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Quark polarimetry with a recursive fragmentation model including the spin degree of freedom

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Abstract. Several estimators of the initial transverse polarization of a quark fragmenting into a jet are proposed and compared using Monte Carlo simulations. These are based on the simplified version of a recursive model [A. Kerbizi et al, Phys. Rev. D97, 074010 (2018)] which includes the quark spin degree of freedom.

1. Introduction

The precise transverse profile of a jet produced by the fragmentation of a quark depends on the polarization vector $\mathbf{S} = (\mathbf{S}_\mathrm{T}, S_L)$. In particular, the Collins effect [1] results in a single-hadron distribution of the form

$$dN_{q_\mathrm{in}\to h+X} = (dz_h/z_h)\, d^2\mathbf{p}_\mathrm{T}\, D_0(z_h, |\mathbf{p}_\mathrm{T}|)\, [1 + |\mathbf{S}_\mathrm{T}|\, A_h(z_h, |\mathbf{p}_\mathrm{T}|)\, \sin\phi_{\mathbf{pS}}]\,; \tag{1}$$

$\mathbf{S}_\mathrm{T} = (S_x, S_y)$ is the *transversity* vector; $S_L = S_z$ is twice the *helicity*; D_0 is the unpolarized fragmentation function, A_h is the Collins analyzing power; $\phi_{\mathbf{pS}} = \phi_\mathbf{p} - \phi_\mathbf{S}$ where $\phi_\mathbf{V}$ is the azimuth of a vector $\mathbf{V}$; p and k_in are the 4-momenta of the observed hadron h and the quark which initiates the jet; $z_h = p^+/k^+_\mathrm{in}$ with $V^\pm = V^0 \pm V^z$. The z-axis is chosen along $\mathbf{k}_\mathrm{in}$. For a two-particle distribution, one may separate the global variables $z_{dh} = z_{h1} + z_{h2}$ and $\mathbf{P}_\mathrm{T} = \mathbf{p}_{1\mathrm{T}} + \mathbf{p}_{2\mathrm{T}}$ of the *di-hadron* $dh = \{h_1, h_2\}$, from the relative variables $\dot{z}_1 = z_{h1}/z_{dh}$ and $\mathbf{R}_\mathrm{T} = (1-\dot{z}_1)\,\mathbf{p}_{1\mathrm{T}} - \dot{z}_1\,\mathbf{p}_{2\mathrm{T}}$. Then

$$\begin{aligned} dN_{q_\mathrm{in}\to h_1+h_2+X} &= d^2\mathbf{P}_\mathrm{T}\, d^2\mathbf{R}_\mathrm{T}\, \frac{dz_{h1}}{z_{h1}}\frac{dz_{h2}}{z_{h2}}\, D_0(\mathcal{X}) \\ \times\ & \left\{1 + |\mathbf{S}_\mathrm{T}|\, [A_{dh\mathbf{P}}(\mathcal{X})\, \sin\phi_{\mathbf{PS}} + A_{dh\mathbf{R}}(\mathcal{X})\, \sin\phi_{\mathbf{RS}}] + S_L\, A_{dh\mathrm{L}}(\mathcal{X})\, \sin\phi_{\mathbf{RP}}\right\} \end{aligned} \tag{2}$$

where $\mathcal{X}$ is the argument list $\{z_{dh}, \dot{z}_1, \mathrm{P}_\mathrm{T}, \mathrm{R}_\mathrm{T}, |\phi_{\mathbf{RP}}|\}$, $A_{dh\mathbf{P}}$ and $A_{dh\mathbf{R}}$ are the *global* and *relative* Collins asymmetries of the pair. $A_{dh\mathrm{L}}$ is a measure of *jet handedness* [2, 3] associated to the quark helicity. Integrating over $\mathbf{P}_\mathrm{T}$ yields

$$dN_{q_\mathrm{in}\to h_1+h_2+X} = d^2\mathbf{R}_\mathrm{T}\, \frac{dz_{h1}}{z_{h1}}\frac{dz_{h2}}{z_{h2}}\, \bar{D}_0(z_{dh}, \dot{z}_1, \mathrm{R}_\mathrm{T})\left\{1 + \mathbf{S}_\mathrm{T}\, \bar{A}_{dh\mathbf{R}}(z_{dh}, \dot{z}_1, \mathrm{R}_\mathrm{T})\, \sin\phi_{\mathbf{RS}}\right\}. \tag{3}$$

$\bar{A}_{dh\mathbf{R}}$ is called the *di-hadron analyzing power*, $\bar{A}_{dh\mathbf{R}} \times \bar{D}_0$ the *interference fragmentation function.* The above asymmetries can serve as quark polarimeters in the measurement of the transversity distribution in the nucleon, $h_1(x)$, or for testing new physics. If $\mathbf{k}_{\rm in}$ is not precisely defined, due to experimental errors or to gluon emission, A_h, $A_{dh\mathbf{P}}$ and $A_{dh\rm L}$ are blurred, but not $A_{dh\mathbf{R}}$ or $\bar{A}_{dh\mathbf{R}}$.

