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Black holes, AdS, and CFTs

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Abstract This brief conference proceeding attempts to explain the implications of the anti-de Sitter/conformal field theory (AdS/CFT) correspondence for black hole entropy in a language accessible to relativists and other non-string theorists. The main conclusion is that the Bekenstein–Hawking entropy S_{BH} is the density of states associated with certain superselection sectors, defined by what may be called the algebra of boundary observables. Interestingly while there is a valid context in which this result can be restated as “ S_{BH} counts all states inside the black hole,” there may also be another in which it may be restated as “ S_{BH} does not count all states inside the black hole, but only those that are distinguishable from the outside.” The arguments and conclusions represent the author’s translation of the community’s collective wisdom, combined with a few recent results.

Keywords Black hole entropy, AdS/CFT

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1 Introduction

In a classic set of lectures Wheeler [1], emphasized the importance of the initial-value problem for quantum gravity. In particular, given any smooth set of initial data, Marolf Physics Department UCSB Santa Barbara, CA 93106, USA
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Fig. 1 Moment of time-symmetry in a bag of gold spacetime

(i) solves the gravitational constraints and (ii) has small curvatures, one expects the complete theory of quantum gravity to contain a state which, in a suitable semi-classical limit, approximates the spacetime generated by evolution from the given initial data. One expects this to hold even if the solution develops singularities in both the far future and the far past. In such regions, quantum effects will be very important in describing the physics of this state. However, there will be a large region of spacetime, including complete initial value surfaces, where the semi-classical approximation holds. It would therefore be a great surprise if quantum gravity somehow contradicts the classical result that such regions of spacetime can exist.

This argument continues to serve as an interesting point of discussion for black hole physics, in particular in the context of the anti-de Sitter/Conformal Field theory correspondence (AdS/CFT) [2]. At the WE-Heraeus-Seminar: *Quantum Gravity: Challenges and Perspectives*, my charge was to discuss and explain the implications of AdS/CFT for black hole entropy and unitarity, a theme I will explore below using examples from [1]. The focus will be on the old question of whether the Bekenstein–Hawking entropy counts *all* states inside black holes or only, in some sense, those states which are distinguishable from the outside. Interestingly, we will see that AdS/CFT suggests that there are reasonable contexts realizing each of these two possibilities, but that both cases are compatible with what may be called a “unitary” description of black hole evaporation.

Let us begin by recalling what have come to be known as Wheeler’s “bag of gold” spacetimes,¹ in which a large $k = +1$ Friedmann–Robertson–Walker (FRW) universe is sewn onto the “back side” of Kruskal extension of the Schwarzschild black hole spacetime so that it is accessed by passing through the Einstein–Rosen bridge. An embedding diagram for a time-symmetric slice is shown in Fig. 1. A detailed recent review of this construction can be found in [3].

Bag of gold spacetimes play an important role in discussions of the meaning of black hole entropy. Indeed, they typify one of the two main classes of examples which suggest that black holes might have an infinite number of internal states (even at finite Planck length ℓ_p) so that in particular the (finite) Bekenstein–Hawking entropy S_{BH} would not be simply the density of such states. The point here is that, for a black hole of any fixed mass M , the FRW interior can be taken to be as large as one likes. Since the time-symmetric slice of a $k = +1$ FRW universe of scale factor a has energy density $\rho \sim 1/\ell_p^2 a^2$, if this energy is in the form of

¹ This term seems to be in frequent though informal use in the relativity community for the spacetimes of Fig. 1. The actual connection to [1] is somewhat subtle, however, as Wheeler’s original use of the term was somewhat different. Wheeler used the term “bag of gold” in [1] to refer to certain singularities that arose in constructing time-symmetric initial data using conformal methods when the conformal factor passed through zero. The method was intended for asymptotically flat spacetimes, but when such zeros appeared they effectively pinched off part of the spacetime, tying it up in a (singular) “bag of gold.” However, the same lectures [1] do introduce the spacetimes of Fig. 1, though the emphasis is on the physics of the FRW side of the Einstein–Rosen bridge as opposed to considering the FRW side as the “inside” of a black hole as we do here.

thermal radiation it contains an entropy $S \sim V\rho^{3/4} \sim \left(\frac{a}{\ell_p}\right)^{3/2}$. Here the symbol \sim means that we have dropped constants of order one. We see that the number of radiation states which can be placed inside the FRW bag diverges as $a \rightarrow \infty$ so that, from the perspective of the asymptotically flat region, an infinite number of states can exist behind the black hole horizon.²

It is interesting to compare the bag of gold spacetime with the other class of examples typically used to argue that black holes might contain an infinite number of internal states. In this second example, one starts with a black hole of given mass M , considers some large number of ways to turn this into a much larger black hole (say of mass M'), and then lets that large black hole Hawking radiate back down to the original mass M . Unless information about the method of formation is somehow erased from the black hole interior by the process of Hawking evaporation, the resulting black hole will have a number of possible internal states which clearly diverges as $M' \rightarrow \infty$. One can also arrive at an arbitrarily large number of internal states simply by repeating this thought experiment many times, each time taking the black hole up to the same fixed mass $M' > M$ and letting it radiate back down to M . We might therefore call this the ‘‘Hawking radiation cycle’’ example. Again we seem to find that the Bekenstein–Hawking entropy does not count the number of internal states.

