SLAC-PUB-1509
(T)
November 1974

Asymptotic Behavior of Form Factors for Two- and Three-Body Bound States II: Spin-1/2 Constituents *

by

C. Alabiso + and G. Schierholz ++
Stanford Linear Accelerator Center
Stanford University
Stanford, Calif. 94305

(Submitted to Physical Review D)

* Work supported by the U.S.Atomic Energy Commission
+ Permanent address: Istituto di Fisica dell' Universita, Parma, Italy
++ Permanent address: II. Institut für Theoretische Physik der Universität, Hamburg, Germany.

Abstract The asymptotic behavior of form factors for two- and three- particle bound states are investigated in the case of spin-$\frac{1}{2}$ constituents in order to shed some light on the underlying structure of the pion and nucleon. Here the Blankenbecler-Sugar approach proves to be a powerful tool for studying dynamics at infinite momentum. For a two-body interaction which for large momentum transfer behaves as $V(q,k) \simeq [(q-k)^2]^{-1-\Delta}$ we obtain for the two- and three-body form factors $F_2 \simeq [Q^2]^{-3/2-\Delta}$ and $F_3 \simeq [Q^2]^{-3-2\Delta}$ respectively in the case of scalar and pseudoscalar couplings and $F_2 \simeq [Q^2]^{-1-\Delta}$ and $F_3 \simeq [Q^2]^{-2-2\Delta}$ for the vector coupling. The experimental pion and nucleon form factors are, e.g., consistently recovered assigning a quark-antiquark and three-quark structure to the pion and nucleon respectively and the quarks interacting via vector-gluon exchange (i.e., $\Delta \to 0$).

I. Introduction

In a recent paper[1] we have investigated the asymptotic behavior of form factors for two- and three-body bound states for spin-zero constituents. It has been proven that the large-momentum-transfer behavior of the hadron form factors provides an excellent means of studying the hadronic constituents and their dynamics. Neglecting the spin of the nucleon and assuming spin-zero constituents of the pion and nucleon, we have shown by consistently looking at the pion and nucleon form factors[2] that the "pion" and "nucleon" are likely to have a two- and three-particle structure respectively (at infinite momentum) and the constituents interacting (relativistically) via a Bethe-Salpeter (BS) kernel $V(q,k)$ which for large momentum transfer $(q-k)^2$ behaves like $V(q,k) \simeq \text{const.}$

This result is very much in favour of the quark model (even though the case of spin-1/2 constituents has yet to be discussed) so that it seems desirable to extend these ideas to the more realistic case of spin-1/2 constituents (i.e., to the "true" pion and nucleon). A similar result has been reported even for spin-1/2 constituents employing dimensional counting techniques.[3] Here, binding corrections are, however, neglected which is rather doubtful and, in fact, leads to a different result in case of spin-1/2 constituents as we shall see.

Unfortunately, the three-body spin-1/2 case is extremely difficult to handle relativistically. Here, the vertex function consists of 16 invariant amplitudes compared to 4 in the two-body case. However, we need not also go into the discussion of the complete BS equation. For the large momentum transfer behavior of the form factors it is sufficient to know the Blankenbecler-Sugar (BLS) vertex function[4] which takes the constituents on the mass-shell.

In this work we shall systematically investigate the large momentum transfer behavior of form factors of two- and three-body s-wave bound states[5] adopting the BLS approach. We are primarily interested in the case of spin-1/2 constituents considering various kinds of interactions. But we will also review the spin-zero case using the BLS equation for two main reasons. First, this may serve as a test of our approach. Secondly, as we shall see later, the form factors become in general a convolution of two nonrelativistic clusters at large momentum transfer which allows a relativistic description of bound state form factors in terms of instantaneous wave functions.

The paper is organized as follows. In Sec.II and Sec.III we shall investigate the large momentum behavior of the BLS vertex functions for two- and three-body s-wave bound states being composed of spin-zero and spin-1/2 constituents. In the case of spin-1/2 constituents we will consider scalar, pseudoscalar and vector couplings. As a by-product relativistic (three-dimensional) Faddeev equations are presented which is an interesting subject by itself. In Sec.IV and Sec.V the asymptotic behavior of form factors for two- and three-body bound states is derived from the asymptotic form of the vertex functions. It turns out that the form factors can be expressed as an integral over a two-dimensional disc in the Breit frame which has a nice physical interpretation. Finally, in Sec.VI we add some concluding remarks.

II. Two-Body Wave Function for Spin-Zero and Spin-1/2 Constituents

The BS equation for the vertex function of two spin-zero particles reads (cf. Fig.1)

$$\phi_{\vec{P}}(q) = i \int d^4k\ V(q,k)\ G_1(\tfrac{1}{2}P + k)\ G_2(\tfrac{1}{2}P - k)\ \phi_{\vec{P}}(k) \qquad \text{(II.1)}$$

where[6] $G^{-1}_{1,2}(p) = p^2-1$. If we write $i\,G_1(\frac{1}{2}P+k)G_2(\frac{1}{2}P-k) = E_2+R_2$ defining

$$E_2 = 2\pi \int ds'\ \frac{1}{s'-s}\ \delta^+(1-(\tfrac{1}{2}P'+k)^2)\ \delta^+(1-(\tfrac{1}{2}P'-k)^2)$$

$$= \pi\ \delta(k_o - \tfrac{1}{2}(\sqrt{1+(\tfrac{1}{2}\vec{P}+\vec{k})^2} - \sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{k})^2})) \qquad \text{(II.2)}$$

$$\times \frac{\sqrt{1+(\frac{1}{2}\vec{P}+\vec{k})^2} + \sqrt{1+(\frac{1}{2}\vec{P}-\vec{k})^2}}{\sqrt{1+(\frac{1}{2}\vec{P}+\vec{k})^2}\ \sqrt{1+(\frac{1}{2}\vec{P}-\vec{k})^2}}\ [(\sqrt{1+(\tfrac{1}{2}\vec{P}+\vec{k})^2}+\sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{k})^2})^2-\vec{P}^2-s]^{-1}$$

where $s = P^2$, $s' = P'^2$ and $\vec{P}' = \vec{P}$, the BS equation can be cast into the equivalent BLS equation[7]

$$\chi_{\vec{P}}(\vec{q}) = \pi \int d^3\vec{k}\ W(\tilde{q},\tilde{k})\ \frac{\sqrt{1+(\frac{1}{2}\vec{P}+\vec{k})^2}+\sqrt{1+(\frac{1}{2}\vec{P}-\vec{k})^2}}{\sqrt{1+(\frac{1}{2}\vec{P}+\vec{k})^2}\ \sqrt{1+(\frac{1}{2}\vec{P}-\vec{k})^2}}$$

$$\times\ [(\sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{k})^2}+\sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{k})^2})^2 - \vec{P}^2-s]^{-1}\ \chi_{\vec{P}}(\vec{k}) \qquad \text{(II.3)}$$

where

$$\chi_{\vec{P}}(\vec{q}) = \phi_{\vec{P}}(\tilde{q})\ ,\quad \tilde{q} = (\tfrac{1}{2}(\sqrt{1+(\tfrac{1}{2}\vec{P}+\vec{q})^2} - \sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{q})^2}),\ \vec{q}) \qquad \text{(II.4)}$$

and

$$W = (1 - V R_2)^{-1} V \quad . \tag{II.5}$$

It is obvious that the BLS equation (II.3) bears all the information of the BS equation. Once the BLS vertex function is known, the BS vertex function is given by the right hand side of Eq.(II.3) simply replacing $\tilde{q}$ by q.

In the spin-zero case we assume the BS kernel having the asymptotic form

$$V(q,k) \underset{(q-k)^2 \to \infty}{\simeq} [(q-k)^2]^{-\theta} \tag{II.6}$$

with $\theta > 0$ which is all we need for our further calculations. It is obvious that $\theta = 1$ corresponds to a $\lambda\phi^3$ interaction whereas the limiting case $\theta \to 0$ corresponds to $\lambda\phi^4$. Later on we are interested in the BLS vertex function for large momenta $\vec{P}$ and $\vec{q}$ so that it is sufficient to consider $W = V$ only because the higher contributions to the BLS kernel (such as VR_2V, etc.) behave at least like $[(q-k)^2]^{-2\theta}$. Hence, Eq.(II.6) leaves us with the BLS interaction

$$V(\tilde{q},\tilde{k}) \underset{(\tilde{q}-\tilde{k})^2 \to \infty}{\simeq} [(\tilde{q}-\tilde{k})^2]^{-\theta}$$

$$=[\tfrac{1}{4}(\sqrt{1+(\tfrac{1}{2}\vec{P}+\vec{q})^2}-\sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{q})^2}-\sqrt{1+(\tfrac{1}{2}\vec{P}+\vec{k})^2}+\sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{k})^2})^2-(\vec{q}-\vec{k})^2]^{-\theta} \quad . \tag{II.7}$$

We could base our further calculations directly on the asymptotic behavior of the BLS kernel W which would have saved us giving any arguments in favour of the approximation $W = V$. However, we believe that starting from the BS interaction kernel is much more transparent as far as the contact to Lagrangian field theory is concerned . But one should bear in mind that both ways would lead to the same conclusions.

