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Fermionic steering in multi-event horizon spacetime

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Abstract We investigate quantum steering of Dirac field for different types of Bell states in Schwarzschild–de Sitter (SdS) spacetime that has a black hole event horizon (BEH) and a cosmological event horizon (CEH). We find that fermionic steerability from Bob to Alice is greater than fermionic steerability from Alice to Bob, while bosonic steerability exhibits the opposite behavior in SdS spacetime. These different properties between fermionic and bosonic steering arise from the differences between Fermi–Dirac statistics and Bose–Einstein statistics. We also find that the Hawking effect of the black hole decreases fermionic steerability. However, the Hawking effect of the expanding universe can enhance fermionic steerability, which differs from the properties of quantum steering in single-event horizon spacetime. Interestingly, we can indirectly protect quantum steering by using appropriate types of Bell states in multi-event horizon spacetime. These conclusions are helpful to guide the task of processing relativistic quantum information for quantum steering in SdS spacetime.

1 Introduction

Early studies of quantum nonlocality focused on symmetric correlations, including entanglement [1] and Bell nonlocality [2–4], where the observers in the system have symmetric states. In 1935, Schrödinger proposed the Einstein–Podolsky–Rosen (EPR) steering to discuss the EPR paradox [5–7], highlighting that EPR steering is a type of asymmetric quantum nonlocality. EPR steering has the property that one observer influences the state of another observer by performing local measurements, and its asymmetric quantum nonlocality has attracted widespread attention [8,9]. In a bipartite system involving Alice and Bob, Alice can steer Bob's state if the assemblage of Bob's conditional states, after Alice's local measurements, cannot be explained by a local hidden state model [8,10]. Based on the asymmetry of quantum steering, all asymmetric configurations in the bipartite system can be divided into three types: two-way steering, one-way steering, and no-way steering. The unique directional applications of EPR steering have been widely discussed and applied in fundamental quantum information processing and asymmetric quantum communication protocols [11–20].

The SdS spacetime constitutes a precise black hole solution derived from Einstein's equations, delineating a static, spherically-symmetric, uncharged black hole formation within the context of a positive cosmological constant. Research on black holes typically emphasizes asymptotically flat spacetime. However, experimental evidence indicates that our universe is undergoing accelerating expansion [21,22], necessitating an acknowledgment of the effects of a positive cosmological constant. We are particularly interested in the SdS spacetime with a positive cosmological constant, because it holds the potential to deepen our understanding of the early expansion stages of the universe. The SdS spacetime also encompasses causally disconnected regions, and an observer can only access the parts of the universe bounded by their respective horizon. In SdS spacetime, two distinct event horizons exist: the black hole horizon and the cosmological horizon. Compared to the single-event horizon spacetime, each event horizon in this spacetime is characterized by its unique temperature [23–28]. This temperature is proportionate to the surface gravity of the respective event horizon [29].

Generally, the investigation of relativistic quantum information primarily concentrates on single-event horizon spacetime [30–78]. Notably, A. Ali et al. were the first to identify a persistent trade-off between first-order coherence and concurrence, highlighting the delicate balance between coherence and entanglement in such spacetimes [43]. Addition-

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ally, the relationship between fermionic entanglement and steering within single-event horizon spacetime has also been explored [67]. W. Wen et al. extended the concept of quantum steering to de Sitter space, originally providing an explanation of its relationship with asymmetry in this single-event horizon spacetime [66]. Interestingly, fermionic steering always survives under the influence of the Hawking effect [67], whereas bosonic steering undergoes sudden death in single-event horizon spacetime [64–66]. As fermionic steering is more suitable for handling relativistic quantum information compared to bosonic steering, we investigate fermionic steering in multi-event horizon spacetime in this paper. Given that the Hawking temperature of the black hole exceeds the cosmological Hawking temperature, the difference between the two temperatures leads to asymmetric steering. Hence, quantum steering exhibits a richer set of properties than quantum entanglement in multi-event horizon spacetime. We can better understand multi-event horizon spacetime through asymmetric steering.

Our model considers two observers, Alice and Bob, who are located at the black hole event horizon (BEH) and the cosmological event horizon (CEH), respectively. In SdS spacetime, fermionic steerability from Bob to Alice consistently exceeds fermionic steerability from Alice to Bob, while bosonic steerability from Bob to Alice is smaller than bosonic steerability from Alice to Bob in multi-event horizon spacetime [79]. These results indicate a stark contrast between fermionic and bosonic steering, which arises from the fundamental differences between Fermi–Dirac statistics and Bose–Einstein statistics in multi-event horizon spacetime. In addition, we want to explore whether there are the same properties of fermionic steering for different types of Bell states in SdS spacetime? This conclusion can help us choose the appropriate type of Bell state to handle relativistic quantum information tasks involving quantum steering. Generally, the Hawking effect of the single-event horizon spacetime reduces fermionic steering [67]. A natural question arises: Can the Hawking effect in multi-event horizon spacetime increase fermionic steerability?

The structure of the paper is as follows. In Sect. 2, we discuss the quantization of the Dirac field in SdS spacetime. In Sect. 3, we introduce the quantification of quantum steering. In Sect. 4, we study fermionic steering for four types of Bell states in multi-event horizon spacetime. The last section is devoted to a brief conclusion.

2 Quantization of Dirac field in SdS spacetime

The SdS metric can describe a static spherically symmetric black hole in de Sitter spacetime in Fig. 1, which is given by

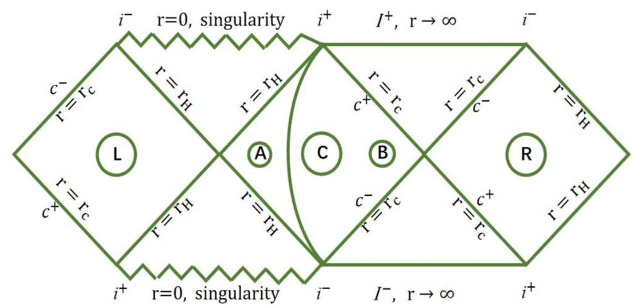

Fig. 1 The Penrose–Carter diagram shows the causal structure of the extended SdS spacetime. $i^{\pm}$ respectively denote the future and past time-like infinities and the infinities $I^{\pm}$ are spacelike. A thermally opaque membrane placed in region $C(r_H < r < r_C)$ cuts it into two subregions: A and B. The regions R and L are time reversed with respect to the region C. All the seven wedges are causally disconnected

[80–83]

$$ds^2 = -\left(1-\frac{2M}{r}-\frac{\Lambda r^2}{3}\right)dt^2+\left(1-\frac{2M}{r}-\frac{\Lambda r^2}{3}\right)^{-1}dr^2+r^2d\Omega^2, \tag{1}$$

where M is the mass of the black hole and Λ is the cosmological constant. Note that $1-\frac{2M}{r}-\frac{\Lambda r^2}{3}=\frac{\Lambda}{3r}(r-r_H)(r_C-r)(r-r_U)$, where r_C and r_H are the locations of the cosmological event horizon (CEH) and black hole event horizon (BEH), respectively. In addition, $r_U=-(r_H+r_C)$ is considered as an unphysical and negative solution [82], because we cannot extend the coordinate range beyond the curvature singularity at $r=0$. In SdS spacetime, if $3M\sqrt{\Lambda}<1$, we can get Killing horizons as

$$r_H=\frac{2}{\sqrt{\Lambda}}\cos\left[\frac{\cos^{-1}(-3M\sqrt{\Lambda})+\pi}{3}\right],$$
$$r_C=\frac{2}{\sqrt{\Lambda}}\cos\left[\frac{-\cos^{-1}(-3M\sqrt{\Lambda})+\pi}{3}\right]. \tag{2}$$