I mentioned above that I will be interested in making contact with AdS/CFT. One of the interesting aspects of AdS/CFT is that the entropy of the dual CFT is finite and agrees with the Bekenstein–Hawking entropy S_{BH} . This then provides contrasting evidence that, in some sense, the Bekenstein–Hawking entropy S_{BH} *does* count the full set of black hole states in the corresponding theory of quantum gravity. We will review this evidence below in Sect. 4, but this foreshadowing motivates a closer examination of the two examples above. Is there any way that they might be reconciled with such a finite-dimensional space of black hole states? Both examples admit ready generalizations to the AdS context: adding a negative cosmological constant to the bag of gold simply results in a $k = +1\Lambda$ -FRW spacetime attached to the back side of an AdS-Schwarzschild black hole, and adding a negative cosmological constant to the Hawking radiation cycle example is straightforward when the black hole of interest is very small compared to the AdS scale. When the black hole is larger than the AdS scale it becomes thermodynamically stable and does not naturally radiate back down to a smaller black hole on its own. However, as we will discuss in Sect. 2, one can nevertheless remove the radiation by acting with certain operators which are readily mapped to the CFT, draining away the Hawking radiation ‘‘by hand’’ until the black hole evaporates back to its original mass.

One notices two important differences between the bag of gold example and that of the Hawking radiation cycle. The first is that the bag of gold construction

² Other examples that we consider to be in the same general class include the Kruskal extension of the Schwarzschild black hole, in which the 2nd asymptotically flat region can be thought of as the limit of a large FRW universe, and the ‘‘monster’’ initial data sets of [3]. While the latter do not contain apparent horizons, they contain a large amount of matter in an extremely deep throat-region on the verge of gravitational collapse. It seems clear that collapse must ensue on a timescale much too short for the matter to leave the throat. A black hole must then result and, while the initial data surface contains no apparent horizon, part of this surface (and most of the entropy) would nevertheless lie behind the event horizon and in that sense be inside the black hole, as in the bag of gold case.

Fig. 2 Rough conformal diagrams for **a** an AdS bag of gold spacetime and **b** an asymptotically AdS black hole made by throwing in matter (*shaded*) through the AdS boundary

involves no Hawking radiation or other quantum processes (except for counting the entropy of thermal radiation). Thus, while the Hawking radiation cycle example might be reconciled with the view that S_{BH} counts the number of internal states if some mechanism could be found by which Hawking radiation would erase the relevant information from the black hole interior, such a reconciliation is not possible for bags of gold.

The other difference is the dramatically different character of the two examples at early times. The bag of gold spacetime contains a past singularity, as do the related examples of footnote 2, while the Hawking radiation cycle example can be formed from smooth initial conditions in an asymptotically flat spacetime. A similar statement can be made about the AdS case. There one may begin the Hawking radiation cycle example with completely empty anti-de Sitter space and proceed by throwing in matter through the AdS boundary. In contrast, there is no obvious way to construct a bag of gold spacetime by simply manipulating perturbative excitations near the AdS boundary.³ See Fig. 2 for a comparison. As we will discuss below, one may therefore interpret these two examples as belonging to different superselection sectors of the theory associated with the algebra of boundary observables.

Below, we first summarize the essentials of AdS/CFT in Sect. 2. For simplicity, we focus on the particular case where the CFT lives on the Einstein static universe spacetime. The main goal is to explain the notion of boundary observables in the asymptotically AdS quantum gravity theory, and we do so through the example of the boundary stress tensor. We also attempt to state the sense in which the conformal field theory is “dual” to a quantum gravity theory with asymptotically anti-de Sitter boundary conditions. We then briefly discuss issues of unitarity and information loss in Sect. 3 before turning to our main discussion of entropy in Sect. 4. The emphasis in Sect. 4 is on making precise, conservative statements. We then further discuss the implications of these statements in Sect. 5. By necessity, Sect. 5 is less precise, but we hope that the desire to flesh out the details will lead to interesting future research. In all sections, we attempt to use language accessible to relativists and practitioners of quantum field theory in curved spacetime.