We are now interested in the large momentum behavior of the BLS vertex function. By means of the consistency argument widely applied in our previous paper[1] we obtain the asymptotic form of the vertex function (generally $\vec{P}$ and $\vec{q}$ large)

$$\chi_{\vec{P}}(\vec{q}) \simeq [\tilde{q}^2]^{-\theta} = [\tfrac{1}{4}(\sqrt{1+(\tfrac{1}{2}\vec{P}+\vec{q})^2}-\sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{q})^2})^2 - \vec{q}^{\,2}]^{-\theta} \tag{II.8}$$

for $\theta > 0$. In case of the $\lambda\phi^4$ theory we take the limit $\theta \to 0$ at the end of the calculation which is analogous to the analytic regularization in perturbation theory.[8]

We like to point out that the asymptotic behavior (II.8) is not uniform in $\vec{q}$. If we write $\vec{q} = x\,\vec{P} + \vec{q}_\perp$, Eq.(II.8) gives $\chi_{\vec{P}}(\vec{q}) \simeq [\vec{q}_\perp^{\,2}]^{-\theta}$ for $-\frac{1}{2} < x < +\frac{1}{2}$ whereas for $x = \pm\frac{1}{2}$ we find $\chi_{\vec{P}}(\vec{q}) \simeq [|\vec{P}+\vec{q}_\perp||\vec{q}_\perp|]^{-\theta}$.

So far the two-body spin-zero vertex function. For spin-1/2 constituents we can now proceed in a similar way. As before we start off with the BS equation (II.1) where $G_i^{-1}(p) = \gamma^{(i)}p-1$ now. Again we write $i\,G_1(\frac{1}{2}P+k)\,G_2(\frac{1}{2}P-k) = E_2 + R_2$ defining[9] (consistently)

$$E_2 = 4\pi\ \delta\left(k_o - \tfrac{1}{2}\left(\sqrt{1+(\tfrac{1}{2}\vec{P}+\vec{k})^2} - \sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{k})^2}\right)\right)\ \Lambda_+^{(1)}(\tfrac{1}{2}\vec{P}+\vec{k})\ \Lambda_+^{(2)}(\tfrac{1}{2}\vec{P}-\vec{k})$$

$$\times \frac{\sqrt{1+(\tfrac{1}{2}\vec{P}+\vec{k})^2} + \sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{k})^2}}{\sqrt{1+(\tfrac{1}{2}\vec{P}+\vec{k})^2}\ \sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{k})^2}} \left[\left(\sqrt{1+(\tfrac{1}{2}\vec{P}+\vec{k})^2} + \sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{k})^2}\right)^2 - \vec{P}^2 - s\right]^{-1} \qquad \text{(II.9)}$$

$$\Lambda_+^{(i)}(\vec{p}) = \gamma^{(i)}p + 1 \quad , \quad \Lambda_+^{(i)}(\vec{p}) = \frac{1}{2}\sum_{r=1}^{2} w_i^r(\vec{p}) \otimes \tilde{w}_i^r(\vec{p})$$

which leads us to the BLS equation

$$\chi_{\vec{P}}^{rs}(\vec{q}) = 4\pi \int d^3\vec{k}\ W^{rs,r's'}(\tilde{q},\tilde{k}) \frac{\sqrt{1+(\tfrac{1}{2}\vec{P}+\vec{k})^2} + \sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{k})^2}}{\sqrt{1+(\tfrac{1}{2}\vec{P}+\vec{k})^2}\ \sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{k})^2}}$$

$$\times \left[\left(\sqrt{1+(\tfrac{1}{2}\vec{P}+\vec{k})^2} + \sqrt{1+(\tfrac{1}{2}\vec{P}-\vec{k})^2}\right)^2 - \vec{P}^2 - s\right]^{-1} \chi_{\vec{P}}^{r's'}(\vec{k}) \ . \qquad \text{(II.10)}$$

Here we have written (the analogue holds for V) .

$$W^{rs,r's'}(\tilde{q},\tilde{k}) = \tilde{w}_1^r(\tfrac{1}{2}\vec{P}+\vec{q})\ \tilde{w}_2^s(\tfrac{1}{2}\vec{P}-\vec{q})\ W(\tilde{q},\tilde{k})\ w_1^{r'}(\tfrac{1}{2}\vec{P}+\vec{k})\, w_2^{s'}(\tfrac{1}{2}\vec{P}-\vec{k}) \qquad \text{(II.11)}$$

and (note that $\chi_{\vec{P}}(\vec{q}) = \phi_{\vec{P}}(\tilde{q})$ is a 4 x 4 matrix in this case)

$$\chi_{\vec{P}}^{rs}(\vec{q}) = \tilde{w}_1^r(\tfrac{1}{2}\vec{P}+\vec{q})\ \tilde{w}_2^s(\tfrac{1}{2}\vec{P}-\vec{q})\,\chi_{\vec{P}}(\vec{q}) \qquad \text{(II.12)}$$

where $r,s,r',s' = 1,2$. Since we have only retained positive energy particles this corresponds to pairs of quarks and time reversed anti-quarks in the language of the quark model for mesons.

In the spin-1/2 case we take the BS kernel of the asymptotic form

$$V(q,k) \underset{(q-k)^2 \to \infty}{\simeq} \Gamma_{(\mu)}^{(1)}\ \Gamma^{(2)(\mu)}[q-k)^2]^{-1-\Delta} \ . \qquad \text{(II.13)}$$

For definiteness we consider the couplings $\Gamma^{(i)} = 1$, $\Gamma^{(i)} = \gamma_5^{(i)}$ and $\Gamma_\mu^{(i)} = \gamma_\mu^{(i)}$ which for $\Delta \to 0$ corresponds (asymptotically) to a $\lambda\bar{\psi}\psi\phi$, $\lambda\bar{\psi}\gamma_5\psi\phi$ and $\lambda\bar{\psi}\gamma_\mu\psi\phi^\mu$ interaction respectively. The asymptotic form of $V^{rs,r's'}(\tilde{q},\tilde{k})$ is then given by

$$V^{rs,r's'}(\tilde{q},\tilde{k}) \underset{(\tilde{q}-\tilde{k})^2 \to \infty}{\simeq} [(\tilde{q}-\tilde{k})^2]^{-1-\Delta}[t(\tfrac{1}{2}\vec{P}+\vec{q},\tfrac{1}{2}\vec{P}+\vec{k})\,t(\tfrac{1}{2}\vec{P}-\vec{q},\tfrac{1}{2}\vec{P}-\vec{k})]^{1/2} \tag{II.14}$$

for the scalar and pseudoscalar coupling where $t(\vec{p},\vec{p}\,') = (\sqrt{1+\vec{p}^{\,2}} - \sqrt{1+\vec{p}\,'^{2}}\,)^2 - (\vec{p}-\vec{p}\,')^2$ and

$$V^{rs,r's'}(\tilde{q},\tilde{k}) \underset{(\tilde{q}-\tilde{k})^2 \to \infty}{\simeq} [(\tilde{q}-\tilde{k})^2]^{-1-\Delta}(\tilde{q}^2)^{1/2}(\tilde{k}^2)^{1/2} \tag{II.15}$$

for the vector coupling. The last two terms in Eqs.(II.14) and (II.15) correspond to the estimate of

$$[\tilde{w}_1^r(\tfrac{1}{2}\vec{P}+\vec{q})\Gamma^{(1)}_{(\mu)}\, w_1^{r'}(\tfrac{1}{2}\vec{P}+\vec{k})][\tilde{w}_2^s(\tfrac{1}{2}\vec{P}-\vec{q})\Gamma^{(2)(\mu)}\, w_2^{s'}(\tfrac{1}{2}\vec{P}-\vec{k})] \ . \tag{II.16}$$

The large momentum behavior of the vertex function $\chi^{rs}_{\vec{P}}(\vec{q})$ can now be traced as in the spin-zero case. It is again sufficient to consider $W = V$ only for the same reasons given before. We obtain

$$\chi^{rs}_{\vec{P}}(\vec{q}) \simeq [\tilde{q}^2]^{-1-\Delta}[t(\tfrac{1}{2}\vec{P}+\vec{q},\tfrac{1}{2}\vec{P})\ t(\tfrac{1}{2}\vec{P}-\vec{q},\tfrac{1}{2}\vec{P})]^{1/2} \tag{II.17}$$

for the scalar and γ_5 interaction and

$$\chi^{rs}_{\vec{P}}(\vec{q}) \simeq [\tilde{q}^2]^{-1-\Delta}\ [t(\tfrac{1}{2}\vec{P}+\vec{q},\tfrac{1}{2}\vec{P}) + t(\tfrac{1}{2}\vec{P}-\vec{q},\tfrac{1}{2}\vec{P})] \tag{II.18}$$

for the vector coupling for $\Delta > 0$. The $\bar{\psi}\,\Gamma_{(\mu)}\psi\,\phi^{(\mu)}$ interactions can be included defining the physical solution as in the spin-zero case (i.e., $\Delta \to 0$ at the end of the calculation).

Some further remarks concerning the asymptotic forms (II.17) and (II.18) are in order. First, we like to recall that the large-momentum behavior is not uniform in $\vec{q}$ as in the spin-zero case. Secondly, in case of the γ_μ coupling the leading asymptotic behavior is governed by the z near $\pm \frac{1}{2}$ ($\vec{k} = z\,\vec{P}+\vec{k}_\perp$) region of integration in Eq.(II.10) in contrast to the spin-zero and scalar and γ_5 calculations. Here, the naive (and wrong) procedure of inverting the limits $\vec{P}$, $\vec{q} \to \infty$ and the integration over $\vec{k}$ leaves us with an infinite integral.

