Kerr-Schild Geometry and Twistorial Structure of Spinning Particles

Alexander Burinskii

NSI Russian Academy of Sciences, B. Tulskaya 52, 115191, Moscow, Russia

Spinning particles and twistors are traditional subjects of the string theory and the present SQS-conference. Usual treatment concerns the pointlike (or stringlike) objects in (super)space-time. The Kerr-Schild spinning particle is *extended* space-time object based on the rotating Kerr-Newman black hole solution which has gyromagnetic ratio, $g = 2$, as that of the Dirac electron (Carter 1969). This solution has a reach twistorial structure which polarizes space-time, so that the all surrounding (super)fields: spinor, electromagnetic, gravitational, (axion and dilaton) are to be aligned with the Kerr principal null congruence formed by twistor field.

The Dirac theory of electron and QED neglect gravity, arguing that it is very small. The Kerr geometry shows that gravity plays essential role which is expanded to the Compton region of electron. Origin of that is the extremely high value of the spin/mass ratio (about 10^{44} in the units $\hbar = c = G = 1$,) which shows that the estimations of the gravitational effects have to be based on the Kerr-Newman solution. Strong twistor polarization of the Kerr space-time induces very strong deformation of electromagnetic (em-) field which has to be aligned with the Kerr congruence. As a result, in the Kerr-Newman space-time with parameters of electron, em-field acquires singular ring of the Compton size $a = S/m = \hbar/2m \sim 10^{22} \sim 10^{-11}$ cm. which is naked, since horizons are absent by $a >> m$. The corresponding source of the Kerr-Newman solution represents string-like structure which matches naturally with gravity and displays especial role of the Compton region.

This space-time has twofold topology with a branch line along the Kerr singular ring which is a closed 'Alice' string. The field around the Kerr string is similar to the field around a heterotic string [1]. Polarization of the em-field near this string may be considered as em-excitation of the string.

Structure of the Kerr-Schild geometry. The Kerr-Newman metric may be represented in the Kerr-Schild form

$$g_{\mu\nu} = \eta_{\mu\nu} - 2Hk_\mu k_\nu, \tag{1}$$

where $\eta_{\mu\nu}$ is auxiliary Minkowski metric and $H = \frac{mr - q^2/2}{r^2 + a^2\cos^2\theta}$. For small mass m metric is flat almost everywhere, for exclusion of a small neighborhood of the Kerr string. Vector field k_μ is tangent to a twisting family of null rays – twistors, forming a vortex which is described by **the Kerr theorem**. This congruence of twistors **polarizes space-time** and determines the form of Kerr-Newman vector potential $A_\mu \sim k_\mu$, and the flow of radiation $T^{\mu\nu} \sim \Phi(x)k^\mu k^\nu$.

The Kerr theorem states that *any holomorphic surface in the projective twistor space* with coordinates Y, $\lambda_1 = \zeta - Yv$, $\lambda_2 = u + Y\bar\zeta$,

(where $2^{\frac{1}{2}}\zeta = x + iy$, $2^{\frac{1}{2}}\bar\zeta = x - iy$, $2^{\frac{1}{2}}u = z - t$, $2^{\frac{1}{2}}v = z + t$ are the null Cartesian coordinates) determines the geodesic and shear-free null congruence in M^4.

Such congruences lead to solutions of the Einstein-Maxwell field equations with metric (1) and aligned with k_μ em-field. The Kerr congruence of twistors is built of a family of straight null lines – the twisting family of photon geodesics. For any holomorphic function

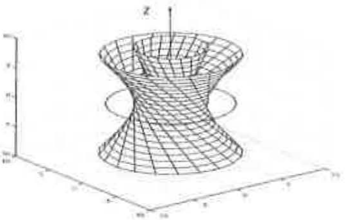

Figure 1: The Kerr singular ring and congruence.