2 Boundary observables and AdS/CFT

As is by now well known, arguments from string theory [2] suggest that at least certain theories of asymptotically anti-de Sitter quantum gravity are in some sense “dual” to the large N limit of certain local conformal field theories. Here N is some measure of the number of CFT fields involved, which in typical cases can be thought of as the rank of a gauge group associated with some gauge field. The correspondence furthermore identifies N with some positive power of ℓ_{AdS}/ℓ_p ; i.e., $\ell_{AdS}/\ell_p \propto N^\alpha$, where ℓ_{AdS} is the AdS scale and the particular value of $\alpha > 0$

³ Though one could, of course, imagine tossing a black hole with a bag of gold in through the boundary.

depends on the particular duality being considered. Many different such dualities are believed to hold, associated in string theory with D3-branes, M2-branes, M5-branes, or with bound states including several types of branes. Each duality relates a CFT to some string or M-theory with boundary conditions which require spacetimes to be asymptotically $\text{AdS}_d \times X$, where X is some compact manifold and the details of d , X , and the strong theory vary with the duality.

However, we will avoid discussing the details of any particular case here. Such details are simply not relevant to our discussion below. Instead, we focus on features common to all AdS/CFT correspondences. In particular, we will ignore the manifold X . Effectively, one may suppose that a Kaluza–Klein reduction has been performed to replace fields on $\text{AdS}_d \times X$ with an infinite tower of massive fields living just on AdS_d . For definiteness, we focus on the case where the CFT is defined on the Einstein static universe spacetime $S^{d-2} \times \mathbb{R}$ below.

Now, we could begin by stating that the CFT Hilbert space is isomorphic to a Hilbert space on the quantum gravity side of the correspondence. However, this comment contains little information since any two separable infinite-dimensional Hilbert spaces are already well-known to be isomorphic. Another statement that might be made is that the algebra of CFT operators is isomorphic to an algebra of quantum gravity operators, and that the above isomorphism of Hilbert spaces can be thought of as an isomorphism of representations of this operator algebra. However, this again provides little information since each of the trivial isomorphisms between two separable Hilbert spaces readily defines an isomorphism of the associated von Neumann algebras, and also of the desired representations. We must clearly be more explicit to give a non-trivial statement of the correspondence.

What gives AdS/CFT content is that we understand something about how physically interesting operators map between the AdS gravity theory and the conformal field theory. Rather than attempt to state this in great generality, it suffices for our purposes to explain a particularly important example. The claim is that the local stress-energy tensor $T_{ij}^{CFT}(y)$ of the CFT maps directly to an object $T_{ij}^{AdS}(y)$ which we will explain below. This $T_{ij}^{AdS}(y)$ is known as the “boundary stress tensor” of the AdS quantum gravity theory [4; 5]. In contrast to the above general statements, this claim contains *vast* amounts of information. For example, from the stress tensor one can construct the Hamiltonian, which on the CFT side has a non-degenerate ground state. Thus AdS/CFT states that the quantum gravity theory has a corresponding ground state and that, in that state, arbitrary correlation functions (i.e., n -point functions, for all n) of $T_{ij}^{AdS}(y)$ agree with the corresponding CFT vacuum correlators of $T_{ij}^{CFT}(y)$. This is now a very specific prediction.

Let us take a moment to remind the reader what is meant by the AdS boundary stress tensor $T_{ij}^{AdS}(y)$. This also provides an opportunity to state more precisely what we mean by asymptotically AdS quantum gravity. For our purposes here, it suffices to use an extremely simple set of boundary conditions. We consider spacetimes M such that:

1. One can attach a boundary $\mathcal{S} \cong R \times S^{d-2}$ to M such that $\tilde{M} = M \cup \mathcal{S}$ is a manifold with boundary.
2. On \tilde{M} , there is a $(d-1)$ -times continuously differentiable metric \tilde{g}_{ab} and a smooth function Ω such that $g_{ab} = \Omega^{-2} \tilde{g}_{ab}$, with $\Omega = 0$ and

$$\tilde{n}_a \equiv \tilde{\nabla}_a \Omega \neq 0 \tag{2.1}$$

at points of \mathcal{S} . We also require that the metric $g_{(0)ij}$ on \mathcal{S} induced by \tilde{g}_{ij} is the Einstein static universe,

$$g_{(0)ij} dy^i dy^j = -dt^2 + \ell_{AdS}^2 d\sigma^2, \quad (2.2)$$

where $d\sigma^2$ is the line element of the unit sphere S^{d-2} and y^i are a set of coordinates on the boundary.

Given such an asymptotically AdS spacetime (g, M) , the corresponding Ω , and an extension of the y^i into the bulk, there is some diffeomorphism one can apply to g_{ab} such that the unphysical metric takes the ‘‘Gaussian normal form’’

$$\tilde{g}_{ab} dx^a dx^b = \ell_{AdS}^2 d\Omega d\Omega + \sum_{n \geq 0} \Omega^n g_{(n)ij} dy^i dy^j + O(\Omega^{d+1}). \quad (2.3)$$