III. Three-Body Wave Function for Spin-Zero and Spin-1/2 Constituents

For the three-body case we shall assume only two-particle forces interacting in a ladder-type pattern as shown in Fig.2 so that the dynamics of the three-body system is governed by the BS interaction kernel introduced before and some relativistic Faddeev equation.[1,10]

Making use of the Faddeev decomposition $\phi=\phi^{(1)}+\phi^{(2)}+\phi^{(3)}$ we can write down a BS type of equation for the various components of the vertex function

$$\phi^{(1)}(p_1,p_2,p_3) = \int d^4p_2'\; T^{(1)}(p_2,p_3;p_2',p_2+p_3-p_2')\; G_2(p_2')\; G_3(p_2+p_3-p_2') \tag{III.1}$$

$$\times\,[\phi^{(2)}(p_1,p_2',p_2+p_3-p_2') + \phi^{(3)}(p_1,p_2',p_2+p_3-p_2')]$$

and similarly for $\phi^{(2)}$ and $\phi^{(3)}$. Here $T^{(1)}$ denotes the two-body BS T-matrix, i.e., $T^{(1)} = V + V\, G_2\, G_3\, T^{(1)}$. Symbolically, this can also be written

$$\begin{pmatrix} \phi^{(1)} \\ \phi^{(2)} \\ \phi^{(3)} \end{pmatrix} = \begin{pmatrix} 0 & T^{(1)}G_2G_3 & T^{(1)}G_2G_3 \\ T^{(2)}G_3G_1 & 0 & T^{(2)}G_3G_1 \\ T^{(3)}G_1G_2 & T^{(3)}G_1G_2 & 0 \end{pmatrix} \begin{pmatrix} \phi^{(1)} \\ \phi^{(2)} \\ \phi^{(3)} \end{pmatrix} \qquad \text{(III.2)}$$

In the following it will prove to be useful to consider the second iteration of the vertex equation (Fig.3)

$$\begin{pmatrix} \phi^{(1)} \\ \phi^{(2)} \\ \phi^{(3)} \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} G_1G_2G_3 \begin{pmatrix} \phi^{(1)} \\ \phi^{(2)} \\ \phi^{(3)} \end{pmatrix} \qquad \text{(III.3)}$$

where we have written

$$\begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} = \qquad \text{(III.4)}$$

$$= \begin{pmatrix} T^{(1)}G_3T^{(2)}+T^{(1)}G_2T^{(3)} & T^{(1)}G_2T^{(3)} & T^{(1)}G_3T^{(2)} \\ T^{(2)}G_1T^{(3)} & T^{(2)}G_3T^{(1)}+T^{(2)}G_1T^{(3)} & T^{(2)}G_3T^{(1)} \\ T^{(3)}G_1T^{(2)} & T^{(3)}G_2T^{(1)} & T^{(3)}G_2T^{(1)}+T^{(3)}G_1T^{(2)} \end{pmatrix}$$

Similar to the two-body case we then define[4] $G_1G_2G_3 = E_3+R_3$, where E_3 has only three-particle singularities, which leads us to the three-body analogue of the BLS equation (II.3)

$$\begin{pmatrix}\phi^{(1)}\\ \phi^{(2)}\\ \phi^{(3)}\end{pmatrix} = \left[1-\begin{pmatrix}V_{11} & V_{12} & V_{13}\\ V_{21} & V_{22} & V_{23}\\ V_{31} & V_{32} & V_{33}\end{pmatrix}R_3\right]^{-1} \begin{pmatrix}V_{11} & V_{12} & V_{13}\\ V_{21} & V_{22} & V_{23}\\ V_{31} & V_{32} & V_{33}\end{pmatrix} E_3 \begin{pmatrix}\phi^{(1)}\\ \phi^{(2)}\\ \phi^{(3)}\end{pmatrix} \qquad \text{(III.5)}$$

Let us now first discuss the spin-zero case. In order to calculate E_3 we introduce c.m. variables $q^{(1)}$ and $k^{(1)}$ by

$$\begin{aligned} p_1 &= \tfrac{1}{3}P + q^{(1)} & & \\ p_2 &= \tfrac{1}{3}P - \tfrac{1}{2}q^{(1)} + k^{(1)} & \qquad k^{(1)} &= \tfrac{1}{2}(p_2 - p_3) \\ p_3 &= \tfrac{1}{3}P - \tfrac{1}{2}q^{(1)} - k^{(1)} & \qquad q^{(1)} &= -\tfrac{1}{3}(p_2 + p_3 - 2p_1) \end{aligned} \qquad \text{(III.6)}$$

Similarly, we define $q^{(2)}$, $k^{(2)}$ and $q^{(3)}$, $k^{(3)}$ by cyclic permutation of the particle indices in Eq.(III.6). Then E_3 may be written (again $s = P^2$, $s' = P'^2$ and $\vec{P}' = \vec{P}$)

$$\begin{aligned} E_3 &= (2\pi)^2 \int ds' \frac{1}{s'-s}\, \delta^+(1 - (\tfrac{1}{3}P'-q^{(1)})^2) \\ &\quad \times \delta^+(1-(\tfrac{1}{3}P' - \tfrac{1}{2}\, q^{(1)}+k^{(1)})^2)\ \delta^+(1-(\tfrac{1}{3}P' - \tfrac{1}{2}\, q^{(1)} - k^{(1)})^2) \\ &= \pi^2\ \delta\Big(q_o^{(1)}-\tfrac{2}{3}\sqrt{1+(\tfrac{1}{3}\vec{P}+\vec{q}^{(1)})^2}+\tfrac{1}{3}\sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}+\vec{k}^{(1)})^2}+\tfrac{1}{3}\sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}-\vec{k}^{(1)})^2}\Big) \\ &\quad \times \delta\Big(k_o^{(1)}-\tfrac{1}{2}\Big(\sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}+\vec{k}^{(1)})^2}-\sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}-\vec{k}^{(1)})^2}\Big)\Big)\ \mathcal{E}(\vec{q}^{(1)},\vec{k}^{(1)}), \end{aligned}$$

$$\begin{aligned} \mathcal{E}(\vec{q}^{(1)},\vec{k}^{(1)}) &= \frac{\sqrt{1+(\tfrac{1}{3}\vec{P}+\vec{q}^{(1)})^2}+\sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}+\vec{k}^{(1)})^2}+\sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}-\vec{k}^{(1)})^2}}{\sqrt{1+(\tfrac{1}{3}\vec{P}+\vec{q}^{(1)})^2}\ \sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}+\vec{k}^{(1)})^2}\ \sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}-\vec{k}^{(1)})^2}} \\ &\quad \times \Big[\Big(\sqrt{1+(\tfrac{1}{3}\vec{P}+\vec{q}^{(1)})^2}+\sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}+\vec{k}^{(1)})^2}+\sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}-\vec{k}^{(1)})^2}\Big)^2-\vec{P}^2-s\Big]^{-1} \end{aligned}$$

(III.7)

and, equivalently for any other choice of variables, i.e., $\vec{q}^{(2)}$, $\vec{k}^{(2)}$ or $\vec{q}^{(3)}$, $\vec{k}^{(3)}$. This gives explicitly

$$\chi^{(1)}_{\vec{P}}(\vec{q}^{(1)},\vec{k}^{(1)}) = \int d^3\vec{q}^{(1)'} \int d^3\vec{k}^{(1)'}\, W_{11}(\tilde{q}^{(1)},\tilde{k}^{(1)};\, \tilde{q}^{(1)'},\tilde{k}^{(1)'})$$

$$\times\, \mathcal{L}(\vec{q}^{(1)'},\vec{k}^{(1)'})\, \chi^{(1)}_{\vec{P}}(\vec{q}^{(1)'},\vec{k}^{(1)'})$$

$$+ \int d^3\vec{q}^{(2)'} \int d^3\vec{k}^{(2)'}\, W_{12}\, (\tilde{q}^{(1)},\tilde{k}^{(1)};\tilde{q}^{(2)'},\tilde{k}^{(2)'})$$

$$\times\, \mathcal{L}(\vec{q}^{(2)'},\vec{k}^{(2)'})\, \chi^{(2)}_{\vec{P}}(\vec{q}^{(2)'},\vec{k}^{(2)'})$$

$$+ \int d^3\vec{q}^{(3)'} \int d^3\vec{k}^{(3)'}\, W_{13}\, (\tilde{q}^{(1)},\tilde{k}^{(1)};\tilde{q}^{(3)'},\tilde{k}^{(3)'})$$

$$\times\, \mathcal{L}(\vec{q}^{(3)'},\vec{k}^{(3)'})\, \chi^{(3)}_{\vec{P}}(\vec{q}^{(3)'},\vec{k}^{(3)'}) \qquad \text{(III.8)}$$

and similarly for $\chi^{(2)}_{\vec{P}}(\vec{q}^{(2)},\vec{k}^{(2)})$ and $\chi^{(3)}_{\vec{P}}(\vec{q}^{(3)},\vec{k}^{(3)})$. Here W_{ij} means the three-body analogue of W with V replaced by V_{ij} and (we now parameterize the Faddeev components $\phi^{(i)}$ in the form $\phi^{(i)}_{\vec{P}}(q^{(i)},k^{(i)})$)

$$\chi^{(i)}_{\vec{P}}(\vec{q}^{(i)},\vec{k}^{(i)}) = \phi^{(i)}_{\vec{P}}(\tilde{q}^{(i)},\tilde{k}^{(i)}),$$

$$\tilde{q}^{(i)} = \left(\frac{2}{3}\sqrt{1+(\tfrac{1}{3}\vec{P}+\vec{q}^{(i)})^2} - \frac{1}{3}\sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(i)}+\vec{k}^{(i)})^2} - \frac{1}{3}\sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(i)}-\vec{k}^{(i)})^2},\, \vec{q}^{(i)}\right),$$

$$\tilde{k}^{(i)} = \left(\frac{1}{2}\left(\sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(i)}+\vec{k}^{(i)})^2} - \sqrt{1+(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(i)}-\vec{k}^{(i)})^2}\right),\, \vec{k}^{(i)}\right). \qquad \text{(III.9)}$$

