PEDAGOGICAL INTRODUCTION TO COLOR CALCULATIONS*

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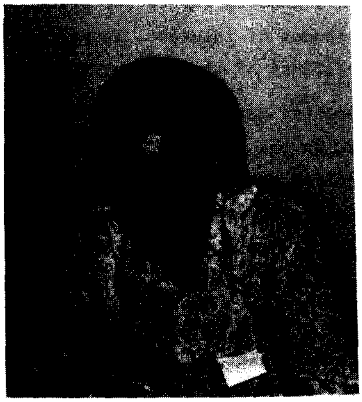

ABSTRACT : An introduction to calculations with color symmetry is provided for non-experts; we proceed mainly by giving examples. The observability of color symmetry, even though only colorless states might exist, is emphasized.

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I. INTRODUCTION

It seems rather likely that for some time to come the most fruitful view of hadrons will be as composites of colored quarks and gluons. While the theory of Quantum Chromodynamics (QCD) is still developing, it is already standard to find "color factors", factors due to the presence of this internal symmetry, quoted in many amplitudes of interest. Indeed, color is most directly observable in terms of the effect of these color factors on transition rates.

While there is nothing particularly new or mysterious about calculating the color factors - it is, in the standard theory, an exercise in conventional SU(3) - some care is required to get it right. There are few places where an uninitiated reader can go to learn easily the necessary techniques. Consequently, it was thought that it would be useful to include an introductory pedagogical treatment as part of the present book, and that is the purpose of this chapter.

Other introductory aspects such as the history and the main tests of color are well covered in other chapters or in references given there, so we will concentrate on calculational techniques. A few other sources are mentioned in references below. References to the original literature should be traced from other chapters and from the reviews mentioned.

First some introduction to the formalism is given in Section II. Then in Section III a number of examples are analyzed, some with simple arguments and some with formal manipulations. We will concentrate on the standard theory, where color is an unbroken symmetry and hadrons are only in color singlet states ; other views and ways to distinguish them are well covered in the articles by Chanowitz and Greenberg in this volume, and by Greenberg and Nelson. Finally, in Section IV some further remarks are given about the observability of color. It is worth emphasizing again that the purpose of the present notes is to give an elementary but fairly complete, pedagogical, introduction to the use of color symmetry in quark physics.

II. FORMALISM

We assume that quarks come in a triplet representation of an unbroken SU(3) color symmetry, and that they interact by exchange of an octet of colored gluons. That there are 3 colors and the symmetry is SU(3) is the standard choice to have three quarks in a baryon, and have them in the totally antisymmetric state required by statistics, and in a color singlet state. We will label the quark colors, as r, g, b (for red, green, blue). The gluons can change quark color, so a r quark can absorb or omit a gluon and become a g quark, etc ...

Some color factors can just be written down, as we will see, but for some calculations we need the Gell-Mann SU(3) λ matrices, so let us recall their properties ; it is probably worthwhile to give essentially a self-contained treatment. They are just the generalization to SU(3) of the Pauli matrices for spin or isospin. Let us think in terms of isospin, with matrices

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

and commutation relations

$$\left[\frac{\tau_i}{2}, \frac{\tau_j}{2}\right] = i\,\varepsilon_{ijk}\,\frac{\tau_k}{2} \tag{2}$$

Combining the commutation relations with the anticommutator

$$\left\{\tau_i, \tau_j\right\} = 2\delta_{ij} \tag{3}$$

one has the useful relation

$$\tau_i\tau_j = \delta_{ij} + i\varepsilon_{ijk}\tau_k \tag{4}$$

Most of the useful properties of the λ matrices can be deduced from the commutation relations and simple arguments based on the isospin analogy. We have :

$$\left[\frac{\lambda_i}{2}, \frac{\lambda_j}{2}\right] = i\,f_{ijk}\,\frac{\lambda_k}{2} \tag{5}$$

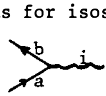
and

$$\left\{\lambda_i, \lambda_j\right\} = 2\,d_{ijk}\,\lambda_k + \frac{4}{3}\,\delta_{ij} \tag{6}$$