The present quark fragmentation models used in simulation codes like PYTHIA are blind to the quark spin degree of freedom, therefore these codes cannot generate Collins- or jet handedness asymmetries. However this situation is changing with the development of recursive models for the fragmentation of polarized quarks. One of them [4, 5, 6] is based on the string fragmentation model and the $^3P_0$ mechanism of $q\bar{q}$ pair creation. It postulates a particular spin-dependent quantum-mechanical amplitude of the elementary splitting process $q \to h + q'$. Implemented in a Monte Carlo code [7], it gives results in reasonable agreement with e^+e^- data from the BELLE collaboration [8] and semi-inclusive deep inelastic scattering data from the COMPASS collaboration [9, 10]. In particular it predicts a large negative A_{h^+} and equally large positive A_{h^-} in u-quark jet. For the moment the model is restricted to jets made of pseudoscalar mesons only. Another model [11], which uses the positivity constraints, has also been proposed.

Quark polarimetry relies on *estimators*, $E_\alpha\big(p_1, Q_1, p_2, Q_2...\big)$ for the component S_α of the polarization. E_α is a function of the 4-momenta p_n and charges Q_n of a set $\{h_1, h_2, ...\}$ of detected particles of the jet. Its expectation value is S_α times a calibration coefficient C_α. The estimated S_α from a set of N experimental events is therefore

$$S_\alpha = \frac{\langle E_\alpha \rangle}{C_\alpha}\,, \text{ with } \langle E_\alpha \rangle = \frac{1}{N}\sum_{\nu=1}^{N} E_\alpha(\nu)\,, \tag{4}$$

$E_\alpha(\nu)$ being the measured value of E_α in the $\nu^{\rm th}$ event. If E_α cannot be measured in this event, the value $E_\alpha(\nu) = 0$ is assigned. The efficiency of an estimator E_α is defined for $S_\alpha = +1$ by

$$\text{Efficiency}\{E_\alpha\} = \hat{E}_\alpha^2\,, \text{ where } \hat{E}_\alpha = \langle E_\alpha \rangle / \sqrt{\langle E_\alpha^2 \rangle} \in [-1, +1]\,. \tag{5}$$

$\hat{E}_\alpha$ is indeed the signal/noise ratio per event. For a given precision the required number of events is inversely proportional to $\hat{E}_\alpha^2$.

A good estimator E_α must be simple, universal (*i.e.*, usable in all types of jet-producing reactions like e^+e^- annihilation, deep inelastic scattering an hight-$\mathbf{p}_{\rm T}$ collisions) and have a large efficiency. Its choice can be guided by a Monte Carlo simulation based on a polarized fragmentation model. In this article we present an example of this method for quark transverse polarization, using the the string+$^3P_0$ model. Examples of estimators for the transversity are given in Section 2. In Section 3 the string+$^3P_0$ model is briefly reviewed, in a simplified version [12]. Simulated results of $\hat{E}$ for several estimators are given and discussed in Section 4.

2. Exemples estimators of transversity

A simple estimator of transversity, based on the Collins effect, is

$$\mathbf{E}_{\rm T} = (E_x, E_y) = \hat{\mathbf{z}} \times \mathbf{p}_{\rm T}/|\mathbf{p}_{\rm T}| = (-\sin\phi_{\mathbf{p}}, \cos\phi_{\mathbf{p}})\,, \tag{6}$$

where $\mathbf{p}$ is the momentum a detected particle of the jet, selected for instance for its charge and its order in rapidity. With two hadrons h_1 and h_2 one can use the relative Collins effect, replacing $\mathbf{p}_{\rm T}$ by $\mathbf{R}_{\rm T}$. An estimator more efficient than (6), inspired by Eq. (13) of [4], could be