The surface gravities of the black hole and expanding universe can be written as

$$\kappa_H=\frac{\Lambda(2r_H+r_C)(r_C-r_H)}{6r_H}=-\sqrt{\Lambda}\left\{\cos\left[\frac{1}{3}\cos^{-1}[3M\sqrt{\Lambda}]+\frac{\pi}{3}\right]-\frac{1}{4\cos\left[\frac{1}{3}\cos^{-1}[3M\sqrt{\Lambda}]+\frac{\pi}{3}\right]}\right\}, \tag{3}$$

$$
\begin{aligned}
-\kappa_C &= \frac{\Lambda(r_H+2r_C)(r_H-r_C)}{6r_C} \\
&= \sqrt{\Lambda}\left\{\frac{1}{4\cos\left[\frac{1}{3}\cos^{-1}[3M\sqrt{\Lambda}]-\frac{\pi}{3}\right]}\right. \\
&\quad \left. -\cos\left[\frac{1}{3}\cos^{-1}[3M\sqrt{\Lambda}]-\frac{\pi}{3}\right]\right\}.
\end{aligned} \tag{4}
$$

The surface gravity of the expanding universe is negative, because repulsive effects are generated by a positive cosmological constant ($\Lambda > 0$). The Hawking temperature of the black hole can be written as $T_H = \frac{\kappa_H}{2\pi}$ and the Hawking temperature of the expanding universe is $T_C = \frac{\kappa_C}{2\pi}$. Since $r_H < r_C$, we can get that the Hawking temperature of the black hole is bigger than the Hawking temperature of the expanding universe ($T_H > T_C$). For the Nariai limit $3M\sqrt{\Lambda} \to 1$, we get $r_H = r_C = \frac{1}{\sqrt{\Lambda}}$, which represents the configuration in the largest black hole and smallest de Sitter space. The mass of the black hole in multi-event horizon spacetime cannot exceed a certain threshold at the given cosmological constant Λ. Therefore, when $3M\sqrt{\Lambda} > 1$, the BEH no longer exists, and a naked curvature singularity occurs.

Since $r = r_H, r_C$ are two coordinate singularities [80], we need two sets of the Kruskal-like coordinates to extend the spacetime

$$
\begin{aligned}
ds^2 = &-\frac{2M}{r}\left|1-\frac{r}{r_C}\right|^{1+\frac{\kappa_H}{\kappa_C}}\left(1+\frac{r}{r_C+r_H}\right)^{1-\frac{\kappa_H}{\kappa_U}} \\
&\times d\bar{\mu}_H d\bar{\nu}_H + r^2 d\Omega^2,
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
ds^2 = &-\frac{2M}{r}\left|\frac{r}{r_H}-1\right|^{1+\frac{\kappa_C}{\kappa_H}}\left(1+\frac{r}{r_C+r_H}\right)^{1+\frac{\kappa_C}{\kappa_U}} \\
&\times d\bar{\mu}_C d\bar{\nu}_C + r^2 d\Omega^2,
\end{aligned} \tag{6}
$$

where $\bar{\mu}_H = -\frac{1}{\kappa_H}e^{-\kappa_H \mu}$, $\bar{\nu}_H = \frac{1}{\kappa_H}e^{\kappa_H \nu}$, $\bar{\mu}_C = \frac{1}{\kappa_C}e^{\kappa_C \mu}$, and $\bar{\nu}_C = -\frac{1}{\kappa_C}e^{-\kappa_C \nu}$ are the Kruskal null coordinates. Here, $\mu = t - r_*$ and $\nu = t + r_*$ are the usual retarded and advanced null coordinates with the radial tortoise coordinate

$$
\begin{aligned}
r_* = &\frac{1}{2\kappa_H}\ln|\frac{r}{r_H}-1| - \frac{1}{2\kappa_C}\ln|1-\frac{r}{r_C}| \\
&+\frac{1}{2\kappa_U}\ln|\frac{r}{r_U}-1|,
\end{aligned} \tag{7}
$$

where κ_U is the surface gravity of the unphysical horizon r_U. In the limit $\Lambda \to 0$, we obtain the $r_C \to \infty$, $\kappa_C \to 0$, $r_H \to 2M$, and $\kappa_H \to \frac{1}{4M}$. For this section, the SdS metric has the same line element as the Schwarzschild metric.

We initially consider a two-mode entangled state shared by Alice and Bob. Here, Alice located at the black hole side detects the Hawking radiation at the temperature T_H in the subregion A, while Bob detects the temperature T_C in the subregion B. Because the SdS spacetime has two event horizons associated with different Hawking temperatures, we separate two event horizons with a thermally opaque membrane [83–85] in the region C ($C = A \bigcup B$) in Fig. 1, which prevents modes from penetrating and confines them in the respective regions.

The massless Dirac equation in a general background spacetime is [86,87]

$$
[\gamma^a e_a^\mu(\partial_\mu + \Gamma_\mu)]\Psi = 0, \tag{8}
$$

where γ^a denotes the Dirac matrices, $\Gamma_\mu = \frac{1}{8}[\gamma^a, \gamma^b]e_a^\nu e_{b\nu;\mu}$ is the spin connection coefficient, and the four-vectors e_a^μ represents the inverse of the tetrad e_μ^a. The quantization in the side of the membrane that faces the black hole in subregion A, is similar to a single-event horizon spacstime [39,88,89]. Therefore, we can quantize the Dirac field using the local modes and Kruskal modes, respectively. Using the Bogoliubov transformations between the creation and annihilation operators of SdS and Kruskal spacetime [89], the Kruskal vacuum state in SdS spacetime can be expressed as

$$
|0_{\kappa_H}\rangle = \cos r|0_A, 0_L\rangle + \sin r|1_A, 1_L\rangle, \tag{9}
$$

where $\cos r = \frac{1}{\sqrt{1-e^{-\frac{\omega}{T_H}}}}$. Similarly, the expression of the Kruskal vacuum state in the expanding universe reads

$$
|0_{\kappa_C}\rangle = \cos\alpha|0_B, 0_R\rangle + \sin\alpha|1_B, 1_R\rangle, \tag{10}
$$

where $\cos\alpha = \frac{1}{\sqrt{1-e^{-\frac{\omega}{T_C}}}}$. Here, $|n_A\rangle$, $|n_L\rangle$, $|n_B\rangle$, and $|n_R\rangle$ ($n \in 0, 1$) characterize the modes in regions A, L, B, and R, respectively. Note that region A is causally disconnected from region L, and region B is causally disconnected from region R. In addition, these excited states are written as

$$
|1_{\kappa_H}\rangle = |1_A, 0_L\rangle, \tag{11}
$$

$$
|1_{\kappa_C}\rangle = |1_B, 0_R\rangle. \tag{12}
$$

In Fig. 2, we show the dependency of the Hawking temperatures T_H and T_C on the mass M of the black hole and cosmological constant Λ, respectively. In Fig. 2a, we can see that the Hawking temperature T_H of the BEH decreases with the mass M of the black hole. However, the Hawking temperature T_C of the CEH first increases and then decreases with the cosmological constant Λ in Fig. 2b. It is shown that the Hawking temperature T_H is greater than the Hawking temperature T_C, which is the cause of the steering asymmetry.

