re}\,\mathrm{Tr}\, U_p \tag{1}$$

is a sum over the elementary plaquette contributions which use the parallel transporter $U_p \equiv U_{ijkl} = U_{ij}U_{jk}U_{kl}U_{li}$. In the weak coupling limit $\beta=2N/g^2 \to \infty$, the lattice system is conjectured to reduce to the continuous Yang-Mills field theory. In the case of asymptotically free theories (e.g. SU(N) with $N \geq 2$) and according to the renormalization group analysis, the dimensioned quantities are expressed in terms of the lattice mass scale

$$\Lambda = a^{-1} (\beta_0 g^2)^{-\beta_1/2\beta_0^2} e^{-1/2\beta_0 g^2} \tag{2}$$

which is to be kept fixed as the lattice spacing $a \to 0$, $g^2 \to 0$. The first two numerical constants $\beta_0 = 11N/48\pi^2$ and $\beta_1 = 17N^2/384\pi^4$ are universal and independent of the regularization of the theory.

Let us discuss first the pure gauge theory with respect to its glueball mass spectrum. The problem is to compute the non-dimensioned quantity m/Λ in the limit $\beta \to \infty$. The usual lattice systems using continuous non-Abelian gauge groups exhibit two different behaviours as β varies.

- A strong coupling regime for low β. In the whole region, strong coupling series yield very good results. This region is naturally confined.

- A weak coupling region. For high enough dimension ($d \geq 5$), the two regions are separated by a first order phase transition, where the physical quantities have a jump. Very precise results are easily obtained using statistical techniques[1]. In particular, the whole phase structure of the system is well described by the mean field approximation and its systematic corrections. However, these high dimensional systems are not of physical interest.

Unfortunately, the situation becomes worse in four dimensions. Now, the weak coupling region is also confined. It is remarkable that asymptotic freedom results are very well reproduced, even at lowest order, in this region. Any physical length ξ is indeed such that $\Lambda\xi$ is almost constant. Between weak and strong coupling regions, there is some evidence for the absence of a phase transition. However, a sharp change in the properties of the system is observed (near β=2.2 for SU(2) and β=5.6 for SU(3)). The different approaches of the determination of physical quantities now lead to the following comments.

- Mean field method is no more directly usable. d=4 appears as the upper critical dimension where the series of mean field corrections diverge. In particular, the predicted first order phase transition is not present at d=4.

- Weak coupling expansions correspond in fact to the usual perturbation theory and is not suited for a correct description of confinement effects.

- It is possible to simulate the system by numerical Monte-Carlo method. Due to time and storage constraints, these very heavy and costful simulations are always performed at the very beginning of the weak coupling region and their results extrapolated to $\beta=\infty$ using asymptotic freedom formula.

- The most interesting method on the theoretical point of view is to use the strong coupling expansion, which naturally lead to confinement. The series[3] are rapidly convergent in the strong coupling region. However, the extrapolation in the weak coupling regime is rather difficult, due to the existence of nearby complex singularities near the cross-over point.

An empirical rule is to extrapolate smoothly the curves obtained by some resummation technique (e.g. Padé approximants) from the strong coupling series, using the asymptotic behaviour. Figure 1, extracted from ref.(4), illustrates this technique for the SU(2) glueball mass. The same analysis performed in SU(3) shows less stability in the results. This suggests the use of more elaborate extrapolation techniques[5].

We want also to point out the problem of rotational invariance. With regular lattices, one think that this symmetry is restored in the weak coupling region, and there is some numerical evidence for it[6]. This can be checked by computing the energy-momentum dispersion of glueballs[7]. The energy E(p) may be expanded for small **p** as

$$a\,E(p) = F(\beta) + a^2F_1(\beta)\Sigma p_i^2 + a^4F_2(\beta)(\Sigma p_i^2)^2 + a^4F_3(\beta)\Sigma p_i^2p_j^2 + \dots \quad (3)$$

In the approach of the physical region, we expect to recover the invariant spectrum $E(p)^2=m^2+\mathbf{p}^2$, hence

$$F(\beta)\to am \qquad 2FF_1\to 1 \qquad -8F^3F_2\to 1 \qquad F^3F_3\to 0 \qquad \dots\dots \quad (4)$$

This program works reasonably well, but suffers from the lack of convergence of the (short) series in four dimensions. Another approach is to compute the off-axis glueball masses and compare to the on-axis masses[8].

We have not discussed the problems related to the introduction of matter fields and, in particular, fermion fields. They are the subject of some contributions in this conference.

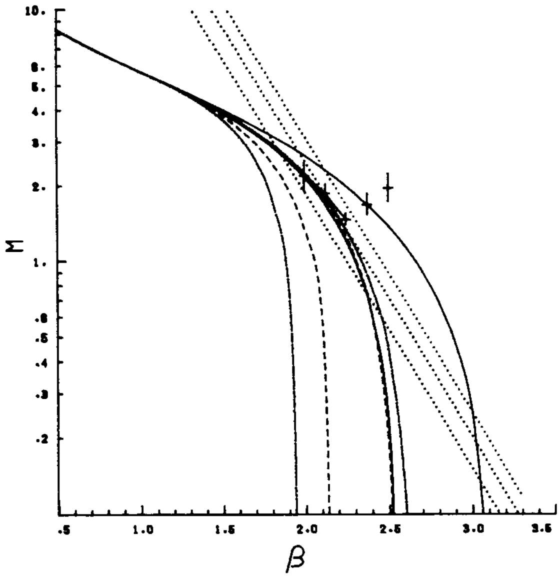

Figure 1.