The second term in (6) has a 3 x 3 unit matrix factor understood. Combining these we find one useful relation analogous to (4),

$$\lambda_i\lambda_j = i\,f_{ijk}\,\lambda_k + d_{ijk}\,\lambda_k + \frac{2}{3}\,\delta_{ij}$$

Many complicated expressions with λ matrices can be handled using this equation. The numbers f_{ijk} and d_{ijk} are the standard SU(3) constants, available in many textbooks. Just as for isospin, at a quark-gluon vertex there is a factor due to color



$$\frac{\lambda^i_{ab}}{2}$$

where a, b are quark color labels, a, b = r, g, b or a, b = 1, 2, 3. When calculating color factors we expect only internal gluons, whose color "polarization" is summed over. Therefore we will find factors such as

$$\sum_i \lambda^i_{ab} \lambda^i_{cd}$$

which need to be evaluated. The appropriate identity is

$$\sum_i \frac{\lambda^i_{ab}}{2} \frac{\lambda^i_{cd}}{2} = \frac{1}{2} \delta_{ad}\delta_{bc} - \frac{1}{6} \delta_{ab}\delta_{cd} \tag{7}$$

To check that this is correct, note the following. We can write down a few λ's by analogy with isospin. We can take

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

merely the Pauli matrices with a third row and column, still traceless and hermitian. Since

$$\tau_i^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

then for i = 1, 2, 3

$$\lambda_i^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In particular, note $\mathrm{Tr}\lambda_i^2 = 2$ for $i = 1, 2, 3$ and so $\mathrm{Tr}\lambda_i^2 = 2$ for any i. One more λ is obvious, the one which is diagonal in isospin, usually called λ_8. Since it has unity in the 11 and 22 positions and is traceless and normalized with $\mathrm{Tr}\lambda_8^2 = 2$, it is

$$\lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \frac{1}{\sqrt{3}} .$$

Now, to check eqn.(7), we can (i) put b = c and sum, giving the matrix product, (ii) put d = a and sum, giving the trace.

Since each term gives 2 and there are 8 terms we get 16, divided by 4 for the two factors of 1/2. The right hand side gives

$$\sum_{ac} \left(\frac{1}{2} \delta_{aa}\delta_{cc} - \frac{1}{6} \delta_{ac}\delta_{ac} \right) = \frac{1}{2} (3)\ (3) - \frac{1}{6} (3) = \frac{9}{2} - \frac{1}{2} = 4$$

as expected.

If we put a = b = c = d = 3 , all but λ_8 have a zero in the 33 position so we get

$$\frac{1}{4} \sum_i \left(\lambda^i_{33}\right)^2 = \frac{1}{4} \left(\frac{4}{3}\right) = \frac{1}{3}$$

and

$$\frac{1}{2} \delta_{33}\delta_{33} - \frac{1}{6} \delta_{33}\delta_{33} = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$$

A third check is to take a = c = 1 , b = d = 2 . Then $\left(\lambda^1_{12}\right)^2 = 1$ and $\left(\lambda^2_{12}\right)^2 = -1$ so both sides should give zero. These three independent checks establish the identity of eqn.(7) since there are only three ways to combine the indices.

Finally, we assume that hadrons are color singlet states. To have normalized wave functions,then, a meson is represented by

$$\frac{1}{\sqrt{3}} \delta_{ab} \tag{8}$$

and a baryon by

$$\frac{1}{\sqrt{6}} \varepsilon_{abc} \quad . \tag{9}$$

Now we proceed to various physical manifestations of color symmetry.

III. EXAMPLES

First, we can write the electromagnetic current as

$$J_\mu^{EM} = \frac{2}{3} \left(\bar{u}_r u_r + \bar{u}_g u_g + \bar{u}_b u_b\right) - \frac{1}{3} \left(\bar{d}_r d_r + \bar{d}_g d_g + \bar{d}_b d_b\right) + \cdots \tag{10}$$

where the dots are similar contributions for other flavors s, c, etc... The normalization of each term is required by the charge of each quark.

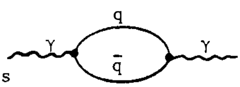
Now let us calculate the ratio

$$R = \sigma_T(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$$

The transition is through an intermediate photon.