The presence of the thermally opaque membrane allows us to understand the influence of one horizon by regarding the other horizon as the boundary [83–85]. To achieve this goal, the following Schrödinger-like equation was addressed in the following manner [90,91]

$$
-\frac{d^2 Z_\pm}{dr^{*2}} + V_\pm = \omega^2 Z_\pm\ , \tag{13}
$$

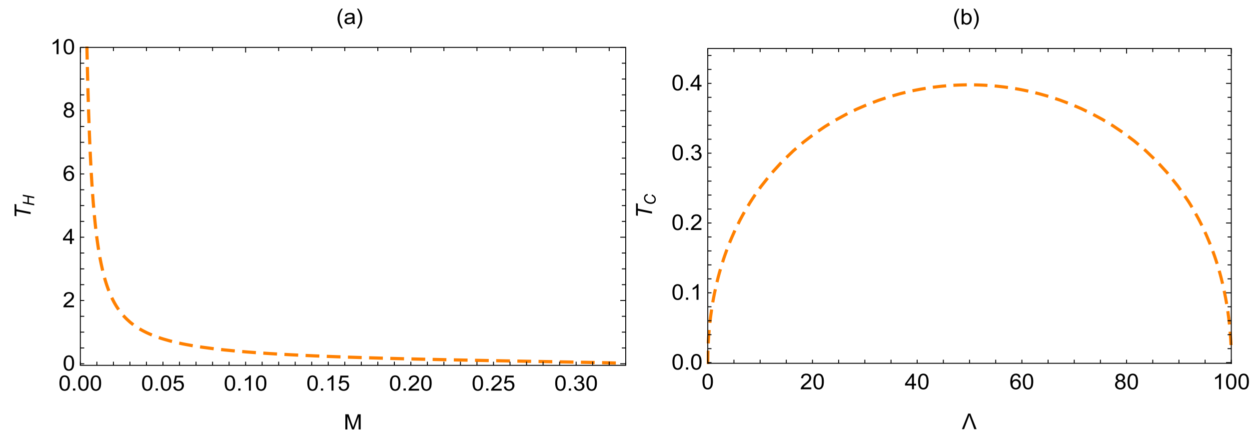

Fig. 2 **a** The Hawking temperature T_H of the BEH as functions of the mass M of the black hole for fixed $\Lambda = 1$. **b** The Hawking temperature T_C of the CEH as functions of the cosmological constant Λ for fixed $M = \frac{1}{30}$

with the effective potentials $V_\pm$

$$V_\pm = W^2 \pm \frac{dW}{dr^*} = \mp K \frac{f(r)\sqrt{f(r)}}{r^2} \pm K \frac{f(r)'\sqrt{f(r)}}{2r} + \frac{K^2 f(r)}{r^2}, \tag{14}$$

where the parameter K can assume positive and negative integer values, given by $K = \pm(l+1)$, with $l = 0, 1, 2, \ldots$. The potentials V_+ and V_- are smooth functions that approach zero near r_H and r_C, respectively, while remaining positive in between. Consequently, this bell-shaped potential serves as the thermally opaque membrane between the BEH and CEH. Modes unable to traverse this thermally opaque membrane become localized near the horizons, leading to their isolation from each other.

3 Quantification of quantum steering

One party of a bipartite quantum system can use local measurement to influence the quantum state of the other party. This is known as quantum steering, which is a type of nonlocal correlation. We consider a density matrix of the X-state ρ_x as

$$\rho_x = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}, \tag{15}$$

where ρ_{ij} is the real element satisfying $\rho_{ij} = \rho_{ji}$. The concurrence of the X-state ρ_x given by Eq. (15) can be specifically shown as [92]

$$C(\rho_x) = 2\max\left\{0, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}\right\}. \tag{16}$$

For a bipartite state ρ_{AB} shared by Alice and Bob, the steering from Bob to Alice can be witnessed if the state is entangled. Its density matrix τ_{AB} is defined as

$$\tau_{AB} = \frac{\rho_{AB}}{\sqrt{3}} + \frac{3-\sqrt{3}}{3}\left(\rho_A \otimes \frac{I}{2}\right), \tag{17}$$

where $\rho_A = \mathrm{Tr_B}(\rho_{\mathrm{AB}})$ and I is the two-dimension identity matrix [93,94]. In a similar way, we can witness the steering from Alice to Bob when the density matrix τ_{BA} defined as

$$\tau_{BA} = \frac{\rho_{AB}}{\sqrt{3}} + \frac{3-\sqrt{3}}{3}\left(\frac{I}{2} \otimes \rho_B\right), \tag{18}$$

is entangled, where $\rho_B = \mathrm{Tr_A}(\rho_{\mathrm{AB}})$. We calculate that the matrix τ_{AB} of the X-state ρ_x can be specifically expressed as

$$\tau_{AB}^x = \begin{pmatrix} \frac{\sqrt{3}}{3}\rho_{11} + g & 0 & 0 & \frac{\sqrt{3}}{3}\rho_{14} \\ 0 & \frac{\sqrt{3}}{3}\rho_{22} + g & \frac{\sqrt{3}}{3}\rho_{23} & 0 \\ 0 & \frac{\sqrt{3}}{3}\rho_{32} & \frac{\sqrt{3}}{3}\rho_{33} + h & 0 \\ \frac{\sqrt{3}}{3}\rho_{41} & 0 & 0 & \frac{\sqrt{3}}{3}\rho_{44} + h \end{pmatrix}, \tag{19}$$

with $g = \frac{3-\sqrt{3}}{6}(\rho_{11}+\rho_{22})$ and $h = \frac{3-\sqrt{3}}{6}(\rho_{33}+\rho_{44})$. Using Eq. (16), the state τ_{AB}^x is entangled, if the state τ_{AB} satisfies inequality

$$|\rho_{14}|^2 > L_a - L_b, \tag{20}$$

or

$$|\rho_{23}|^2 > L_c - L_b, \tag{21}$$

where

$$L_a = \frac{2-\sqrt{3}}{2}\rho_{11}\rho_{44} + \frac{2+\sqrt{3}}{2}\rho_{22}\rho_{33} + \frac{1}{4}(\rho_{11}+\rho_{44})(\rho_{22}+\rho_{33}),$$

$$L_b = \frac{1}{4}(\rho_{11}-\rho_{44})(\rho_{22}-\rho_{33}),$$

$$L_c = \frac{2+\sqrt{3}}{2}\rho_{11}\rho_{44} + \frac{2-\sqrt{3}}{2}\rho_{22}\rho_{33} + \frac{1}{4}(\rho_{11}+\rho_{44})(\rho_{22}+\rho_{33}). \tag{22}$$

Therefore, the steering from Bob to Alice is thus witnessed. In the same way, we find that the steering from Alice to Bob can be witnessed via one of the inequality

$$|\rho_{14}|^2 > L_a + L_b, \tag{23}$$

or

$$|\rho_{23}|^2 > L_c + L_b. \tag{24}$$

According to Eqs. (20)–(24), the steerability from Alice to Bob is found to be

$$S^{A\rightarrow B} = \max\left\{0, \frac{8}{\sqrt{3}}[|\rho_{14}|^2 - L_a - L_b], \frac{8}{\sqrt{3}}[|\rho_{23}|^2 - L_c - L_b]\right\}, \tag{25}$$

and the steerability from Bob to Alice reads

$$S^{B\rightarrow A} = \max\left\{0, \frac{8}{\sqrt{3}}[|\rho_{14}|^2 - L_a + L_b], \frac{8}{\sqrt{3}}[|\rho_{23}|^2 - L_c + L_b]\right\}, \tag{26}$$

where the coefficient $\frac{8}{\sqrt{3}}$ is to ensure that the steerability of the maximally entangled state is 1.

