

OPTIMIZING THE FILLING FACTOR IN HIGH ENERGY COLLIDERS

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Abstract

The energy of a hadron collider is proportional to the field of the main bending dipoles and to the length of the arcs available to them. A way to increase energy without increasing the field or making a longer tunnel is to have a larger filling factor (fraction of the arcs covered by dipoles). In this paper we discuss three possible paths to increase the filling factor, namely having longer spacing between quadrupoles, using a 60° phase advance optics rather than 90°, and spreading the quadrupole gradient in the dipoles, i.e. going for a combined function magnet.

INTRODUCTION

The energy of a hadron collider is proportional to the curvature radius and to the field in the main dipoles. All requirements for accelerators are given in terms of integrated fields: the limits given by technology and the constraints in space determine the balance between stronger fields and longer magnets. An significant parameter is the filling factor of the lattice, i.e. the fraction of the arcs covered by the magnetic length of the dipoles. For high energy accelerators using the superconducting magnet technology at its limits, as LHC for Nb-Ti [1,2] or the FCC-hh study for Nb₃Sn [3,4], a small (order of few percent) reduction of the dipole field, obtained by filling the lattice with longer bending fields, can bring to a significant reduction in the cost of the magnet system. The aim of this paper is to review the main equations ruling the filling factor.

The simpler way to fill more the accelerator with dipoles is to have longer cells: this allows to reduce the space covered by quadrupoles, and also to make them shorter since their strength is proportional to the inverse of the cell length. Longer cells imply larger beta functions and therefore a larger magnet aperture; however, the aperture scales with a fraction of the square root of the cell length. The cell length is usually fixed by the beam optics team, but the implications on the magnet design and on their cost are so significant that a global optimization to minimize the cost of the whole system should be done.

Changing the phase advance from 90° to 60° also allows reducing by 30% the quadrupole length, keeping the same beam size. Recently, a study has been focussed on spreading the quadrupolar components in the dipole [5], aka combined optics, as done in low energy machines [6]. The combined optics was also proposed for the LHC [7]: this option has the obvious interest of removing the cell quadrupole magnets; even though these magnets are not the driving factor of a high energy collider cost, their removal significantly simplifies the system and the magnet production.

In this paper we discuss the implications of optimizing the filling factor using these strategies; the LHC has a 80% filling factor in the arcs, and it is shown that lattices with 85-90% could be achieved. Its implications for the FCC-hh are particularly interesting since the marginal cost of the field is very high. We show that further increase of the cell length, together with a change to 60° phase advance, allows reducing the FCC-hh dipole field of 5-7% keeping the same energy and machine size.

ARC FILLING FACTOR EQUATIONS

Let us consider a FODO lattice [8-10], having a quadrupole spacing L , and N_d dipoles between two quadrupoles, with magnetic length l_d . The arc filling factor is defined as the fraction of the cell covered by dipoles:

$$\chi = \frac{N_d l_d}{L}. \quad (1)$$

We assume a structure similar to the LHC lattice [1], see Fig. 1 left; to estimate the filling factor we have to consider the magnetic length of the quadrupole l_q , the spacing between the dipoles i_{d-d} and the spacing between dipoles and quadrupoles i_{q-d} . The LHC lattice has $\chi=0.803$. The driving parameters are steered by the following elements:

- The dipole cold mass length is limited by the European regulations on maximum length of regular trucks (15 m). By cold mass we denote the LHe vessel containing the magnet and correctors. This assumes that cryostatting is done in loco, as for the LHC, and that the cryostated magnet can exceed 15 m.
- The difference in length between the cold mass and the dipole magnetic length (see Fig. 1, right) is given by the coil heads (200 mm on each side), plus the space given to spool pieces (110 mm for the MCS, 66 mm for the MCDO), plus the space required at the end of the cold mass (54 mm on each side). The dipole spacing i_{d-d} includes these elements, plus the length of the interconnection between the cold masses (500 mm, for a total of 1.36 m in the LHC case).
- The quadrupole-dipole spacing i_{q-d} includes the same elements, but the instead of spool pieces we have longer lattice correctors (dipole orbit correctors, chromatic sextupoles, tuning quadrupoles, skew quadrupole, Landau octupoles), requiring a total of 2 m, split half on one side and half on the other one, thus giving $i_{d-d}=2.34$ m in the LHC case.