It has been shown that the total rate to hadrons can be calculated as if there were a transition to free quarks (which then turn into hadrons). Then $\gamma \to q\bar{q}$ and

$$\sigma \sim \Sigma|M|^2 \sim \sum_{\text{colors}}$$



so will all colors contributing equally we get a factor of N_c .

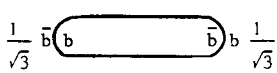
We can see the same result and practice with the formalism. The photon quark vertex for quark colors a, b is given by δ_{ab} , and the color singlet wave function is $\delta_{ab}/\sqrt{3}$, so

$$M \sim \delta_{ab}\delta_{ab}/\sqrt{3} = \delta_{bb}/\sqrt{3} = \sqrt{3}$$

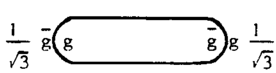
so the cross section is larger by $M^2 \sim 3$. If there had been N_c colors this would become $M \sim \sqrt{N_c}$, $\sigma \sim N_c$, so R gives a direct measurement of the numbers of colors as well as the presence of such a symmetry.

Next let us consider the Drell-Yan process, where a $\bar{q}q$ pair from two interacting hadrons annihilate to a lepton pair via an intermediate photon. Again, we are only considering the color factor, which here is expected to be 1/3. More extensive discussion can be found in the chapter by Quigg. Let us derive this factor a couple of ways.

Consider color singlet mesons, $(\bar{r}r + \bar{g}g + \bar{b}b)/\sqrt{3}$. Then when blue quarks annihilate we have



with the circled quarks annihilating. For green quarks, similarly, the contribution is



The remaining quarks are blue in one case and green in the second, so they do not interfere. Consequently, we square before adding. The amplitude for each color is 1/3, so the rate for each color is 1/9 , so the total rate has $1/9 + 1/9 + 1/9 = 1/3$, an overall factor of 1/3 from color.

We can also view the Drell-Yan process in terms of the conventional formalism. Ignoring color we would write the proton structure function

$$F_2^{ep}(x) = x\left[\frac{4}{9}\left(u(x)+\bar{u}(x)\right)+\frac{1}{9}\left(d(x)+\bar{d}(x)\right)+\frac{1}{9}\left(s(x)+\bar{s}(x)\right)\right]$$

where $u(x)$, etc ... is the distribution of u quarks with a fraction x of the proton momentum in the scaling region. Including color we would instead write

$$F_2^{ep}(x) = x\sum_c\left[\frac{4}{9}\left(u_c(x)+\bar{u}_c(x)\right)+\frac{1}{9}\left(d_c(x)+\bar{d}_c(x)\right)+\frac{1}{9}\left(s_c(x)+\bar{s}_c(x)\right)\right]$$

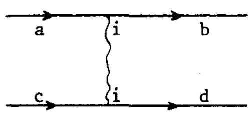
and

$$u_c(x) = u(x)/3 \quad , \quad c = r, g, b$$

since the measured $u(x)$ summed over color.
For other color models the last equation might not hold, with possibly a variation depending on x or q^2 . The Drell-Yan rate depends on the product of structure functions from each hadron, and the annihilation is due to one pair of colored quarks, so a factor $1/N_c^2$ comes into the rate. However, there are N_c more pairs than without color, so the final factor is $1/N_c$ as above.

Now we turn to calculating color factors for several processes where it is most convenient to use the formalism of SU(3) λ matrices summarized above.

First consider qq scattering by single octet gluon exchange. Consider the diagram



$$M = \frac{1}{4}\sum_i \lambda^i_{ab}\lambda^i_{cd}$$

Each vertex has a factor $\lambda^i/2$, and the virtual gluon color state is summed. The initial and final states can be thought of as having the quarks come from different hadrons, perhaps to undergo a hard scattering to large $p_\perp$. Then the cross section is

$$\sigma \sim |M|^2 = \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\sum_{abcd}\left(\sum_i \lambda^i_{ab}\lambda^i_{cd}\right)\left(\sum_j \lambda^{j*}_{ab}\lambda^{j*}_{cd}\right)\frac{1}{16}$$