Asymmetric steering, evidenced by the fact that $S^{A\rightarrow B}$ and $S^{B\rightarrow A}$ are often unequal, has sparked our research interest. Therefore, quantum steering can be distinguished into three cases: (i) no-way steering, where the state is non-steerable in any direction; (ii) two-way steering, where the state is steerable in both directions; and (iii) one-way steering, where the state is steerable in only one direction. The last case reflects the asymmetric nature of quantum steering. To measure the steering asymmetry, the steering asymmetry between mode A and mode B can be defined as

$$S^{\Delta}_{AB} = \left|S^{A\rightarrow B} - S^{B\rightarrow A}\right|. \tag{27}$$

4 Fermionic steering for four types of Bell states in SdS spacetime

The four types of Bell states, which form an orthonormal basis of the two-qubit system [95] between Kruskal mode κ_H and Kruskal mode κ_C, can be described as

$$|\Psi^{1,\pm}_{AB}\rangle = \frac{1}{\sqrt{2}}(|0_{\kappa_H}\rangle|0_{\kappa_C}\rangle \pm |1_{\kappa_H}\rangle|1_{\kappa_C}\rangle), \tag{28}$$

$$|\Psi^{2,\pm}_{AB}\rangle = \frac{1}{\sqrt{2}}(|0_{\kappa_H}\rangle|1_{\kappa_C}\rangle \pm |1_{\kappa_H}\rangle|0_{\kappa_C}\rangle), \tag{29}$$

where the modes A and B are observed by Alice and Bob, respectively. Then, we specify that Alice is located at the BEH in subregion A of C and Bob is located in subregion B of C.

In SdS spacetime, the initial modes A and B under two-mode squeezing transformations of Eqs. (9)–(12) are mapped into four sets of modes: the modes A and B in regions A and B, respectively; the modes $\bar{A}$ and $\bar{B}$ in regions L and R, respectively. Therefore, the complete system is given by

$$|\Psi^{1,\pm}_{ALBR}\rangle = \frac{1}{\sqrt{2}}(\cos r\cos\alpha|0000\rangle + \cos r\sin\alpha|0011\rangle + \sin r\cos\alpha|1100\rangle + \sin r\sin\alpha|1111\rangle \pm |1010\rangle), \tag{30}$$

$$|\Psi^{2,\pm}_{ALBR}\rangle = \frac{1}{\sqrt{2}}(\cos r|0010\rangle + \sin r|1110\rangle \pm \cos\alpha|1000\rangle \pm \sin\alpha|1011\rangle). \tag{31}$$

Because the exterior region is causally disconnected from the interior region of the black hole and expanding spacetime, Alice and Bob cannot access modes that belong to the regions R and L. Therefore, we take the trace over modes $\bar{B}$ and $\bar{C}$ and derive the density matrix between Alice and Bob as

$$\rho^{1,\pm}_{AB} = \frac{1}{2}\begin{pmatrix} \cos^2 r\cos^2\alpha & 0 & 0 & \pm\cos r\cos\alpha \\ 0 & \cos^2 r\sin^2\alpha & 0 & 0 \\ 0 & 0 & \sin^2 r\cos^2\alpha & 0 \\ \pm\cos r\cos\alpha & 0 & 0 & \sin^2 r\sin^2\alpha + 1 \end{pmatrix}, \tag{32}$$

$$\rho^{2,\pm}_{AB} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2 r & \pm\cos r\cos\alpha & 0 \\ 0 & \pm\cos r\cos\alpha & \cos^2\alpha & 0 \\ 0 & 0 & 0 & \sin^2 r + \sin^2\alpha \end{pmatrix}. \tag{33}$$

Employing Eqs. (25) and (26), we can get quantum steering from Alice to Bob of $\Psi^{1,\pm}_{AB}$ and $\Psi^{2,\pm}_{AB}$ as

$$S^{A\rightarrow B}_{1,\pm} = \max\left\{0, \cos^2\alpha(\cos^2 r - \frac{2}{\sqrt{3}}\cos^2 r\sin^2 r\sin^2\alpha - \frac{1}{\sqrt{3}}\sin^2\alpha - \frac{1}{\sqrt{3}}\sin^2 r)\right\}, \tag{34}$$

$$S^{A\rightarrow B}_{2,\pm} = \max\left\{0, \cos^2\alpha(\cos^2 r - \frac{1}{\sqrt{3}}\sin^2\alpha - \frac{1}{\sqrt{3}}\sin^2 r)\right\}. \tag{35}$$

From Eqs. (34) and (35), we can see that quantum steering from Alice to Bob depends not only on the mass M of black hole, but also on the cosmological constant Λ, which indicates that the Hawking radiation of the SdS spacetime will affect the $A \to B$ steerability. To check if the quantum steerability is symmetric in multi-event horizon spacetime, we can define steering asymmetry as

$$S^{\Delta}_{1,\pm} = \left| S^{A\to B}_{1,\pm} - S^{B\to A}_{1,\pm} \right|, \quad S^{\Delta}_{2,\pm} = \left| S^{A\to B}_{2,\pm} - S^{B\to A}_{2,\pm} \right|, \tag{36}$$

where quantum steering from Bob to Alice of $\Psi^{1,\pm}_{AB}$ and $\Psi^{2,\pm}_{AB}$ is found to be

$$S^{B\to A}_{1,\pm} = \max\left\{0, \cos^2 r(\cos^2\alpha - \frac{2}{\sqrt{3}}\cos^2\alpha \sin^2 r \sin^2\alpha - \frac{1}{\sqrt{3}}\sin^2 r - \frac{1}{\sqrt{3}}\sin^2\alpha)\right\}, \tag{37}$$

$$S^{B\to A}_{2,\pm} = \max\left\{0, \cos^2 r(\cos^2\alpha - \frac{1}{\sqrt{3}}\sin^2 r - \frac{1}{\sqrt{3}}\sin^2\alpha)\right\}. \tag{38}$$

From the analysis of Eqs. (34)–(38), we find that quantum steering exhibits distinct properties for different types of Bell states between Alice and Bob in multi-event horizon spacetime. In contrast, quantum correlations in these states display consistent properties in single-event horizon spacetime, as only Bob is affected by the gravitational effect [30–34].

Figure 3 depicts the relationship between quantum steering, steering asymmetry, and the mass M of the black hole for different types of Bell states. We can find that, for $\omega = 1$, quantum steering first decreases and then reaches a fixed value with the decrease of the mass M of the black hole, meaning that the Hawking effect of the black hole reduces quantum steerability. In the Nariai limit, where $3M\sqrt{\Lambda} \to 1$, quantum steerability reaches its maximum 1, unaffected by the gravitational effect of SdS spacetime. At this limit, the coincidence of the two horizons results in their temperatures approaching zero ($T_H = T_C = 0$), which implies $\cos r = \cos\alpha = 1$. It is shown that fermionic steering $B \to A$ is always greater than fermionic steering $A \to B$, indicating that the observer with the lower temperature has stronger steerability than the other one. However, bosonic steering $B \to A$ is always smaller than bosonic steering $A \to B$ [79]. Therefore, the phenomenon of the bosonic steering exhibits the opposite behavior to fermionic steering in multi-event horizon spacetime. These findings indicate that fermionic steering contrasts sharply with bosonic steering, due to the differences between Fermi–Dirac and Bose–Einstein statistics in SdS spacetime. Based on the characteristics of fermionic and bosonic steering in curved spacetime, we can utilize the appropriate type of particle steering to manage relativistic quantum information tasks.

