A SHORT REVIEW OF THE CDF ELECTROMAGNETIC AND HADRONIC SHOWER SIMULATION

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The purpose of this paper is to describe the electromagnetic and hadronic shower simulation[1] used by CDF. We feel that this parametrization is relatively simple and reproduces the test beam data quite accurately. We encourage the reader to look at the detailed comparisons for both the electromagnetic[2] and hadronic[3] showers.

Showers that occur "upstream" of tracking chambers must be simulated by full cascade models, since each daughter particle of the interaction is potentially observable by the tracking device. However, for situations where individual tracks from the shower are not seen, as in a calorimeter, we chose to make simplified models of showers that reproduce the properties of showers measured in test beams. These models need to be computed quickly so that high statistic studies can be made to evaluate acceptances and backgrounds.

Electromagnetic (EM) showers can be simulated by a Monte Carlo program like EGS[4], but because of computer time this is not practical for us. D. Wagoner and D. Judd[5] estimate (depending on geometry and other factors) that EGS requires approximately 1000 VAX-780 CPU seconds for a 500 GeV shower, while our simulation requires 0.2 seconds. EGS produces a detailed cascade shower, in which a large number of secondary particles are produced, with a recipe containing the essential physics ingredients such as Bremsstrahlung and pair production. In our parametrization the centroid of the shower is considered as a neutral particle travelling in the same direction as the particle immediately before showering. The physics is put in by parametrization of the test beam data. We have also simulated minimum ionizing particles and nuclear interactions[6], but they are not discussed in this paper.

The energy resolution of a calorimeter is usually given as $\sigma(\mathrm{GeV})/E(\mathrm{GeV}) = R/\sqrt{E(\mathrm{GeV})}$ (R=.23 $\mathrm{GeV}^{1/2}$ for the CDF plug detector). The sampling fluctuations depend on the square of R. Values of R^2 are given in Table I for the 7 calorimeters of the CDF detector. In addition for sophisticated event analysis one needs to understand both the longitudinal and transverse shower development. Below, we discuss these distributions for both EM and hadron showers.

For an electron, the EM shower starts as soon as the particle enters a dense material. For a photon, the start of the shower is determined by the exponential probability distribution (The exponent is 9/7 the number of radiation lengths traversed.). Physically the assymmetry is due to the fact that the charged electron immediately starts Bremsstrahlung while the gamma must convert first.

The longitudinal development of the electromagnetic shower is expressed as[7] :

$$\frac{dE}{dL} = K \times L^{\alpha-1} e^{-\beta L}$$

where L is the number of radiation lengths traversed since the shower started, $\langle\alpha\rangle$ =2.1+ 0.56×ln E(GeV), $\langle\beta\rangle$= 0.5, and $K = E\times\beta^{\alpha}/\Gamma(\alpha)$ is a normalization constant that insures that the integral equals the total incoming particle energy[2].

We wish to not only reproduce the average longitudinal shower profile, but also the fluctuations of the energy depositied in each segment of the calorimeter. Fitting the longitudinal shower profile measured in the test beam on an event-by-event basis, we find that α and β are Gaussian distributed about their mean values. We also find that α and β are correlated with an energy-independent correlation coefficient C = 0.83 where C is defined as:

$$C = \frac{\langle\alpha\beta\rangle}{[\,(\langle\alpha^2\rangle - \langle\alpha\rangle^2)\,(\langle\beta^2\rangle - \langle\beta\rangle^2)\,]^{0.5}}$$

In the EM shower counter, α and β are determined for each shower through the following perscription:[2]

Mean values and sigmas of α and β are determined for the incident particle energy and two eigenvalues $(\sigma d_1)^2$ and $(\sigma d_2)^2$ and a unitary matrix U are obtained by diagonalizing the matrix M.

$$M = \begin{pmatrix} \sigma_\alpha^2 & C\sigma_\alpha\sigma_\beta \\ C\sigma_\alpha\sigma_\beta & \sigma_\beta^2 \end{pmatrix}$$

$$D = \begin{pmatrix} (\sigma d_1)^2 & 0 \\ 0 & (\sigma d_2)^2 \end{pmatrix} = U^{\dagger} M U$$

with $\sigma_{\alpha} = 0.5$ and $\sigma_{\beta} = 0.051$.