$$\mathbf{E}'_{\rm T} = z_h\, \hat{\mathbf{z}} \times \mathbf{p}_{\rm T}/(\mathbf{p}_{\rm T}{}^2 + \lambda\langle \mathbf{p}_{\rm T}{}^2\rangle)\,, \tag{7}$$

where λ is a parameter of the order of unity. This estimator takes advantage of the increase of $A_h(z_h, |\mathbf{p}_\mathrm{T}|)$ with z_h and its vanishing for $|\mathbf{p}_\mathrm{T}| \to 0$ or ∞. Theoretically, the most efficient estimator of S_α is the derivative $\partial/\partial_{S_\alpha}$ of the spin asymmetry. Rewriting the square bracket in Eq.(1) as $[1 - A_h(z_h, |\mathbf{p}_\mathrm{T}|)\,\mathbf{S}\cdot(\hat{\mathbf{z}}\times\mathbf{p}_\mathrm{T})/|\mathbf{p}_\mathrm{T}|]$, we deduce

$$\mathbf{E}_\mathrm{T}(\text{optimal}) = -A_h(z_h, |\mathbf{p}_\mathrm{T}|)\,\hat{\mathbf{z}}\times\mathbf{p}_\mathrm{T}/|\mathbf{p}_\mathrm{T}|\,. \tag{8}$$

To have an efficiency close to the optimal one, it not necessary to know the function $A_h(z_h, |\mathbf{p}_\mathrm{T}|)$ precisely. $\hat{E}_\alpha(\text{optimal}) - \hat{E}_\alpha$ is indeed of the second order in $E_\alpha(\text{optimal}) - E_\alpha$.

One can improve the polarimetry by gathering informations from all detected particles h_n of the jet. The *multi-hadron* estimator

$$\mathbf{E}^\mathrm{m}_\mathrm{T} = \sum_n Q_n\, z_{hn}\,\hat{\mathbf{z}}\times\mathbf{p}_{n\mathrm{T}}\,/\,(\mathbf{p}^2_{n\mathrm{T}} + \lambda\langle{\mathbf{p}_\mathrm{T}}^2\rangle)\,, \tag{9}$$

which generalizes (7), takes into account the fact that the sign of the Collins asymmetry is linked to the charge Q of the hadron.

3. The recursive *string* + $^3P_0$ model

This model is a generalization of the Lund model of string fragmentation [13], but starting with amplitudes instead of probabilities and including the quark spin degree of freedom, represented by Pauli spinors. It can also be formulated as a multiperipheral model with quark propagators and quark-hadron vertex functions being 2×2 matrices. In the *ladder* approximation, one obtains the Markov chain

$$q_1 \to h_1 + q_2\,,\quad q_2 \to h_2 + q_3\,,\quad \textit{etc.}, \tag{10}$$

which can be generated by the Monte Carlo method. The *splitting function* F of the elementary process $q \to h + q'$, defined by

$$dN(q\to h+q') = F_{q\to h+q'}(Z, \mathbf{p}_\mathrm{T}, \mathbf{k}_\mathrm{T}, \mathbf{S}_q)\, d^2\mathbf{p}_\mathrm{T}\, dZ/Z \quad (Z = p^+/k^+)\,, \tag{11}$$

depends on the polarization $\mathbf{S}_q$ of q and is of the form

$$F_{q\to h+q'}(Z, \mathbf{p}_\mathrm{T}, \mathbf{k}_\mathrm{T}, \mathbf{S}_q) = \mathrm{Tr}\left[T_{q\to h+q'}(Z, \mathbf{p}_\mathrm{T}, \mathbf{k}_\mathrm{T})\ \rho(q)\ T^\dagger_{q\to h+q'}(Z, \mathbf{p}_\mathrm{T}, \mathbf{k}_\mathrm{T})\right]. \tag{12}$$

$\rho(q) = (\mathbf{1} + \mathbf{S}_q\cdot\boldsymbol{\sigma})/2$ is the spin density matrix of quark q. The *splitting matrix* T cannot be chosen arbitrarily because the model must be symmetric under the reversal of the multiperipheral quark line (the "left-right" symmetry of the Lund model). Its general form is given in Eqs. (35) and (50) of [7]. Once h, q', Z and $\mathbf{p}_\mathrm{T}$ have been drawn, one calculates the spin density matrix of q' for the next iteration by

$$\rho(q') = \left[T_{q\to h+q'}(Z, \mathbf{p}_\mathrm{T}, \mathbf{k}_\mathrm{T})\ \rho(q)\ T^\dagger_{q\to h+q'}(Z, \mathbf{p}_\mathrm{T}, \mathbf{k}_\mathrm{T})\right] / \,\mathrm{Tr}\,[\text{idem}] \tag{13}$$