F, solution $Y(x^\mu)$ of the equation $F(Y, \lambda_1, \lambda_2) = 0$ determines congruence of null lines by the 1-form

$$e^3 = du + \bar{Y}d\zeta + Yd\bar{\zeta} - Y\bar{Y}dv \qquad (2)$$

and the null vector field $k_\mu dx^\mu = P^{-1}e^3$. Function Y is projective spinor, $Y = \phi_2/\phi_1$, so $k_\mu = \bar{\phi}_{\dot\alpha}\bar{\sigma}_\mu^{\dot\alpha\alpha}\phi_\alpha$ in spinor form.

Solutions considered in the original work [2] have a quadratic in Y function

$$F \equiv \phi(Y) + (qY + c)\lambda_1 - (pY + \bar{q})\lambda_2 \qquad (3)$$

where $\phi = a_0 + a_1 Y + a_2 Y^2$.

In this case congruences may be found in explicit form and correspond to the Kerr solution up to Lorentz boost, orientation of spin, and shift of the origin.

Methods developed in the papers [3] allow one to fix explicit values of these parameters corresponding to concrete values of the boost and orientation of angular momentum.

Coefficients a_0, a_1, a_2 define orientation of angular momentum and the constants $p, q, \bar{q}, c$ are related to Killing vector of the solution. They determine function $P = pY\bar{Y} + qY + \bar{q}\bar{Y} + c$.

In particular, for the Kerr-Newman solution at rest $p = c = 2^{-1/2}$, $q = \bar{q} = 0$ and $P = 2^{-1/2}(1 + Y\bar{Y})$. Spin S is oriented along z axis for $a_0 = a_2 = 0$, and $a_1 = -ia$.

Recently, *the light-like analog of the Kerr-Newman solution* we considered in [4]. The total mass-energy $E = m$ of this solution is finite and concentrated near the light-like front $z + t = 0$. In the light-like limit the rest-mass m_0 of the corresponding stationary Kerr-Newman solution has to tend to zero, providing the finite value for the relativistic mass-energy $E = m = m_0/\sqrt{1 - (v/c)^2}$ in the limit $v \to c$. So, the limit $v \to c$ is to be related with the limit $m_0 \to 0$.

If angular momentum of the Kerr solution is oriented along the boost direction, the Kerr singular ring will be orthogonal to the boost and will not be subjected to Lorentz contraction. Thus, during the limiting procedure its radius will be $a = a_0$. Assuming that $m_0 \to 0$ by $v \to c$, and using the Kerr metric relation $J = m_0 a_0$, we have to set simultaneous $a_0 \to \infty$, or to shift singular ring of the corresponding Kerr solution to infinity. Setting $a = a_0\sqrt{1 - (v/c)^2}$ as a 'relativistic' parameter $a = J/m$ we conserve during the limit the Kerr relation $J = m_0 a_0 = ma$.

The parameters of the above Kerr generating function F were taken as a prototype of the boosted solution. We have $\phi = -iaY$ which corresponds to orientation of the angular momentum in the z-direction, and we direct collinearly the light-like boost which will be described by the parameters $p = q = \bar{q} = 0$, $c = 1$ corresponding to the null Killing direction $\hat{K} = \partial_u$.

It leads to the twistor congruence which depend explicitly on the light-like coordinate $v = 2^{-1/2}(z - t)$ The functions F and P of the Kerr theorem take in this case the simple form $F = -iaY + (\zeta - Yv)$, $P = 1$, and solution of the equation $F = 0$ yields

$$Y = \zeta/(ia + v) \ , \tag{4}$$

and function $Y(x)$ determines the principal null congruence e^3 by the relation (2). One sees that congruence has a non-trivial coordinate dependence and has a non-zero expansion θ and twist ω, where $Z = \theta + i\omega = (v - ia)/[v^2 + a^2]$, where Z is also determined by generating function F of the Kerr theorem,

$$Z^{-1} = \tilde{r} = -\partial_Y F = v + ia. \tag{5}$$

One sees that expansion tends to zero only near the front plane $v = 0$, (or $z = t$,) where the twist ω is maximal. In the vicinity of the axis z, where $Y\bar{Y} \to 0$, and far from the front plane, congruence tends to simple form $e^3 = du$ corresponding to pp-wave solutions. In spite of the very nontrivial form of the congruence, the equations turn out to be very simple and solvable. As a result, metric is determined by the Kerr-Schild ansatz (1) with function h given by

$$h = [mv - \frac{1}{2}A(Y)\bar{A}(\bar{Y})]/(v^2 + a^2), \tag{6}$$

where $Y = \frac{x+iy}{2^{1/2}(v+ia)}$.