The so called Fefferman–Graham coefficients $g_{(n)ij}$ for $n < d - 1$ are determined by $g_{(0)ij}$ and the Einstein equations, though $g_{(d-1)ij}$ contains new information [6] which defines the boundary stress tensor [4; 5]:

$$T_{ij}^{AdS}(y) = \frac{d-1}{16\pi G} g_{(d-1)ij}(y). \quad (2.4)$$

We refer to [4; 5] for a detailed explanation of why this object is called the boundary stress tensor, though we comment briefly that if ξ_{boundary}^i is a conformal Killing field of the Einstein static universe and C is any Cauchy surface of \mathcal{S} , the expression

$$\int_C \sqrt{g_0} T_{ij}^{AdS} \xi_{\text{boundary}}^i n_C^j, \quad (2.5)$$

with n_C^j the unit future-directed timelike normal to C with respect to $g_{(0)ij}$, gives the conserved charge⁴ associated with the asymptotic symmetry that acts on \mathcal{S} via ξ_{boundary}^i . In particular, $T_{ij}^{AdS}(y)$ turns out to be essentially the electric part of the Weyl tensor at \mathcal{S} [7], which is known to give the appropriate conserved charges [9; 10].

Because the coefficients $g_{(d)ij}$ are defined in terms of the Gaussian normal form (2.3), $T_{ij}^{AdS}(y)$ does not depend on the extension of the coordinates y^i into the bulk. Instead, $T_{ij}^{AdS}(y)$ is a tensor on the boundary spacetime \mathcal{S} . As a result, $T_{ij}^{AdS}(y)$ defines an *observable*. By this we mean that it is invariant under all gauge transformations of the theory. We shall not go into the details of this story here, but merely sketch an outline that should be familiar to most relativists e.g., from the asymptotically flat context. Beginning with the usual gravitational symplectic structure (see e.g., [11; 12]), one defines infinitesimal gauge transformations as tangent vectors to the covariant phase space which yield degenerate directions of the symplectic structure. One then finds that diffeomorphisms are only gauge transformations if they are generated by vector fields ξ^a which are smooth on \tilde{M} , vanish on \mathcal{S} , and preserve the above notion of asymptotically AdS spacetimes. See e.g., [7; 13] for details. Since gauge transformations act trivially on \mathcal{S} , they also act trivially on any tensor on \mathcal{S} such as $T_{ij}^{AdS}(y)$. Thus we refer to $T_{ij}^{AdS}(y)$ as a boundary observable. Note that, in contrast, diffeomorphisms of \tilde{M} which preserve the space of asymptotically AdS metrics but which act non-trivially on \mathcal{S} define asymptotic symmetries which act non-trivially on observables.

In a similar way, Fefferman–Graham-like expansions of any other fields also define boundary observables. The set of such observables for all bulk fields is one of the primary ingredients of the AdS/CFT dictionary. The dictionary that maps such observables to local CFT observables is understood in each example of AdS/CFT, at least at leading order in the $1/N$ expansion.

Other CFT observables (such as Wilson loops) can also be mapped to AdS observables using more stringy ingredients (see e.g., [14]). The details will not be important here, but we briefly mention that they again lead to AdS observables associated with certain regions of the boundary, and which transform in a natural way under asymptotic symmetries. Furthermore, unless the regions R_1 and R_2 on \mathcal{S} associated with two such observables are connected by causal curves, the corresponding CFT observables commute. As more observables are added to the dictionary, these important properties are expected to hold in each case. Thus the AdS/CFT dictionary generally relates CFT observables to what may reasonably be called boundary observables of the quantum gravity theory.

We will not need the details of any further such constructions below, nor will it be necessary to give a precise statement of the full set of boundary observables. Instead, it will suffice to suppose merely that there is some algebra of CFT observables \mathcal{D}_{CFT} and some algebra of quantum gravity observables \mathcal{D}_{AdS} , for

⁴ Though sometimes with a choice of zero-point different from that typically chosen by relativists. See e.g., [7; 8] for discussions of this point.

which this dictionary has been stated; i.e., we have been given a particular bijective map $\phi_{AdS/CFT} : \mathcal{D}_{CFT} \leftrightarrow \mathcal{D}_{AdS}$. The AdS/CFT conjecture is the claim that, for every state ρ_{CFT} in the CFT, there is some state ρ_{AdS} in the asymptotically AdS quantum gravity theory such that the restriction⁵ of ρ_{AdS} to \mathcal{D}_{AdS} agrees with the image under $\phi_{AdS/CFT}$ of the restriction of ρ_{CFT} to \mathcal{D}_{CFT} .

As one gains more control over the AdS/CFT dictionary and the algebra \mathcal{D}_{AdS} becomes larger, the above claim becomes increasingly powerful and the space of states ρ_{AdS} which might correspond to a given ρ_{CFT} becomes smaller. In fact, it is quite plausible that the sort of AdS boundary observables defined above via Fefferman-Graham expansions are already dual to a complete set of CFT operators. As we will shortly explain, this will be true if something like an ergodicity conjecture holds for the CFT, which one might expect to hold due to its strongly interacting nature.