Since the SU(3) matrices are hermition, $\lambda^{j*}_{ab} = \lambda^j_{ba}$, and we have summed over all final quark colors and averaged (giving $1/3$ for each initial quark) over all initial quark colors. Since the initial quarks are in color singlet hadrons the probability of finding a quark of given color is $1/3$ $\left(\sum_b\left(\delta_{ab}/\sqrt{3}\right)\left(\delta_{a'b}/\sqrt{3}\right)=1/3\delta_{aa'}\right.$ for mesons, $\sum_{bc}\left(\frac{1}{\sqrt{6}}\varepsilon_{abc}\right)\left(\frac{1}{\sqrt{6}}\varepsilon_{a'bc}\right)=1/3\delta_{aa'}$, for baryons$\left.\right)$, so in averaging over the initial state we have a factor $1/3$ for each quark ; it is like spin states with an unpolarized beam if we start with color singlet hadrons. Using eqn. 7, we get

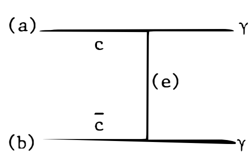
$$\sigma = \frac{1}{9} \sum_{abcd}\left(\frac{1}{2}\delta_{ad}\delta_{cb} - \frac{1}{6}\delta_{ab}\delta_{cd}\right)\left(\frac{1}{2}\delta_{bc}\delta_{ad} - \frac{1}{6}\delta_{ad}\delta_{cd}\right)$$

$$= \frac{1}{9}\left(\frac{1}{4}(3)(3) - \frac{1}{12}(3) - \frac{1}{12}(3) + \frac{1}{36}(3)(3)\right) = \frac{2}{9}$$

For a physical situation the relevant calculation is more complicated (see Feynman and Field for more discussion). One must add the crossed diagram where quark b comes from the lower vertex. The size of the interference term depends on the scattering angle, and is different for different color symmetries. For a given physical process one must add the relevant diagrams with spin and color factors, and explicitly calculate the cross section.

Next, let us examine the effect of color on several decay rates. We ignore changes due to electromagnetic and strong coupling strengths. In all cases when we speak of the effect of the color factor, it should always be interpreted as meaning a comparison of processes, such as the effect relative to positronium or of two different final states.

a) Consider the expected rate for the charmonium state $\eta_c \to 2\gamma$, as shown, with color labels in parentheses,

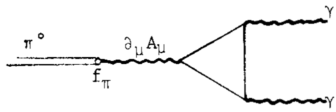


Formally, the matrix element has $\left(\delta_{ab}/\sqrt{3}\right)$ for the initial wave function, and δ_{ae}, δ_{be} at the quark-γ vertices. Then

$$M = \delta_{ab}\delta_{ae}\delta_{be}/\sqrt{3} = \sqrt{3}$$

so the rate is enhanced a factor of 3 by color (compared to the analogous rate for parapositronium).

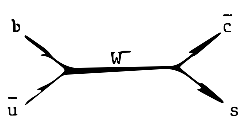
b) Next consider $\pi^\circ \to 2\gamma$. Here, as reviewed in detail by Chanowitz, it is well known that the expected enhancement due to color is 3 in amplitude, 9 in rate. But what is different from $\eta_c \to 2\gamma$ just above ? It is the role of PCAC and the triangle anomaly that is different. The π° case is viewed not as a quark annihilation but as



shown here, where the π° couples to the divergence of the axial current, and three currents are coupled to the quark loop. The coupling of π° to $\partial_\mu A_\mu$ is via the measured decay factor f_π , which then effectively includes the color wave function normalization. The coupling of the axial current and two vector currents to the quark loop is normalized by the currents. Clearly with three colors there are three loops and the amplitude is enhanced by 3, the rate by 9.

Comparison of (a) and (b) illustrates nicely that one must pay some attention to the detailed dynamical situation when working out color factors.