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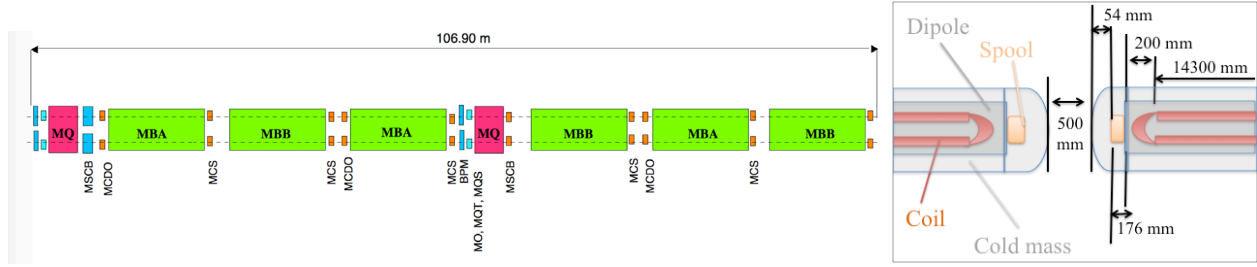


Figure 1: LHC arc cell (left): dipoles (MB), quadrupoles (MQ), sextupole (MCS) octupole/decapole (MCDO) spool pieces, chromatic sextupoles (MCS), tuning normal and skew quadrupoles (MQT, MQS) and Landau octupole (MO); space between two dipoles (right), with detail on coil ends, cold masses, spool pieces and interconnection.

The simplest way to increase the filling factor is to have a longer cell. One gains twice, since there are less quadrupoles, and they become shorter as their strength scales with the quadrupoles spacing L : for a 90° phase advance one has in a simplified model (thin lens quadrupole, no focusing from dipoles and no space between magnets)

$$G l_q \approx \sqrt{2} \frac{B\rho}{L}. \quad (2)$$

Therefore, given the gradient that can be provided by the magnet technology (for instance, 200 T/m in 28 mm aperture radius for the LHC), longer L requires smaller l_q . A drawback is the aperture increase imposed by the larger beta functions that scales with L :

$$\beta_F \approx (\sqrt{2} + 2)L. \quad (3)$$

The beam size is proportional to the square root of the beta function, i.e. to \sqrt{L} . A fraction of the aperture (typically 2/3) scales with the beam size, the rest is being needed for beam screen, cold bore and tolerances.

Note that a 60° phase advance lattice has the advantage of requiring a 30% lower gradient with a very similar maximum beta function (see Table 1): the use of this optics has been explored for the FCC [11] and experimented in the LHC [12,13]

$$G l_q \approx \frac{B\rho}{L}. \quad (4)$$

Finally, we consider a combined function lattice where the quadrupole is spread in the dipoles, and is changing sign every N_d dipoles [5], i.e. over a length L . The gradient G is related to the phase advance μ via

$$\cosh \left[\sqrt{\frac{G}{B\rho}} L \right] \cos \left[\sqrt{\frac{G}{B\rho}} L \right] = \cos \mu. \quad (5)$$

Therefore the solution for 90° phase advance is

$$\cosh \left[\sqrt{\frac{G}{B\rho}} L \right] \cos \left[\sqrt{\frac{G}{B\rho}} L \right] = 0 \rightarrow G = \frac{\pi^2}{4} \frac{B\rho}{L^2} \quad (6)$$

If combined function lattice were adopted for the LHC, the required gradient to add to the dipoles would be 25 T/m over three consecutive dipoles, and -25 T/m over the next three (see also [7], pg 4, Eq. (9) where the alternative layout for the 7 TeV LHC is achieved via a 28 T/m gradient added to a 7.9 T bending field and 85% of filling factor). Note that for a 60° phase advance, the gradient is 30% lower than for the previous case:

$$\cosh \left[\sqrt{\frac{G}{B\rho}} L \right] \cos \left[\sqrt{\frac{G}{B\rho}} L \right] = \frac{1}{2} \rightarrow G \approx (1.32)^2 \frac{B\rho}{L^2}. \quad (7)$$

The maximum beta functions are located in the center of the focusing dipole; they are 80% lower than the corresponding values of separate cell lattice, for the same cell length (see Table 1). Therefore the aperture requirements of the combined are less stringent than the separate cell lattice. For a fair comparison between the two options, one should consider for the combined function 20% longer cell, giving the same beam size.

Table 1: Gradient Requirement and Maximum Beta Functions for Separate and Combined Function Lattices

| | μ | $l_q G/B\rho$ | $G/B\rho$ | β_{max} |
|----------|------------|---------------|---------------------|-----------------|
| Separate | 90° | $\sqrt{2}/L$ | | $(\sqrt{2}+2)L$ |
| | 60° | $1/L$ | | $2\sqrt{3}L$ |
| Combined | 90° | | $(\pi/2)^2/L^2$ | $\sim 2.7L$ |
| | 60° | | $\sim (1.32)^2/L^2$ | $\sim 2.8L$ |

A combined function dipole has the drawback of “consuming” part of the superconductor properties to produce a quadrupole component. The spread quadrupole increases the dipole peak field by

$$\Delta B_p = \lambda G r. \quad (8)$$

In this equation, r is the aperture radius, and λ is the overshoot of the quadrupolar field in the coil that can be estimated as 1.15 [14]. Therefore this contribution has to be removed from the gain in the main dipolar field that can be induced by a larger filling factor. Combined function magnets have been realized for JPARC [15], with dipolar and a quadrupolar components of the same order of magnitude. For this application, the quadrupolar component is a second order perturbation on the main component: for an FCC with $L=100$ m and 16 T main field, the quadrupolar gradient would be 41 T/m, i.e. 4% of the dipole field at the reference radius of 17 mm. two-in-one magnets with small asymmetries in the coil to create a geometric quadrupolar component were proposed in [16], and the HL-LHC D2 adopted this approach [17,18] having a 2% of quadrupole in the single coil, to compensate the cross-talk arising from the double aperture configuration. This case shows that one can build dipoles with adequate tolerances to achieve these field quality goals.