To understand this last point, consider just the algebra of observables generated by the CFT stress tensor $T_{ij}^{CFT}(y)$. Since this algebra contains the Hamiltonian, it contains the projection onto the CFT vacuum. But one expects to be able to create rather general superpositions of energy eigenstates by using a tensor test field $\sigma^{ij} : \mathcal{S} \rightarrow \mathbb{C}$ to construct a unitary operator $U[\sigma^{ij}] = \exp(\int_{\mathcal{S}} \sqrt{g_{(0)}} \sigma^{ij} T_{ij}^{CFT})$ with which one may act on the CFT vacuum. Acting further with functions of the Hamiltonian H and taking linear superpositions plausibly generates all states in the CFT Hilbert space.⁶ We shall therefore assume that \mathcal{D}_{CFT} contains a complete set of observables in our discussion below. (In any case, including the Wilson loop observables mentioned above would certainly make \mathcal{D}_{CFT} complete.) It is then clear that two distinct CFT states $\rho_{CFT}^1, \rho_{CFT}^2$ cannot both map to the same state ρ_{AdS} .

On the other hand, we defer any discussion of whether each ρ_{CFT} has a unique ρ_{AdS} to Sect. 5. Nevertheless, it will be convenient to use the term ‘‘AdS vacuum’’ to refer to a particular state ρ_{AdS}^0 whose correlators agree under $\phi_{AdS/CFT}$ with those of the CFT vacuum. If there is more than one such state, for now we simply pick one, say ρ_{AdS}^0 , and call it the AdS vacuum. We may use this ρ_{AdS}^0 to extend $\phi_{AdS/CFT}$ to a one-to-one map from CFT states to AdS states: it maps the CFT vacuum to the AdS vacuum, and it maps the action of \mathcal{O} on the CFT vacuum to the action of $\phi_{AdS/CFT}(\mathcal{O})$ on the AdS vacuum.

As a brief aside, we note that while we have fixed $g_{(0)ij}$ to be the metric on the Einstein static universe, one may in fact generalize the discussion to any smooth Lorentz-signature metric on which quantum field theory may be defined. For any $g_{(0)ij}$, the dual CFT is given by the same Lagrangian⁷ as for the Einstein static universe, but built instead from the metric $g_{(0)ij}$. The Fefferman–Graham expan-

⁵ Here we use the language of algebraic quantum field theory in which one thinks of a state as a positive linear functional on an algebra. One may then restrict any state to a subalgebra, e.g. to \mathcal{D}_{AdS} , the subalgebra of AdS quantum gravity operators for which the details of the dictionary have been stated.

⁶ The only case in which one expects this not to happen occurs when there is some symmetry that leaves the stress tensor invariant; e.g., global charge rotations. In such a case one also needs to include in \mathcal{D}_{CFT} some operator not invariant under this symmetry. This is not difficult in specific examples and proceeds in direct analogy to our discussion of $T_{ij}^{CFT}(y)$ and $T_{ij}^{AdS}(y)$ above.

⁷ Any renormalization ambiguities associated with passing to a general $g_{(0)ij}$ on the CFT side of the correspondence turn out to have direct analogues on the AdS side.

sion (2.3) and the construction of $T_{ij}^{AdS}(y)$ generalizes readily to this case, though when d is odd an additional logarithmic term must be added to (2.3) for general $g_{(0)ij}$. The boundary stress tensor also receives contributions from this logarithmic term. Such a term was not included in (2.3) because the coefficient of this logarithm happens to vanish for the Einstein static universe; see e.g., [4; 13] for details. For simplicity, we avoid considering such general $g_{(0)ij}$ and confine ourselves to the Einstein static universe case below.

Since they are dual to local fields in the CFT, one might expect $T_{ij}^{AdS}(y)$ -like boundary observables (defined by Fefferman–Graham expansions of general bulk fields) to act much like local fields themselves. This does indeed turn out to be the case; see e.g., [15]. In particular, by smearing local boundary observables with positive- and negative-frequency functions on the boundary, one can define operators that act like creation and annihilation operators [16]. It is precisely these operators which may be interpreted as “throwing particles into the AdS space through the boundary,” or as removing such particles through the boundary. This interpretation follows from a close connection between such operators and deformations of the AdS boundary conditions [17]. For example, it turns out that acting with $T_{ij}^{AdS}(y)$ is equivalent to deforming the boundary metric $g_{(0)ij}$ at the event y . The point here is that a single state of the deformed theory can be used to construct *two* states of the undeformed theory. There is a “retarded state” defined by noting that the two theories are identical to the past of y , as well as a corresponding “advanced state.” The advanced state is then the action of $T_{ij}^{AdS}(y)$ on the retarded state, in what is essentially an example of the Schwinger variational principle [18; 19; 20; 21], see [22] for details.