We are now interested in the large-momentum behavior of the BLS vertex functions $\chi^{(i)}_{\vec{P}}(\vec{q}^{(i)},\vec{k}^{(i)})$. For this purpose we keep only the first order term (in the expansion) of W_{ij} in the BLS equation (III.8) following the same arguments as before, i.e., $W_{ij} = V_{ij}$ We put also $T^{(i)}=V$ since they have the same high momentum behavior up to logarithms. The two-body BS potential V is taken to be of the (same) form (II.6). Hence, we are left with the kernel (in the simplest parametrization)

$$V_{13}(\tilde{q}^{(1)},\tilde{k}^{(1)};\tilde{q}^{(2)'},\tilde{k}^{(2)'}) = V(\tilde{k}^{(1)}, \tfrac{1}{2}\tilde{q}^{(1)} + \tilde{q}^{(2)'})$$

$$\times \frac{1}{(\frac{1}{3}P-\tilde{q}^{(1)}-\tilde{q}^{(2)'})^2-1} V(-\tilde{q}^{(1)} - \tfrac{1}{2}\tilde{q}^{(2)'}, \tilde{k}^{(2)'}),$$

$$V_{12}(\tilde{q}^{(1)},\tilde{k}^{(1)};\tilde{q}^{(3)'},\tilde{k}^{(3)'}) = V(\tilde{k}^{(1)}, -\tfrac{1}{2}\tilde{q}^{(1)} - \tilde{q}^{(3)'})$$

$$\times \frac{1}{(\frac{1}{3}P-\tilde{q}^{(1)}-\tilde{q}^{(3)'})^2-1} V(-\tilde{q}^{(1)} - \tfrac{1}{2}\tilde{q}^{(3)'}, \tilde{k}^{(3)'}),$$

$$V_{23}(\tilde{q}^{(2)},\tilde{k}^{(2)};\tilde{q}^{(1)'},\tilde{k}^{(1)'}) = V(\tilde{k}^{(2)}, -\tfrac{1}{2}\tilde{q}^{(2)} - \tilde{q}^{(1)'})$$

$$\times \frac{1}{(\frac{1}{3}P-\tilde{q}^{(2)}-\tilde{q}^{(1)'})^2-1} V(-\tilde{q}^{(2)} - \tfrac{1}{2}\tilde{q}^{(1)'}, -\tilde{k}^{(1)'}),$$

$$V_{11} = V_{12} + V_{13}, \quad V_{22} = V_{21} + V_{23}, \quad V_{33} = V_{31} + V_{32}, \qquad \text{(III.10)}$$

whose large momentum behavior can easily be deduced from Eq.(II.6). The particular sets of relative momenta are related by

$$q^{(2)} = -\tfrac{1}{2}q^{(1)} + k^{(1)}, \quad k^{(2)} = -\tfrac{3}{4}q^{(1)} - \tfrac{1}{2}k^{(1)},$$

$$q^{(3)} = -\tfrac{1}{2}q^{(1)} - k^{(1)}, \quad k^{(3)} = \tfrac{3}{4}q^{(1)} - \tfrac{1}{2}k^{(1)} \qquad \text{(III.11)}$$

V_{21}, V_{31}, V_{32} are obtained from (III.10) by permutation of indeces.

The asymptotic behavior of the BLS vertex function can now be read off from the BLS equation (III.8). For $\theta > 0$ the only consistent solution has the asymptotic form

$$\chi_{\vec{P}}^{(i)}(\vec{q}^{(i)},\vec{k}^{(i)}) \simeq [(-\tfrac{1}{2}\tilde{q}^{(i)}+\tilde{k}^{(i)})^2]^{-\theta}[\tilde{q}^{(i)2}]^{-\theta}[(\tfrac{1}{3}P-\tilde{q}^{(i)})^2]^{-1}$$

$$+ [(\tfrac{1}{2}\tilde{q}^{(i)}+\tilde{k}^{(i)})^2]^{-\theta}\ [\tilde{q}^{(i)2}]^{-\theta}[(\tfrac{1}{3}P-\tilde{q}^{(i)})^2]^{-1} \qquad \text{(III.12)}$$

(for the limiting case $\theta \to 0$ see the two-body case). As we would expect from the two-body calculations the asymptotic behavior (III.12) is not uniform in $\vec{q}^{(i)}$, $\vec{k}^{(i)}$. If we write $\vec{q}^{(i)} = x\,\vec{P} + \vec{q}_{\perp}^{(i)}$ and $\vec{k}^{(i)} = z\,\vec{P} + \vec{k}_{\perp}^{(i)}$ we find $\chi_{\vec{P}}^{(i)}(\vec{q}^{(i)},\vec{k}^{(i)}) \simeq |\vec{P}|^{-1-2\theta}$ for $x = -\frac{1}{3}$ and/or $z = \pm(\frac{1}{3} - \frac{1}{2}x)$ whereas $\chi_{\vec{P}}^{(i)}(\vec{q}^{(i)},\vec{k}^{(i)}) \simeq$ const. elsewhere.

The spin-1/2 case (when treated in form of the BLS equation) is very much the same as the spin-zero calculation. The only problem arises from the somewhat more subtle spin structure of the BLS kernel. We shall consider a spin-1/2 bound state, and the two-body interaction be of the form (II.13). Formally the integral equations (III.2)-(III.3) and (III.5) remain the same but now $G_i^{-1}(p) = \gamma^{(i)}p-1$ and

$$E_3 \to E_3\, \Lambda_+^{(1)}(\tfrac{1}{3}\vec{P}+\vec{q}^{(1)})\ \Lambda_+^{(2)}(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}+\vec{k}^{(1)})\ \Lambda_+^{(3)}(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}-\vec{k}^{(1)}) \qquad \text{(III.13)}$$

and similarly for any other parameterization (i.e., in terms of $q^{(2)}$, $k^{(2)}$ or $q^{(3)}$, $k^{(3)}$). The projection operation $\Lambda_+^{(i)}$ can be absorbed into the wave function and the kernel following Sec.II. Writing

($\chi^{(i)}_{\vec{P},M}(\vec{q}^{(i)}, \vec{k}^{(i)}) = \phi^{(i)}_{\vec{P},M}(\tilde{q}^{(i)}, \tilde{k}^{(i)})$ now where $\phi^{(i)}_{\vec{P},M}(q^{(i)},k^{(i)})$ is the corresponding BS vertex function; M denotes the spin of the bound state)

$$\chi^{(1)rsm}_{\vec{P},M}(\vec{q}^{(1)},\vec{k}^{(1)}) = \tilde{w}^{r}_{1}(\tfrac{1}{3}\vec{P}+\vec{q}^{(1)})\ \tilde{w}^{s}_{2}(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}+\vec{k}^{(1)})$$

$$\times\ \tilde{w}^{m}_{3}(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}-\vec{k}^{(1)})\chi^{(1)}_{\vec{P},M}(\vec{q}^{(1)},\vec{k}^{(1)})\ ,$$

$$W^{rsm,r's'm'}_{11}(\tilde{q}^{(1)},\tilde{k}^{(1)};\tilde{q}^{(1)'},\tilde{k}^{(1)'}) = \tilde{w}^{r}_{1}(\tfrac{1}{3}\vec{P}+\vec{q}^{(1)})\tilde{w}^{s}_{2}(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}+\vec{k}^{(1)})$$

$$\times\ \tilde{w}^{m}_{3}(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}-\vec{k}^{(1)})\ W_{11}(\tilde{q}^{(1)},\tilde{k}^{(1)};\tilde{q}^{(1)'},\tilde{k}^{(1)'})w^{r'}_{1}(\tfrac{1}{3}\vec{P}+\vec{q}^{(1)'})$$

$$\times\ w^{s'}_{2}(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)'}+\vec{k}^{(1)'})w^{m'}_{3}(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)'}-\vec{k}^{(1)'}), \qquad \text{(III.14)}$$

and similarly for the other components, we are led to the integral equation (e.g.)