Here we use a simpler version [12] of the model than the one applied in [7]. It corresponds to the choice (c) for the function $\check{g}(\epsilon^2_h)$ below Eq.(50) of [7]. Then the auxiliary matrix function $\hat{u}_q(\mathbf{k}_\mathrm{T})$ introduced in Eq.(46) of [7] is the unit matrix times a normalization factor. We restrict ourselves to the emission of pseudo-scalar mesons, where the vertex function is simply $\Gamma_h = \sigma_z$ (the 2×2 analogue of γ_5). Instead of Eq.(51) of [7] we obtain

$$\begin{aligned} F_{q',h,q}(Z, \mathbf{p}_\mathrm{T}, \mathbf{k}_\mathrm{T}, \mathbf{S}_q) &= |C_{q',h,q}|^2\, N_a^{-1}(\epsilon^2_h)\ \left[(1-Z)/\epsilon^2_h\right]^a\ \exp(-b_\mathrm{L}\epsilon^2_h/Z - b_\mathrm{T}\mathbf{k}'^2_\mathrm{T}) \\ &\times\ \mathrm{Tr}\left[(\mu + \sigma_z\boldsymbol{\sigma}\cdot\mathbf{k}'_\mathrm{T})\,\sigma_z\,\rho(q)\,\sigma_z\,(\mu^* + \boldsymbol{\sigma}\cdot\mathbf{k}'_\mathrm{T}\sigma_z)\right] \end{aligned} \tag{14}$$

with $\epsilon_h = [m_h^2 + \mathbf{p}_\mathrm{T}{}^2]^{1/2}$, $\mathbf{k}'_\mathrm{T} = \mathbf{k}_\mathrm{T} - \mathbf{p}_\mathrm{T}$ and

$$N_a(\epsilon_h^2) = \int_0^1 \frac{dZ}{Z} \left(\frac{1-Z}{\epsilon_h^2}\right)^a \exp(-b_\mathrm{L}\,\epsilon_h^2/Z)\,. \tag{15}$$

a and b_L correspond to the parameters a and b of the Lund model. b_T governs the width of quark transverse momentum. $C_{q',h,q}$ is proportional to the internal $(\bar{q}'q)$ wave function of the hadron h in flavor space. It acts upon the relative abundances of the hadron species. The second line of Eq.(14) makes the difference with the spin-blind Lund model. The factor $\boldsymbol{\sigma}\cdot\mathbf{k}'_\mathrm{T}$ is inspired by the $^3P_0$ wave function $\propto \boldsymbol{\sigma}\cdot\mathbf{k}$ of a $(q'\bar{q}')$ state. μ is a complex parameter having the dimension of a mass. The qualitative results of the classical string+$^3P_0$ mechanism of $q\bar{q}$ pair production are reproduced by taking $\mathrm{Im}(\mu) > 0$ [4].

4. Numerical results for the estimators of transverse polarization

We took the same parameters $a = 0.9$, $b_\mathrm{L} = 0.5$ GeV^{-2}, $b_\mathrm{T} = 5.17$ GeV^{-2} and $\mu = (0.42{+}i0.76)$ GeV as used in [7] and the same subroutines for the generation of the quark flavors and meson species. In principle we should have re-ajusted the parameters for the present version of the model. We think, however, that il will not change our qualitative comparisons between estimators. We generated 20 hadrons per event, without taking into account the finite mass of the intial string, but made a cut $z_h \geq z_{h,\mathrm{min}}$. We also introduced a Gaussian-distributed *primordial transverse momentum* $\mathbf{k}_\mathrm{T}^\mathrm{prim}$ of r.m.s. K. The $\mathbf{p}_\mathrm{T}$'s of all the mesons in a jet were shifted by $z_h\,\mathbf{k}_\mathrm{Tprim}$. It can as well simulate the effect of gluon radiation or an experimental error in the definition of the jet axis. We have compared the following estimators of the transverse polarization S_y:

$E^\pm = p_x/|\mathbf{p}_\mathrm{T}| = \cos\phi_\mathbf{p}$ of the fastest positive (for E^+) or negative (for E^-) hadron,
$E^\mathrm{m} = \sum_n Q_n\,p_{nx}/|\mathbf{p}_{n\mathrm{T}}|$, where n labels the n^th detected hadron,
$E'^\pm = z_h\,p_x/(\mathbf{p}_\mathrm{T}^2 + 0.5|\mu|^2 + 0.5\,z_h^2\,K^2)$ of the fastest positive or negative hadron,
$E'^\mathrm{m} = \sum_n Q_n\,z_{hn}\,p_{nx}/(\mathbf{p}_{n\mathrm{T}}^2 + 0.5|\mu|^2 + 0.5\,z_{hn}^2\,K^2)$,
$E^\mathrm{R} = R_x/|\mathbf{R}_\mathrm{T}|$, where $\mathbf{R} = (z_{h^-}\,\mathbf{p}_{h^+} - z_{h^+}\,\mathbf{p}_{h^-})/(z_{h^+} + z_{h^-})$ for the fastest h^+ and h^-,
$E'^\mathrm{R} = z_{h^+}\,z_{h^-}\,R_x/(\mathbf{R}_\mathrm{T}^2 + 1.0)$.

The unprimed E's are of the type of (6). $E'^\pm$ and E'^m are of the type of (7) with $\lambda = 0.5$, assuming $\langle\mathbf{p}_\mathrm{T}^2\rangle \sim |\mu|^2 + (z_h\,K)^2$. E^m and E'^m are *multi-hadron* estimators. E^R and E'^R, based on the relative Collins effect or di-hadron asymmetry, are not sensitive to K.

Table 1 shows the "root efficiencies" $\hat{E} = \langle E\rangle/\sqrt{\langle E^2\rangle}$, obtained in a simulation of 10^5 jets from u quarks fully polarized along $+\hat{\mathbf{y}}$. The r.m.s. of $\mathbf{p}_\mathrm{T}$ (in GeV) is also indicated. Three values of K were considered. We retained only particles with $z_h \geq z_{h,\mathrm{min}} = 0.1$. This restriction makes some estimators unavailable in parts of the events (then they are assigned the value 0). The fraction of events where the estimator E is available is given in the last line. The mean charged multiplicities are $\langle N^+\rangle = 1.06$ and $\langle N^-\rangle = 0.58$.

Discussion of the results.
- For K =1 GeV the efficiencies of all estimators are significantly reduced, except for the di-hadron ones E^R and E'^R which are not affected by the primordial $\mathbf{k}_\mathrm{T}$. Note that an increase of $\hat{E}$ by a factor 1.5 reduces the required number of events by a factor 2.25 to get the same precision.
- Replacing an E by the corresponding E' increases the efficiency, particularly for E^R.
- The *multi-hadron* estimators E^m or E'^m have the largest efficiencies, for all values of K.

Table 1. Root effeciencies $\hat{E}$ of several estimators (see text).

K(GeV)	rms($\mathbf{p}_\mathrm{T}$)	$\hat{E}^+$	$\hat{E}'^+$	$\hat{E}^-$	$\hat{E}'^-$	$\hat{E}^m$	$\hat{E}'^m$	$\hat{E}^\mathrm{R}$	$\hat{E}'^\mathrm{R}$
0	0.733	0.171	0.188	- 0.107	- 0.132	0.181	0.215	0.118	0.157
0.5	0.751	0.156	0.166	- 0.104	- 0.126	0.172	0.197	0.118	0.157
1.0	0.801	0.128	0.132	- 0.096	- 0.113	0.154	0.168	0.118	0.157
available	fraction:	0.838	0.838	0.516	0.516	1.0	1.0	0.457	0.457

5. Conclusion

Our study shows that quark polarimetry can be significantly improved by a judicious choice of the estimator. This choice can be first guided by Monte-Carlo simulations based on a theoretical model. The final optimization should be made using equation like (8) with experimental data. The choice may also be influenced by the experimental conditions like the resolution and the acceptance of the detectors, but then the estimator is no more "universal".

In the simulation we assumed that the quark flavor is u. A d-quark would give approximately opposite asymmetries. When the flavor is not *a priori* known, the polarization estimator has to be coupled with a flavor estimator.

The choice of estimator will be still more critical for the polarimetry of quark helicity based on jet handedness. This effect is predicted by the existing theoretical models [4, 7, 11]. To avoid the blurring by the primordial transverse momentum, the handedness estimator should involve *three* hadrons [2, 3]. This makes the number of variables rather large and the estimator choice more difficult than for transversity.

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