The fact that one may construct such boundary creation and annihilation operators has two implications for our black hole discussion. First, it means that one may think of the Hawking radiation cycle experiment from the introduction in terms of applying boundary creation and annihilation operators in alternating cycles and examining the resulting states. Second, it tells us much about the vacuum state ρ_{AdS}^0 . Since the CFT vacuum is annihilated by the CFT annihilation operators, the boundary annihilation operators must also annihilate ρ_{AdS}^0 . Thus, ρ_{AdS}^0 is a state for which no energy can be removed by changing the boundary conditions. One thus expects this state to be simply empty AdS space in the classical limit $\ell_p/\ell_{AdS} \rightarrow 0$; i.e., $N \rightarrow \infty$. We shall assume that this is the case in our discussions below.

3 Unitarity and AdS/CFT

Having reviewed some essential features of AdS/CFT, let us now ask what this correspondence has to say about unitarity in the quantum gravity theory. We will take as given that the CFT is a well-defined local quantum field theory with a self-adjoint Hamiltonian. Furthermore, H is one of the operators in \mathcal{D}_{CFT} for which we understand the dictionary. Thus, at least the boundary observables \mathcal{D}_{AdS} must evolve unitarily under the action of the AdS Hamiltonian; i.e., the operators defined by the asymptotic behavior of bulk fields evolve unitarily.

On the one hand, this statement may seem quite surprising. Unitarity is of course associated with conservation of information. Suppose that one creates a state by acting on the vacuum with one of our boundary observables, perhaps

$T_{ij}^{AdS}(y)$. This essentially amounts to creating a graviton in the bulk, near the AdS boundary. This graviton then propagates deeper into the bulk and might fall into a black hole. How then can the information in this graviton still be available on the boundary at later times? Here it is especially interesting to consider the unitary evolution over *short* periods of time, too short, say, for there to be any possibility for the information to return to the boundary in Hawking radiation. The unitarity of the CFT seems to suggest that this information must nevertheless remain present in operators in \mathcal{D}_{AdS} at *any* later time.

On closer examination, however, this statement need not be a surprise after all. In fact, it has been recently argued that such properties naturally arise in any theory of quantum gravity [24; 25]. The essential point is that, as already noted above, the gravitational Hamiltonian is a pure boundary term, and thus lies in the algebra of boundary observables. This Hamiltonian generates time translations on the AdS boundary that are precisely the image under $\phi_{AdS/CFT}$ of CFT time-translations. Furthermore, while we have little control over the Hamiltonian of non-perturbative quantum gravity, one may check [24] that this Hamiltonian is a self-adjoint operator at each order in perturbation theory, *even about a black hole background*. I.e., there appear to be no obstacles to this Hamiltonian remaining self-adjoint in the full quantum theory

So then, it appears that information encoded in our graviton is simultaneously available at two different locations: at the point deep in the bulk (perhaps within a black hole) to which the graviton has traveled, and also at the boundary. We refer the reader to [25] for a detailed discussion of just how the desired information might be recovered by a suitable boundary observer, and for a resolution of what might seem to be potential paradoxes. As explained in [24], there is no claim that this information has in any way been copied into duplicate qubits. Such a duplication would violate the quantum “no xerox theorems” [26; 27]. Instead, there remains a single qubit of quantum information, but one finds that both boundary operators and operators deep in the bulk can be sensitive to the same qubit. The reader should consult [24; 25] for further details.

As a final comment, it is interesting to note that no issues of “baby universes” (see e.g., [28]) arose in our discussion above. Since the argument for unitarity depends only on the region near the boundary, it is independent of whether or not baby universes form in gravitational collapse far from the boundary. Suppose in particular that baby universes do exist, that they can be present in the initial quantum state, and that they contain additional degrees of freedom not present in the original asymptotic region. The conclusion of the above argument is that these new baby universe degrees of freedom simply do not mix with boundary observables under the boundary time evolution. This is reminiscent of the superselection effects noted in certain other discussions of baby universes [29; 30; 31].

So, in retrospect, much of what AdS/CFT has to say about unitarity follows directly from natural extrapolations of bulk gravitational physics. Does AdS/CFT teach us anything new? It most certainly does, in at least three ways. First, it confirms the above extrapolations. Second, it tells us about the microscopic density of states which, when combined with unitarity, makes additional interesting predictions (see [32; 33; 34; 35; 36]). We will briefly discuss this density of states in Sect. 4. Third, in specific examples AdS/CFT tells us how to construct additional complete sets of boundary observables. We argued in Sect. 2 that the algebra \mathcal{D}_{CFT}

is likely to be complete simply because it contains the stress tensor (and perhaps a few other fields charged under certain symmetries). However, when the CFT is a gauge theory, another complete set of observables immediately suggests itself: spacelike Wilson loops at each time. As noted above, these operators can also be translated to the AdS side of the correspondence using stringy degrees of freedom (see [14]). One thus learns that such stringy boundary observables at each time are sufficient to generate \mathcal{S}_{AdS} .