Random numbers d_1' (d_2') are then extracted from Gaussian distributions with sigmas σd_1 (σd_2). Then α and β are given by:

$$\alpha = \langle \alpha \rangle + \Delta\alpha \,, \; \beta = \langle \beta \rangle + \Delta\beta$$

with

$$\begin{pmatrix} \Delta\alpha \\ \Delta\beta \end{pmatrix} = U \begin{pmatrix} d_1' \\ d_2' \end{pmatrix}$$

Thus the average energy deposited in the k th segment is

$$\overline{E_k} = \int K(E) \times L^{\alpha-1} \times e^{-\beta L} \, dL$$

where the integral is over the depth interval of segment k.

To correctly account for fluctuations we use

$$E_k = \overline{E_k} + R\, r_k \sqrt{\overline{E_k}} = \overline{E_k} + R^2 r_k \sqrt{(\overline{E_k}/R^2)}$$

where R is the resolution paramter, and r_k is a random number which is Gaussian distributed about 0 with sigma equal to 1. If the energy deposited in the segment is small, Poisson-like distributions are used as the fluctuations in the second term instead of Gaussian distributions. The dimensionless quantity (E_k/R^2) is called the number of equivalent particles and is the number on which the statistics is based.

The transverse profile is parametrized as a 2-component Gaussian[8]

$$dE/_{dr} = K_T \{ A\, e^{-r^2/2\sigma_n^2} + (1-A)\, e^{-r^2/2\sigma_w^2} \}$$

The 2 components are considered to represent a narrow and a wide component of the shower profile. "A" is the splitting of the energy between the 2 components (we use A = 0.6). The normalization constant can be expressed in terms of the constant used in the longitudinal parametrization.

$$K_T = \frac{K \times L^{\alpha-1} \times e^{-\beta L}}{2\pi[\, A\sigma_n^2 + (1-A)\sigma_w^2 \,]}$$

The sigma for the narrow component is linearly dependent on the total number of radiation lengths (L_0) traversed by the shower[9]

$$\sigma_n = B_n L_0 X_{eq}/E_c$$

where X_{eq}(cm) is the effective radiation length of the calorimeter, and E_c(MeV) is the critical energy (when the energy of a secondary particle is less than this it will no longer be detected by the calorimeter). The values of E_c(MeV/cm) are given in Ref. 9.

The test beam data gives B_n = 0.389 (MeV/cm) with a sigma 0.055. The sigma of the wide component is

$$\sigma_w = A_w X_{eq}/E_c$$

where the fitted value is A_w =8.19 (Mev/cm), there is no sigma associated with this value. The wide component is empirically determined not to depend on the number of radiation lengths traversed.

In general the k th segment will be divided into j subsegments in the transverse plane by integrating the transverse profile over the geometrical "tower" boundaries. Thus the energy into j th subsegment is:

$$E_{jk} = f_{jk}\overline{E_k} + R r_l \sqrt{f_{jk}\overline{E_k}}$$

where f_{jk} is the fraction of the transverse energy in the k th segment that is in the j th subsegment and where R is the resolution parameter and r_l is a random number which is Gaussian distributed about 0 with sigma equal to 1.

The quality of the agreement between test beam data and the shower model can be seen in Fig. 1. Shown here are the energy distributions for the 3 depth segments of the plug EM calorimeter, and the energy deposition in the plug hadron calorimeter for 100 GeV e^-. Test beam data (solid) and MC (dotted) are shown.

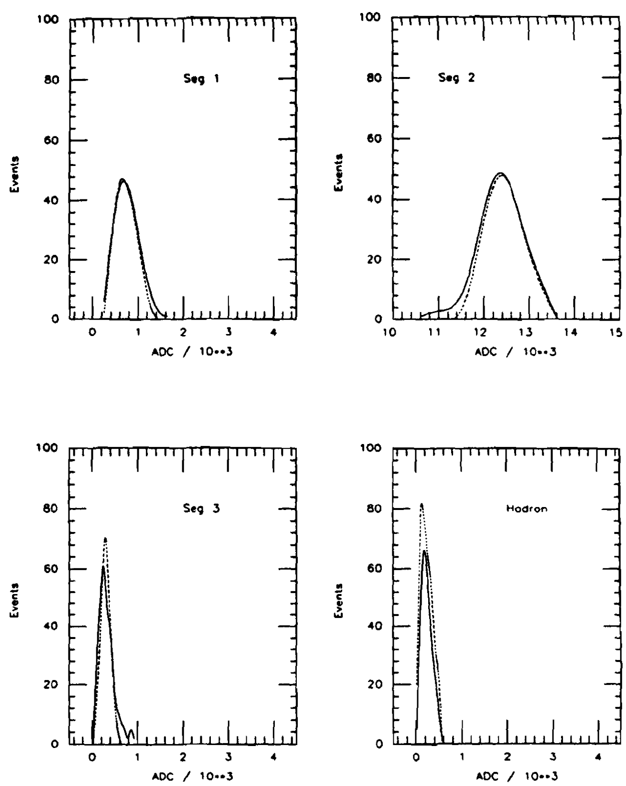

Fig. 1

Comparison of pulse height distributions observed at three longitudinal segments of the EM endplug calorimeter and the hadron endplug calorimeter for 100 GeV e^-. The data (Monte Carlo) is the solid (dashed) line (from Ref. 2).