$$\chi^{(1)rsm}_{\vec{P},M}(\vec{q}^{(1)},\vec{k}^{(1)}) =$$

$$= \sum_{i=1}^{3}\int d^{3}\vec{q}^{(i)'}\int d^{3}\vec{k}^{(i)'}\ W^{rsm,r's'm'}_{1i}(\tilde{q}^{(1)},\tilde{k}^{(1)};\tilde{q}^{(i)'},\tilde{k}^{(i)'})$$

$$\times\ \mathcal{L}(\vec{q}^{(i)'},\vec{k}^{(i)'})\ \chi^{(i)r's'm'}_{\vec{P},M}(\vec{q}^{(i)'},\vec{k}^{(i)'})\ . \qquad \text{(III.15)}$$

As far as the asymptotic behavior of the BLS kernel (cf. Eq. (III.15)) is concerned we may focus our attention on V_{ij} (with $T^{(i)} = V$), e.g.,

$$V_{13}^{rsm,r's'm'}(\tilde{q}^{(1)},\tilde{k}^{(1)};\tilde{q}^{(2)'},\tilde{k}^{(2)'}) \simeq \tilde{w}_1^r(\tfrac{1}{3}\vec{P}+\vec{q}^{(1)})\Gamma^{(1)}_{(\mu)}\; w_1^{r'}(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(2)'}-\vec{k}^{(2)'})$$

$$\times \tilde{w}_2^s(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}+\vec{k}^{(1)})\;\Gamma^{(2)}_{(\nu)}\; w_2^{s'}(\tfrac{1}{3}\vec{P}+\vec{q}^{(2)'})\;\tilde{w}_3^m(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}-\vec{k}^{(1)})$$

$$\times\; \Gamma^{(3)(\nu)}(\gamma^{(3)}(\tfrac{1}{3}P-\tilde{q}^{(1)}-\tilde{q}^{(2)'})+1)\Gamma^{(3)(\mu)}w_3^{m'}(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(2)'}+\vec{k}^{(2)'})$$

$$\times[(\tilde{k}^{(1)}-\tfrac{1}{2}\tilde{q}^{(1)}-\tilde{q}^{(2)'})^2]^{-1-\Delta}[(\tilde{q}^{(1)}+\tfrac{1}{2}\tilde{q}^{(2)'}+\tilde{k}^{(2)'})^2]^{-1-\Delta}[(\tfrac{1}{3}P-\tilde{q}^{(1)}-\tilde{q}^{(2)'})^2-1]^{-1}$$

(III.16)

which asymptotically becomes

$$V_{13}^{rsm,r's'm'}(\tilde{q}^{(1)},\tilde{k}^{(1)};\tilde{q}^{(2)'},\tilde{k}^{(2)'}) \simeq [(\tilde{k}^{(1)}-\tfrac{1}{2}\tilde{q}^{(1)}-\tilde{q}^{(2)'})^2]^{-1-\Delta}$$

$$\times\;[(\tilde{q}^{(1)}+\tfrac{1}{2}\tilde{q}^{(2)'}+\tilde{k}^{(2)'})^2]^{-1-\Delta}\;[(\tfrac{1}{3}P-\tilde{q}^{(1)}-\tilde{q}^{(2)'})^2]^{-1} \qquad \text{(III.17)}$$

$$\times\;[t(\tfrac{1}{3}\vec{P}+\vec{q}^{(1)},\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(2)'}-\vec{k}^{(2)'})\;t(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}+\vec{k}^{(1)},\;\tfrac{1}{3}\vec{P}+\vec{q}^{(2)'})$$

$$\times\;t(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(1)}-\vec{k}^{(1)},\xi_p(\tfrac{1}{3}\vec{P}-\vec{q}^{(1)}-\vec{q}^{(2)'}))$$

$$\times\;t(\xi_p(\tfrac{1}{3}\vec{P}-\vec{q}^{(1)}-\vec{q}^{(2)'}),\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(2)'}+\vec{k}^{(2)'})]^{1/2}$$

for the scalar and γ_5 coupling and

$$V_{13}^{rsm,r's'm'}(\tilde{q}^{(1)},\tilde{k}^{(1)};\tilde{q}^{(2)}),\tilde{k}^{(2)'}) \simeq [(\tilde{k}^{(1)}-\tfrac{1}{2}\tilde{q}^{(1)}-\tilde{q}^{(2)'})^2]^{-1-\Delta}$$

$$\times\;[(\tilde{q}^{(1)}+\tfrac{1}{2}\tilde{q}^{(2)'}+\tilde{k}^{(2)'})^2]^{-1-\Delta}[(\tfrac{1}{3}P-\tilde{q}^{(1)}-\tilde{q}^{(2)'})^2]^{-1}$$

$$\times\;(\tilde{k}^{(1)2})^{\frac{1}{2}}\;(\tilde{k}^{(2)'2})^{\frac{1}{2}}\;[t(\tfrac{1}{3}\vec{P}+\vec{q}^{(1)},\xi_p(\tfrac{1}{3}\vec{P}-\vec{q}^{(1)}-\vec{q}^{(2)'}))$$

$$\times\;t(\tfrac{1}{3}\vec{P}+\vec{q}^{(2)'},\xi_p(\tfrac{1}{3}\vec{P}-\vec{q}^{(1)}-\vec{q}^{(2)'}))]^{1/2} \qquad \text{(III,18)}$$

for the γ_μ interaction. In the spin factors the internal line $\frac{1}{3}P-\tilde{q}^{(1)}-\tilde{q}^{(2)'}$ appears on the mass shell which provides the leading contribution. Here, ξ_p means the signature of the parallel component of the vector $\frac{1}{3}\vec{P}-\vec{q}^{(1)}-\vec{q}^{(2)'}$.

In case of the scalar and γ_5 coupling the asymptotic behavior of the BLS vertex function can now be read off from the asymptotic form of the BLS kernel (III.17) giving ($\Delta > 0$; for $\Delta \to 0$ see the two-body case)

$$
\begin{aligned}
\chi_{\vec{P}}^{(i)rsm}(\vec{q}^{(i)},\vec{k}^{(i)}) \simeq\; & [(-\tfrac{1}{2}\tilde{q}^{(i)}+\tilde{k}^{(i)})^2]^{-1-\Delta}[\tilde{q}^{(i)2}]^{-1-\Delta} \\
& \times [(\tfrac{1}{3}P-\tilde{q}^{(i)})^2]^{-1}\ [t(\tfrac{1}{3}\vec{P}+\vec{q}^{(i)},\ \tfrac{1}{3}\vec{P})\,t(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(i)}+\vec{k}^{(i)},\ \tfrac{1}{3}\vec{P}) \\
& \times t(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(i)}-\vec{k}^{(i)},\ \tfrac{1}{3}\vec{P}-\vec{q}^{(i)})\,t(\tfrac{1}{3}\vec{P}-\vec{q}^{(i)},\tfrac{1}{3}\vec{P})]^{\frac{1}{2}} \\
& + [(\tfrac{1}{2}\tilde{q}^{(i)}+\tilde{k}^{(i)})^2]^{-1-\Delta}[\tilde{q}^{(i)2}]^{-1-\Delta}[(\tfrac{1}{3}P-\tilde{q}^{(i)})^2]^{-1} \\
& \times [t(\tfrac{1}{3}\vec{P}+\vec{q}^{(i)},\ \tfrac{1}{3}\vec{P})\ t(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(i)}-\vec{k}^{(i)},\ \tfrac{1}{3}\vec{P}) \\
& \times t(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(i)}+\vec{k}^{(i)},\ \tfrac{1}{3}\vec{P}-\vec{q}^{(i)})\ t(\tfrac{1}{3}\vec{P}-\vec{q}^{(i)},\ \tfrac{1}{3}\vec{P})]^{\frac{1}{2}}
\end{aligned}
\tag{III.19}
$$

It can easily be checked that this is a consistent solution which means, naively, that the integral (III.15) remains finite once the $\vec{q}^{(i)}$, $\vec{k}^{(i)}$ dependence of the kernel is taken out of the integral. For the γ_μ coupling this simple procedure does not work as we already found in the two-body spin-1/2 case. The asymptotic behavior one would read off from the kernel leads to an infinite integral when applied consistently. As in the two-body case, if we invert the limits and the integration, we end up with a divergent integral. Correctly, we obtain

$$\chi_{\vec{P}}^{(i)rsm}(\vec{q}^{(i)},\vec{k}^{(i)}) \simeq [(-\tfrac{1}{2}\tilde{q}^{(i)}+\tilde{k}^{(i)})^2]^{-1-\Delta}\,[\tilde{q}^{(i)2}]^{-1-\Delta}$$

$$\times\,[(\tfrac{1}{3}P-\tilde{q}^{(i)})^2]^{-1}[t(\tfrac{1}{3}\vec{P}+\vec{q}^{(i)},\tfrac{1}{3}\vec{P})\,t(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(i)}+\vec{k}^{(i)},\tfrac{1}{3}\vec{P})$$

$$+\,t^2(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(i)}-\vec{k}^{(i)},\tfrac{1}{3}\vec{P})]\;+\;[(\tfrac{1}{2}\tilde{q}^{(i)}+\tilde{k}^{(i)})^2]^{-1-\Delta}[\tilde{q}^{(i)2}]^{-1-\Delta}$$

$$\times\,[(\tfrac{1}{3}P-\tilde{q}^{(i)})^2]^{-1}[t(\tfrac{1}{3}\vec{P}+\vec{q}^{(i)},\,\tfrac{1}{3}\vec{P})\,t(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(i)}-\vec{k}^{(i)},\,\tfrac{1}{3}\vec{P})$$

$$+\,t^2(\tfrac{1}{3}\vec{P}-\tfrac{1}{2}\vec{q}^{(i)}+\vec{k}^{(i)},\,\tfrac{1}{3}\vec{P})]\quad. \qquad \text{(III.20)}$$

IV. Form Factor for Two- and Three-Body Bound States: Spin-Zero Constituents

We now come to use the estimates of the BLS vertex functions and calculate the asymptotic behavior of the form factors for two- and three-body bound states. In the spinless case we will recover the results of our earlier work based on the BS equation which may be considered a test of the BLS approach. Apart from this, we expect the BLS vertex function giving quite generally the right asymptotic behavior of the form factors when written in the Breit frame. We shall see that this circumstance also has a nice physical interpretation.