THE LHC AND THE FCC CASES

We first consider the case of the LHC arc, applying the scaling outlined in the previous section to compute how the filling factor would improve by (i) increasing the number of dipoles per half cell from 3 to 10, (ii) changing the phase advance from 90° to 60° , and (iii) considering a combined function lattice. Results are shown in Fig. 2: one can see that doubling the cell length one increases the filling factor by 7-8%, and a 60° phase advance gives a 1% more than the 90° optics. This would allow increasing LHC beam energy by 500 GeV keeping the same dipole field, or reducing the dipole field needed for 6.8 TeV operation from 8.1 T to 7.5 T. Above 10 dipole per half cell, the filling factor saturates to $\sim 90\%$, without further improving as the dominant parameters are the interconnections. The combined lattice requires for the same collider energy the same dipole field, but with the advantage of removing the need of manufacturing the quadrupoles. Making the comparison with the same beam size, one finds that the combined function allows another $\sim 1\%$ of field reduction.

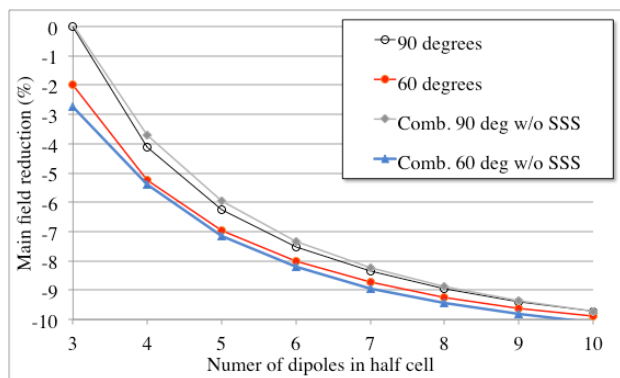


Figure 2: Main field reduction induced by increase of filling factor, LHC case (90° cell), and three options (60° cell, and combined function with 90° or 60°).

The same computation shown for LHC is applied to the baseline of FCC, that consists of a half cell with 6 dipoles, a distance between dipole magnetic lengths of 1.5 m, and a quadrupole length of 7.0 m. Further increasing the number of dipoles per half cell from 6 towards 10 increases the filling factor by 5-10%, thus allowing a 5-10% reduction of the main field (see Fig. 3). The 60° phase advance allows a further field reduction of 1-2%; the combined function cell has similar performances to the regular cell with lumped quadrupoles, and presents the advantage of fully removing the quadrupoles. Comparing the combined optics function to the separate optics keeping the same beam size, one has a further $\sim 1\%$ of field reduction.

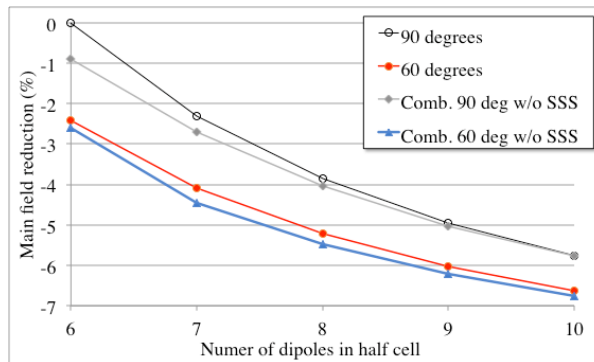


Figure 3: Main field reduction induced by increase of filling factor, FCC case (90° cell), and three options (60° cell, and combined function with 90° or 60°).

CONCLUSION

For a high energy collider as the LHC, or for the planned FCC, the filling factor in the arcs is a significant parameter that should be optimized to minimize the global cost of the system. In the LHC and in the planned FCC the arc filling factor is close to 80%. In this paper we have shown that values as large as 90% could be reached, thus allowing a reduction of the dipole field or a higher energy. The first option is particularly important for the FCC-hh, where the marginal cost of one additional tesla in the 15 T range is very large.

In order to achieve filling factors in the range of 85%-90%, having long cells is beneficial since one has less and shorter quadrupoles. Reducing the phase advance from 90° to 60° degrees allows to further reduce the quadrupole length by 30%. Finally, a combined function lattice allows to remove the need of manufacturing cell quadrupoles, and allows a further gain in the filling factor of 1%.

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