Hadronic showers are also modeled as a neutral particle travelling in the same direction as the particle immediately before showering, with longitudinal and transverse energy density profiles. The start of a hadronic shower is determined by keeping track of the number of interaction lengths traversed by the particle and using an exponential probability distribution where energy-and particle species-dependent interaction cross sections are used[6].

The longitudinal parametrization of hadronic showers is[10]:

$$dE/dZ = K\{f_1 \times W \times L^{\alpha-1} \times e^{-\beta L} + f_2 \times (1-W) \times l^{\gamma-1} e^{-\delta l}\}$$

where L(l) is the number of radiation (interaction) lengths traversed since the shower started. Interaction lengths are measured in terms of the pion interaction length (21.27 cm in Fe)[11]. The constants f_1 and f_2 depend on the type of calorimeter the shower is evolving in. We use $f_1 = 1.0$ (1.15) and $f_2 =$ 0.40(0.90) for an electromagnetic (hadronic) calorimeter. The choice of these constants depends on the observation that hadrons create less pulse height per GeV of deposited energy than do electrons. The gain of electrons is set so that the electromagnetic showers in an electromagnetic calorimeter yield the incident energy. The gain of the hadron shower is set so that hadron showers that were minimum-ionizing in the EM calorimeter yielded the incident energy. The fraction of the energy that is electromagnetic is W. W is chosen to have a uniform probability between 0.01 and 0.40 . Above 0.40 the probability is given as a Gaussian with a mean of 0.40 and a sigma of 0.25. This distribution is truncated at 0.99. Physically, W is related to the fractional energy content of the "prompt" π^0 's in the initial nuclear interaction that starts the shower. The average π/e response (W=0.4) in an electromagnetic calorimeter is:

$$(\pi/e) = (Wf_1 + (1-W)f_2)/f_1 = .4\times 1 + .6\times .4 = .64$$

Yoh and Wickland[12] have observed that the electromagnetic plus hadron energy is nonlinear as a function of E_{EM}.

Fig. 2 shows the average E_{EM} versus E_{HAD} for 50 GeV π's hitting the calorimeter. Both test beam and MC are shown. The above fit reproduces this behaviour.

The parameters of the hadron shower model are determined to be[3] :

$\alpha = 1.0 + 0.36 \times E$ for E < 10. GeV

$\alpha = 0.62 + 0.31 \times \ln E$ for E > 10. GeV

$\beta = 0.22$

$\gamma = \alpha$

$\delta = 0.81 - 0.024 \times \ln E$

Our treatment of the fluctuations in the shower is a little different than in the pure electromagnetic case.The test beam data used in the fit of the EM shower constants was from the plug EM calorimeter, where each each sample in depth was read out, allowing for a detailed study of the correlations of α and β. For the fit of the hadron shower constants, the central EM and hadron calorimetry test beam data was used. Here, there was no detailed longitudinal profile information available, preventing a sophisticated analysis of the correlations between the various hadron shower constants. For this reason, the hadron shower constants are fixed at their average values, and the longitudinal profile is fluctuated by a random multiplicative factor.

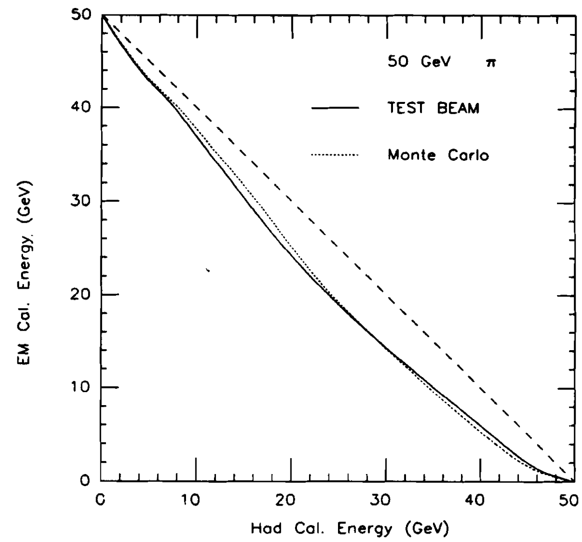

Fig. 2

Energy peak values in the central hadron calorimeter, E_{HAD}(peak), at fixed energy in the central EM calorimeter, plotted versus E_{EM} for 50 GeV π. Test beam data and Monte Carlo are compared. The dashed straight line corresponds to $E_{EM} + E_{HAD} = 50$ GeV. Note the observed nonlinearity (from Ref. 3).