(c) Consider a hypothetical heavy quark b of charge -1/3 , coupled to the u quark in a weak isospin doublet (Cahn and Ellis). The nonleptonic decay of a $b\bar{u}$ meson state to $s\bar{c}$ would proceed via a direct channel intermediate vector boson W^-, as shown



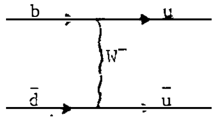
According to our rules we have

$$M \sim \Sigma\left(\delta_{ab}/\sqrt{3}\right)\delta_{ab}\,\delta_{cd} \sim \sqrt{3}\ \delta_{cd}$$

$$\sigma \sim 3\ \Sigma\ \delta_{cd}\delta_{cd} = 9$$

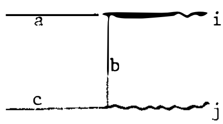
so the rate is enhanced by 9 . If the $b\bar{u}$ meson decays into a lepton pair such as $\mu\nu_\mu$, there is no color factor for the final state and the enhancement is 3 (the reverse of $(e^+e^- \to \text{hadrons})$).

(d) Alternatively, consider the hypothetical $b\bar{d}$ meson going into $u\bar{u}$. Then the W is a t-channel exchange,



and since the coupling at each vertex is just the Kronecher delta coupling of the current, the color factor is just unity in the rate.

(e) Finally, consider the QCD calculation for the total hadronic decay of η_c , $\eta_c \to 2$ gluons. Since the two gluons are assumed to have unit probability to become hadrons, it is sufficient to just calculate the annihilation graph to two gluons. The amplitude for the figure shown for quark colors a, b, c and gluons i, j is



$$M \sim \frac{\delta_{ac}}{\sqrt{3}} \frac{\lambda^i_{ab}}{2} \frac{\lambda^j_{bc}}{2} = \frac{1}{4\sqrt{3}} \mathrm{Tr}\left(\lambda^i \lambda^j\right)$$

Now $\mathrm{Tr}\, \lambda^i \lambda^j$ is zero if $i \neq j$ and 2 if $i = j$, as shown above, so

$\sigma \sim \frac{4}{48} \sum_{ij} \delta_{ij}\delta_{ij} = \frac{1}{12} \sum_i \delta_{ii} = \frac{8}{12} = \frac{2}{3}$. The complete amplitude is given by the sum of this process and the crossed diagram. In this case since $M \sim \delta_{ij}$ the two gluons are identical particles and the interference and counting of states is the same as for final photons. Thus we can take the QED parapositronium calculation and just multiply by the color factor of 2/3 (and redefine the coupling strength) to get the expected η_c decay rate. For the three gluon annihilation of ψ relative to orthopositronium it is more complicated to show that the counting is the same as for photons.

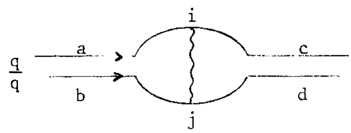
(f) Finally, consider a heavy lepton decay. The expected mechanism is $L \to \nu_L + W$, $W \to xy$ and one must sum over all pairs x, y that couple to the appropriate W . Presumably one has $x, y = \mu\nu_\mu$, $e\nu_e$, $c\bar{s}$, $u\bar{d}$ plus any additional contributions from heavy quarks or leptons permitted by energy conservation. Since each color couples to the current independently, there is one contribution from each lepton, plus N_c from each quark pair. Thus the branching ratio to a single channel is expected to be $1/(2+2N_c)$ if all the above are allowed. For the decay of the SPEAR heavy lepton presumably $c\bar{s}$ is not allowed by energy conservation, so 3 colors predict $1/(2+3) = 1/5$, as discussed by Chanowitz.

Next we turn to some aspects of the influence of color on hadron spectroscopy. Some of these details are available in Jackson (1976) but they are included here for completeness ; the connections to the original literature can also be traced from his lectures as well as the articles in this volume.

If there were a coupling g_o in the absence of color, the single gluon exchange force gives an effective coupling g between q and $\bar{q}$, for color singlet mesons,

$$g^2 = \sum_{ij\,abcd} \frac{\delta_{ab}}{\sqrt{3}} \left(g_o \frac{\lambda^i_{ac}}{2}\right)\left(g_o \frac{\lambda^j_{db}}{2}\right) \frac{\delta_{cd}}{\sqrt{3}}$$

where the labeling is as shown. Notice



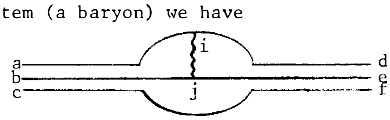
the order of indices (think of $\bar{q}$ as q in the other direction).