The two-body BS form factor reads in the Breit frame (Fig.4)

$$F(Q^2) \simeq \int d^4k \; \phi_{-\vec{P}}(k-\tfrac{1}{2}Q) \frac{1}{(\frac{1}{2}P+k-Q)^2-1} \frac{1}{(\frac{1}{2}P+k)^2-1} \frac{1}{(\frac{1}{2}P-k)^2-1} \phi_{\vec{P}}(k) \tag{IV.1}$$

where $Q = (0,2\vec{P})$. The single particle propagators can (in the Breit frame) also be written $G_1(\frac{1}{2}P+k-Q)G_1(\frac{1}{2}P+k)G_2(\frac{1}{2}P-k) = K_2 + L_2$ where (in the spirit of the BLS approach; see Eq.(II.2))

$$K_2 = 2\pi^2 \int ds' \frac{1}{s'-s} \delta^+((\tfrac{1}{2}P'+k-Q)^2-1)\delta^+((\tfrac{1}{2}P'+k)^2-1)\delta^+((\tfrac{1}{2}P'-k)^2-1)$$

$$= \frac{\pi}{4|\vec{P}|} \delta(k_P - \tfrac{1}{2}|\vec{P}|) \; E_2 \; , \; k_P = \frac{\vec{k}\cdot\vec{P}}{|\vec{P}|} \quad . \tag{IV.2}$$

By construction K_2 has only two-particle singularities while L_2 evidently is accompanied by three-particle singularities. For $\vec{P} \to \infty$ the contribution arising from L_2 now vanishes relative to the (leading) K_2 contribution, i.e.,

$$G_1(\tfrac{1}{2}P+k-Q) \; G_1(\tfrac{1}{2}P+k) \; G_2(\tfrac{1}{2}P-k) \underset{\vec{P}\to\infty}{=} K_2 \tag{IV.3}$$

which makes the BLS approach a very useful tool for studying the asymptotic behavior of form factors of composite systems (note that Eq.(IV.3) makes implicit use of the interchangeability of the limit $\vec{P} \to \infty$ and the integration over k (Eq.(IV.2)) which, however, is justified for the cases we are going to consider here).

For large $\vec{P}$ the form factor then reads

$$F(Q^2) \simeq \frac{\pi^2}{8\vec{P}^2} \int d^2\vec{k}_\perp \; \chi_{-\vec{P}}(\vec{k}_\perp - \tfrac{1}{2}\vec{P})[1+\vec{k}_\perp^2]^{-1} \; \chi_{\vec{P}}(\vec{k}_\perp + \tfrac{1}{2}\vec{P}) \tag{IV.4}$$

where only the BLS vertex function enters (remember $\phi_{\vec{P}}(\tilde{k}) = \chi_{\vec{P}}(\vec{k})$). Physically this means that at large $\vec{P}$ the form factor becomes a convolution of two nonrelativistic clusters (i.e., having a definite number of constituents) which (later on) will allow an interpretation of our results in terms of the (nonrelativistic) quark model. Eq.(IV.4) also supports the suggestion of Licht and Pagnamenta[11] that, given a nonrelativistic wave function, the form factor is most adequately represented in the Breit frame because at large $\vec{P}$ the interaction can take place instantaneously. Geometrically the form factor (IV.4) can be interpreted as two flat (Lorentz-contracted) discs penetrating through each other.

Taking now the estimate (II.8) of the BLS vertex function the asymptotic behavior of the form factor for a bound state of two spin-zero constituents becomes

$$F(Q^2) \simeq [Q^2]^{-1-\theta} \tag{IV.5}$$

just as in the BS calculation.[1]

In the three-body case we can proceed in the same way. The BS form factor is written (Fig.5)

$$F(Q^2) \simeq \int d^4q^{(1)} \int d^4k^{(1)} \phi_{-\vec{P}}(q^{(1)} - \tfrac{2}{3}Q, k^{(1)}) \frac{1}{(\frac{1}{3}P + q^{(1)} - \frac{2}{3}Q)^2 - 1}$$

$$\times \frac{1}{(\frac{1}{3}P + q^{(1)})^2 - 1} \; \frac{1}{(\frac{1}{3}P - \frac{1}{2}q^{(1)} + k^{(1)})^2 - 1} \; \frac{1}{(\frac{1}{3}P - \frac{1}{2}q^{(1)} - k^{(1)})^2 - 1} \; \phi_{\vec{P}}(q^{(1)}, k^{(1)}) \tag{IV.6}$$

where the photon is coupled to particle 1. Similar to the two-body case we define $G_1(\frac{1}{3}P+q^{(1)}-\frac{2}{3}Q)G_1(\frac{1}{3}P+q^{(1)})G_2(\frac{1}{3}P-\frac{1}{2}q^{(1)}+k^{(1)}) \times$ $\times\, G_3(\frac{1}{3}P-\frac{1}{2}q^{(1)}-k^{(1)}) = K_3 + L_3$ where

$$K_3 = 4\pi^3\int ds' \frac{1}{s'-s}\, \delta^+((\tfrac{1}{3}P'+q^{(1)}-\tfrac{2}{3}Q)^2-1)\, \delta^+((\tfrac{1}{3}P'+q^{(1)})^2-1)$$

$$\times\, \delta^+((\tfrac{1}{3}P'-\tfrac{1}{2}q^{(1)}+k^{(1)})^2-1)\, \delta^+((\tfrac{1}{3}P'-\tfrac{1}{2}q^{(1)}-k^{(1)})^2-1)$$

$$= \frac{3\pi}{4|\vec{P}|}\, \delta(q_P^{(1)}-\tfrac{2}{3}|\vec{P}|)\, E_3\ ,\quad q_P^{(1)} = \frac{\vec{q}^{(1)}\cdot\vec{P}}{|\vec{P}|}\ . \tag{IV.7}$$

Here K_3 has only three-particle singularities while L_3 contains all the other time orderings.

As in the two-body case we now have

$$G_1(\tfrac{1}{3}P+q^{(1)}-\tfrac{2}{3}Q)G_1(\tfrac{1}{3}P+q^{(1)})G_2(\tfrac{1}{3}P-\tfrac{1}{2}q^{(1)}+k^{(1)})G_3(\tfrac{1}{3}P-\tfrac{1}{2}q^{(1)}-k^{(1)}) \underset{\vec{P}\to\infty}{=} K_3 \tag{IV.8}$$

so that the form factor (IV.6) becomes for large $\vec{P}$ $(\phi_{\vec{P}}(\tilde{q}^{(1)},\tilde{k}^{(1)}) = = \chi_{\vec{P}}(\vec{q}^{(1)},\vec{k}^{(1)}))$

$$F(Q^2) \simeq \frac{3\pi^3}{8\vec{P}^2}\int d^2\vec{q}_\perp^{(1)} \int d^3\vec{k}^{(1)}\ \chi_{-\vec{P}}(\vec{q}_\perp - \tfrac{2}{3}\vec{P},\vec{k}^{(1)})$$

$$\times\, [\sqrt{1+(\tfrac{1}{2}\vec{q}_\perp^{(1)}-\vec{k}^{(1)})^2}\ \sqrt{1+(\tfrac{1}{2}\vec{q}_\perp^{(1)}+\vec{k}^{(1)})^2} \tag{IV.9}$$

$$\times\, (\sqrt{1+(\tfrac{1}{2}\vec{q}_\perp^{(1)}-\vec{k}^{(1)})^2} + \sqrt{1+(\tfrac{1}{2}\vec{q}_\perp^{(1)}+\vec{k}^{(1)})^2})]^{-1}\ \chi_{\vec{P}}(\vec{q}_\perp^{(1)}+\tfrac{2}{3}\vec{P},\vec{k}^{(1)}).$$

Here, again, the asymptotic behavior is completely described by the BLS vertex function. Geometrically Eq.(IV.9) allows the same interpretation as the two-body form factor but with particle 2 replaced by the center-of-mass of particles 2 and 3.

If we now insert the asymptotic form (III.12) of the three-body vertex function in Eq.(IV.9) we obtain for the asymptotic behavior of the form factor for a bound state of three spin-zero constituents

$$F(Q^2) \simeq [Q^2]^{-2-2\theta} \tag{IV.10}$$

in agreement with the BS calculation.[1]

So far we have only recovered what we knew already before. In the next section our prologue will, however, meet its expenses.