Strong coupling expansion for the glueball mass. Different curves correspond to different resummations of the series. Dotted lines are extrapolations in the weak coupling region. Monte Carlo data also appear. (figure from ref (4)).

2 - A ROTATIONAL INVARIANT MODEL.

We want now to present a calculation performed on a random lattice. The randomness is expected to preserve from the beginning the rotational invariance. One hopes to avoid the preceding problems; furthermore, as the occurence of the cross-over separating the strong- and weak-coupling regimes is perhaps related to the restoration of the rotational invariance, there is some hope to obtain a better convergence of the strong coupling series up to the physical region. Another motivation is the possibility of a more convincing comparison of a computed spectrum with the experimental one, where particles are identified by their spin content. Finally, the impossibility of setting chiral fermions without species multiplication[(9)] might be overcome in a Euclidean invariant regularization.

The model is defined on a d-dimensional random distribution of sites with a density ρ. The standard random model[10] introduces the definition of nearest neighbours; as a consequence, practical evaluations of observables are almost untractable. We introduce interactions between any two points of the lattice, with a Gaussian factor of range b. The physical limit is obtained for fixed value of ρb^d, as $b \to 0$ where locality is restored. Gauge fields U(x,y) are introduced for any couple of points and the action is

$$S = \frac{\beta}{\rho^3 b^d} \Sigma\, 3^{d/2} \pi^{-d} b^{-2d} \exp\left(- \frac{(x-y)^2+(y-z)^2+(z-x)^2}{b^2}\right) \times$$
$$\times \; \mathrm{Tr}(U(x,y)U(y,z)U(z,x)) \; + \frac{2}{\rho^2 b^2} \Sigma\, \bar{q}(x)\Gamma(x,y)U(x,y)q(y) \quad (5)$$

where fermionic fields for quarks have been introduced.

As a first step in a *systematic* strong coupling expansion, the calculation of the particle spectrum is performed

- at β=0 where the action is linear in U.
- at the leading order in 1/N, which allows to use a steepest descent method already applied for regular lattice systems[11].
- at leading order in $1/\rho b^d$

We refer for details to[2]. The steps of the calculation are as follows. First, using the linearity in U, the gauge fields U are integrated over. The resulting action only depends on the mesonic local composite fields $\bar{q}(x)q(x)$ and baryonic composite fields q(x)q(x)q(x). Then it is linearized into these composite fields using the standard integral transformations which introduce new effective mesonic and baryonic fields with conjugated "mean fields". This linearization allows now the integration over the Grassmannian quark fields, and the remaining integrations can be estimated using the steepest descent method.

J^{PC}	I	Particle	Theoretical mass in MeV	Experimental mass in MeV
0^{-+}	1	π	140 (input)	140
0^{-+}	0	η_1 $(u\bar{u}+d\bar{d})$	140	550 (η)
0^{-+}	0	η_2 $(s\bar{s})$	689	958 (η')
0^{-}	1/2	K	495 (input)	495
1^{--}	1	ρ	770 (input)	770
1^{--}	0	ω	770	783
1^{--}	0	φ	997	1020
1^{-}	1/2	K^*	889	892
0^{-+}	1	π'	1502	1300
0^{-+}	0	η_1' $(u\bar{u}+d\bar{d})$	1502	-
0^{-+}	0	η_2' $(s\bar{s})$	1578	1440
0^{-}	1/2	K'	1539	1400?
1^{--}	1	ρ'	1150	1250? , 1600
1^{--}	0	ω'	1150	-
1^{--}	0	φ'	1285	1680
1^{-}	1/2	K'^*	1218	1650?

TABLE : mesonic spectrum with our random lattice model.

Calculations are nevertheless intricate and rather long; the interested reader will find details in[2], and we only quote here the table giving the mesonic spectrum of this model. Note that the scale b is fixed by a fit to the spectrum computed at β=0 and appears as an effective interaction range. Its value is to be adjusted at each value of β and is expected to vanish when β grows indefinitely. Here, b = 0.45fm. We have introduced three species of quarks and have used the masses of the ρ, π and K to fit our parameters. Note that the π appears as the Goldstone particle with a vanishing mass when the chirality is restored by setting the quark mass to zero. In this kind of models, there is a degeneracy between particles of different isospin. In particular, the η is degenerated with the π, which indicates the absence of U(1)-anomaly. However, this is in fact expected at leading order in 1/N. The calculation of various corrections will lift this degeneracy. It is now important to evaluate these corrections to check the correct chiral structure of our model.

We also mention that the baryon spectrum is also computed within the same approximations. We only mention here that it seems satisfactory, although the ratio m_Δ/m_N is a bit low. All details can be found in ref.(2).

3 - CONCLUSION

In spite of various pitfalls, strong coupling methods have achieved remarkable results. It seems that longer series might offer a real alternative to Monte-Carlo simulations.

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