From Fig. 3, we also find that, for $\omega = 0.4$, fermionic steering from Alice to Bob undergoes a sudden death as the mass of the black hole M decreases. This represents a crucial transition in the quantum system, shifting from two-way steering to one-way steering, driven by the Hawking effect. Notably, this maximal asymmetry marks the exact transition point between two-way and one-way steering in multi-event horizon spacetime. In addition, we obtain the condition of maximal steering asymmetry that is a point of the sudden death of fermionic steering $A \to B$. For two-way steering, it is interesting to find that steering asymmetry for $\Psi^{1,\pm}_{AB}$ equals steering asymmetry for $\Psi^{2,\pm}_{AB}$

$$S^{\Delta}_{1,\pm} = S^{\Delta}_{2,\pm} = \frac{1}{\sqrt{3}}\left|(\cos^2 r - \cos^2\alpha)(\sin^2 r + \sin^2\alpha)\right|. \tag{39}$$

For one-way steering, we obtain $S^{\Delta}_{1,\pm} = S^{B\to A}_{1,\pm}$ and $S^{\Delta}_{2,\pm} = S^{B\to A}_{2,\pm}$. From the Eqs. (34), (35), (37), and (38), we can see that quantum steering of $\Psi^{1,\pm}_{AB}$ is always less than quantum steering of $\Psi^{2,\pm}_{AB}$ in SdS spacetime. In practical applications, we should choose quantum steering of Bell states $\Psi^{2,\pm}_{AB}$ to handle relativistic quantum information tasks.

In Fig. 4, we plot quantum steering and steering asymmetry as functions of the cosmological constant Λ for different types of Bell states. From Fig. 4, we can see that, with the increase of the cosmological constant Λ, quantum steering first decreases to its minimum value and then returns to its initial value in the Nariai limit. This suggests that quantum steering in SdS spacetime can be strengthened and protected by the Hawking effect of the expanding universe. Since the temperature T_H is greater than the temperature T_C, the steering from B to A is always greater than the steering from A to B. In Fig. 4e, we find that quantum steering first suffers from the sudden death and then the sudden birth with the increase of Λ. In other words, quantum system first transitions from two-way steering to one-way steering in $S^{A\to B}$ sudden death, and then from one-way steering to no-way steering in $S^{B\to A}$ sudden death. Next, it transitions from no-way steering to one-way steering in $S^{B\to A}$ sudden birth, and finally from one-way steering to two-way steering in $S^{A\to B}$ sudden birth. For $\Lambda = 0$, the SdS spacetime degenerates into the Schwarzschild spacetime, resulting in $\Psi^{1,\pm}_{AB} = \Psi^{2,\pm}_{AB}$. Therefore, quantum steering of $\Psi^{1,\pm}_{AB}$ equals quantum steering of $\Psi^{2,\pm}_{AB}$ in Schwarzschild spacetime.

In multi-event horizon spacetime, the relationship between fermionic steering and entanglement is both closely related and distinct in its expression. Quantum entanglement serves as a key resource for quantum steering, which enables one party (Alice) to nonlocally influence the state of another party (Bob) through local measurements. In the specific case of SdS spacetime, fermionic steering is a quantum phenomenon that is influenced by both the gravitational environment and the underlying entanglement structure of the system. Notably, while fermionic steering is completely suppressed by the Hawking effect in SdS spacetime, quantum entanglement can persist in multi-event horizon spacetime [81].

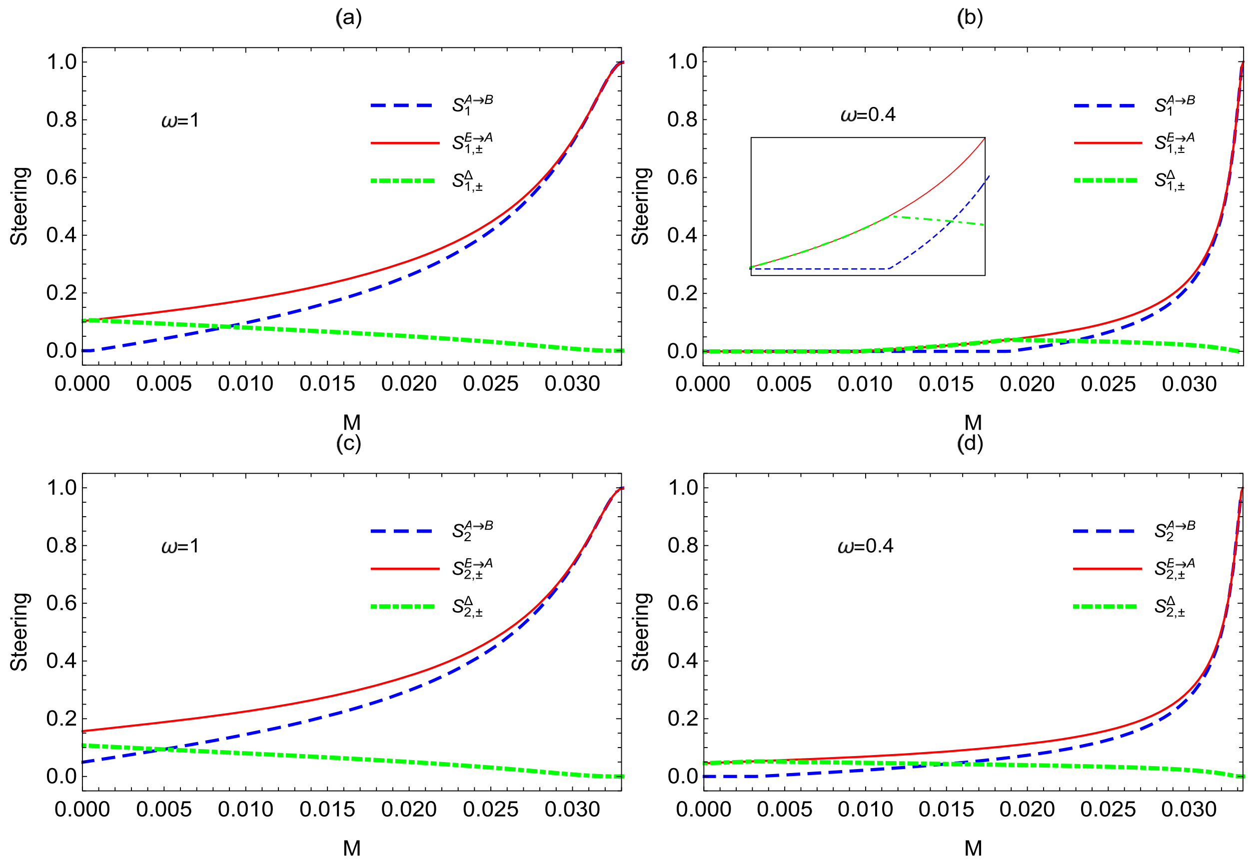

Fig. 3 Quantum steering and steering asymmetry as functions of the mass M of the black hole for fixed $\Lambda = 100$

5 Conclusions

The gravitational influence of a black hole and an expanding universe on fermionic steering and its asymmetry for different types of Bell states in SdS spacetime has been investigated. This spacetime features both a black hole event horizon (BEH) and a cosmological event horizon (CEH). By positioning a thermally opaque membrane between these two horizons, two independent thermal equilibrium regions are established. Our model consists of two modes: the mode A, observed by Alice at the BEH; the mode B, observed by Bob at the CEH. Since the Hawking temperature of the black hole is higher than that of the expanding universe, fermionic steering from Bob to Alice is greater than steering from Alice to Bob. Conversely, bosonic steering exhibits opposite characteristics, which can be attributed to the differences between Fermi–Dirac statistics and Bose–Einstein statistics [79]. Furthermore, the Hawking effect of the black hole reduces fermionic steerability, while the Hawking effect of the expanding universe can enhance it. This indicates that the Hawking effect of the expanding universe is not always detrimental to fermionic steering. This conclusion provides a more comprehensive understanding of the Hawking effect via fermionic steering. The gravitational effect can completely destroy fermionic steering in multi-event horizon spacetime, whereas fermionic steering can persist indefinitely in single-event horizon spacetime [67]. Finally, we have found that quantum steering of Bell states $\Psi_{AB}^{1,\pm}$ is always less than quantum steering of Bell states $\Psi_{AB}^{2,\pm}$ in SdS spacetime. Therefore, selecting the appropriate Bell state is crucial for effectively managing relativistic quantum information tasks involving quantum steering.

