

Bandhead Energies of two quasiparticle bands in ^{188}Re

Harpreet Kaur¹, Karmjeet Kaur¹, Manpreet Kaur¹, Sushil Kumar^{1*}, Sukhjeet Singh¹ and A.K. Jain^{1,2}

¹Department of Physics, Akal University Talwandi Sabo, Bathinda, Punjab-151302, India

²AINST, Amity University, Noida- 201313, India

*Email: sushil.rathi179@gmail.com

Introduction

It is now well-established that the phenomenon of odd-even staggering or signature effects observed in odd-odd deformed nuclei in rare-earth and actinide region is the implication of Coriolis mixing among large number bands with $\Delta K=1$ [1]. Recently, Shneidman et al. [2] pointed out the vital role of Coriolis mixing on the lifetime of isomeric states in heavy nuclei and suggested that the Coriolis mixing can explain the enhancement of isomeric decay strength by three orders of magnitude. Generally, in order to perform Coriolis mixing calculations, we need to estimate the location of bandhead energies of two quasiparticle (2qp) bands present in the basis space of Particle Plus Rotor Model. Out of all the 2qp bands taking part in Coriolis mixing calculations, it is often necessary to estimate the bandhead energies of certain unidentified bands, particularly the bands whose odd-even staggering can be transmitted to high-K bands through Coriolis couplings. In present work, we have calculated the bandhead energies of two quasiparticle bands that are observed and predicted the same for yet unobserved bands in odd-odd ^{188}Re [3] nuclide using a semi-empirical two quasiparticles plus rotor model approach.

Theoretical Framework

The bandhead energies of particular two quasiparticle rotational bands can be written as [4]:

$$E = E(\Omega_p) + E(\Omega_n) + \Delta E_{rot} + \Delta E_a + V_{pn} \quad (1)$$

where $E(\Omega_p)$ and $E(\Omega_n)$ are the single protons and neutron energies. These energies are adopted from neighboring odd-A nuclei if available, otherwise these energies are estimate

using the procedure suggested by Jain *et al.* [5]. The ΔE_{rot} is a rotational correction term, ΔE_a and V_{pn} are diagonal contributions of the Coriolis interaction for $K=1/2$ bands and of neutron-proton residual interactions, respectively. The general form of above-mentioned terms is given below [6]:

$$\Delta E_{rot} = \frac{\hbar^2}{2\mathfrak{I}_{o-o}} K - \left(\frac{\hbar^2}{2\mathfrak{I}_p} \Omega_p + \frac{\hbar^2}{2\mathfrak{I}_n} \Omega_n \right) \approx -2 \frac{\hbar^2}{2\mathfrak{I}} \Omega_{<} \delta_{KK-} \quad (2)$$

$$\Delta E_a = \frac{\hbar^2}{2\mathfrak{I}_{o-o}} a_p a_n \delta_{\Omega_p, 1/2} \delta_{\Omega_n, 1/2} \delta_{K,0} \quad (3)$$

$$V_{np} = - \left(\frac{1}{2} - \delta_{\Sigma,0} \right) E_{GM} + (-1)^J E_N \delta_{K,0} \quad (4)$$

In above equations, \mathfrak{I}_{o-o} , \mathfrak{I}_p and \mathfrak{I}_n are the empirical inertia parameter of neighboring odd-odd, odd-proton and odd-neutron nuclei, respectively. The inertia parameters of different 2qp bands are estimated using difference of first two experimentally observed energy levels and if not available then average inertia 12.0 keV for K_- and 12.5 keV for K_+ is assumed which is based on the fact that the average value inertia in rare-earth nuclides is in the range of 10.0-13.0 keV. The proton decoupling (a_p) and neutron decoupling (a_n) parameters are calculated using Nilsson model [7]. The Gallagher Moszkowski (GM) splitting (E_{GM}) and Newby shift (E_N) energies are calculated using following semi-empirical relations:

$$E_{GM} = E_{K_-}^{\text{exp}} - E_{K_+}^{\text{exp}} + \frac{\hbar^2}{2\mathfrak{I}_{K_+}} K_+ - \frac{\hbar^2}{2\mathfrak{I}_{K_-}} K_- + (E_N - E_a) \delta_{K,0} \quad (5)$$

$$E_N = (-1)^{I+1} \left[\frac{1}{2} (E_{I+1}^{\text{exp}} - E_I^{\text{exp}}) \right] - 2 \frac{\hbar^2}{2\mathfrak{I}_{K_{\perp}}} (I+1) + \frac{\hbar^2}{2\mathfrak{I}} a_p a_n \delta_{\Omega_p, 1/2} \delta_{\Omega_n, 1/2} \delta_{K, 0} \quad (6)$$

Results and Discussion

In case of ^{188}Re , the two quasiparticle rotational bands are formed due to the coupling of odd proton orbital i.e., 5/2[402], 9/2[514] with 3/2[512], 9/2[505], 7/2[503], 11/2[615], 1/2[510] neutron orbitals. In order to calculate the energy of possible two quasiparticle rotational bands, we hereby performed simple but effective semi-empirical two quasiparticle rotor model calculations. In present formulation, the energies of given two quasiparticle states can be estimated using the superposition of one quasiparticle states in neighboring nuclides and the magnitude of residual interactions (GM Splitting) among odd-odd particles. The values of GM splitting used in present calculations are extracted from experimental data of nuclide of interest and if not available then these energies are taken from neighboring nuclides. The results of present model calculation are given in Table 1. The

RMS deviation among experimental and calculated energies is 130.10 keV which is reasonable as compared to 92.64 keV in case of ^{186}Re [8]. On the basis of present calculations, we also predicted the energies of GM partners whose energies are yet unknown for future experimental endeavors.

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References

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Table 1: Comparison of experimental and calculated energies of intrinsic 2qp states in ^{188}Re [3]

Nilsson Configuration	K^{π}	E_{sp}	E_{np}	E_{res}	$E_{\text{exp.}}$	$E_{\text{cal.}}$
5/2 ⁺ [402] _π ⊗3/2 ⁻ [512] _v	4 ⁻	0.0	0.0	229.75	182.76	276.74
	1 ⁻				0	0
5/2 ⁺ [402] _π ⊗1/2 ⁻ [510] _v	3 ⁻	0.0	145.85	107.75	169.45	262.96
	2 ⁻				256.93	350.44
5/2 ⁺ [402] _π ⊗11/2 ⁺ [615] _v	8 ⁺	0.0	410.06	226.04	-	440.06
	3 ⁺				439.76	609.06
5/2 ⁺ [402] _π ⊗7/2 ⁻ [503] _v	6 ⁻	0.0	350.43	175.58	172.08	393.03
	1 ⁻				290.67	511.62
5/2 ⁺ [402] _π ⊗9/2 ⁻ [505] _v	7 ⁻	0.0	364.22	199.32	-	587.74
	2 ⁻				205.35	338.09
9/2 ⁺ [514] _π ⊗1/2 ⁻ [510] _v	5 ⁺	125	145.85	130.88	360.89	359.58
	4 ⁺				493.72	492.4
9/2 ⁺ [514] _π ⊗3/2 ⁻ [512] _v	6 ⁺	125	0.0	240.12	-	398.38
	3 ⁺				230.92	125.14
9/2 ⁺ [514] _π ⊗7/2 ⁻ [503] _v	8 ⁺	125	350.43	302.00	-	459.16
	1 ⁺				-	673.16
9/2 ⁺ [514] _π ⊗11/2 ⁺ [615] _v	10 ⁻	125	410.06	249.22	-	557.81
	1 ⁻				609.01	699.81