# Scalar Field Mass Generation in the Gauge Theory $SU(2)\otimes U(1)\otimes Z_2$

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**Abstract**. A minimal extension of the Standard Model with the gauge grup $SU(2)\otimes U(1)\otimes Z_2$ has been built. In this theory, there are three scalar fields, namely two doublet Φ and η and one singlet Φs. The scalar field mass of Φs is found to be greater than the mass of η and the scalar field mass of η is greater than the mass of Φ (mΦs > mη > mΦ).

## 1. Introduction

Basic interactions in the universe are divided into four types, namely electromagnetic, weak, strong, and gravitational interactions [1]. Electromagnetic, weak, strong interactions have been incorporated, and are summarized in the theory called the Particle Physics Standard Model with the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ . In the Standard Model it is assumed that there is only one Higgs boson particle whose role is to generate the mass of the particles in the Standard Model. As times goes by, particle physicists try to detect the existence of the Higgs boson. At this time, the ATLAS team was succeeded in discovering the Higgs boson particle at the LHC [2]. In spite of being successful, the Standard Model is said to be incomplete because it is unable to explain several problems. Among the problems encountered by the Standard Model were the imbalance between particles and antiparticles [3], mass hierarchy of three generations of leptons and quarks [4], dark matter problems [5], and oscillations of neutrinos [6, 7]. Therefore, in order to overcome those problems, physicists try to expand the Standard Model. One of the extension model of the Standard Model is the minimal extension of Standard Model with the gauge group $SU(2)_L\otimes U(1)_Y \otimes Z_2$ [8]. This model was proposed by Ernest Ma, which contains two scalar fields namely the Standard Model doublet scalar field Φ and introduces the new doublet scalar field η. The scalar field Φ is transformed according even transformation of $Z_2$ (Φ → Φ), while the scalar field η is oddly transformed (η → - η). We add a singlet scalar field Φs into the previous model. Based on the symmetry of Z2, we define the transformation of the scalar fields as follows

$$Z_2 : \Phi \rightarrow \Phi,\ \Phi s \rightarrow \Phi s,\ \eta \rightarrow -\eta$$

The mass of each scalar field must first be generated since it will later play a role in generating the mass of fermionic and boson particles in the minimal extension of the Standard Model.

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## 2. Minimal Extention of the Standard Model

A minimal extension of the Standard Model introduces three scalar fields. The scalar potential of this model is given by

$$
\begin{aligned}
V = &\frac{1}{2}\mu_1^2\Phi^\dagger\Phi + \frac{1}{2}\mu_2^2\eta^\dagger\eta + \frac{1}{4}\lambda_1(\Phi^\dagger\Phi)^2 + \frac{1}{4}\lambda_2(\eta^\dagger\eta)^2 \\
&+ \frac{1}{2}\lambda_3(\Phi_s^\dagger\Phi_s)^2 + \frac{1}{2}\lambda_4(\Phi^\dagger\Phi)(\eta^\dagger\eta) + \frac{1}{2}\lambda_5(\Phi^\dagger\eta)(\eta^\dagger\Phi) \\
&+ \frac{1}{2}\lambda_6(\Phi_s^\dagger\Phi_s)(\Phi^\dagger\Phi) + \frac{1}{2}\lambda_7(\Phi_s^\dagger\Phi_s)(\eta^\dagger\eta) + \lambda_8(\Phi^\dagger\Phi)\Phi_s \\
&+ \lambda_9(\eta^\dagger\eta)\Phi_s + \frac{1}{4}\lambda_{10}\left[(\Phi^\dagger\eta)^2 + hc\right].
\end{aligned}
\tag{1}
$$

Table 1 shows the particle types in the minimal extension Standard Model which consists of fermions and scalar fields.