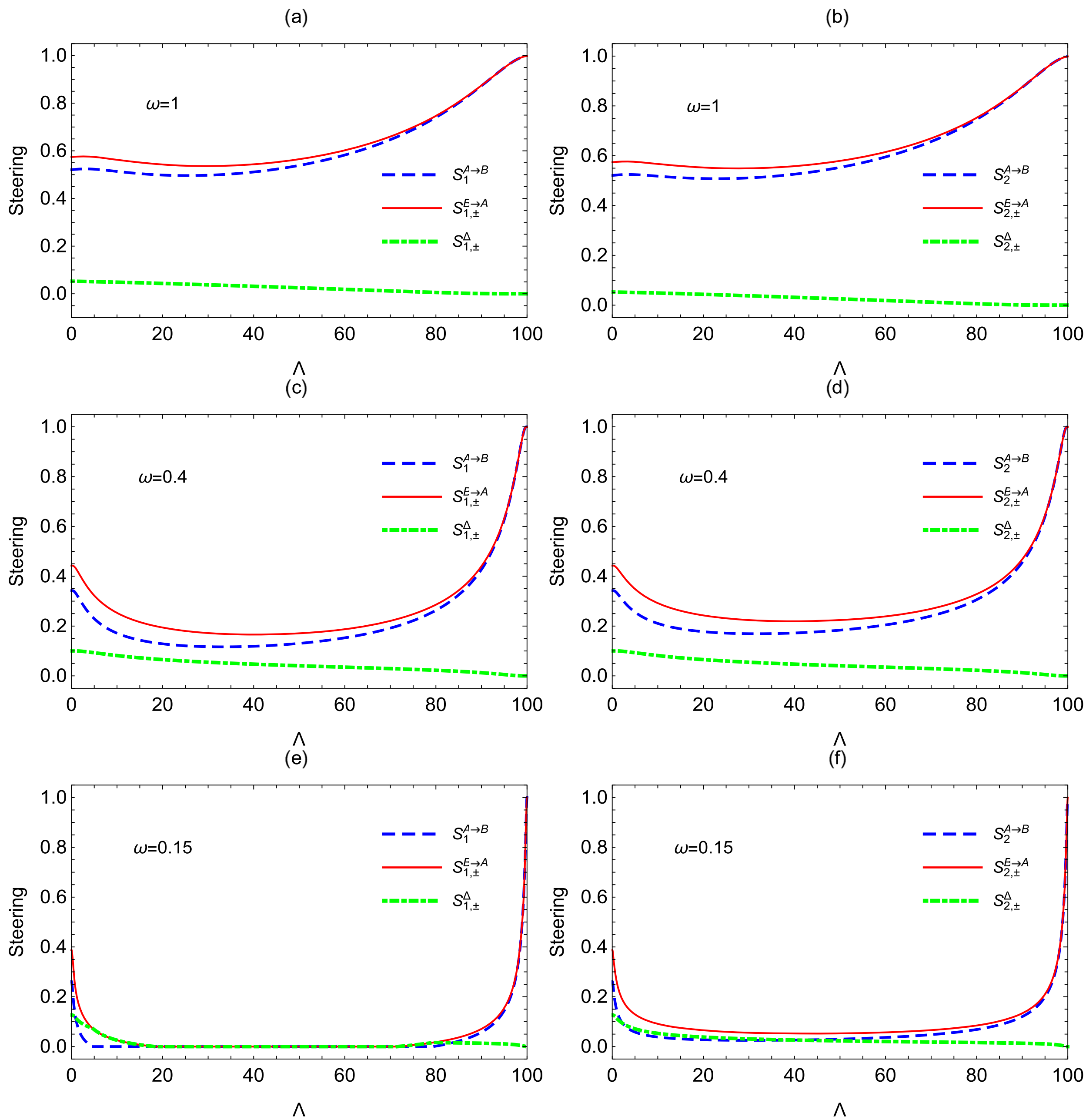

Fig. 4 Quantum steering and steering asymmetry as functions of the cosmological constant Λ for fixed $M = 1/30$