4 The Bekenstein–Hawking Entropy in AdS/CFT

Having reviewed the basics of the AdS/CFT correspondence, and after our brief aside on unitarity, we may now return to the main question raised in the introduction: What, precisely, does AdS/CFT have to say about the entropy of black holes?

We first note that our unitarity discussion above suggests how the Hawking radiation cycle example from the introduction might be reconciled with the idea that black holes have a finite number of internal states given by S_{BH} . Consider the version in which we start with empty AdS space and proceed by applying various boundary operators. We have seen that any information sent into the AdS space through the boundary does, at least in a certain sense, remain accessible on the boundary. Thus, there is simply no sharp division between information “inside the black hole” and information “outside”. In particular, boundary operators *can* act on qubits associated with information sent into the black hole. By their very nature as annihilation operators, their action can decrease the space of possible black hole states. Thus, instead of increasing each time, the number of states associated with a black hole of given mass remains the same in each cycle of the experiment.

We can now address what AdS/CFT tells us about entropy. The basic ingredients of this story are simply the ability to count states in the CFT, and the map $\phi_{AdS/CFT}$ that takes states and observables from one theory to the other. Performing a precise counting of CFT states is generally difficult in strongly coupled CFTs, though simple estimates give a number of states in rough agreement with the Bekenstein–Hawking entropy of AdS black holes. The good news is that one *can* do a precise counting (in the limit of large excitations) for 1 + 1 CFTs which satisfy a property known as modular invariance. Since the 1 + 1 CFTs that arise in AdS₃ dualities turn out to satisfy this property, Cardy’s formula [37] gives the leading growth of the entropy with energy and angular momentum:

$$S(E, J) = 2\pi\sqrt{c(\ell_{AdS}E + J)/12} + 2\pi\sqrt{\bar{c}(\ell_{AdS}E - J)/12}. \quad (4.1)$$

Here c and \bar{c} are respectively the left- and right-moving central charges. As described in [38], (4.1) agrees precisely with the Bekenstein–Hawking entropy of the corresponding BTZ black holes in AdS₃. We will therefore assume that, if a similarly precise counting of CFT states were possible in other dimensions, the results would again agree with the Bekenstein–Hawking entropy of the relevant black hole so long as (i) the semi-classical gravity approximation is valid and (ii) one is at appropriate E, J , or other charges so that S_{BH} is much larger than the entropy of other black holes or of non-black hole states.

So then, what does this counting tell us about quantum gravity states on the AdS side of the correspondence? The image of $\phi_{AdS/CFT}$ in the space of AdS quantum gravity states is precisely the set of states obtained by acting on the vacuum with \mathcal{D}_{AdS} . Thus, AdS/CFT tells us that the density of such black holes states is given by S_{BH} .

Now, this statement may not yet appear to be very useful from the bulk point of view. After all, we defined \mathcal{D}_{AdS} to be the set of boundary operators for which an AdS/CFT dictionary is understood. What we need to make the statement useful is a more gravitational characterization of this algebra. Recall, however, that we suggested that a sort of ergodicity result might hold in the CFT that would naturally make \mathcal{D}_{CFT} equivalent to the algebra generated by the stress tensor (and perhaps a few other local observables), and we stated that we would assume this property below. Since we assume this property to hold for \mathcal{D}_{CFT} , the map $\phi_{AdS/CFT}$ allows us to carry it over directly to the AdS side so that it also holds for \mathcal{D}_{AdS} ; i.e., it follows that \mathcal{D}_{AdS} is the algebra generated by the AdS boundary stress tensor (and in some cases a few additional boundary observables).

We now have a precise technical statement, but it may yet convey little intuition. Let us therefore recall that, as noted in Sect. 3, any AdS operator which might be called a boundary observable should evolve unitarily under boundary time-translations. In parallel with our ergodicity assumption for the CFT, it is natural to expect all such boundary observables to mix with each other under this evolution.⁸ This argues that, at least so long as we consider spacetimes having only a single boundary,⁹ it is useful to think of \mathcal{D}_{AdS} as the algebra of *all* boundary observables. In this sense, AdS/CFT states that *the Bekenstein–Hawking entropy gives the density of states created by acting with boundary observables on the vacuum*. Or, more succinctly, one might say that it gives the density of boundary observable states in the superselection sector defined by the vacuum.

5 Discussion: Where is the bag of gold?

We have attempted to state precisely what AdS/CFT implies about black hole entropy. We found a precise characterization of the Bekenstein–Hawking entropy as the density of black hole states which lie in the same superselection sector as the vacuum, where the notion of superselection sector was defined by a certain observable algebra \mathcal{D}_{AdS} . This \mathcal{D}_{AdS} can in turn be defined as the algebra generated by the boundary stress tensor (and perhaps a few additional operators), and we argued that it is best thought of as the algebra of “all boundary observables”.