**Table 1**. Particles accompanied by a fundamental representation in the minimal extension of the Standard Model

| Particle Type | Particles | $SU(2)_L \otimes U(1)_Y \otimes Z_2$ |
|---|---|---|
| Fermionic | $l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ | $(\mathbf{2}, -1, +)$ |
| | $e_R$ | $(\mathbf{1}, -2, +)$ |
| | $\nu_R$ | $(\mathbf{1}, 0, -)$ |
| | $q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$ | $(\mathbf{2}, +\frac{1}{3}, +)$ |
| | $u_R$ | $(\mathbf{1}, +\frac{4}{3}, +)$ |
| | $d_R$ | $(\mathbf{1}, -\frac{2}{3}, +)$ |
| Scalar field | $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ | $(\mathbf{2}, +1, +)$ |
| | $\eta = \begin{pmatrix} \eta^+ \\ \eta^- \end{pmatrix}$ | $(\mathbf{2}, +1, -)$ |
| | $\Phi_s$ | $(\mathbf{1}, 0, +)$ |

The vacuum expectation value of the three scalar fields in table 1 are defined as

$$
\Phi_0 \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \upsilon_\phi \end{pmatrix}, \quad \eta_0 \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \upsilon_\eta \end{pmatrix}, \quad \Phi_s \equiv \left( \upsilon_{\phi_s} \right), \tag{2}
$$

where $\upsilon_\varphi$, $\upsilon_\eta$, and $\upsilon_{\varphi s}$ are the vacuum expectation values for the scalar fields $\Phi$, $\eta$ and $\Phi_s$ respectively. By substituting eq.(2) into eq.(1), the scalar potential value in the vacuum state is obtained in the following equation

$$
\begin{aligned}
V = &\frac{1}{2}\mu_1^2\upsilon_\phi^2 + \frac{1}{2}\mu_2^2\upsilon_\eta^2 + \frac{1}{2}\mu_s^2\upsilon_{\phi_s}^2 + \frac{1}{4}\lambda_1\upsilon_\phi^4 + \frac{1}{4}\lambda_2\upsilon_\eta^4 \\
&+ \frac{1}{4}\lambda_3\upsilon_{\phi_s}^4 + \frac{1}{2}\lambda_4\upsilon_\phi^2\upsilon_\eta^2 + \frac{1}{2}\lambda_5\upsilon_\phi^2\upsilon_\eta^2 + \frac{1}{2}\lambda_6\upsilon_{\phi_s}^2\upsilon_\phi^2 \\
&+ \frac{1}{2}\lambda_7\upsilon_{\phi_s}^2\upsilon_\eta^2 + \lambda_8\upsilon_\phi^2\upsilon_{\phi_s} + \lambda_9\upsilon_\eta^2\upsilon_{\phi_s} + \frac{1}{2}\lambda_{10}\upsilon_\phi^2\upsilon_\eta^2.
\end{aligned}
\tag{3}
$$

From eq.(3) we get the minimum value of scalar potential and the vacuum expectation value (VEV) for each scalar field as given in the following equation

$$\begin{aligned}
\upsilon_\phi &= \pm\sqrt{\frac{-\mu_1^2 - \upsilon_\eta^2(\lambda_4+\lambda_5+\lambda_{10}) - \lambda_6\upsilon_{\phi_s}^2 - 2\lambda_8\upsilon_{\phi_s}}{\lambda_1}},\\
\upsilon_\eta &= \pm\sqrt{\frac{-\mu_2^2 - \upsilon_\phi^2(\lambda_4+\lambda_5+\lambda_{10}) - \lambda_7\upsilon_{\phi_s}^2 - 2\lambda_9\upsilon_{\phi_s}}{\lambda_2}},\\
\upsilon_{\phi_s} &= \pm\sqrt{\frac{-\mu_s^2 - \lambda_6\upsilon_\phi^2 - \lambda_7\upsilon_\eta^2 - \frac{\lambda_8\upsilon_\phi^2}{\upsilon_{\phi_s}} - \frac{\lambda_9\upsilon_\eta^2}{\upsilon_{\phi_s}}}{\lambda_3}}.
\end{aligned} \tag{4}$$

The vacuum expectation value for the scalar field $\Phi_s$ in eq.(4) will be assumed to be greater than the vacuum expectation value of η, and the vacuum expectation value for the scalar field η is assumed to be greater than the vacuum expectation value of Φ ($\upsilon_{\varphi s} >> \upsilon_\eta >> \upsilon_\varphi$).