Then

$$g^2 = \frac{1}{12}\, g_o^2 \sum_{ij} \lambda^i_{bc}\lambda^j_{cb} = \frac{1}{12}\, g_o^2 \sum_{ij} \mathrm{Tr}\left(\lambda^i\lambda^j\right) = \frac{2}{12}\, g_o^2 \sum_{ij} \delta_{ij} = \frac{4}{3}\, g_o^2$$

For vector, octet gluons this corresponds to an attractive force, giving binding for mesons.

For the qqq system (a baryon) we have



$$g^2 = \sum_{\substack{abc\\def}} \sum_{ij} \frac{\varepsilon_{abc}}{\sqrt{d}} \left(g_o \frac{\lambda^i_{ad}}{2}\right)\left(g_o \frac{\lambda^j_{be}}{2}\right) \frac{\varepsilon_{def}}{\sqrt{6}}\, \delta_{cf}$$

$$= \frac{g_o^2}{24} \sum_{\substack{ab\\de}} \sum_{ij} \lambda^i_{ad}\lambda^j_{be} \left(\delta_{ad}\delta_{be} - \delta_{ae}\delta_{bd}\right)$$

$$= \frac{-g_o^2}{24} \sum_{de} \sum_{ij} \lambda^i_{ed}\lambda^j_{de} = \frac{-g_o^2}{24} \sum_{ij} \mathrm{Tr}\left(\lambda^i\lambda^j\right) = -\frac{2}{3}\, g_o^2$$

This also gives an attractive force, half the strength of the meson binding force Thus the electromagnetic coupling strength α in the photon exchange interaction gets replaced by $(4/3)\alpha_s$ for gluon exchange in a meson and $(2/3)\alpha_s$ for gluon exchange in a baryon.

Another useful way to get this result is as follows (de Grand et al, 1975). Consider quark and antiquark bound together in a color singlet state $|0>$. Then we must have for any i

$$\left(\lambda^i_q + \lambda^i_{\bar{q}}\right)|0> = 0 \quad .$$

Taking a scalar product with λ_q , we have for this situation

$$\sum_i \lambda^i_q \lambda^i_{\bar{q}} = -\sum_i \left(\lambda^i_q\right)^2 = -16 \quad .$$

With numerical factors $1/4$ since $\lambda/2$ goes at each vertex and $1/\sqrt{3}$ for initial and final states, this gives $-4/3$ as before. For the baryon

$$\left(\lambda^i_{q1} + \lambda^i_{q2} + \lambda^i_{q3}\right)|0> = 0 \quad .$$

Taking scalar products with λ_{q1} , λ_{q2} , λ_{q3} in turn and combining equations gives

$$\sum_i \lambda^i_{q1} \lambda^i_{q2} = -\frac{1}{2} \sum_i \left(\lambda^i_q\right)^2 = -8$$

or half of the meson result, giving the factor $-2/3$ from above.

The $q\bar{q}$ interaction due to single gluon exchange is proportional to the scalar product $\lambda_q \cdot \lambda_{\bar{q}}$. We change just seen that this gives an attractive force (i.e., a minus sign) for color singlet meson (and baryon) states. If we write

$$\left(\lambda_q + \lambda_{\bar{q}}\right)^2 = \lambda_q^2 + \lambda_{\bar{q}}^2 + 2\lambda_q \cdot \lambda_{\bar{q}}$$

and solve for $\lambda_q \cdot \lambda_{\bar{q}}$ (following the common procedure with spin or isospin) we have

$$2\lambda_q \cdot \lambda_{\bar{q}} = \left(\lambda_q + \lambda_{\bar{q}}\right)^2 - \lambda_q^2 - \lambda_{\bar{q}}^2$$

In the color singlet state $\left(\lambda_q + \lambda_{\bar{q}}\right)^2 = 0$ and we have our result above. For higher color states we would need to know the eigenvalue of $\left(\lambda_q + \lambda_{\bar{q}}\right)^2$, i.e. the analogue of $J^2 = J(J+1)$, but it is sufficient for our purposes to just note that $\left(\lambda_q + \lambda_{\bar{q}}\right)^2$ is positive, so that forces mediated by color gluon exchange will always have the color singlet states lying lowest. Consequently, even if quarks and gluons were not confined it would make sense to have the observed hadrons in color singlet states.