Where then, does this leave Wheeler’s bag of gold and its kin? Semiclassical considerations suggest that the density of such states is far higher than the Bekenstein–

Hawking entropy. There are thus two logical alternatives: (i) the semi-classical states do not all correspond to distinct, well-defined states of the full quantum gravity theory or (ii) these states (or at least, most of them) are not in the boundary-

⁸ Again, there are possible exceptions due to conservation laws. We shall not mention such possible exceptions further. As in the CFT discussion, one expects such exceptions to be easily dealt with by using a few additional charged boundary observables to generate the full algebra along with the stress tensor.

⁹ If more than one boundary is present, the gravity theory will have separate Hamiltonians generating time translations along each boundary. See e.g., [25] for a review of this point.

observable superselection sector containing the vacuum. A survey of AdS/CFT researchers would no doubt find supporters both of (i) and of (ii), though option (ii) seems to have been more explicitly discussed in the AdS/CFT literature; see e.g., [32] in the limit where the internal FRW space becomes and asymptotically flat spacetime and [39] for the case where the FRW spacetime is de Sitter space.

We also find option (ii) to be more plausible and focus on this alternative below. In particular, as noted above, at the classical level no mechanism is known to create such a bag of gold from empty AdS space by acting with boundary observables. However, because bags of gold can classically be smoothly deformed to configurations which *can* clearly be created by the action of boundary operators, one might naturally ask why bag-of-gold states cannot arise from the action of boundary operators via some quantum tunneling process. Indeed, instantons which appear to describe tunneling to bag-of-gold-like states exist [40; 41; 42]. However, since they are not smooth the issue remains unclear. I have no clear answer to this question, other than that AdS/CFT appears to predict that such tunneling is not possible and that understanding this prediction from the AdS gravity point of view remains an important open problem.

In some sense, option (ii) implies that the full quantum gravity theory contains additional observables not found in \mathcal{D}_{AdS} . I.e., the algebra of boundary observables is *not* complete. However, it is worth contemplating this statement in more detail. Consider, for example, the special case in which the FRW region in the bag of gold is replaced by a second asymptotically anti-de Sitter region. It is no surprise that such regions cannot be created by boundary observables associated with the original boundary. In fact, it would not surprise a semi-classical physicist to learn that such states are not part of the same Hilbert space as spacetimes with a single asymptotic region. Because they correspond to different boundary conditions, they are naturally associated with different classical phase spaces and thus with different quantum Hilbert spaces. One might say that they correspond to different superselection sectors of the theory. We have also seen that, with respect to the algebra of boundary observables, the bag of gold spacetimes also lie in a new superselection sector. So then, might it be reasonable to treat finite-sized bags of gold as being on a similar status to a second asymptotically AdS region? That is, might one be able to *define* a reasonable theory of quantum gravity with what amounts to some quantum generalization of a boundary condition that excludes bags of gold? The idea is that, in analogy with boundary conditions, other definitions would exist for which bags of gold would in fact be allowed, or perhaps even required, but that nevertheless bag-free definitions would remain possible.

A detailed analysis of this question would take us far beyond the scope of this work. However, the existence of such a definition seems quite reasonable on the basis of known semi-classical physics. We already noted that we expect no process of throwing in particles through the boundary to create such a bag of gold. In particular, even if we throw in a very advanced scientist and a well-equipped laboratory, no action of the scientist described by classical gravity will create the bag of gold. Furthermore, as discussed above, it seems that quantum tunneling also does not create such states. Thus, it seems plausible that bag-of-gold operators are not needed for a reasonable self-contained theory of quantum gravity. In fact, by focussing on quanta which can be thrown in through the boundary, we have argued that we need only consider states created from the vacuum by the action of

boundary observables. In other words, there does appear to be a reasonable, self-contained theory of asymptotically AdS quantum gravity in which the boundary observables form a complete set of operators. In such a theory, the Bekenstein–Hawking entropy *would* count the total number of black hole states, and bags of gold simply do not arise.

On the other hand, we have not yet addressed those definitions of the AdS quantum gravity theory that *do* allow bags of gold or their relatives. The current literature [32; 39] suggests that such theories are dual to a product theory, with one factor being the usual CFT and the other being some new set of degrees of freedom. The implication is then that the full theory decomposes into superselection sectors with respect to our boundary observables, *all* of which are isomorphic to the one containing the vacuum. In this context one might say that the Bekenstein–Hawking entropy counts the density of black hole states in any superselection sector. Thus, in what some readers may consider an ironic twist, it may be fair to say that in AdS/CFT the Bekenstein–Hawking entropy counts “not the full set of states describing the black hole interior, but only those states which are distinguishable from the outside,” a point of view which has long been championed within the relativity community. However, a detailed discussion of the product theory point of view raises many questions about the experience of observers who fall into the black holes, as well as other questions of local physics not readily expressed in terms of boundary observables. As is so often the case, we end with the conclusion that understanding how to describe such local bulk observations remains one of the most interesting open questions in studies of the AdS/CFT correspondence, and in quantum gravity more generally.

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