## 3. Scalar Field Mass Generation

The scalar field mass of Φ, η and Φs can be generated by performing expansion around the vacuum expectation values of each scalar field. The form of the expansion is shown as follows

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \upsilon_\phi + h_\phi \end{pmatrix}, \quad \eta = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \upsilon_\eta + h_\eta \end{pmatrix}, \quad \Phi_s = \left( \upsilon_{\phi_s} + h_{\phi_s} \right), \tag{5}$$

where $h_\varphi$, $h_\eta$, and $h_{\varphi s}$ is the amount of disturbance in the vacuum expectation value of each scalar field.

As seen from the fundamental representation in table 1, the scalar field Φ is the Standard Model Higgs field so the υφ value is the VEV value for the Higgs boson in the Standard Model. If an expansion is carried out around the vacuum expectation value of scalar field, it will produce a scalar potential containing the scalar field mass term, the interaction term between three scalar fields, and the interaction term between four scalar fields [9]. By substituting eq.(5) into eq.(1), we get the mass term for each scalar field which contain $h^2_\varphi$, $h^2_\eta$, and $h^2_{\varphi s}$ respectively

$$h_\phi^2\left(\frac{\mu_1^2}{2} + \frac{3\lambda_1\upsilon_\phi^2}{2} + \frac{\lambda_4\upsilon_\eta^2}{2} + \frac{\lambda_5\upsilon_\eta^2}{2} + \frac{\lambda_6\upsilon_{\phi_s}^2}{2} + \lambda_8\upsilon_{\phi_s} + \frac{\lambda_{10}\upsilon_\eta^2}{2}\right), \tag{6}$$

$$h_\eta^2\left(\frac{\mu_2^2}{2} + \frac{3\lambda_2\upsilon_\eta^2}{2} + \frac{\lambda_4\upsilon_\phi^2}{2} + \frac{\lambda_5\upsilon_\phi^2}{2} + \frac{\lambda_7\upsilon_{\phi_s}^2}{2} + \lambda_9\upsilon_{\phi_s} + \frac{\lambda_{10}\upsilon_\phi^2}{2}\right), \tag{7}$$

$$h_{\phi_s}^2\left(\frac{\mu_s^2}{2} + \frac{3\lambda_3\upsilon_{\phi_s}^2}{2} + \frac{\lambda_6\upsilon_\phi^2}{2} + \frac{\lambda_7\upsilon_\eta^2}{2}\right), \tag{8}$$

and the following mixed terms containing hφhη, hφhφs and hηhφs

$$h_\varphi h_\eta(2\lambda_4\upsilon_\varphi\upsilon_\eta + 2\lambda_5\upsilon_\varphi\upsilon_\eta + 2\lambda_{10}\upsilon_\varphi\upsilon_\eta) + h_\varphi h_{\varphi s}(2\lambda_6\upsilon_{\varphi s}\upsilon_\varphi + 2\lambda_8\upsilon_\varphi) + h_\eta h_{\varphi s}(2\lambda_7\upsilon_{\varphi s}\upsilon_\eta + 2\lambda_9\upsilon_\eta). \tag{9}$$

The mixed terms will be ignored as it has been done in [10].

By comparing the mass terms of the particles with the mass terms of each scalar field, the scalar field mass is obtained as follows

$$\begin{aligned} m_{h_\phi} &= \sqrt{2\lambda_1 v_\phi^2}, \\ m_{h_\eta} &= \sqrt{2\lambda_2 v_\eta^2}, \\ m_{h_{\phi_s}} &= \sqrt{2\lambda_3 v_{\phi_s}^2 - \frac{\lambda_8 v_\phi^2}{v_{\phi_s}} - \frac{\lambda_9 v_\eta^2}{v_{\phi_s}}}. \end{aligned} \quad (10)$$

Based on the assumption of eq.(4), the scalar field mass of $\Phi_s$ is greater than the mass of η and the scalar field mass of η is greater than the mass of Φ ($m_{\Phi s} > m_\eta > m_\Phi$).

## 4. Conclusion

Minimal extension of the Standard Model is a model that extends the Standard Model. The scalar field mass of Φ generated in this expansion model matches the one in the Standard Model, while the added scalar field of η and $\Phi_s$ has a mass greater than Φ.

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