V. Form Factor for Two- and Three-Body Bound States: Spin-1/2 Constituents

Now we come to our main (new) results. Let us first consider the two-body case. Decomposing the loop propagators into K_2 and L_2 where K_2 now is the spin-1/2 analogue of Eq.(IV.2) (for the general procedure see Eq.(III.13)) we again can show that only K_2 contributes to the asymptotic value of the form factor. Hence, we obtain for large $\vec{P}$

$$F(Q^2) \simeq \frac{\pi^2}{8\vec{P}^2} \int d^2\vec{k}_\perp \; \chi^{rs}_{-\vec{P}}(\vec{k}_\perp - \tfrac{1}{2}\vec{P})\,[1+\vec{k}_\perp^2]^{-1} \; \chi^{rs}_{\vec{P}}(\vec{k}_\perp + \tfrac{1}{2}\vec{P}) \tag{V.1}$$

making use of the definition (II.12) and $\tilde{w}^m(-\vec{P})\gamma_o\, w^n(\vec{P}) \simeq 2|\vec{P}|\delta_{mn}$ as $\vec{P} \to \infty$.

The asymptotic form of the BLS vertex function, Eqs. (II.17) and (II.18), inserted in Eq. (V.1) then gives

$$F(Q^2) \simeq [Q^2]^{-\frac{3}{2}-\Delta} \tag{V.2}$$

for the scalar and γ_5 interaction and

$$F(Q^2) \simeq [Q^2]^{-1-\Delta} \tag{V.3}$$

for the γ_μ coupling. For the moment we note that the large-momentum behavior of the bound-state form factor for spin-1/2 constituents depends apparently on the nature of the two-body forces.

In the three-body case we consider the spin-averaged form factor. Employing the same technique as before we find

$$\begin{aligned} F(Q^2) \simeq \; & \frac{3\pi^3}{32\vec{P}^2} \sum_{M,M'} \int d^2\vec{q}_\perp^{(1)} \int d^3\vec{k}^{(1)}\, \chi^{rsm}_{-\vec{P},M}(\vec{q}_\perp - \tfrac{2}{3}\vec{P}, \vec{k}^{(1)}) \\ & \times \Big[\sqrt{1+(\tfrac{1}{2}\vec{q}_\perp^{(1)}-\vec{k}^{(1)})^2}\, \sqrt{1+(\tfrac{1}{2}\vec{q}_\perp^{(1)}+\vec{k}^{(1)})^2} \\ & \times \Big(\sqrt{1+(\tfrac{1}{2}\vec{q}_\perp^{(1)}-\vec{k}^{(1)})^2} + \sqrt{1+(\tfrac{1}{2}\vec{q}_\perp+\vec{k}^{(1)})^2}\Big)\Big]^{-1} \chi^{rsm}_{\vec{P},M}(\vec{q}_\perp^{(1)} + \tfrac{2}{3}\vec{P}, \vec{k}^{(1)}) \,. \end{aligned} \tag{V.4}$$

Inserting the asymptotic form of the BLS vertex function, Eqs.(III.19) and (III.20), this gives the asymptotic behavior

$$F(Q^2) \simeq [Q^2]^{-3-2\Delta} \tag{V.5}$$

for scalar and pseudoscalar exchange and

$$F(Q^2) \simeq [Q^2]^{-2-2\Delta} \tag{V.6}$$

for the γ_μ coupling.

Here we have the same result that the asymptotic behavior of the form factor depends on the dynamics of the two-body system. This is in contrast to what the dimensional counting rules[3] are trying to make us believe. We agree with that analysis only in case of the vector coupling $(\Delta \to 0)$ while in case of the scalar and pseudoscalar coupling we are off by half a (one) power of Q^2 in the two- (three-) body form factor.

The two-body form factor has recently also been discussed by Ezawa[12] for vector gluon exchange $(\Delta \to 0)$. He obtains a monopole behavior in agreement with the dimensional counting rules and our result.

VI. Conclusions

We have given a thorough discussion of the asymptotic behavior of two- and three-body bound state form factors for both spin-zero and spin-1/2 constituents. Our calculations avoid some of the crucial assumptions inherent in the dimensional analysis and, as a matter of fact, lead to a different result for scalar and pseudoscalar couplings.

In order to match the (experimental) asymptotic behavior of the pion and nucleon form factors[2] our results suggest that the pion and nucleon have an underlying (non-relativistic) quark-antiquark and three-quark structure respectively at infinite momentum where the quarks interact via a vector-gluon exchange (being accomplished by taking the limit $\Delta \to 0$ at the end of the calculation).

Our investigation was, of course, not exclusively guided by the aim of verifying a quark-like structure of the pion and nucleon. We have covered a large scale of possible two-body interactions which has a wide range of applications in few-body problems. Because of the intimate relation of the large momentum transfer behavior of the form factors and the two-body interaction this (e.g., the deuteron form factor) might give some information about the particle (e.g., the nucleon-nucleon) forces at small distances.

The BLS approach has proven to be a powerful tool for studying infinite momentum dynamics. In a forthcoming paper[13] we shall complete our survey of constituent structures of the hadrons by systematically looking at the deep inelastic pion and nucleon structure functions, especially its threshold behavior, adopting the same technique.

Acknowledgement

We like to thank R. Blankenbecler and S. J. Brodsky for useful discussions and S.D. Drell for the warm hospitality he extended to us at SLAC.

References

1. C.Alabiso and G.Schierholz; Phys. Rev. D10, 960 (1974)

2. See for example: N.Silvestrini; Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, 1972, ed. by J.D.Jackson and A.Roberts (NAL, 1973), Vol.4, p.1; P.N.Kirk et al., Phys. Rev. D8, 63 (1973)

3. V.A.Matveev, R.M.Muradyan and A.N.Tavkhelidze; Nuovo Cimento Lett. 7, 719 (1973), S.J.Brodsky and G.R.Farrar; Phys. Rev. Lett. 31, 1153 (1973), and Caltech Report Nr. CALT 68-441 (1974)

4. R.Blankenbecler and R.Sugar, in Proceedings of the Twelfth International Conference on High Energy Physics, Dubna, 1964, edited by Ya.A.Smorodinskii (Atomizdat, Moskow, 1966); Phys. Rev. 142, 1051 (1966)

5. Effects of orbital angular momentum have been discussed by M. Ciafaloni; Phys. Rev. 176, 1898 (1968), D. Amati, L. Caneschi and R. Jengo; Nuovo Cimento 58A, 783 (1968)

6. The mass of the constituents is assumed to be $m = 1$.

7. Usually the approximation $W = V$ is called the Blankenbecler-Sugar equation. But here we refer to Eq.(II.3) as well as the BLS equation.

8. E. Speer; J. Math. Phys. 9, 1403 (1968)

9. See also R. H. Thompson; Phys. Rev. D1, 110 (1970)

10. For the relativistic three-body problem see quite generally D. Stoyanov and A. N. Tavkhelidze; Phys. Lett. 13, 76 (1964), V. P. Shelest and D. Stoyanov; Phys. Lett. 13, 256 (1964), R. Blankenbecler and R. Sugar; Ref.4, V. A. Alessandrini and R. L. Omnes ; Phys. Rev. 139, B167 (1965), V. M. Vinogradov; Yad. Fiz. 14, 1091 (1971) [Sov. Journ. Nucl. Phys. 14, 609 (1972)]

11. A. L. Licht and A. Pagnamenta; Phys. Rev. D2, 1150 (1970), ibid. D2, 1156 (1970)

12. Z. F. Ezawa; Cambridge Report Nr. DAMTP 74/5 (1974)

13. C. Alabiso and G. Schierholz; Stanford Linear Accelerator Center Report, unpublished.

Figure Captions

Fig.1 The BS equation for the vertex function of a two-body bound state.

Fig.2 Ladder-type diagrams for a system of three particles interacting via a two-body interaction.

Fig.3 The once-iterated Faddeev equation for the vertex function of a three-body bound state. The wavy lines represent the two-body BS T-matrix in the ladder approximation.

Fig.4 The electromagnetic form factor in the ladder approximation for a two-body bound state.

Fig.5 The electromagnetic form factor in the ladder approximation for a three-body bound state.