The longitudinal profile is fluctuated by multiplication by a random scale factor, SCALE. The reciprocal of scale is a truncated Gaussian between 0.01 and infinity. The Gaussian distribution has a mean of 0.60 and a sigma of 0.45. When scale is calculated, only values in the range 0.2 to 1.5 are accepted.

The above fluctuation changes the shape of the longitudinal profile. The distribution of the fluctuation was chosen to fit observed hadronic shower energy flow distributions.The data used in the fit were from the E616 experiment at Fermilab[13] and the WA-1 experiment at CERN[14]. The normalization constant is again determined by integrating the longitudinal profile function:

$$K(E,W) = E/[S\times(f_1 W\times(\Gamma(\alpha)/\beta^{\alpha}) + f_2(1-W)\times(\Gamma(\gamma)/\delta^{\gamma}))]$$

where S is the average value of the scale factor. In the same way as for the EM case, we find the average energy in the k th segment by integrating the longitudinal profile. We have the same expression for the energy in the k th segment:

$$E_k = \overline{E_k} + R^2 r_k \sqrt{(\overline{E_k}/R^2)}$$

and again, the energy is split into "tower" subsegments, via integrating over the transverse shower profile, with statistical fluctuations of each subsegment done independently.

The quality of agreement between the hadron shower model and measured test beam data are shown in Fig. 3 and 4. Fig. 3 shows the hadron calorimeter energy distribution for 50 GeV π's that were minimum ionizing passing through the front EM calorimeter. Fig. 4 shows the EM calorimeter energy distribution for all incident pions. The minimum ionizing peak is truncated to emphasize the rest of the distribution.

The transverse profile has 2 components: one associated with the electromagnetic part; and one with the hadronic part. The electromagnetic part is a Gaussian that is identical to the narrow part of the pure electromagnetic shower. The hadronic part is a Gaussian with sigma:

$$\sigma_H = A_H + B_H L$$

where A_H = 6.45 (there is no sigma associated with this value) and B_H =0.07 with a sigma of 0.02, and L is the length in gm/cm^2.

Recently H. Jensen[15] has investigated the non-linearity in the (pulse height/energy) versus energy for pion showers. This non-linearity is inserted into the simulation by multiplying the longitudinal parametrization by a function g(E). At the present time this function is not parametrized but is tabulated numerically. Fig. 5 shows the normalized pulse height per GeV versus energy of the incident π. The curve is a fit to a compilation of data from several different experiments. There is a striking non-linearity in the data. The point E_π = 50 GeV is chosen as our normalization point. The turnover in the curve at very low pion energy is understood as due to pions ranging out by dE/dx without nuclear interactions.

In summary we have tried to indicate how both electromagnetic and hadronic showers are simulated in the CDF detector. There is much additional information the reader may wish to know such as the number of radiation lengths, absorption length, segmentation, and other properties of the 7 calorimeters in the CDF detector, these are given in Ref. 1. The agreement between test beam data and the simulation is given Ref. 2 and Ref. 3. Much as the UA1 parametrization[10] was a good starting point for us, we hope that our parametrization will be a good starting point for the detectors of the SSC.