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References

1. R. Horodecki, P. Horodecki, M. Horodecki, K. Horodecki, Quantum entanglement. Rev. Mod. Phys. **81**, 865 (2009)
2. J.F. Clauser, M.A. Horne, A. Shimony, R.A. Holt, Proposed experiment to test local hidden-variable theories. Phys. Rev. Lett. **23**, 880 (1969)
3. A. Aspect, P. Grangier, G. Roger, Experimental tests of realistic local theories via Bell's theorem. Phys. Rev. Lett. **47**, 460 (1981)
4. N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Bell nonlocality. Rev. Mod. Phys. **86**, 419 (2014)
5. A. Einstein, B. Podolsky, N. Rosen, Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. **47**, 777 (1935)
6. E. Schrödinger, Discussion of probability relations between separated systems. Math. Proc. Cambr. Philos. Soc. **31**, 555 (1935)
7. E. Schrödinger, Probability relations between separated systems. Math. Proc. Cambr. Philos. Soc. **32**, 446 (1936)
8. R. Uola, A.C.S. Costa, H.C. Nguyen, O. Gühne, Quantum steering. Rev. Mod. Phys. **92**, 015001 (2020)
9. O. Gühne, E. Haapasalo, T. Kraft, J.P. Pellonpää, R. Uola, Colloquium: Incompatible measurements in quantum information science. Rev. Mod. Phys. **95**, 011003 (2023)
10. H.M. Wiseman, S.J. Jones, A.C. Doherty, Steering, entanglement, nonlocality, and the Einstein–Podolsky–Rosen Paradox. Phys. Rev. Lett. **98**, 140402 (2007)
11. P.J. Cavalcanti, J.H. Selby, J. Sikora, T.D. Galley, A.B. Sainz, Post-quantum steering is a stronger-than-quantum resource for information processing. npj Quantum Inf. **8**, 76 (2022)
12. V. Händchen, T. Eberle, S. Steinlechner, A. Samblowski, T. Franz, R.F. Werner, R. Schnabel, Observation of one-way Einstein–Podolsky–Rosen steering. Nat. Photon. **6**, 596 (2012)
13. Y. Fan, C. Jia, L. Qiu, Quantum steering as resource of quantum teleportation. Phys. Rev. A **106**, 012433 (2022)
14. E. Medina-Guerra, P. Kumar, I.V. Gornyi, Y. Gefen, Quantum state engineering by steering in the presence of errors. Phys. Rev. Res. **6**, 023159 (2024)
15. S. Armstrong, M. Wang, R.Y. Teh, Q. Gong, Q. He, J. Janousek, H.A. Bachor, M.D. Reid, P.K. Lam, Multipartite Einstein–Podolsky–Rosen steering and genuine tripartite entanglement with optical networks. Nat. Phys. **11**, 167 (2015)
16. N. Tischler, F. Ghafari, T.J. Baker, S. Slussarenko, R.B. Patel, M.M. Weston, S. Wollmann, L.K. Shalm, V.B. Verma, S.W. Nam, H.C. Nguyen, H.M. Wiseman, G.J. Prydel, Conclusive experimental demonstration of one-way Einstein–Podolsky–Rosen steering. Phys. Rev. Lett. **121**, 100401 (2018)
17. C. Branciard, E.G. Cavalcanti, S.P. Walborn, V. Scarani, H.M. Wiseman, One-sided device-independent quantum key distribution: security, feasibility, and the connection with steering. Phys. Rev. A **85**, 010301 (2012)
18. L. Maximilian Seifert, K. Beyer, K. Luoma, W.T. Strunz, Quantum steering on IBM quantum processors. Phys. Rev. A **105**, 042413 (2022)
19. T. Gehring, V. Händchen, J. Duhme, F. Furrer, T. Franz, C. Pacher, R.F. Werner, R. Schnabel, Implementation of continuous-variable quantum key distribution with composable and one-sided-device-independent security against coherent attacks. Nat. Commun. **6**, 8795 (2015)
20. K. Bartkiewicz, A. Černoch, K. Lemr, A. Miranowicz, F. Nori, Temporal steering and security of quantum key distribution with mutually unbiased bases against individual attacks. Phys. Rev. A **93**, 062345 (2016)
21. A.G. Riess et al., Observational evidence from supernovae for an accelerating universe and a cosmological constant. AJ. **116**, 1009 (1998)
22. S. Perlmutter, G. Aldering, G. Goldhaber, R.A. Knop, P. Nugent, P.G. Castro, S. Deustua, S. Fabbro, A. Goobar, D.E. Groom et al., Measurements of Ω and Λ from 42 high-redshift supernovae. AJ **517**, 565 (1999)
23. K. Goswami, K. Narayan, Small Schwarzschild de Sitter black holes, the future boundary and islands. J. High Energy Phys. **05**, 016 (2024)
24. S.E. Aguilar-Gutierrez, S. Baiguera, N. Zenoni, Holographic complexity of the extended Schwarzschild–de Sitter space. J. High Energy Phys. **05**, 201 (2024)
25. R. Bousso, S.W. Hawking, Pair creation of black holes during inflation. Phys. Rev. D **54**, 6312 (1996)
26. S. Shankaranarayanan, Temperature and entropy of Schwarzschild–de Sitter space-time. Phys. Rev. D **67**, 084026 (2003)
27. T.R. Choudhury, T. Padmanabhan, Concept of temperature in multi-horizon spacetimes: analysis of Schwarzschild–De Sitter metric. Gen. Relativ. Gravit. **39**, 1789 (2007)
28. Y. Qiu, J. Traschen, Black hole and cosmological particle production in Schwarzschild de Sitter. Class. Quantum Gravity **37**, 135012 (2020)
29. G.W. Gibbons, S.W. Hawking, Cosmological event horizons, thermodynamics, and particle creation. Phys. Rev. D **15**, 2738 (1977)
30. I. Fuentes-Schuller, R.B. Mann, Alice falls into a black hole: entanglement in noninertial frames. Phys. Rev. Lett. **95**, 120404 (2005)
31. W. Liu, C. Wen, J. Wang, Lorentz violation alleviates gravitationally induced entanglement degradation. (2024). arXiv:2410.21681
32. J. Maldacena, G.L. Pimentel, Entanglement entropy in de Sitter space. J. High Energy Phys. **02**, 038 (2013)
33. M. Montero, E. Martín-Martínez, The entangling side of the Unruh–Hawking effect. J. High Energy Phys. **07**, 006 (2011)
34. P.M. Alsing, I. Fuentes-Schuller, R.B. Mann, T.E. Tessier, Entanglement of Dirac fields in noninertial frames. Phys. Rev. A **74**, 032326 (2006)
35. Y. Yang, J. Jing, Z. Tian, Probing cosmic string spacetime through parameter estimation. Eur. Phys. J. C **82**, 688 (2022)
36. S. Sen, A. Mukherjee, S. Gangopadhyay, Entanglement degradation as a tool to detect signatures of modified gravity. Phys. Rev. D **109**, 046012 (2024)
37. M.M. Du, H.W. Li, Z. Tao, S.T. Shen, X.J. Yan, X.Y. Li, W. Zhong, Y.B. Sheng, L. Zhou, Basis-independent quantum coherence and its distribution under relativistic motion. Eur. Phys. J. C **84**, 838 (2024)
38. S. Xu, X.K. Song, J.D. Shi, L. Ye, How the Hawking effect affects multipartite entanglement of Dirac particles in the background of a Schwarzschild black hole. Phys. Rev. D **89**, 065022 (2014)
39. E. Martín-Martínez, L.J. Garay, J. León, Unveiling quantum entanglement degradation near a Schwarzschild black hole. Phys. Rev. D **82**, 064006 (2010)
40. A.J. Torres-Arenasa, Q. Dong, G.H. Sun, W.C. Qiang, S.H. Dong, Entanglement measures of W-state in noninertial frames. Phy. Lett. B **789**, 93 (2019)
41. S.M. Wu, X.W. Fan, R.D. Wang, H.Y. Wu, X.L. Huang, H.S. Zeng, Does Hawking effect always degrade fidelity of quantum teleportation in Schwarzschild spacetime? J. High Energy Phys. **11**, 232 (2023)
42. S.M. Wu, C.X. Wang, D.D. Liu, X.L. Huang, H.S. Zeng, Would quantum coherence be increased by curvature effect in de Sitter space? J. High Energy Phys. **02**, 115 (2023)