One can try to use this light-like analog of the Kerr-Newman solution to describe gravitational field of a photon. In this case the Kerr relation $J = ma$ takes the form $E = J/a$, where $J = \hbar$. Comparison with the photon relation $E = \hbar\nu$ shows that characteristic size of this solution a corresponds to de Broglie wavelength of the photon.

Therefore, the twistorial structure of the Kerr geometry allows to describe an extended structure of the massive and light-like spinning particles. In the massive case this twistorial structure forms a complex string which is controlled by the solutions of the Dirac equations [5].

The case of linear in Y generating function F of Kerr theorem is important for the problem of gravitational (and electromagnetic) interactions of spinning particles. The corresponding Kerr-Schild treatment of multi-particle solutions has to be based on the Kerr theorem with generating functions F of different degrees in Y, including the linear ones and of higher orders [6]. Twistorial structure of a Kerr-Newman particle is described by a quadratic in Y generating function $F = F_2(Y)$, (3), and this case was investigated in details. Contrarily, the solutions with generating functions of first degree in Y, $F = F_1(Y)$ have not paid considerable attention, in spite of their important physical meaning - relations to the light-like spinning particles. The general case of higher degrees in Y was considered in [6]. It was shown that multi-particle Kerr-Schild solutions lead to a multi-sheeted twistor space which may be split into simple one- and two-sheeted blocks corresponding to a set of the light-like and massive particles. Machinery of the Kerr-Schild formalism shows that the gravitational and electromagnetic fields of interacting particles form a singular string along their *common* twistor line, fixed by the set

$$\{x^{\mu} : Y_1(x^{\mu}) = Y_2(x^{\mu})\}. \tag{7}$$

In other words, solutions acquire a propagator $\sim \frac{1}{Y_1(x^{\mu}) - Y_2(x^{\mu})}$, which is exhibited in the form of a singular pp-wave beam between the particles.

This treatment is close related to the suggested by Nair [7] and renewed by Witten [8] twistor-string approach to perturbative gauge theory of quantum scattering, where the traditional quantum treatment in momentum space gets a natural generalization to twistor space with corresponding twistorial generalizations of the wave functions and amplitudes of scattering. The gauge bosons are described by curves of first degree in twistor space, and the corresponding plane waves are replaced by twistor null planes. This approach has got great attention last years leading to drastic simplifications some of of the Feynman graphs. The approach on the base of Kerr theorem is more informative, since the description of wave function by a single twistor null plane is replaced here by section of a twistor bundle, and twistorial description of the wave functions contains important coordinate information which is necessary to incorporate gravity in quantum theory.

References

[1] A. Burinskii, Phys. Rev. **D 52** (1995)5826; **D68**, 105004 (2003); **D70**, 086006 (2004).

[2] G.C. Debney, R.P. Kerr and A. Schild, J. Math. Phys. **10** 1842 (1969).

[3] A.Burinskii and G. Magli, Phys.Rev.**D 61**044017 (2000).

[4] A. Burinskii, *Kerr-Schild photon-like metric solutions,* arXive:0704.3184 [gr-qc].

[5] A. Burinskii, *Complex Kerr geometry, twistors and the Dirac electron,* arXive:0710.4249 [hep-th].

[6] A. Burinskii, Grav. & Cosmol. **11**, 301 (2005); **12**, 119 (2006); IJGMMP 4, 437 (2007), hep-th/0510246.

[7] V.P. Nair, Phys. Lett. **B214**, 215 (1988).

[8] E. Witten, Comm. Math. Phys. **252**, 189 (2004).