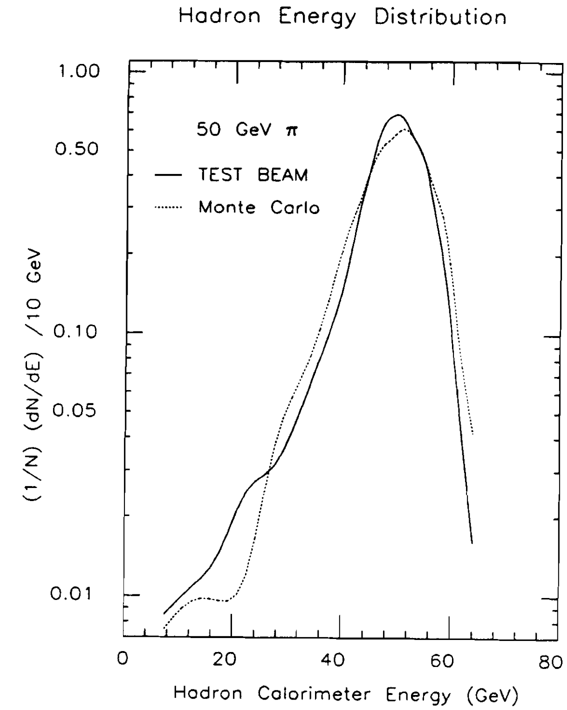

Fig. 3

Distribution of the energy, E_{HAD}, measured in the central hadron calorimeter, normalized to unit area for 50 GeV π minimum ionizing in the central EM calorimeter. The test beam data and Monte Carlo are compared (from Ref. 3).

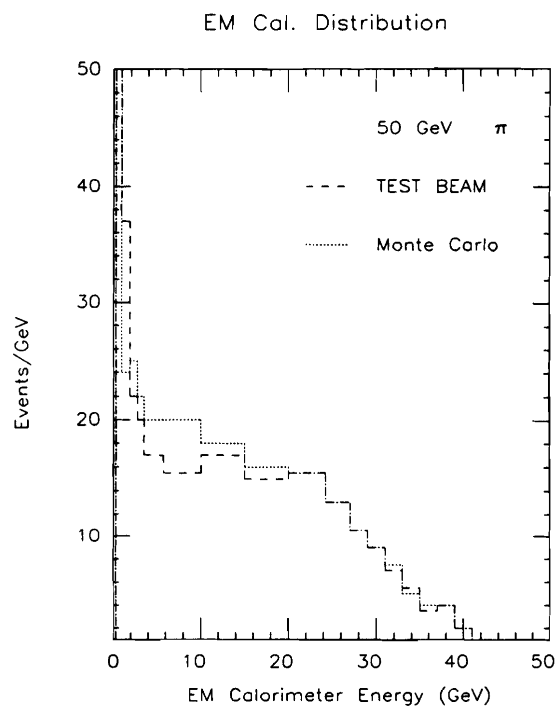

Fig. 4

Distribution of the energy, E_{EM}, measured in the central EM calorimeter, with arbitrary normalization, for 50 GeV π (either interacting or not interacting in the central EM calorimeter). Test beam data and Monte Carlo are compared (from Ref. 3).

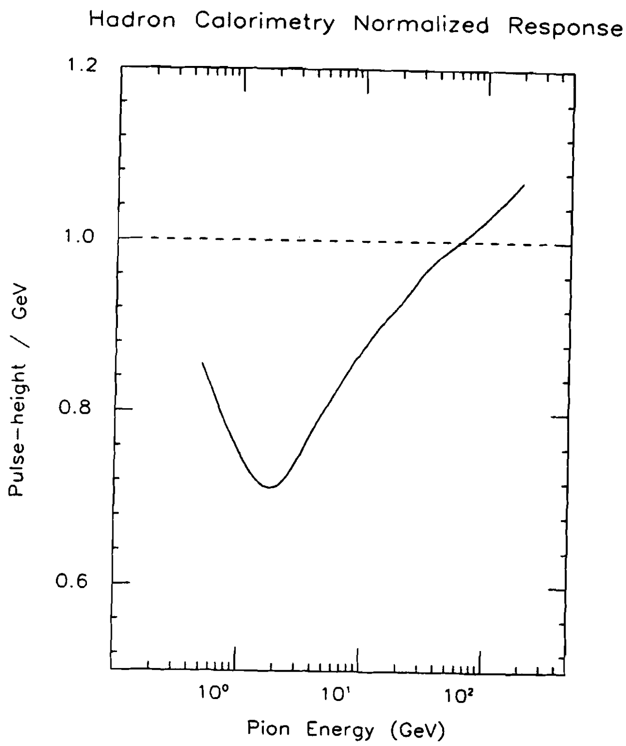

Fig. 5

This figure shows normalized pulse-height per GeV in hadron calorimeters as a function of energy of the incident π. The graph has been normalized so that pulse height is equal to energy at 50 GeV (from Ref.15).

Table I $R^2 = (\text{Response})^2$

DETECTOR	EM	HADRON
CENTRAL EM	0.02	0.20
CENTRAL H	0.10	0.50
PLUG EM	0.05	0.20
PLUG H	0.60	1.2
FORWARD EM	0.05	0.20
FORWARD H	0.70	1.4
WALL H	0.50	1.0

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