43. A. Ali, S. Al-Kuwari, M. Ghominejad, M.T. Rahim, D. Wang, S. Haddadi, Quantum characteristics near event horizons. Phys. Rev. D **110**, 064001 (2024)
44. W.C. Qiang, G.H. Sun, Q. Dong, S.H. Dong, Genuine multipartite concurrence for entanglement of Dirac fields in noninertial frames. Phys. Rev. A **98**, 022320 (2018)
45. Z. Tian, J. Jing, How the Unruh effect affects transition between classical and quantum decoherences. Phy. Lett. B **707**, 264 (2012)
46. E.G. Brown, K. Cormier, E. Martín-Martínez, R.B. Mann, Vanishing geometric discord in noninertial frames. Phys. Rev. A **86**, 032108 (2012)
47. H. Miriam Reji, H.S. Hegde, R. Prabhu, Conditions for separability in multiqubit systems with an accelerating qubit using a conditional entropy. Phys. Rev. A **110**, 032403 (2024)
48. E. Martín-Martínez, I. Fuentes, Redistribution of particle and antiparticle entanglement in noninertial frames. Phys. Rev. A **83**, 052306 (2011)
49. D.E. Bruschi, A. Dragan, I. Fuentes, J. Louko, Particle and antiparticle bosonic entanglement in noninertial frames. Phys. Rev. D **86**, 025026 (2012)
50. D.E. Bruschi, J. Louko, E. Martín-Martínez, A. Dragan, I. Fuentes, Unruh effect in quantum information beyond the single-mode approximation. Phys. Rev. A **82**, 042332 (2010)
51. W.M. Li, S.M. Wu, Bosonic and fermionic coherence of N-partite states in the background of a dilaton black hole. J. High Energy Phys. **09**, 144 (2024)
52. T.Y. Wang, D. Wang, Entropic uncertainty relations in Schwarzschild space-time. Phys. Lett. B **855**, 138876 (2024)
53. S.M. Wu, H.S. Zeng, Genuine tripartite nonlocality and entanglement in curved spacetime. Eur. Phys. J. C **82**, 4 (2022)
54. B.N. Esfahani, M. Shamirzaie, M. Soltani, Reduction of entanglement degradation in Einstein–Gauss–Bonnet gravity. Phys. Rev. D **84**, 025024 (2011)
55. D. Ahn, Unruh effect as a noisy quantum channel. Phys. Rev. A **98**, 022308 (2018)
56. S. Bhattacharya, H. Gaur, N. Joshi, Some measures for fermionic entanglement in the cosmological de Sitter spacetime. Phys. Rev. D **102**, 045017 (2020)
57. Z. Tian, J. Jing, A. Dragan, Analog cosmological particle generation in a superconducting circuit. Phys. Rev. D **95**, 125003 (2017)
58. Y. Pan, B. Zhang, Influence of acceleration on multibody entangled quantum states. Phys. Rev. A **101**, 062111 (2020)
59. W.G. Brenna, R.B. Mann, E. Martín-Martínez, Anti-Unruh phenomena. Phys. Lett. B **757**, 307 (2016)
60. Z. Tian, L. Wu, L. Zhang, J. Jing, J. Du, Probing Lorentz-invariance-violation induced nonthermal Unruh effect in quasi-two-dimensional dipolar condensates. Phys. Rev. D **106**, L061701 (2022)
61. Z. Liu, J. Zhang, R.B. Mann, H. Yu, Does acceleration assist entanglement harvesting? Phys. Rev. D **105**, 085012 (2022)
62. Y. Zhou, J. Hu, H. Yu, Entanglement dynamics for Unruh-DeWitt detectors interacting with massive scalar fields: the Unruh and anti-Unruh effects. J. High Energy Phys. **09**, 088 (2021)
63. T. Liu, J. Jing, J. Wang, Satellite-based quantum steering under the influence of spacetime curvature of the earth. Adv. Quantum Technol. **1**, 1800072 (2018)
64. B. Hu, C. Wen, J. Wang, J. Jing, Gaussian quantum steering under the influence of a dilaton black hole. Eur. Phys. J. C **81**, 925 (2021)
65. J. Wang, H. Cao, J. Jing, H. Fan, Gaussian quantum steering and its asymmetry in curved spacetime. Phys. Rev. D **93**, 125011 (2016)
66. C. Wen, J. Wang, J. Jing, Quantum steering for continuous variable in de Sitter space. Eur. Phys. J. C **80**, 78 (2020)
67. S.M. Wu, H.S. Zeng, Fermionic steering and its monogamy relations in Schwarzschild spacetime. Eur. Phys. J. C **82**, 716 (2022)
68. S. Elghaayda, A. Ali, S. Al-Kuwari, M. Mansour, Physically accessible and inaccessible quantum correlations of Dirac fields in Schwarzschild spacetime. Phys. Lett. A **525**, 129915 (2024)
69. G.W. Mi, X. Huang, S.M. Fei, T. Zhang, Impact of the Hawking effect on the fully entangled fraction of three-qubit states in Schwarzschild spacetime. Ann. Phys. (Berl) 2400308 (2024)
70. Z. Tian, X. Liu, J. Wang, J. Jing, Dissipative dynamics of an open quantum battery in the BTZ spacetime. arXiv:2409.09259
71. Y. Wu, Z. Tian, Geometric phase assisted detection of Lorentz-invariance violation from modified dispersion at high energies. arXiv:2409.09257
72. Z. Tian, J. Jing, J. Du, Direct characteristic-function tomography of the quantum states of quantum fields. Sci. China Phys. Mech. Astron. **66**, 110412 (2023)
73. H. Dolatkhah, A. Czerwinski, A. Ali, S. Al-Kuwari, S. Haddadi, Tripartite measurement uncertainty in Schwarzschild space-time. Eur. Phys. J. C **84**, 1162 (2024)
74. C. Liu, Z. Long, Q. He, Quantum coherence and quantum Fisher information of Dirac particles in curved spacetime under decoherence. Phys. Lett. B **857**, 138991 (2024)
75. L.J. Henderson, R.A. Hennigar, R.B. Mann, A.R.H. Smith, J. Zhang, Anti-Hawking phenomena. Phys. Lett. B **809**, 135732 (2020)
76. Z. Tian, J. Du, Probing low-energy Lorentz violation from high-energy modified dispersion in dipolar Bose–Einstein condensates. Phys. Rev. D **103**, 085014 (2021)
77. S. Bhattacharya, S. Chakrabortty, H. Hoshino, S. Kaushal, Background magnetic field and quantum correlations in the Schwinger effect. Phys. Lett. B **811**, 135875 (2020)
78. X. Liu, Z. Tian, J. Jing, Entanglement dynamics in κ-deformed spacetime. Sci. China Phys. Mech. Astron. **67**, 100411 (2024)
79. S.M. Wu, J.X. Li, X.W. Fan, W.M. Li, X.L. Huang, H.S. Zeng, Gaussian quantum steering in multi-event horizon spacetime. Eur. Phys. J. C **84**, 176 (2024)
80. S. Bhattacharya, N. Joshi, Entanglement degradation in multi-event horizon spacetimes. Phys. Rev. D **105**, 065007 (2022)
81. Q. Liu, S.M. Wu, C. Wen, J. Wang, Quantum properties of fermionic fields in multi-event horizon spacetime. Sci. China Phys. Mech. Astron. **66**, 120413 (2023)
82. S.A. Major, Energy for accelerating observers in black hole spacetimes. Phys. Rev. D **105**, 104050 (2022)
83. S. Bhattacharya, Particle creation by de Sitter black holes revisited. Phys. Rev. D **98**, 125013 (2018)
84. A. Gomberoff, C. Teitelboim, de Sitter black holes with either of the two horizons as a boundary. Phys. Rev. D **67**, 104024 (2003)
85. Y. Sekiwa, Thermodynamics of de Sitter black holes: thermal cosmological constant. Phys. Rev. D **73**, 084009 (2006)
86. D.R. Brill, J.A. Wheeler, Interaction of neutrinos and gravitational fields. Rev. Mod. Phys. **29**, 465 (1957)
87. D. Garfinkle, G.T. Horowitz, A. Strominger, Erratum: Charged black holes in string theory. Phys. Rev. D **45**, 3888 (1992)
88. J. Wang, J. Jing, H. Fan, Quantum discord and measurement-induced disturbance in the background of dilaton black holes. Phys. Rev. D **90**, 025032 (2014)
89. J. Wang, Q. Pan, J. Jing, Entanglement redistribution in the Schwarzschild spacetime. Phys. Lett. B **692**, 202 (2010)
90. A. Aragón, R. Bécar, P.A. González, Y. Vásquez, Massive Dirac quasinormal modes in Schwarzschild–de Sitter black holes: anomalous decay rate and fine structure. Phys. Rev. D **103**, 064006 (2021)
91. X. Zhang, M. Wang, J. Jing, Quasinormal modes and late time tails of perturbation fields on a Schwarzschild-like black hole with a global monopole in the Einstein-bumblebee theory. Sci. China Phys. Mech. Astron. **66**, 100411 (2023)

92. S.M. Hashemi Rafsanjani, M. Huber, C.J. Broadbent, J.H. Eberly, Genuinely multipartite concurrence of N-qubit X matrices. Phys. Rev. A **86**, 062303 (2012)
93. D. Das, S. Sasmal, S. Roy, Detecting Einstein–Podolsky–Rosen steering through entanglement detection. Phys. Rev. A **99**, 052109 (2019)
94. K. Zhang, J. Wang, Asymmetric steerability of quantum equilibrium and nonequilibrium steady states through entanglement detection. Phys. Rev. A **104**, 042404 (2021)
95. M.A. Nielsen, I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2010)