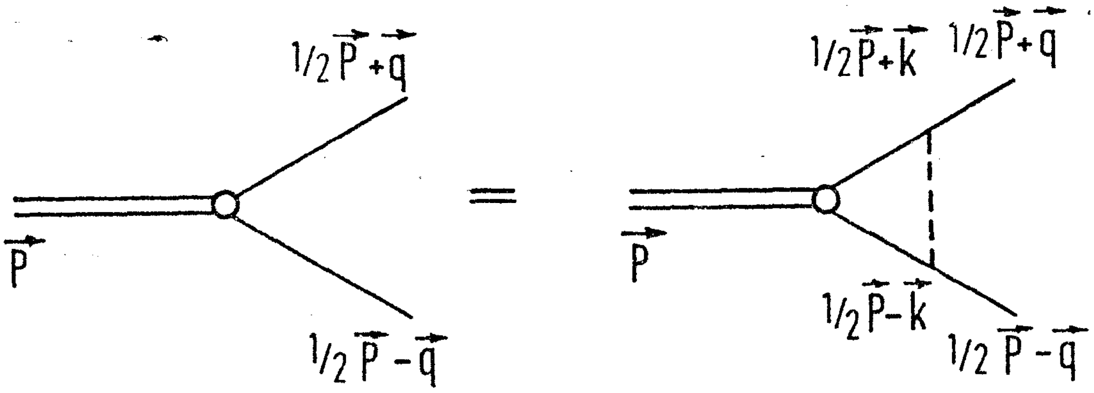

$\vec{P}$
$1/2\,\vec{P}+\vec{q}$
$1/2\,\vec{P}-\vec{q}$
$=$
$\vec{P}$
$1/2\,\vec{P}+\vec{k}$
$1/2\,\vec{P}+\vec{q}$
$1/2\,\vec{P}-\vec{k}$
$1/2\,\vec{P}-\vec{q}$

Fig.1

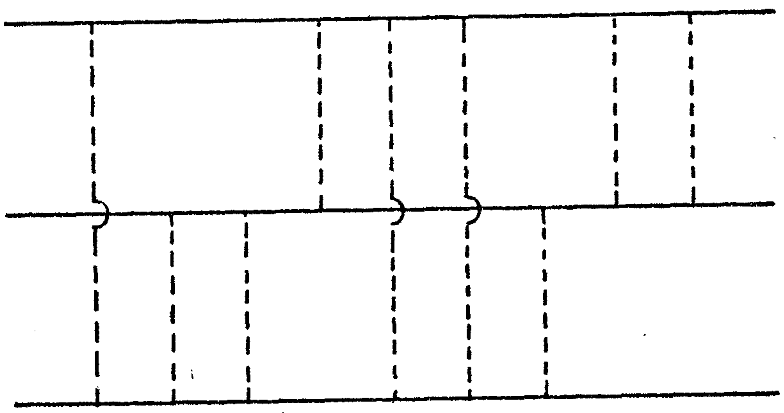


Fig.2

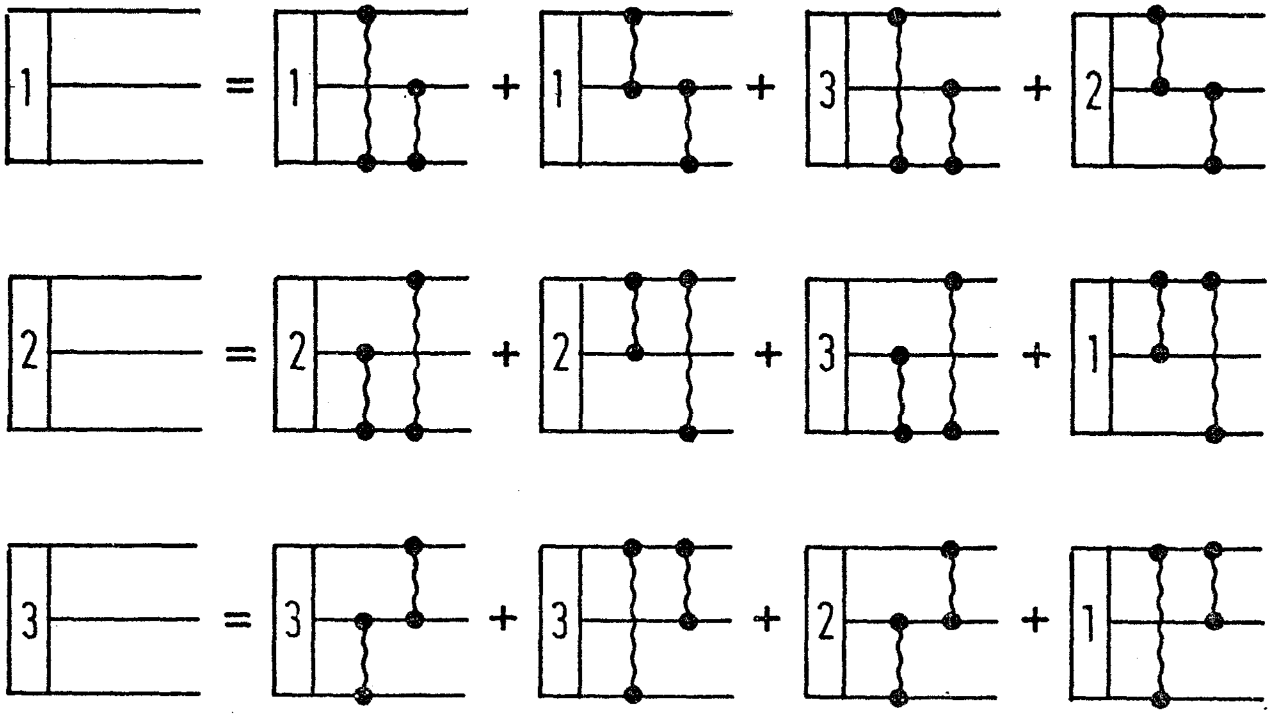
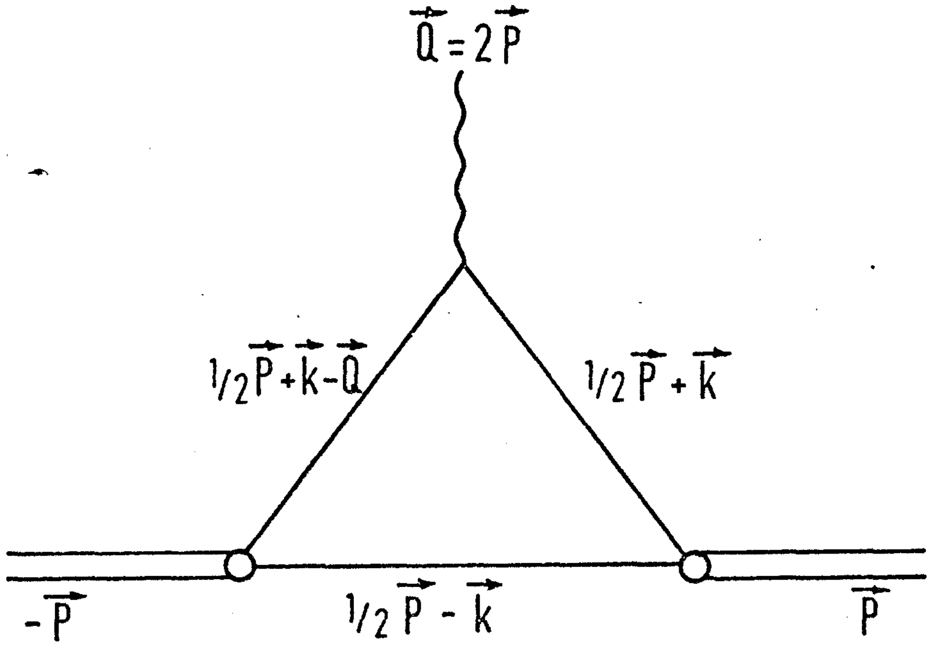
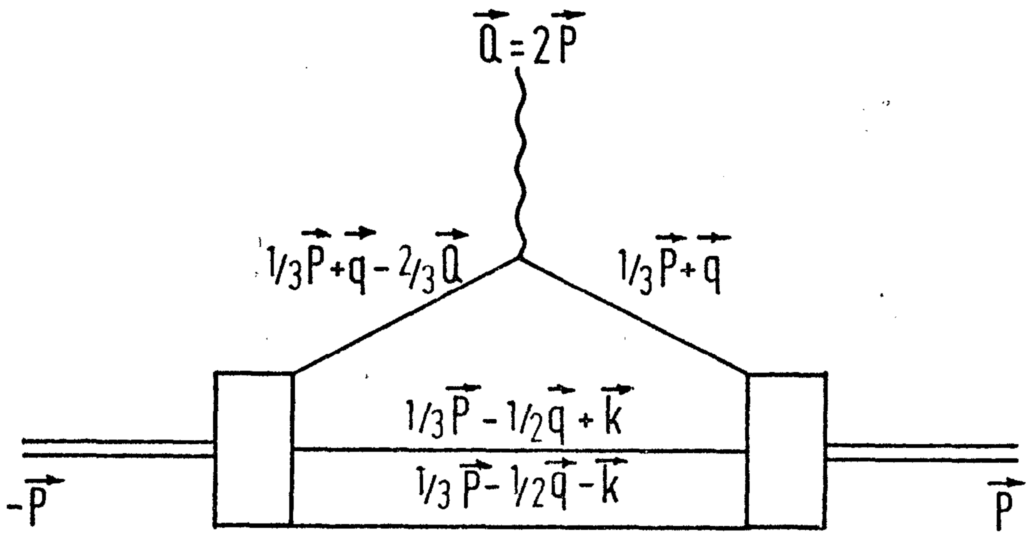


Fig.3



$\vec{Q} = 2\vec{P}$
$1/2\,\vec{P} + \vec{k} - \vec{Q}$
$1/2\,\vec{P} + \vec{k}$
$-\vec{P}$
$1/2\,\vec{P} - \vec{k}$
$\vec{P}$

Fig.4



$\vec{Q} = 2\vec{P}$
$1/3\,\vec{P} + \vec{q} - 2/3\,\vec{Q}$
$1/3\,\vec{P} + \vec{q}$
$1/3\,\vec{P} - 1/2\,\vec{q} + \vec{k}$
$1/3\,\vec{P} - 1/2\,\vec{q} - \vec{k}$
$-\vec{P}$
$\vec{P}$

Fig.5