Similarly, consider a color singlet hadron and ask what is the force between it and another quark Q . We can write the interaction as

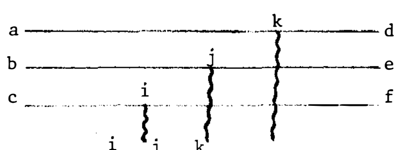
$$M \sim \sum_{\substack{q,\bar{q} \\ \text{in hadron}}} \lambda_Q \cdot \lambda_{q(\bar{q})}$$

and rearranging gives

$$M \sim \lambda_Q \cdot \left(\sum_{q,\bar{q}} \lambda_{q(\bar{q})} \right) .$$

But the quantity in parentheses will vanish for a color singlet hadron, so the existence of only $q\bar{q}$ and qqq systems can perhaps be understood dynamically, in terms of the properties of the color gluon forces. See Greenberg's chapter for more discussion on this saturation question.

Two other ways in which the color properties of the forces may affect dynamics are interesting. First, since hadrons are to be taken as color singlets, and one can get a singlet from $3 \otimes \bar{3} = 1 \oplus 8$ or $3 \otimes (3 \otimes 3) = 1 + \ldots$, the $(3 \otimes 3)$ diquark in the baryon must transform as a $\bar{3}$ as far as color forces can tell. This may help clarify the dynamical role that diquark systems seem to play and the similarity of the Regge behavior of mesons and baryons. Second, it has been remarked (Nussinov, 1976) that a dynamical symmetry property of the Pomeron may arise from color. If one builds the Pomeron from multigluon exchange, the possibility of odd charge conjugation (C) exchange naturally arises in the 3-gluon contribution. But due to color this odd C part of the exchange vanishes identically. It is given by



$$M \sim \Sigma\, f_{ijk}\, \lambda_{cf}^{i}\, \lambda_{be}^{j}\, \lambda_{ad}^{k}\, \varepsilon_{abc}\, \varepsilon_{def}$$

where the antisymmetric SU(3) numbers f_{ijk} project out the odd C part of the gluon exchange and all indices are summed. Since i, j and e, f and b, c are all summed over we can interchange them $i \leftrightarrow j$, $c \leftrightarrow b$, $e \leftrightarrow f$. After doing so the three λ's are unchanged, and the quantity $f_{ijk}\varepsilon_{abc}\varepsilon_{def}$ has had three sign changes, one from each of its totally antisymmetric pieces. Thus $M \to -M$ so $M = 0$.

IV. OBSERVABILITY OF COLOR

To conclude we want to briefly emphasize the ways in which the existence of a color symmetry may be observable. While the elegance and richness of QCD may suffice to convince many theorists of its relevance or even truth, one often hears -- particularily for the unbroken, "hidden" color symmetry -- questions about the meaning of having a new set of degrees of freedom that are not directly observable. Because states of non-zero color may exist, is color unobservable ?

Of course not. We are familiar with procedures for observing non-abelian groups such as angular momentum or isospin from selection rules and transition rates. If π and N could only interact in $I = 3/2$ states we would learn it from the 9/1 ratio of elastic scattering of π^+p and π^-p . Similarly, we have mentioned a number of observable consequences of color above (see especially Chanowitz's article for additional examples). In addition to the basic need to antisymmetrize three quarks in a baryon, these include the predicted factor of N_c increase in $R = \sigma_T(e^+e^-)/\sigma(e^+e^- \to \mu^+\mu^-)$, the factor of N_c decrease in the Drell-Yan cross section, a number of changes in decay transition rates and branching ratios, such as a factor N_c^2 in nonleptonic weak decays with s-channel intermediate vector bosons and no modification for t-channel ones. Other color dependent predictions are available for a number of processes involving photons, and for scaling violations in deep inelastic ν, e, μ reactions (see the Chanowitz review). Many of these test both the presence of an internal symmetry and the actual degree of symmetry, the number of colors. In the near future both our experimental and our theoretical understanding of these tests will become clear, and the experimental status of color symmetry will be settled by several independent measurements.

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